Analysis of the $A_{zz}$ measurement in $np$ scattering at $T_{\text{lab}} = 67.5$ MeV

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Abstract

We analyze a recent experiment in which the spin-correlation parameter $A_{zz}$ in $np$ scattering at $T_{\text{lab}} = 67.5$ MeV was measured. The $I = 0$ phase parameters can now be determined much more accurately in a single-energy analysis at 50 MeV. The value found for the $^3S_1 - ^3D_1$ mixing parameter $\varepsilon_1$ is in excellent agreement with modern potential-model predictions.

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Recently, a measurement of the spin-correlation parameter \( A_{zz} \) in \( np \) scattering at 67.5 MeV was reported [1]. In that Letter the authors claimed that their determination of the \( ^3S_1-^3D_1 \) mixing parameter \( \varepsilon_1 \) at 50 MeV is in disagreement with modern \( NN \) potential-model predictions. When true, this result would be very disturbing. Because we just finished a preliminary version of the Nijmegen 0–350 MeV \( NN \) partial-wave analysis in which no strange behavior of the \( \varepsilon_1 \) was found, we decided to have a better look at this experiment at 67.5 MeV. Here, we give a brief report of our results in which we refute the claim of the Basel group [1]: We find the \( \varepsilon_1 \) mixing parameter to be in excellent agreement with modern potential predictions. A complete and detailed discussion of our \( NN \) analysis will be deferred to a future paper.

The \( \varepsilon_1 \) mixing parameter and the \( ^1P_1 \) phase shift have always been very difficult to determine accurately. The problem is that only higher-order spin observables such as spin-correlation parameters are sensitive to \( \varepsilon_1 \). Measurements of such observables have been scarce and the data that are available are often also sensitive to the \( ^1P_1 \) phase shift [2]. The \( ^1P_1 \) phase shift can be determined from the \( np \) differential cross-section data, but these data are often not accurate enough to fix it within a reasonably small uncertainty. Selection of different sets of cross-section data that are available can result in very different values for \( ^1P_1 \). Especially for the data in the 50-MeV region, this fact has been amply touched upon [2, 3]. In order to pin down the \( ^1P_1 \) phase shift in this energy region, we are apparently in need of more accurate \( np \) cross-section data, both at extreme forward and extreme backward angles.

Below 400 MeV, the few \( np \) spin-correlation parameters that were available until recently are \( A_{yy} \) data. They consist of 4 data at 23.1 MeV [4], 8 data at 50.0 MeV [5, 6], 10 data at 181.0 MeV [7], and 35 data at 220.0 and 325.0 MeV [8]. The \( A_{yy} \) measurement at \( \theta_{\text{cm}} = 90^\circ \) at 13.7 MeV by Schöberl et al. [9] is of particular importance, due to the insensitivity of \( A_{yy} \) to the \( ^1P_1 \) phase shift at this scattering angle. With these data the \( \varepsilon_1 \) mixing parameter can now be reasonably well determined in a multienergy (m.e.) partial-wave analysis. However, in a single-energy (s.e.) analysis at 50 MeV or 100 MeV it is still poorly determined. In order to improve this situation, we are in need of more accurate spin-correlation data in this energy range. Recently, there have been 45 measurements of \( A_{yy} \) at 9 energy bins centered at 19 to 50 MeV, but these (preliminary) data have only been presented in Ref. [10]. Another very important experiment is the recent measurement of \( A_{zz} \) at 20 scattering angles at 67.5 MeV [1]. Its importance lies in the fact that the correlations between \( \varepsilon_1 \) and \( ^1P_1 \) for the \( A_{yy} \) and \( A_{zz} \) spin-correlation parameters are of opposite sign. This eliminates a possible bias in the determination of \( \varepsilon_1 \).

In this Brief Report, we present the results of our m.e. partial-wave analysis (PWA), where we include the new 67.5-MeV data. The data are analyzed in a combined PWA, including all \( pp \) and \( np \) scattering data below \( T_{\text{lab}} = 350 \) MeV. In the s.e. analysis of the 50-MeV region, the \( pp \) phase parameters are accurately known. This is mainly due to the presence of an extremely accurate \( pp \) analyzing-power experiment at 50.04 MeV [11], which was analyzed in an earlier publication by our group [12]. We show that the quality of the \( np \) data in this energy region was poor, in that the determination of especially the \( I = 0 \) phase parameters in the s.e. analysis lead to results which were unsatisfactory. Inclusion of the new 67.5-MeV data gives a considerable improvement. Therefore, this \( A_{zz} \) experiment, together with the 50.04 MeV \( pp \) analyzing-power experiment [11], is very important in that it provides us with a fairly complete set of \( NN \) scattering data around 50 MeV.
Our way of analyzing the NN scattering data is extensively discussed elsewhere [13, 14], so here we will not go into any particular details. In our analyses we solve the relativistic Schrödinger equation, where the well-known long-range interaction is incorporated by a potential tail. The short-range interaction is parametrized with an energy-dependent boundary-condition model. The boundary-condition parametrization is used for the lower partial waves with total angular momentum \( J \leq 4 \). For the intermediate partial waves \( (5 \leq J \leq 8) \) we use the phase shifts and mixing parameters of the one-pion-exchange (OPE) potential plus the heavy-boson-exchange contributions of the Nijmegen soft-core NN potential [15]. All higher partial waves are given by the OPE phase parameters.

The \( np \) data base is not rich and accurate enough to determine both the \( I = 0 \) and the \( I = 1 \) partial waves. Therefore, in our analyses the \( I = 0 \) lower partial waves are searched for, whereas the \( np I = 1 \) partial waves are obtained from the \( pp I = 1 \) partial waves, after correcting them for Coulomb distortion and mass difference effects. The only exception is the \( ^1S_0 \) \( np \) partial wave, which is parametrized independently of the \( pp \) data. In the combined analysis, all NN scattering data are analyzed simultaneously, so the \( I = 1 \) lower partial waves are not only determined by the \( pp \) data, but also by the \( np \) data.

Our data base contains all \( pp \) and \( np \) scattering data below 350 MeV, published in a regular physics journal as of 1955. The data were carefully pruned on the basis of certain rejection criteria (for more details, see Refs. [13, 14]). Prior to the \( A_{zz} \) experiment, our \( np \) data base contained 2421 scattering data. The \( pp \) data base contains 1766 scattering data. In total, we need 51 parameters to parametrize the energy dependence of the boundary conditions. Taking into account the floated normalization parameters which are to be fitted, we are left with 3850 degrees of freedom. We reach \( \chi^2_{\text{min}} = 4171.1 \), consisting of \( \chi^2_{\text{min}}(pp) = 1771.4 \) and \( \chi^2_{\text{min}}(np) = 2399.7 \).

The \( A_{zz} \) measurement also involved two analyzing-power measurements [1]. Using the model parameters of our m.e. PWA without these data, we predict \( \chi^2 = 66.2 \) for these 54 data. The different contributions including the normalization uncertainties are \( \chi^2(P_{\text{beam}}) = 9.88 \) for 12 \( p(n, n)p \) analyzing-power data, \( \chi^2(P_{\text{target}}) = 23.05 \) for 19 \( p(n, p)n \) analyzing-power data, and \( \chi^2(A_{zz}) = 33.29 \) for the 20 spin-correlation data. Refitting the model parameters and including these data, we find \( \chi^2(P_{\text{beam}}) = 9.22 \), \( \chi^2(P_{\text{target}}) = 22.21 \), and \( \chi^2(A_{zz}) = 14.77 \), which means a drop of 20.0 on these data. The \( \chi^2 \) on the other data in our data base rises with 1.0. Therefore, the new 67.5-MeV data are in excellent agreement with our PWA.

Over the last decade there has been an addition of many precise \( pp \) and \( np \) scattering data to the world data base, which means that the energy behavior of the phase shifts and mixing parameters, as determined in a m.e. PWA, is very well known. However, one still must do s.e. analyses to obtain estimates for the errors on the phase parameters within some particular energy bin, where one must bear in mind the results of the m.e. analysis. An important criterion for the quality of a s.e. analysis is that it has to agree with the results of the corresponding m.e. analysis. This implies that the s.e. values for the phase parameters should be scattered statistically around the curve representing the m.e. values. So a s.e. analysis without an accompanying m.e. analysis can be misleading. For example, the absence of spin-correlation data in the 100-MeV region makes that the \( \varepsilon_1 \) cannot be determined very accurately in a s.e. analysis in this energy region. However, the available spin-correlation data at the adjoining energies at 50 and 150 MeV make that the energy behavior of \( \varepsilon_1 \) is fixed rather well in the m.e. analysis. This means that also at 100 MeV, \( \varepsilon_1 \)
is in fact much more accurately determined by the data than the s.e. result would suggest. Therefore, a s.e. analysis only provides what we would say is an upper limit for the errors on the phase parameters.

In order to obtain an estimate for the errors on the phase parameters, we have also performed s.e. analyses at 50 MeV with and without the new 67.5-MeV data. In these s.e. analyses we analyze the \( pp \) and \( np \) scattering data between 35.0 and 75.0 MeV. This amounts to 244 \( pp \) scattering data and 270 \( np \) scattering data. Here we omitted the Harwell \( np \) differential cross-section data [16] because they do not survive our rejection criteria, which is in agreement with earlier analyses [2, 3]. (They were also not included in the analysis of the Basel group [1]). The 67.5-MeV data contribute with 54 to the \( np \) data. The \( pp \) \( ^1S_0 \) phase shift and the \( np \) phase parameters up to total angular momentum \( J = 2 \) (except for the \( ^3F_2 \)) are searched for by adding a constant to be added to the energy-dependent boundary condition of the m.e. fit, which ensures a proper energy dependence for the phase parameters. The differences between the \( pp \) and \( np \) \( I = 1 \) phase parameters are fixed at the values as obtained in our m.e. analysis. All other phase parameters are fixed at their m.e. value.

In the second and third columns of Table I we present the m.e. and s.e. \( np \) phase parameters and the \( pp \) \( ^1S_0 \) phase shift as obtained in the analysis without the 67.5-MeV data. The errors on the \( I = 1 \) phase parameters (except the \( np \) \( ^1S_0 \)) shown in the upper half of Table I are rather small. This is due to the fact that the corresponding \( pp \) phase parameters are accurately known [12], and the \( np \) \( I = 1 \) phase parameters in our analyses are obtained from the \( pp \) phase parameters after correcting them for Coulomb and mass-difference effects, where we also allow for a possible difference between the neutral- and charged-pion nucleon coupling constants [17, 18].

The \( \varepsilon_1 \) is very ill-determined; the difference between the m.e. result and the s.e. result is more than 6 standard deviations, which is unacceptably large. We have not been able to pinpoint a specific group of data which causes this aberrant behavior. We also included preliminary values of the Karlsruhe \( A_{yy} \) data [10] (which were included in the analysis of the Basel group), but this did not change the result for the \( \varepsilon_1 \). We therefore redid the s.e. analysis, now fixing the \( \varepsilon_1 \) at its m.e. value. The results are presented in the fourth column of Table I. The result is still not satisfactory in that some of the s.e. phase shifts \( (^1S_0(np),^1P_1,^3D_2) \) are more than 3 standard deviations off when compared with their m.e. values. These results demonstrate that a s.e. analysis cannot be very useful if it cannot be compared with an accompanying m.e. analysis. The fact that it is mainly the \( I = 0 \) phase parameters which are ill-determined, reflects that the information stored in the \( np \) data in this energy region is rather poor.

Inclusion of the 67.5-MeV data gives a considerable improvement. The results for the phase parameters are presented in the last two columns of Table I. The phase parameters from the m.e. analysis including these data do not differ very much from those of the m.e. analysis without these data, demonstrating that the energy behavior of the phase parameters was already pretty well-determined before the inclusion of the 67.5-MeV experiment. The differences between the m.e. and the s.e. phase parameters are now within one standard deviation. The \( I = 1 \) phase parameters did not change very much, because they are mainly determined by the \( pp \) scattering data. The \( ^1S_0(np) \) phase shift and the \( I = 0 \) phase parameters, however, are now determined much more accurately.
Supported by the fact that the m.e. values for the phase parameters do not change very much and are determined by the \(NN\) data as a whole, we believe that for the Nijmegen analyses the “best” value for a particular phase parameter is the value as obtained in the m.e. analysis, rather than the value as obtained in the s.e. analysis. Our m.e. result for the mixing parameter at 50 MeV is \(\varepsilon_1 = 2.2^\circ \pm 0.1^\circ\), whereas our s.e. result reads \(\varepsilon_1 = 2.4^\circ \pm 0.5^\circ\), where the s.e. error provides an upper bound for the true error. The true error is likely to be smaller. We therefore quote our result as \(\varepsilon_1 = 2.2^\circ\), with an error somewhat smaller than 0.5\(^\circ\). This is substantially lower than the result of the Basel group who find \([1] \varepsilon_1 = 2.9^\circ \pm 0.3^\circ\). There are several possibilities which could give rise to such a difference. First of all, we include all data in the 35–75 MeV energy range, whereas the Basel group studied the 32–68 MeV energy range and included a free normalization parameter for every experiment. They also include the Karlsruhe \(A_{yy}\) data \([10]\), whereas we do not since these have not been published in a regular physics journal. Next to the phase parameters with \(J \leq 2\) they also search the \(^1F_3\) phase shift and the \(\varepsilon_3\) mixing parameter, where the phase parameters are assumed to be linear over the energy range studied. Moreover, in their analysis the \(I = 1\) \(np\) phase parameters are obtained from the corresponding \(pp\) phase parameters correcting them for Coulomb effects only. In Ref. \([17]\) it is demonstrated that the Coulomb and mass-difference effects are of the same order of magnitude, so the latter corrections should not be neglected; they will influence the values found for the phase parameters.

Our present result \(\varepsilon_1 = 2.2^\circ\) with an error somewhat smaller than 0.5\(^\circ\) is not much different from the prediction of modern \(NN\) potential models, in contrast to the result of the Basel group. The Nijmegen soft-core potential \([15]\) gives \(\varepsilon_1 = 2.27^\circ\), the parametrized Paris potential \([19]\) gives \(\varepsilon_1 = 1.89^\circ\), and the full Bonn potential \([20]\) gives \(\varepsilon_1 = 2.08^\circ\). These values are in excellent agreement with our determination. In Fig. 1, we plotted \(\varepsilon_1\) as determined in our s.e. and m.e. PWA’s up to \(T_{lab} = 200\) MeV, together with various potential-model predictions. This figure clearly refutes the claim of the Basel group \([1]\) that the value of \(\varepsilon_1\) (especially at 50 MeV) is significantly higher than the potential-model predictions. For the \(^1P_1\) phase shift, the agreement is less satisfactory. The value of the Nijmegen potential with \(\delta(^1P_1) = -8.65^\circ\) is smaller than our m.e. \((-9.77^\circ\)) and s.e. \((-9.52^\circ\)) results, whereas the results of the Paris and Bonn potentials with \(\delta(^1P_1) = -10.95^\circ\) and \(\delta(^1P_1) = -10.48^\circ\), respectively, are higher.

Summarizing, we have analyzed the new \(A_{zz}\) data at 67.5 MeV. These data are in excellent agreement with the other \(NN\) scattering data in our 0–350 MeV data base. The importance of this experiment lies in the fact that it provides us with a fairly complete set of \(NN\) data in the 50-MeV region. Especially the \(I = 0\) phase parameters can now be determined much more accurately. However, the \(\varepsilon_1\) mixing parameter is in good agreement with modern potential predictions. This in contrast to the claim of the Basel group. The accuracy with which the \(I = 0\) phase parameters can be determined suggests that similar experiments should be valuable in the 100-MeV region. However, we want to stress the fact that the phase parameters at 100 MeV are already rather accurately fixed in the m.e. analysis, due to the accuracy of the \(NN\) scattering data in the adjoining energy regions below and above 100 MeV. Still, such experiments would improve the quality of the s.e. analysis.

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REFERENCES


TABLE I. Multienergy (m.e.) and single-energy (s.e.) np phase parameters in degrees at \( T_{\text{lab}} = 50 \) MeV. The \( pp^1S_0 \) phase shift is also given. The results are for the analyses without and with the inclusion of the 67.5-MeV data. In the s.e. (ii) analysis the \( \varepsilon_1 \) is fixed at its m.e. value. \( N_{\text{df}} \) denotes the number of degrees of freedom.

<table>
<thead>
<tr>
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<th>Without 67.5-MeV data</th>
<th>With 67.5-MeV data</th>
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<td></td>
<td>m.e.</td>
<td>s.e. (i)</td>
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<tr>
<td>( \chi^2 )</td>
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<tr>
<td>( N_{\text{df}} )</td>
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<td>461</td>
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<tr>
<td>( ^1S_0(pp) )</td>
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<tr>
<td>( ^1S_0(np) )</td>
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<td>42.50±1.70</td>
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<tr>
<td>( ^1D_2 )</td>
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<td>1.69±0.01</td>
</tr>
<tr>
<td>( ^3P_0 )</td>
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<td>10.10±0.11</td>
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FIGURES

FIG. 1. Mixing parameter $\varepsilon_1$ in degrees versus $T_{\text{lab}}$ in MeV. Black dots: single-energy result; solid curve: multi-energy result; dash-dotted curve: Nijmegen potential; dotted curve: Paris potential; dashed curve: Bonn potential.