Evidence for a Large Breaking of Charge Independence in the $NN$ Interaction

V.G.J. Stoks, P.C. van Campen, T.A. Rijken, and J.J. de Swart

Institute for Theoretical Physics, University of Nijmegen, Nijmegen, The Netherlands

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Abstract

In an energy-dependent phase-shift analysis of all $np$ scattering data below $T_{\text{lab}} = 30$ MeV, we find a large breaking of charge independence in the $\pi NN$ coupling constants $g_0^2$ and $g_c^2$. For 495 degrees of freedom we obtain $\chi^2_{\text{min}} = 533.8$ and $\Delta g^2 \equiv g_c^2 - g_0^2 = 1.28 \pm 0.15$. A combined analysis, including also all $pp$ scattering data below $T_{\text{lab}} = 30$ MeV, results in $\chi^2_{\text{min}} = 847.2$ for 840 degrees of freedom. We then obtain $\Delta g^2 = 1.62 \pm 0.27$. The effect of such large values for $\Delta g^2$ is most strikingly seen in the $^3P$ waves. The $np$ $^3P$ phase shifts at $T_{\text{lab}} = 25$ MeV are much larger than those of other phase-shift analyses.
Since the charge independence of the NN interaction was first clearly formulated [1], it has been an important assumption in nuclear physics. That this isospin symmetry is not exact, but is certainly broken because of the electromagnetic force between the protons, because of the proton-neutron mass difference, and also because of the difference between the neutral- and charged-pion masses, has been clear for a long time (see, e.g., Ref. [2] and references cited therein). However, this breaking has always been assumed to be rather small.

From the $\pi^\pm p$ scattering data the coupling constant $g_c$ of the charged pions to the nucleons has been known rather accurately for a long time. At present the recommended value is $g_c^2 = 14.3 \pm 0.2$ or $f_c^2 = 0.079(1)$, where the pseudoscalar coupling constant $g_c^2$ and the pseudovector coupling constant $f_c^2$ are related according to

$$g_c^2 = [(M_p + M_n)/m_c]^2 f_c^2 = 181.029 f_c^2,$$

with $m_c$ the charged-pion mass, and $M_p$ and $M_n$ the proton and neutron masses, respectively.

On the other hand, it has always been very hard to obtain an accurate value for the $pp\pi^0$ coupling constant $g_0 \equiv g(pp\pi^0)$ (see, e.g., the different values as given in Table I of Ref. [4]). Therefore, there has never been any clear evidence for a charge-independence breaking (CIB) in the pion-nucleon coupling constants.

Recently this situation has changed because Bergervoet et al. have been able to determine this neutral coupling constant $g_0^2$ in a phase-shift analysis of all $pp$ scattering data below $T_{\text{lab}} = 350$ MeV [4]. The value found is

$$g_0^2 = 13.1 \pm 0.1 \quad \text{or} \quad f_0^2 = 0.0725(6),$$

where $g_0^2 = (2M_p/m_c)^2 f_0^2 = 180.780 f_0^2$. Comparing this result with the value for the charged coupling constant $g_c$, we observe a CIB in the $\pi NN$ coupling constants of $\Delta g^2 \equiv g_c^2 - g_0^2 = 1.2 \pm 0.2$. This large CIB in the $\pi NN$ coupling constants should have important consequences for the charge independence of the NN interaction, since this must then be broken to a much larger extent than previously assumed.

In this Letter we demonstrate that charge independence of the $NN$ interaction is indeed broken, and not only in the $^1S_0$ states. This breaking will be illustrated for the $^1S_0$ and $^3P$ waves. We will assume that charge symmetry in the $\pi NN$ coupling constants is still valid, which implies that $g(pp\pi^0) = -g(nn\pi^0) = g_0$. We will assume that charge independence is broken by the electromagnetic interaction, by the proton-neutron mass difference, by the difference between charged- and neutral-pion masses, and also by the difference between $g_0^2$ and $g_c^2$. The remaining part of the $NN$ interaction we assume to be charge independent.

The effect of the CIB in the $\pi NN$ coupling constants can best be understood as follows. The one-pion exchange (OPE) potential for $pp$ scattering can be written as

$$V_{\text{OPE}}(pp) = g_0^2 V_\pi(m_0),$$

where $m_0$ denotes the neutral-pion mass. The OPE potential for $np$ scattering in the $I = 1$ state can then be written as

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\[ V_{\text{OPE}}(np) = -g_0^2 V_\pi(m_0) + 2g_c^2 V_\pi(m_c) , \]

where \( m_c \) denotes the charged-pion mass. When we neglect for a moment all effects due to mass differences, this becomes

\[ V_{\text{OPE}}(np) = -g_0^2 V_\pi + 2g_c^2 V_\pi = (g_0^2 + 2\Delta g^2)V_\pi . \]

With the aforementioned values for \( g_0^2 \) and \( g_c^2 \), the OPE potential for \( np \) scattering in the \( I = 1 \) state is found to be stronger than the OPE potential for \( pp \) scattering by \( 2\Delta g^2/g_0^2 \approx 18\% \). This is indeed a large effect, and our including the small proton-neutron and charged- and neutral-pion mass differences as in Eq. (2) will not change this result qualitatively.

We have performed energy-dependent phase-shift analyses on all published \( NN \) scattering data below \( T_{\text{lab}} = 30 \) MeV. We start with a multienergy phase-shift analysis of the \( pp \) scattering data below 30 MeV using \( g_0^2 = 13.1 \), based on the \( pp \) analysis as already performed by our group [5]. We obtain \( \chi^2_{\min}(pp) = 344.5 \) for 345 degrees of freedom.

Next we do a multienergy phase-shift analysis of the \( np \) scattering data below 30 MeV. A full account of this analysis will be given in a forthcoming paper [6], so we will now only discuss the parametrization of the lower partial waves. The \( I = 0 \) lower partial waves (\( J \leq 2 \)) are parametrized by the \( P \) matrix at \( r = b = 1.4 \) fm, where the \( P \) matrix is the logarithmic derivative of the wave function at \( r = b \). For \( r > b \), we take the OPE potential into account exactly. For the coupled \( ^3S_1 + ^3D_1 \) channels, the \( P \) matrix is sufficiently well parametrized by three parameters: For the diagonal element in the \( ^3S_1 \) channel we take two parameters (i.e., linear in \( k^2 \), the c.m. momentum squared), for the off-diagonal element a constant is taken, and the diagonal element in the \( ^3D_1 \) channel is given the free \( P \)-matrix value at \( T_{\text{lab}} = 0 \) MeV of \( l + 1 = 3 \). The \( P \) matrix for the \( ^1P_1 \) channel is taken to be a constant and the \( ^3D_2 \) channel is given the free \( P \)-matrix value.

The \( I = 1 \) \( P \) matrix for the \( ^1S_0 \) channel is parametrized by two parameters. The other \( I = 1 \) lower-partial-wave phase shifts (\( ^3P_J \) and \( ^1D_2 \)) are taken from our \( pp \) phase-shift analysis after correcting them not only for electromagnetic and mass-difference effects, but also for the difference between \( g_0^2 \) and \( g_c^2 \). The CIB-corrected \( np \) phase shifts are parametrized as follows. With some realistic \( NN \) potential model \( V_{\text{nuc}} \), \( pp \) phase shifts are calculated by solving the radial Schrödinger equation for the potential \( V = V_{\text{nuc}} + V_{\text{em}} \), where the electromagnetic potential \( V_{\text{em}} \) consists of the relativistic Coulomb potential [7] and the vacuum-polarization potential [8]. The \( np \) phase shifts can then be calculated for various values of \( \Delta g^2 \) by solving the radial Schrödinger equation for the potential \( V = V_{\text{nuc}} + 2\Delta g^2V_\pi \), where neutron-proton and neutral-charged-pion mass differences are taken into account explicitly. The differences between these \( np \) and \( pp \) phase shifts as obtained with this \( NN \) potential model are then used to correct the \( pp \) phase shifts as obtained in our \( pp \) phase-shift analysis. Therefore, the \( ^3P_J \) and \( ^1D_2 \) \( np \) phase shifts in our analysis are parametrized by \( \Delta g^2 \). The CIB corrections to the phase shifts were calculated with the Nijmegen soft-core \( NN \) potential [9], and also with the parametrized Paris \( NN \) potential [10] with almost the same results. However, including the corrections as calculated with the Paris potential in the \( np \) analysis results in a \( \chi^2_{\min} \) which is 18 higher than the \( \chi^2_{\min} \) as obtained with the corrections determined with the Nijmegen potential, so the latter corrections have been used for parametrizing the \( ^3P_J \) and \( ^1D_2 \) phase shifts.
All higher partial waves ($J > 2$) for both $I = 0$ and $I = 1$ are taken to be pure OPE, including the explicit charge splitting $\Delta g^2$ and mass differences. With the parametrizations as described above, our $np$ phase-shift analysis therefore contains six $P$-matrix parameters and $\Delta g^2$, which is used as a parameter by our fixing $g^2_0$ and leaving $g^2_3$ as a free parameter. We have convinced ourselves that $\Delta g^2$ is almost independent of the precise value of $g^2_0$, so in the following we use $g^2_0 = 13.1$, the result of our $pp$ phase-shift analysis of all $pp$ scattering data below $T_{\text{lab}} = 350$ MeV [4].

In our $np$ phase-shift analysis with 495 degrees of freedom we then reach $\chi^2_{\text{min}}(np) = 533.8$ and we find

$$\Delta g^2 = 1.28 \pm 0.15 \quad \text{or} \quad \Delta f^2 = 0.0070(8),$$

where $\Delta f^2 = f^2_3 - f^2_0$. The error is obtained from the $\chi^2$-rise-by-one rule. We note the excellent agreement with the expected value of $\Delta g^2 = 1.2 \pm 0.2$. If we perform an $np$ analysis where $\Delta g^2 \neq 0$ for the $^3P$ waves only, we obtain $\chi^2_{\text{min}} = 533.7$ with $\Delta g^2 = 1.31 \pm 0.15$ in these $^3P$ waves. From this we can conclude that the result of $\Delta g^2 \neq 0$ is almost totally due to the CIB in the $^3P$ waves.

In Fig. 1 we have plotted $\chi^2$ of our $np$ phase-shift analysis as a function of $\Delta g^2$ (dashed line). We see that $\Delta g^2 = 0$ (no CIB in the $\pi NN$ coupling constants) corresponds to $\chi^2_{\text{min}} = 583$. The drop of 50 in $\chi^2_{\text{min}}$ when we vary $\Delta g^2$ shows the significance (7 standard deviations) of this effect. This drop is almost totally due to a better description of the $np$ polarization data (including CIB in the $\pi NN$ coupling constants causes a drop of 40 for these data).

We have also performed a combined $pp$ and $np$ phase-shift analysis. This means that the CIB-corrected $I = 1$ phase shifts ($^3P_J$ and $^1D_2$) are then not only determined by the $pp$ scattering data, but also by the $np$ scattering data. We now obtain $\chi^2_{\text{min}} = 847.2$ for 840 degrees of freedom (i.e., $\chi^2$ per degree of freedom is 1.01), and

$$\Delta g^2 = 1.62 \pm 0.27 \quad \text{or} \quad \Delta f^2 = 0.0088(15).$$

Again we have agreement with the expected value of $\Delta g^2 = 1.2 \pm 0.2$ (about 1.2 standard deviations).

This result of $\chi^2_{\text{min}} = 847.2$ should be compared with the previous value of $\chi^2_{\text{min}}(pp) + \chi^2_{\text{min}}(np) = 878.3$, where we added the results of the separate $pp$ and $np$ analyses. The drop in $\chi^2$ of 31.1 is due to the following. The CIB corrections cannot change the spin-orbit combination of the $np$ $^3P$ waves (for definitions of spin-orbit, central, and tensor combinations of phase shifts see, e.g., Ref. [5]), since the OPE potential does not contain a spin-orbit part and the spin-orbit part of the electromagnetic potential can be neglected in this energy range. The only way to accomplish a change in the spin-orbit combination of the $np$ $^3P$ waves is, therefore, to change the spin-orbit combination of the $pp$ $^3P$ waves. This is exactly what happens. In the combined analysis, the spin-orbit combination of the $pp$ $^3P$ waves is changed and the central and tensor combinations remain the same, causing a rise of $\Delta \chi^2 = 0.9$ on the $pp$ scattering data. This change in the $pp$ $^3P$ waves then enables a change in the spin-orbit combination of the $np$ $^3P$ waves, causing an enormous drop of $\Delta \chi^2 = -32.0$ on the $np$ scattering data.

In order to make an easy comparison possible, we have plotted in Fig. 1 the $\chi^2_{\text{min}}$ for the combined $pp$ and $np$ analysis as a function of $\Delta g^2$, after subtracting $\chi^2_{\text{min}}(pp) = 344.5$ of the
pp analysis (solid line). From the results mentioned above we see that there is a definite CIB present in the $NN$ interaction.

In order to demonstrate the effect of this CIB in the phase shifts we have chosen the specific values $g_0^2 = 13.1$ and $g_2^2 = 14.4$.

In Fig. 2 we plot the $pp$ and $np$ phase shifts of the corresponding combined analysis for the $^1S_0$ and $^3P$ phase shifts. We note the substantial $np-pp$ phase-shift difference for the $^3P_0$ phase shift at 25 MeV of $10.9° - 8.7° = 2.2°$. We have also plotted the phase shifts given by the phase-shift analysis of Arndt, Hyslop, and Roper [11] at 10 and 25 MeV (Ref. [11] does not include the $^1S_0$ phase shift at 10 MeV). The charge splitting in the $^3P_0$ phase shift at 25 MeV in their analysis amounts to only $8.57° - 8.25° = 0.32°$. A similar CIB of $7.95° - 7.67° = 0.28°$ is given by Bohannon, Burt and Signell [12]. On the other hand, Bystricky, Lechanoine-Leluc, and Lehar [13] find a splitting of $7.59° - 7.67° = -0.08°$. This means that our $^3P_0$ $np$ phase shift at 25 MeV differs from 25% to 40% with that of other phase-shift analyses.

Summarizing, in energy-dependent phase-shift analyses of all $pp$ and $np$ scattering data below $T_{lab} = 30$ MeV, we find a definite breaking of charge independence in the $\pi NN$ coupling constants. The results are in good agreement with $g_0^2 = 13.1 \pm 0.1$ as found in a $pp$ phase-shift analysis and $g_2^2 = 14.3 \pm 0.2$ as found from $\pi N$ scattering. This CIB gives rise to $np$ $^3P$ phase shifts which are very much different from earlier analyses.

Finally, we would like to point out that the results as described in this Letter do not have their origin in the fact that we use a different data set compared to other analyses (we think our data set is much more complete), but only in the following:

(1) We make use of a more powerful method of analyzing the data ($P$ matrix with a potential tail).

(2) We correct the $I = 1$ $^3P_J$ and other phase shifts (except for the $^1S_0$), as obtained from our $pp$ phase-shift analysis, for use in the $np$ phase-shift analysis, not only for the Coulomb interaction (as is mostly done in other analyses) and mass-difference effects (as is done in some analyses), but also for the $\Delta g^2 = g_c^2 - g_0^2$ pion-nucleon coupling-constants difference.
REFERENCES

FIGURES

FIG. 1. $\chi^2$ as a function of $\Delta g^2$ for the $np$ phase-shift analysis (dashed line) and for the combined analysis after subtracting $\chi^2 = 344.5$ of the $pp$ analysis (solid line).

FIG. 2. Effects of the CIB corrections on the phase shifts of the combined analysis with $g_0^2 = 13.1$ and $g_c^2 = 14.4$. Dashed lines: $pp$ phase shifts; solid lines: $np$ phase shifts. Filled circles and open squares denote the $pp$ and $np$ phase shifts as given by Arndt, Hyslop, and Roper (Ref. [11]), respectively.