

# Determination of the $pp\pi^0$ -Coupling Constant and Breaking of Charge Independence\*

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## Abstract

In a phase shift analysis of all  $pp$  scattering data below  $T_{\text{lab}} = 350$  MeV, where  $\chi^2/\text{d.o.f.} = 1.07$  is reached, the long and intermediate range  $pp$  interaction has been studied. Using as intermediate range interaction the Nijmegen potential, we find for the  $pp\pi^0$ -coupling constant  $f_0^2 = (72.5 \pm 0.6) \cdot 10^{-3}$  or  $g_0^2 = 13.1 \pm 0.1$ ) and for the  $\pi^0$ -mass  $134.7 \pm 2.1$  MeV. Even when we take account of the model dependence due to the potential tail, this value of  $f_0^2$  is significantly lower than the value of the charged coupling constant in  $\pi N$  scattering, indicating a large breaking of charge independence.

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\*Published in Phys. Rev. Lett. **59**, 2255–2258 (1987)

The coupling of the neutral pion fields  $\phi$  to the proton field  $\psi$  is described by either the pseudoscalar (PS) or the pseudovector (PV) phenomenological vertex functions  $\mathcal{L}_{\text{PS}}$  or  $\mathcal{L}_{\text{PV}}$ , where  $\mathcal{L}_{\text{PS}} = g_0(4\pi)^{1/2}(\bar{\psi}i\gamma_5\psi)\phi$  and  $\mathcal{L}_{\text{PV}} = (f_0/m_+)(4\pi)^{1/2}(\bar{\psi}i\gamma_\mu\gamma_5\psi)\partial^\mu\phi$ . To make the PV coupling constant  $f_0$  dimensionless it is customary[1] to introduce in  $\mathcal{L}_{\text{PV}}$  the charged-pion mass  $m_+$ . These different vertex functions give rise to the same one-pion-exchange (OPE) potential between protons, provided that one has  $g_0^2 = (2M/m_+)^2 f_0^2$ , where  $M$  is the proton mass.

Differences between these two phenomenological vertex functions show up when one looks at the vertex  $\bar{p}p \rightarrow \pi^0$ . When one considers the underlying quark picture,[2] then it is unbelievable that the vertices  $p \rightarrow p\pi^0$  and  $\bar{p}p \rightarrow \pi^0$  can both be described by these simple vertex functions with the same  $g_0$  or  $f_0$ , even if these couplings are modified by form factors describing the spatial extension of the hadrons. Especially  $\mathcal{L}_{\text{PS}}$  is unbelievable because it predicts a very strong  $\bar{p}p \rightarrow \pi^0$  vertex. That  $\mathcal{L}_{\text{PS}}$  is unbelievable does not mean that  $\mathcal{L}_{\text{PV}}$  is correct. We think that both expressions are only valid approximations in a very restricted kinematic domain.

The coupling of charged pions to the nucleons is described by the charged coupling constant  $f_c$ , where  $f(pn\pi^+)f(np\pi^-) = 2f_c^2$ . When one assumes charge independence for the pion-nucleon interaction, then one has  $f_0^2 = f_c^2$ . However, charge independence of the strong interactions is only an approximate symmetry, because it is broken by the presence of the electroweak interactions and by the mass difference of the up and down quarks. In the past it was believed that this breaking of charge independence was small, because it was assumed to be mainly of electromagnetic origin.[3] A recent calculation,[4] where one tries to include also the quark mass difference, gives  $f_0^2$  smaller than  $f_c^2$  by 7% to 10%.

The charged coupling constant  $f_c$  is determined rather precisely in  $\pi N$  scattering, where one seems to agree on  $f_c^2 = (79 \pm 1) \times 10^{-3}$ .[1] The best place to determine the neutral coupling constant  $f_0$  is probably in  $pp$  scattering. In Table different determinations of  $f_0^2$  are listed.

The tensor character is an important feature of the OPE potential. In the phase shifts, the long-range OPE tensor potential can best be seen from the tensor combination of the triplet odd waves ( ${}^3P, {}^3F$ ). The first indication for a very low value of the neutral coupling constant  $g_0^2 \approx 13$  or  $f_0^2 \approx 0.072$  we[11] got from a single-energy phase-shift analysis of  $pp$  analyzing power (polarization) data at 10 MeV.[12] These data give a much smaller tensor combination  $\Delta_{\text{T}}$  of  ${}^3P$  phase shifts than the present best  $NN$  potentials, because all these potentials have too large a value of  $f_0^2$ . Later studies[13] showed that within a potential model it is impossible to obtain such a small  $\Delta_{\text{T}}$  with reasonable values for the  $\rho$ - and  $\omega$ -coupling constants and  $f_0^2$  in the neighborhood of 0.079. Later, in a much more precise  $pp$  analyzing-power experiment[14] at 9.85 MeV confirmed the low value of  $\Delta_{\text{T}}$ . This experiment gives  $\Delta_{\text{T}} = -0.933 \pm 0.007$  while the Nijmegen soft-core potential[15] gives  $\Delta_{\text{T}} = -0.98$  and the parametrized Paris potential[16] gives  $\Delta_{\text{T}} = -1.01$ .

The value of  $f_0^2$  presented here is a result of our phase-shift analysis of all  $pp$  data with  $T_{\text{lab}} < 350$  MeV.[17] It is a continuation of the lower-energy analyses performed by our group.[10, 18] We use a method which is sensitive to the long and intermediate range ( $r > b$ ) of the  $pp$  interaction. This allows us to check in a quantitative way the long- and intermediate-range parts of any  $pp$  potential. We used it to check the OPE potential, to determine the neutral-pion coupling constant, and to compare the long- and intermediate-

range parts ( $r > 1.4$  fm) of the soft-core Nijmegen potential and the parametrized Paris potential.[16]

The data set for  $T_{\text{lab}} < 30$  MeV is extensively discussed in Ref. [10]; that for  $T_{\text{lab}} > 30$  MeV is roughly speaking, a combination of the data sets used in the analyses of Arndt and co-workers[19] and the data lists published by Bystricky and Lehar,[20] whereof the data with too high[10]  $\chi^2$  values are rejected. This leaves us with 1234 scattering observables. Of all groups of data, 26 have an experimentally undetermined normalization, so for a correct model without any adjustable parameters one expects the  $\chi^2$  value:  $\langle \chi^2 \rangle = 1208 \pm 49$ .

The method of analysis is about the same as in our 0–30-MeV analysis.[10] The lower partial waves (with  $J \leq 4$ ) are parametrized by means of an energydependent  $P$  matrix at  $r = b$  and for  $r > b$  a potential tail  $V_L = V_{\text{em}} + V_{\text{nuc}}$ . Here  $V_{\text{em}}$  is the electromagnetic potential, consisting of the modified relativistic Coulomb potential[21] and the vacuum polarization potential.[22] The longest-range part of  $V_{\text{nuc}}$  is the OPE potential

$$V_{\text{OPE}} = \frac{1}{3} f_0^2 \frac{M}{E} \left( \frac{m}{m_+} \right)^2 \frac{e^{-mr}}{r} \left[ \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + S_{12} \left( 1 + \frac{3}{mr} + \frac{3}{(mr)^2} \right) \right] \quad (1)$$

where  $m$  is the  $\pi^0$  mass and  $E = (M^2 + k^2)^{1/2}$  with  $k$  the c.m. relative momentum.

In the highest partial waves ( $J \geq 10$ ), which are very insensitive to the short-range interaction, we use the phaseshifts due to the  $V_{\text{em}}$  and  $v_{\text{OPE}}$ . A correction is determined from the lower partial waves by optimal mapping techniques.[23]

Since the parametrization of the short-range interaction ( $r < b$ ) is purely phenomenological, the number of  $P$ -matrix parameters is determined by the criterion that the description of the data does not improve significantly if one parameter is added. Counting also the pion coupling constant, we need 28 parameters, which is not too different from the number of parameters used in other multienergy phase-shift analyses[19, 24] in this energy range.

The long-range interaction depends on  $f_0^2$ . In fact, it is this dependence which allows us to determine  $f_0^2$ . All realistic models for the  $pp$  interaction include the OPE potential as the longest-range part, but they differ in the description of the shorter-range forces which are due to heavier and/or higher-order boson exchange (HBE). Therefore we have included in  $V_{\text{nuc}}$  the HBE of some modern potentials. Our method of analysis is especially suited to measurement of the quality of potential tails ( $r > 1.4$  fm) via the attained minimal  $\chi^2$  in the analysis.

As possible choices for  $V_{\text{nuc}}$  we have considered the following: (i)  $V_{\text{nuc}} = V_{\text{OPE}}$ . The change in  $V_{\text{OPE}}$  due to a form factor as in the Nijmegen soft-core potential[15] is of no influence, since its effect is of short range. (ii)  $V_{\text{nuc}} = V_{\text{OPE}} + V_{\text{HBE}}^{\text{N}}$ , where  $V_{\text{HBE}}^{\text{N}}$  is the non-OPE part of the Nijmegen soft-core potential.[15] (iii)  $V_{\text{nuc}} = V_{\text{OPE}}^{\text{s}} + V_{\text{HBE}}^{\text{N}}$ , where  $V_{\text{OPE}}^{\text{s}}$  is the static OPE potential [leaving out the factor  $M/E$  in Eq. 1]. (iv)  $V_{\text{nuc}} = V_{\text{OPE}} + V_{\text{HBE}}^{\text{P}}$ , where  $V_{\text{HBE}}^{\text{P}}$  is the non-OPE part of the parametrized Paris potential.[16]

For each potential tail, the  $P$ -matrix parameters and  $f_0^2$ , that affects all partial waves, have been adjusted in a least-squares fit to the data. The results for  $\chi_{\text{min}}^2$  and  $f_0^2$  are given in Table . For the cases (ii), (iii), and (iv) we used  $b = 1.4$  fm. Taking only the OPE potential tail in  $V_{\text{nuc}}$  appeared not to be reasonable for  $b = 1.4$  fm. This indicates that HBE forces are not negligible outside 1.4 fm. Therefore we used  $b = 1.8$  fm in case (i) and also the number of  $P$ -matrix parameters was increased by one to get a more reasonable fit to the

data. Even then the description is the least good: The  $\chi_{\min}^2$  remains about 20 higher than with the other tails. The tail of the Nijmegen potential is seen to be somewhat  $\Delta\chi^2 = 6.6$ ) better than the tail of the Paris potential. The value of  $f_0^2$  found with the Paris potential deviates also (by about 3 standard deviations) from the others, which are very consistent.

The model dependence due to the chosen potential tail gives an estimate for the systematic error in the determinations of  $f_0^2$ . The energy at which the results for  $f_0^2$  in cases (ii) and (iii) imply the same OPE is about 9 MeV, indicating the importance of the analyzing-power data around 10 MeV[12, 14] in this determination. This importance can more clearly be seen from a fit to all data minus these analyzing-power data. This raises  $f_0^2$  by about  $1.4 \times 10^{-3}$  and enlarges the error in the determination of  $f_0^2$  by about 50%.

In order to show that we really look at the OPE potential, characterized by its exchanged mass and its specific spin dependence, we have checked the consistency between different subsets of all partial waves in the determination of  $f_0^2$  and also determined the  $\pi^0$  mass in the same way as we determined  $f_0^2$ .

To save computer time, the tests on the consistency between the partial waves have been done with a matrix representation of the data. The results are for  $V_{\text{nuc}} = V_{\text{OPE}} + V_{\text{HBE}}^{\text{N}}$ . Introducing different coupling constants for the spin-triplets  $f_{\text{T}}$  and for the spin-singlets  $f_{\text{S}}$ , we obtain  $f_{\text{T}}^2 = (72.5 \pm 0.6) \times 10^{-3}$  and  $f_{\text{S}}^2 = (74 \pm 2) \times 10^{-3}$ . This result indicates the importance of the spin-triplet waves in the determination. When we next introduce different coupling constants for the  ${}^3P$  waves  $f({}^3P)$  and all other partial waves  $f(\text{rest})$ , we find  $f^2({}^3P) = (72.2 \pm 0.6) \times 10^{-3}$   $f^2(\text{rest}) = (73.8 \pm 0.9) \times 10^{-3}$ . Also, for the other potential tails, the values from the different subsets of partial waves are rather consistent. We see that the  ${}^3P$  waves are very important in the determination of  $f_0^2$ .

In our judgment the determination of the  $\pi^0$  mass from the  $pp$  scattering data is a crucial test. The mass as well as the coupling can be determined from the potential tail, but only the mass is accurately known. We find  $m = 134.7 \pm 2.1$  MeV, in complete agreement with the more accurate value  $m_0 = 134.9642 \pm 0.0038$  MeV.[25] In Fig. we sketch the  $\chi^2$  surface as a function of  $m$  and  $f_0^2$ . A strong correlation between  $f_0^2$  and  $m$  is seen. Because of the correlation, the correct value found for  $m$  supports the value found for  $f_0^2$ .

Let us summarize and discuss our results. In our study of the long and intermediate range of the  $pp$  interaction we find that the data, which are described with a  $\chi^2/N_{\text{d.o.f.}} \approx 1.07$ , favor the tail of the soft-core Nijmegen potential[15] over the tail of the parametrized Paris potential[16] by 2.5 standard deviations. Using the tail of the Nijmegen potential for the description of the forces with intermediate range, we find for the neutral pion-proton coupling constant  $f_0^2 = (72.5 \pm 0.6) \times 10^{-3}$  or  $g_0^2 = 13.1 \pm 0.1$ . We quote here the value for the fit with the lowest  $\chi_{\min}^2$ . The error given is purely statistical. From Table we get an impression of the model dependence of our result, which gives then an estimate of a possible systematic error. No other systematic errors have been found in this analysis, because the results with the subsets of all partial waves are consistent and also the mass of the exchanged  $\pi^0$  is in excellent agreement with its rest mass. Our result for  $f_0^2$  is in fair agreement (see Table ) with earlier determinations, except with the value quoted by Kroll,[9] who used forward dispersion relations. Our value for  $f_0^2$  is smaller and much more precise than these earlier determinations. It deviates significantly[26] from the value  $f_c^2 = (79 \pm 1) \times 10^{-3}$  or  $g_c^2 = 14.3 \pm 0.2$  for the charged coupling constant. This indicates a large breaking of the charge independence or SU(2)-isospin symmetry. This breaking is of the same order

of magnitude as a very recent estimate in a simple quark model, where it is due to the mass difference between the up and down quarks.[4] This large SU(2) symmetry breaking of the pion-nucleon coupling constants leads to the expectation of even larger SU(3)-flavor symmetry[27] breaking of the meson-baryon coupling constants.

We wish to thank B. J. VerWest for sending us the data set used in the analyses of Arndt and coworkers.[19] Part of this work was included in the research program of the Stichting voor Fundamenteel Onderzoek der Materie (FOM) with financial support from the Nederlandse Organisatie voor Zuiver-Wetenschappelijk Onderzoek (ZWO).

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## TABLES

TABLE I. The neutral coupling constant  $10^3 \times f_0^2$ . (a) and (b) indicate different data sets

Bugg <sup>a</sup>		$75.2 \pm 3.9$
MacGregor, Arndt, and Wright <sup>b</sup>		$81.4 \pm 4.6$
Breit <i>et al.</i> <sup>c</sup>		$73.1 - 81.1$
Bugg <i>et al.</i> <sup>d</sup>		$77.8 \pm 3.6$
Kroll <sup>e</sup>		$80.3 \pm 2.2$
Bergervoet <i>et al.</i> <sup>f</sup>	(a)	$80.0 \pm 6.6$
	(b)	$74.1 \pm 5.5$
Present work		$72.5 \pm 0.6$

<sup>a</sup>Ref. [5]

<sup>b</sup>Ref. [6]

<sup>c</sup>Ref. [7]

<sup>d</sup>Ref. [8]

<sup>e</sup>Ref. [9]

<sup>f</sup>Ref. [10]

TABLE II. Results for the different potential tails.

$V_{\text{nuc}}$	$\chi_{\text{min}}^2$	$10^3 \times f_0^2$
$V_{\text{OPE}}$	1288.9	$71.9 \pm 0.8$
$V_{\text{OPE}} + V_{\text{HBE}}^{\text{N}}$	1266.7	$72.6 \pm 0.6$
$V_{\text{OPE}}^{\text{s}} + V_{\text{HBE}}^{\text{N}}$	1265.9	$72.5 \pm 0.6$
$V_{\text{OPE}}^{\text{s}} + V_{\text{HBE}}^{\text{P}}$	1273.3	$74.6 \pm 0.6$

## FIGURES

FIG. 1. Ellipses of constant  $\chi^2$  in the  $(m, f^2)$  plane with optimal adjustment of the  $P$ -matrix parameters. Solid ellipse: 69% confidence region ( $\Delta\chi^2 = 2.4$ ). Dashed ellipse: 95.5% confidence region ( $\Delta\chi^2 = 6.2$ ). Filled circle with vertical bar: value and error bar for  $f_0^2$  (with  $m$  fixed). Open circle with horizontal bar: value and error bar for  $m$  (with free  $f_0^2$ ). Open circle with vertical bar: value and error bar for  $f_c^2$  from  $\pi N$  scattering