THEF-NIJM 87.22

Soft-Core Hyperon-Nucleon Potential Model*

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Typeset using REVTEX

*Contribution to the XIth European Conference on Few-Body Problems in Physics, Fontevraud (France), 1987
Published in the proceedings: Few-Body Systems, Suppl. **2**, 514–518 (1987)

I. INTRODUCTION

For nucleon-nucleon *(NN)* scattering it is shown in [1] that a soft-core one-boson-exchange *(OBE)* model can describe the wealthy and precise *NN*-data very well. Only 13 parameters were used. Most of these parameters are coupling constants, mixing angles or *F/(F+D)*-ratio's. These physical parameters can be checked in principle with results found for other reactions. The model was partly based on regge-pole theory [2], therefore the proper theoretical framework for it is the analytic S-matrix theory. Recently [3] within this context the full details on the derivation of the Lippmann-Schwinger equation have been worked out for unequal mass scattering. This enables us to extend the model of [1] to baryon-baryon scattering.

We treat here the reactions: (i) the coupled channels $\Lambda p \rightarrow \Lambda p,\ \Sigma^+ n,\ \Sigma^0 p$; (ii) the coupled channels $\Sigma^- p \rightarrow \Sigma^- p,\ \Sigma^0 n,\ \Lambda p$; and (iii) the single channel $\Sigma^+ p \rightarrow \Sigma^+ p$. For these channels there are experimental data at low energies (see *e.g.* [4]). The *YN*- interaction is described by momentum-dependent potentials and the multichannel Schrödinger equation

$$\left[-\frac{1}{2M_{red}}\nabla^2 + V - \left(\nabla^2 \frac{\phi}{2M_{red}} + \frac{\phi}{2M_{red}}\nabla^2\right) + M\right]\Psi = E\Psi \tag{1}$$

is solved numerically employing the Green transformation [1]. In this equation M_{red} is the reduced mass-matrix, M the rest mass-matrix, V the potential matrix in channel space, and Ψ the wave function vector in channel space.

The *YN*-potentials are given by the following exchanges: (i) the pseudoscalar meson nonet $\pi,\ \eta,\ \eta',\ K$; (ii) the vector meson nonet $\rho,\ \varphi,\ K^\star,\ \omega$; and (iii) the scalar meson nonet $\delta,\ S^\star,\ \kappa,\ \varepsilon$; (iv) The 'diffractive' contribution from $P,\ f,\ f',\ A_2$, and $K^{\star\star}$. The pomeron couplings are taken to be universal, *i.e.* independent of the baryon masses. We note that the 'diffractive' exchanges give gaussian and dominantly repulsive contributions to the potentials in all channels and justify in part the use of hard-cores in the earlier work of the Nijmegen group. In a *QCD*-picture the pomeron can be viewed as a substitute for two- and multi-gluon exchange [5].

The basis of our potentials in momentum space is described in detail in [6], section II. In this work we follow this reference for the definition of our potentials. In [3] it is shown that these potentials also can be derived in the context of the analytic S-matrix approach for unequal mass scattering. The configuration space potentials are obtained via Fourier transformation, which is described in detail in [1]. The *YN*-potentials are of the form

$$\begin{aligned} V = \{ & V_C(r) + V_\sigma(r)\, \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T(r) S_{12} + V_{SO}(r)\, \vec{L} \cdot \vec{S} + \\ & + V_{ASO}(r)\, \frac{1}{2}\, (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \vec{L} + V_{Q_{12}}(r) Q_{12} \} \cdot P \end{aligned} \tag{2}$$

where the operators S_{12} and Q_{12} are the tensor and quadratic spin-orbit operators (see *e.g.* [1] for the definition). The exchange operator $P = 1$ for hypercharge $Y = 0$ exchange and $P = -P_x P_\sigma$ for $Y \neq 0$ exchange ($K,\ K^\star,\ \kappa,\ K^{\star\star}$) where P_x and P_σ are the space and spin exchange operators [6].

The potentials V_C etc. are derived from the evaluation of the *OBE*-contributions in momentum space [1, 3, 6, 7]. Then, the Fourier transformation can be carried through analytically.

Like in previous works [6, 8] on YN we use *SU(3)*-symmetry for the coupling constants. *SU(3)*-breaking is introduced by allowing mixing for mesons belonging to different *SU(3)*-irreps ($\eta-\eta'$, $\omega-\varphi$, $\varepsilon-S^\star$, $P-f$) and charge-symmetry-breaking (csb) due to $\Lambda\Sigma$-mixing which introduces a $\pi-$, $\rho-$, $\delta-$, and A_2-potential in the ΛN channel. Further breaking is introduced by using the physical masses of the baryons and mesons in the potentials (see [6, 8]).

Finally we add that the calculations of the YN-channels with the present model w.r.t. coulomb, two-particle kinematics etc. have been performed treating the details exactly the same way as was done for model D and F [6, 8]. For the masses of the baryons and mesons we have used the same values.

II. FORM FACTORS, CHANNELS, STATES, AND SU(3)

The form factors at the baryon-baryonmeson vertices are, according to regge-pole theory [2, 3], of the exponential type *i.e.* $F(\vec{k}^2) = \exp(-\vec{k}^2/\Lambda^2)$. These form factors guarantee a soft behavior of the potentials in configuration space at small distances.

In model F [8] the *SU(3)*-irreps in the BB-channels are the basis for the scheme of the short range parameters the 'hard cores'. We follow the same scheme here except that the role of the 'hard cores' is taken over by the form factors. In these form factors the behavior is controlled by the cut-off mass Λ. For a review of the *SU(3)* content of the states involved see [8]. The cut-off mass in the $\{27\}$ is fixed in the NN-fit. In summary, we have in YN three free cut-off parameters: the $\{8_s\}-$, $\{8_a\}-$, and the $\{10\}-$ cut-off mass. Here $\{8_s\}$ and $\{8_a\}$ refer respectively to the cut-off's in the $\{27\}+\{8_s\}-$ and $\{10^\star\}+\{8_a\}-$ states.

Although from the different weigths of the $\{27\}$ and the $\{8_s\}$ irreps in the $\Lambda\Lambda$ and the $\Sigma\Sigma$ channel it could be justified to use different cut-off masses, it turned out that this was unnecessary. An excellent fit could be obtained with only one form factor for both channels. It also turned out that for a fit to the Σ^+p data we can use the same cut-off mass for the $\{27\}$ as obtained in the NN-analysis [1].

In Table I we summarize the form factor prescription and the values of the form factor masses which have emerged from the YN-fit.

III. COUPLING CONSTANTS

The values of the nucleon-nucleon coupling constants have been discussed in [1]. The handling of *SU(3)* for the pseudoscalar and vector mesons has been amply discussed in [6, 8]. Here we discuss briefly the treatment of the scalar mesons and the 'diffractive' exchange.

In the scalar meson nonet the physical ε- and $S^\star$-meson are described in terms of the *SU(3)*-singlet ε_0 and -octet state $S^\star_0$ using a single mixing angle θ_S

$$\begin{aligned} |\varepsilon\rangle &= \cos\ \theta_S\ |S^\star_0\rangle - \sin\ \theta_S\ |\varepsilon_0\rangle \\ |S^\star_0\rangle &= \sin\ \theta_S\ |S^\star_0\rangle + \cos\ \theta_S\ |\varepsilon_0\rangle \end{aligned} \tag{3}$$

Note that with this convention, the ideal mixing angle in the $q^2\bar{q}^2$-picture of the scalar mesons is $\theta_S = 35.3^o$ [9]. Using these expressions the couplings can readily be expressed

I	Channels	States	*SU(3)* irreps	Parameter
1	NN	${}^1S_0,{}^3P,{}^1D_2,...$	$\{27\}$	Λ_{27} = 964.52 MeV
0		${}^3S_1,{}^1P_1,{}^3D,...$	$\{10^\star\}$	$\Lambda_{10^\star}$ = 964.52 MeV
$\frac{1}{2}$	$\Lambda N, \Sigma N$	${}^1S_0,{}^3P,{}^1D_2,...$	$\{27\}+\{8_s\}$	Λ_{8s} = 775.00 MeV
		${}^3S_1,{}^1P_1,{}^3D,...$	$\{10^\star\}+\{8_a\}$	Λ_{8a} =1248.00 MeV
$\frac{3}{2}$	ΣN	${}^1S_0,{}^3P,{}^1D_2,...$	$\{27\}$	Λ_{27} = 964.52 MeV
		${}^3S_1,{}^1P_1,{}^3D,...$	$\{10\}$	Λ_{10} =1210.00 MeV

TABLE I. Form factors for the different waves in $Y = 1$ and $Y = 2$ baryon-baryon scattering and the cut-off masses of the *YN*-fit

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in terms of the singlet coupling g_1, the octet coupling g_8, the *F/(F+D)*-ratio α_S, and the mixing angle θ_S.

For the 'diffractive' exchanges we take the 'bare' pomeron as a *SU(3)*-singlet. The tensor nonet contains the f_0 and the f_0' which are respectively the *SU(3)*-singlet and octet state. Exact *SU(3)* and unitarity cause a strong mixing between the 'bare' pomeron and f_0. We describe this system by P_0, which is obviously an *SU(3)*-singlet. Medium strong breaking then gives mixing of P_0 and f_0', leading to the physical pomeron P and f. In the *NN*-analysis the combination

$$g_P^2 = g_{PNN}^2 + g_{fNN}^2 = g_1^2 + \frac{1}{3}\,(4\alpha_D - 1)^2\, g_8^2 \tag{4}$$

has been fixed. There also $g_8 = g_{A_2NN}$ is fitted. From the expression for g_P^2 one sees that g_1 and α_D can be written in terms of g_P, g_{A_2NN} and an angle that we call ψ_D. One has

$$g_1 = \cos\,(\psi_D)\; g_P \quad , \quad (4\alpha_D - 1)/\sqrt{3} = \sin\,(\psi_D)\; g_P/g_{A_2NN} \tag{5}$$

So, in the *YN*-analysis we have for the diffractive contributions one extra free parameter, the angle ψ_D. Another possible relevant free parameter would be θ_D. Since we have used the same mass m_P for all diffractive exchanges, the results are independent of θ_D.

IV. RESULTS

1. Parameters. The *NN*-analysis of [1] is the basis for the *OBE*-coupling constants we used. In the calculations reported here, *SU(3)*-relations are assumed for the pseudo-vector

mesons		{1}	{8}	*F/(F+D)*	angles
pseudoscalar	f	0.18455	0.27204	$\alpha_{PV} = 0.355^{\star)}$	$\theta_{PS} = -23.00^o$
vector	g	2.52934	0.89147	$\alpha_V^e = 1.0$	$\theta_V = 37.50^o$
	f	0.97982	3.76255	$\alpha_V^m = 0.275^{\star)}$	
scalar	g	3.77486	1.27734	$\alpha_S = 1.27741$	$\theta_S = 41.26^{o\ \star)}$
diffractive	g	2.85507	0.44372	$\alpha_D = 1.02267$	$\psi_D = 15.50^{o\ \star)}$

TABLE II. Coupling constants, *F/(F+D)*-ratio's, mixing angles etc. The values with $\star)$ have been determined in the fit to the *YN*-data. The other parameters are theoretical input or determined by the fitted parameters and the constraint from the *NN*-analysis.

couplings of the pseudo-scalar mesons, for the Pauli couplings of the vector mesons, for the coupling of the scalar mesons, and for the pomeron and tensor meson contributions. We have analysed the low energy *YN*-data (see *e.g.* [4, 6, 8] for a description) for all *YN*-channels simultaneously: Λp-, $\Sigma^- p$-, and $\Sigma^+ p$-data. An excellent solution was found which appears qualitatively even better than the Nijmegen hard-core potentials [6, 8].

The value found for α_{PV} agrees very well with the determination in weak interactions [10] and that for α_V^m is in full accordance with relativistic *SU(6)* [11]. Note here that α_V^e has not been fitted, but is theoretical input. Another important free parameter is the scalar mixing angle θ_S. The fit appears to be very sensitive to this parameter. We obtained $\theta_S = 41.26^o$. This is rather close to the ideal mixing for scalar mesons, mentioned above. In the region where the data can be fitted successfully the $\Sigma^- p$ elastic and inelastic cross sections dependance on θ_S is rather steep. For the angle ψ_D we found 15.5^o.

In Table II we have listed the information on the coupling constants. Here all couplings refer to rationalized couplings *i.e.* they should be understood to be $g/\sqrt{4\pi}$.

2. Λp-Scattering. The low energy Λp scattering data can be fitted excellently by various sets of s-wave singlet and triplet scattering lengths. We found a significant influence from the coupled ΣN-channels, even on the scattering lengths. The fit to the Λp-data is displayed in Figure 1. We find a total $\chi^2 = 3.6$ for the 12 Λp-cross-sections. The fit has for the Rehovoth-Heidelberg data [12] $\chi^2 = 1.1$ and for the Maryland data [13] $\chi^2 = 2.5$. The p-waves have been included in the fit, but are rather unimportant. In our solution the $^3S_1$ (Λp)-phase shift does not resonate below the $\Sigma^0 n$-threshold: the $^3S_1$-eigenphase shift reaches 64^o at threshold. The scattering lengths $a(\Lambda p)$ and effective ranges $r(\Lambda p)$ are

$$\begin{array}{ll} {}^1S_0: & a_s(\Lambda p) = -2.696\text{fm}\ ,\ r_s(\Lambda p) = 2.951\text{fm} \\ {}^3S_1: & a_t(\Lambda p) = -1.507\text{fm}\ ,\ r_t(\Lambda p) = 3.039\text{fm} \end{array}$$

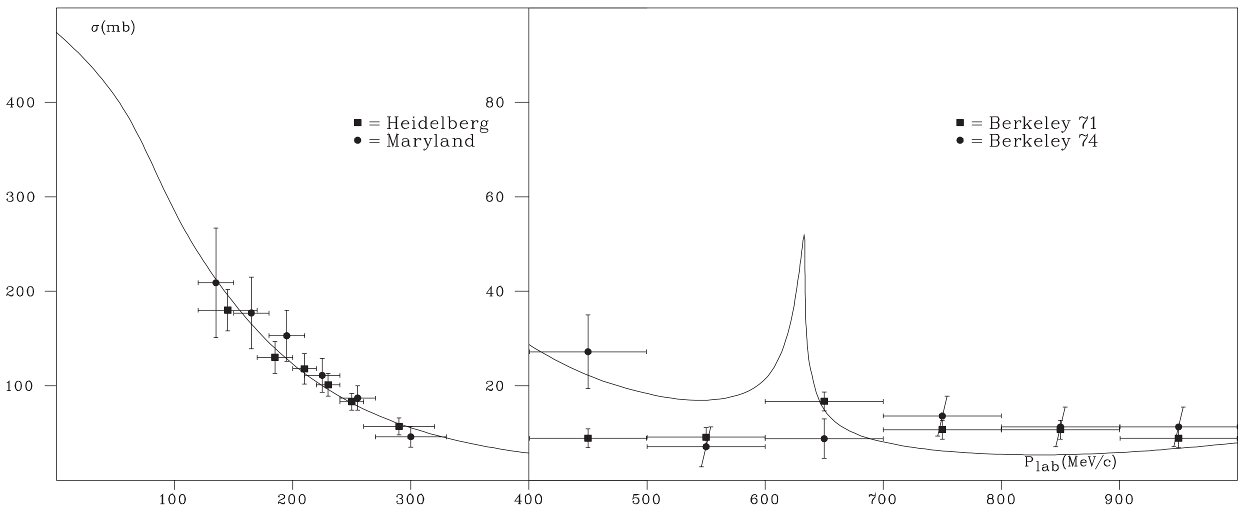

FIG. 1. Λp cross-sections and experimental data

Note that the difference between the singlet and the triplet scattering lengths is larger than in our former analyses using the Nijmegen hard-core models D and F. The effects of the charge-symmetry breaking seem rather small.

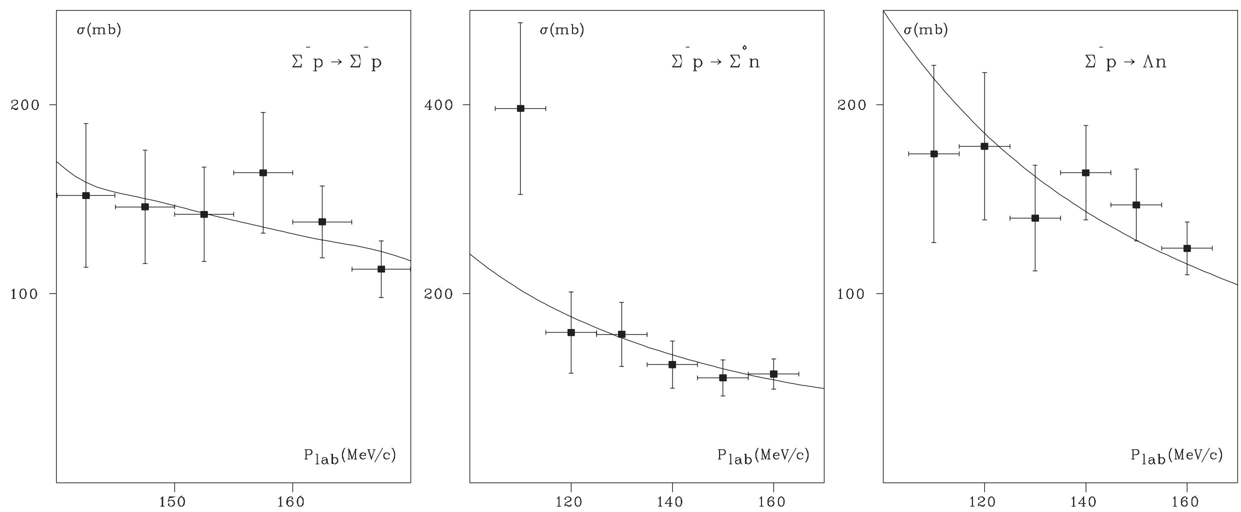

FIG. 2. $\Sigma^{-}p$ elastic and inelastic cross-sections and experimental data

3. $\Sigma^{-}p$-Scattering. The fit to the $\Sigma^{-}p$-cross sections is given in fig. 2. For 18 cross section data [14, 15] we have $\chi^2 = 12.8$, where the $\Sigma^0 n$-cross section at p_{lab}= 110 MeV contributes 6.1. At the same time the capture ratio at rest has the value $r_R = 0.4676$, which is almost equal to the average experimental value $r_R = 0.468 \pm 0.010$ [4].

4. $\Sigma^{+}p$-Scattering. The form factor needed for ${}^1S_0$ $(\Sigma^{+}p)$ could be taken equal to the ${}^1S_0$ (pp) form factor. Despite this imposed equality, we obtain a perfect match to the

Σ^+p-cross section data [15]: $\chi^2 = 0.1$. The scattering lengths and effective ranges are

$$\begin{array}{llllllll} {}^1S_0: & a_s(\Sigma^+p) & = & -3.713\text{fm}\ , & r_s(\Sigma^+p) & = & 3.255\text{fm} \\ {}^3S_1: & a_t(\Sigma^+p) & = & 0.299\text{fm}\ , & r_t(\Sigma^+p) & = & -20.58\text{fm} \end{array}$$

All the p-wave phase shifts are rather small, in contrast to the D and F model, which have large ${}^1P_1$-phases around p_{lab}= 420 MeV.

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