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Notes on Relativistic πN-amplitudes with Dynamical Pair Suppression

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Abstract

The analysis of the πN-amplitudes described in these notes are based on the formulation of Relativistic Quantum Field Theory as developed by Kadyshevsky. Here, in contrast to the usual with Feynman graphs, the particles in the Kadyshevsky-graphs remain on-mass-shell. This is exploited by introducing phenomenological vertex form factors which for the matrix-elements suppress the transitions between the positive and the negative energy solutions in a covariant way. These kind of form factors are easily handled in the Kadyshevsky-formalism, and can be shown rigorously to be effective. This is impossible in the usual treatment using Feynman graphs. Therefore, pair-suppression can be introduced phenomenologically and covariantly, and is accessible for an analysis using a fit to the meson-nucleon data. We apply these ideas to the πN-system and demonstrate explicitly the covariance, frame-independence, crossing-symmetry, charge-conjugation invariance, and causal properties of the system, even with 'absolute pair-suppression'.
We also examine the Kadyshevsky-integral-equation and show that the solutions are frame-independent etc. when the interaction-kernel is.

I. INTRODUCTION

In these notes we describe, see also [1–3], the Lorentz-invariant formulation of (absolute) baryon-antibaryon pair-suppression on the level of baryons and mesons. The main pupose is to give some additional background material to the references alluded to above. For the readability of these notes and for pedagogical reasons, we include some parts which also have been given in the references alluded to above.
We derive the soft-core meson-nucleon potentials in an alternative way as the derivation given in [5, 6], henceforth referred to as I. The basis for this alternative is provided by using the formulation of relativistic quantum field theory (RQFT) as given by Kadyshevsky [7–10]. In this formulation, in contrast to the usual one using Feynman propagators, the particles stay always on the mass-shell. The consequence of this is that there is no four-momentum conservation at the vertices. Besides non-conservation of energy, as happens in the Lippmann-Schwinger equation for the intermediate states, there is also non-conservation of three-momentum, in general. The motive for taking this formulation RQFT as the starting point is that for meson-baryon we have composite particles and not elementary ones. Therefore, a relativistic formulation of a theory using phenomenological form factors which suppress the transition between positive and negative energy states *for the matrix elements* is desirable. Below we will give an example of such form factors, the consequences of which can be proved easily in the on-mass-shell formulation.

Phenomenologically, a covariant form of pair-suppression could be introduced in first instance for example as follows: Consider the following πNN-vertex in momentum space

$$\Gamma_5'(p',p) = \exp[a(\gamma p' - M)]\Gamma_5(p',p)\ \exp[a(\gamma p - M)]\ , \tag{1.1}$$

where a is a phenomenological parameter and $\Gamma_5(p',p)$ is the PV- or the PS-vertex. By taking a large and positive, transition between positive and negative energy solutions can be suppressed strongly in a covariant way to any degree [1]
Since in the Kadyshevsky formulation of RQFT all particles, in particularly the nucleons, are on the mass-shell in both the initial-, final-, and intermediate-states, this suppression is clearly effective and covariant.
Note that factors like in (1.1) are beyond control in a general Feynman graph. In that case, the pair-terms i.e. the so-called Z-diagrams can be neglected. Of course, the QCD and non-relativistic quark-model arguments in favor of this, are quite general and hold also for the vector-, scalar-, etc. mesons. In passing, we note that we propose to have pair-suppression for all low energy hadronic processes, e.g. including nuclear Compton-scattering [11]. In that situation, a calculation with the Thompson-equation, neglecting the coupling to negative-energy states, becomes more physical than a calculation with the Bethe-Salpeter equation without off-mass-shell suppression.

We view (1.1) as a genuine vertex factor, and not as originating from a suppression factor on the full-baryon propagator, like in Bruckner and Gell-Mann [12]. From the point of view of the $^3P_0$- pair-creation model [13] one has for a triple meson vertex MMM and for a meson-nucleon vertex NNM the creation of a single quark-antiquark pair which together

[1] In fact the above form of the vertex is unsatisfactory, because it also suppresses $\bar{v}\Gamma_5 v$ matrix elements. The deficiency is easily avoided, as we show in section II A. Also, it appears that (1.1) is too complicated, and so we have to resort to more simple forms, e.g. an approximate form in case $a > 0$ and large.

with an overlap integral determines the strength of the vertices. Therefore, the MMM- and NNM-vertex are of comparable strength. [2] In the case of the nucleon-antinucleon pair vertex one needs a double quark-antiquark pair-creation, and the overlap integrals are more complicated, which means small. This would be a natural explanation of "pair-suppression".

Since in the Kadyshevsky formalism each 'κ-ordered' graph is by itself covariant, one can assign a factor $\gamma^2 << 1$ to each baryon-anti-baryon creating vertex, where γ denotes the vacuum creation of a $Q\bar{Q}$-pair [14]. We emphasize once more that this does not destroy the covariance and is in the spirit of the large $1/N$-expansion picture as given by Witten [15]. However, this is not necessarily Lorentz-invariant, because it will be in general frame-dependent. To analyze the frame-dependence of the amplitudes we employ here the method of Gross and Jackiw [18]. This leads to the introduction of 'compensating' interaction Hamiltonians in order to obtain causal and Lorentz-invariant amplitudes. Such a procedure is quite general and applies to any particle theory defined in terms of Feynman- or Kadyshevsky-graphs.
An alternative is the application of the Takahashi-Umezawa(TU)-method [17], which uses the Tomonaga-Schwinger formalism [16], and is described in these notes in Appendix E, see also [1]. This general field-theoretical method of deriving the interaction Hamiltonian given an interaction Lagrangian, which might involve derivatives of the fields, gives the same 'compensating' interaction Hamiltonians. Therefore, the TU-method gives a very beautiful confirmation of the results found with the Gross-Jackiw analysis.
Of course, the TU-method is based more heavily on the canonical formalism and the introduction of the Dirac-interaction-picture than the Gross-Jackiw method.

The vertex of type (1.1) amounts in configuration space to the introduction of higher order derivatives in field theory, i.e. non-local interactions, which notoriously poses several difficult problems. As a fundamental Lagrangian this has not been possible, so far. However, recently it appeared that string-theories contain non-local factors of the form $\exp(-\alpha p^2)$, which cause loops to converge in Euclidean space. This fact is a good reason to look again to non-local interactions. For a discussion of these matters, see e.g. Evens et al [19].

We find that strong pair-suppression requires such a non-local modification of the standard point vertices employed in phenomenological models. At the end of this note, we include a discussion and brief review of the difficulties with non-local field theories.

The purpose of this note is to demonstrate that pair-suppression can be implemented in a way which is covariant and frame-independent, i.e. Lorentz invariant. Also, we want to derive the effective extra Hamiltonian ΔH_I, which makes the second-order matrix elements

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[2] The typical overlap integral I_0 in the $^3P_0$-model [13] for meson-nucleon coupling is, for all particles at rest,

$$I_0(N,N;M) = \mathcal{N}\left[\frac{2\pi}{3R_N^2+R_M^2}\right]^{3/2} \quad , \quad \mathcal{N} = \left(3\sqrt{6}/4\right)\left(\frac{R_N^2 R_M}{\pi^{3/2}}\right)^{3/2} \quad ,$$

which gives for ("effective") nucleon and meson radii $R_N = R_M = 0.7$ fm the overlap $I_0 = 0.87$ (fm)$^{3/2}$. Combined with the pair-creation constant $\gamma_0 \approx 1.5$ (fm)$^{-3/2}$ leads to an overall so-called 'rationalized' pair-creation constant $\gamma/\sqrt{4\pi} \sim \gamma_0 I_0/\sqrt{4\pi} \approx 0.42 < 1$.
If we put the "effective pair-creation" radius as $R_0 \approx R_M \approx R_N$, we have that $I_0 = (3/4)[6/8\pi^{3/2}]^{1/2} R_0^{3/2} = 0.275 R_0^{3/2}$, showing that I_0 is decreasing for $R_0 \to 0$.

covariant and analytic, while maintaining the pair-suppression. [3] *It will be shown that a (strong) pair-suppression leads to a (strong) suppression of the transitions between positive and negative energy states for the* <u>*matrix elements*</u> . The meaning of this will become clear in the course of these notes, and see4 also [1].

The contents of these notes is as follows. In section II we give the interaction Lagrangian with pair-suppression, being hermitean and charge-conjugation invariant for any type of vertex. In section III we review the $J^P = (1/2)^+$-resonance and exchange, with PS-coupling, and find in an ad-hoc way a frame independent, i.e. Lorentz-invariant, expression for the πN on-energy-shell amplitude. Then, by finding a correction term we make the amplitude causal. In section IV we perform a Gross-Jackiw analysis for the 2nd-order R-product, which accomplishes a derivation of the correction term found in section III. This procedure leads to the establishment of a non-causal (n.c.) correction Hamiltonian $\Delta H_I(n.c.)$, which is used in section V to extend the amplitude corrections in sections III and IV to the off-energy-shell amplitude. This $\Delta H_I(n.c.)$ turns out to be non-local, i.e. of the type $\Delta\mathcal{H}_{(n.c.)}(x,y)$.
In section VI we make some remarks on baryon-exchange, and (off-energy-shell) su-crossing symmetry. Next, in section VII, we apply the procedure to for the $J^P = (1/2)^+$-resonance and exchange, with PV-coupling. In section VIII we examine our procedures to the $J^P = (3/2)^+$ resonance and exchange amplitudes. Applying our prescription, found in section V, to eliminate the non-causal part of the amplitudes. We find that only subtractions of non-causal parts in the amplitudes is not sufficient in this case, but there

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[3] The same propagator emerges in the following, not charge-conjugation invariant, infinitely dense medium: Imagine a medium where all anti-nucleon states are filled, i,e, the Fermi energy of the anti-nucleons $\bar{p}_F = \infty$, and that for nucleons $p_F = 0$. An example would be (symmetric) anti-nuclear matter of infinite density. Then, in πN-scattering pair-production is Pauli-blocked. Denoting the ground-state by $|\Omega\rangle$, one has, see e.g. [20],

$$S_F(x-y) = -i\langle\Omega|T[\psi(x)\bar{\psi}(y)]|\Omega\rangle ,$$

which gives in momentum space [20]

$$S_F(p;p_F,\bar{p}_F) = \frac{\not{p}+M}{2E_p}\left\{\frac{1-n_F(p)}{p_0-E_p+i\epsilon}+\frac{n_F(p)}{p_0-E_p-i\epsilon} - \frac{1-\bar{n}_F(p)}{p_0+E_p-i\epsilon}-\frac{\bar{n}_F(p)}{p_0+E_p+i\epsilon}\right\} . \tag{1.2}$$

At zero temperature $T=0$ the non-interacting fermion functions $n_F, \bar{n}_F$ are defined by

$$n_F = \begin{cases} 1, & |\mathbf{p}| < p_F \\ 0, & |\mathbf{p}| > p_f \end{cases} , \quad \bar{n}_F = \begin{cases} 1, & |\mathbf{p}| < \bar{p}_F \\ 0, & |\mathbf{p}| > \bar{p}_f \end{cases} .$$

In the medium scetched above, clearly $n_F(p) = 0$ and $\bar{n}_F(p) = 1$, which leads to a propagator $S_{ret}(p;0,\infty)(x-y)$ like in (5.9).
This (academic) example may perhaps convince a sceptical reader that a perfect relativistic model with 'absolute pair-suppression' is feasible indeed. For example, consider πN scattering in a medium background as described above. Then, when computing e.g. the nucleon-exchange contribution the amplitudes are clearly Lorentz-invariant.

remain so-called 'contact-terms' (c.t.'s). These c.t.'s are identified and related to the remaining frame-dependence after eliminating the non-causal terms, again employing the Gross-Jackiw analysis. We show that this analysis leads to a correction Hamiltonian $\Delta H_I(c.t.)$, by which these c.t.'s can be removed. In section IX, armed with the knowledge of the previous sections, we show that the on-energy-shell solutions of the Kadyshevsky integral equation are frame independent, i.e. Lorentz-invariant, provided the interaction kernel is. In section X we close with some discussion and outlook, and mention prospects and problems of non-local field theories. In particularly, we formulate in retrospect of the material in these notes the main motivations for the use of the Kadyshevsky formalism.

In Appendix A we state, for completeness, the Kadyshevsky rules for the computation of the Kadyshevsky graphs. In Appendix B we describe the second quantization for the Kadyshevsky quasi-particles. In Appendix C the relativistic off-enegy-shell invariant amplitudes for baryon-exchange and baryon-resonance graphs with $J^P = 1/2^+, 3/2^+$ are given. In Appendix D we list some kinematic identities. In Appendix E the Takahashi-Umezawa interaction theory is described and applied to e.g. the case with "absolute pair-suppression". It is shown to result in the same interaction Hamiltonian as obtained via the Gross-Jackiw method. In Appendix H the Takahashi-Umezawa interaction theory is described within the BMP-framework [21] of Axiomatic S-matrix theory.

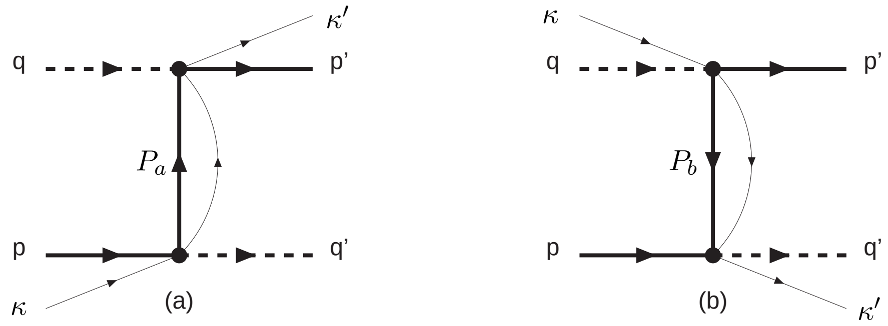

FIG. 1: $J^P = \frac{1}{2}^+$ *Baryon-exchange*

II. PAIR-SUPPRESSION

A. Effective Vertex Pair-suppression

Consider the Kadyshevsky graph (b) of Fig. 1 which, because of the intermediate baryon, has a matrix element of the form

$$M^{(b)}(p_f, p_i) \sim \bar{u}(\mathbf{p}') \left[\Gamma_0 \left(-\not{P} + M_B\right) \Gamma_0\right] u(\mathbf{p}) \ , \tag{2.1}$$

which comes from the $S(-)(P)$-propagator. We write

$$-\not{P} + M_B = -2M_B \sum_s V(\mathbf{P}, s) \, \bar{V}(\mathbf{P}, s) \tag{2.2}$$

where $V(\mathbf{P},s)$ denotes the Dirac-spinor for baryon B.
The inclusion of 'strong-pair-suppression' is effectuated by making the substitution

$$\Gamma_0 \to \Gamma(p_f,p_i) = \frac{g}{2}\Big\{\exp\left[\widetilde{a}\left(\not{p}_f - M_f\right)\right]\ \Gamma_0\ \exp\left[\widetilde{a}\left(\not{p}_i - M_i\right)\right] + \exp\left[-\widetilde{a}\left(\not{p}_f + M_f\right)\right]\ \Gamma_0\ \exp\left[-\widetilde{a}\left(\not{p}_i + M_i\right)\right]\Big\}\ ,\quad \widetilde{a} = \frac{a}{2M} > 0 \tag{2.3}$$

where the subscripts i and f denote respectively the incoming and outgoing baryon at the vertex, and the 'bare-vertex' satisfies $\gamma^0\Gamma_0^\dagger\gamma_0 = \Gamma_0$.
Then, in (2.3) one gets, for on-shell fermions, and large $a > 0$ pair-suppression [4]

$$\begin{aligned}
\bar{u}(p_f)\ \Gamma(p_f,p_i)\ u(p_i) &= \frac{g}{2}\left(1+e^{-2a}\right)\ \bar{u}(p_f)\ \Gamma_0(p_f,p_i)\ u(p_i)\ ,\\
\bar{v}(p_f)\ \Gamma(p_f,p_i)\ v(p_i) &= \frac{g}{2}\left(1+e^{-2a}\right)\ \bar{v}(p_f)\ \Gamma_0(p_f,p_i)\ v(p_i)\ ,\\
\bar{v}(p_f)\ \Gamma(p_f,p_i)\ u(p_i) &= g\ e^{-a}\ \bar{v}(p_f)\ \Gamma_0(p_f,p_i)\ u(p_i)\ ,\\
\bar{u}(p_f)\ \Gamma(p_f,p_i)\ v(p_i) &= g\ e^{-a}\ \bar{u}(p_f)\ \Gamma_0(p_f,p_i)\ v(p_i)\ .
\end{aligned} \tag{2.6}$$

Then, for the matrix element $M^{(b)}(p_f,p_i)$ we get

$$M^{(b)}(p_f,p_i) \sim e^{-2a}\ \left[\bar{u}(\mathbf{p},s_f)\Gamma_0 V(P,s)\right]\left[\bar{V}(\mathbf{P},s)\Gamma_0 u(P,s_i)\right]\ . \tag{2.7}$$

ie. the result of the substitution (2.3) is simply a **strong pair-suppression** factor for large positive a for the contribution of graph (b).

From the perspective of *charge-conjugation invariance* it is easily seen that the vertex (2.3) is a good choice. Under charge-conjugation C one has

$$C\ :\ \psi \to C\ \widetilde{\bar{\psi}}\ \ ,\ \ \bar{\psi} \to -\widetilde{\psi}\ C^{-1}\ \ ,\ \ C^{-1}\gamma^\mu\ C = -\widetilde{\gamma^\mu}\ . \tag{2.8}$$

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[4] An alternative form would be

$$\Gamma_0 \to \Gamma(p',p) = \frac{g}{2}\Big\{\left(\frac{1}{\widetilde{a}\not{p}'+b}\right)^n\ \Gamma_0\ \left(\frac{1}{\widetilde{a}\not{p}+b}\right)^n + \left(\frac{1}{-\widetilde{a}\not{p}'+b}\right)^n\ \Gamma_0\ \left(\frac{1}{-\widetilde{a}\not{p}+b}\right)^n\Big\}\ . \tag{2.4}$$

Requiring that $a+b=1$, and $-a+b=X$, with large $X>0$, one gets

$$\begin{aligned}
\bar{u}\Gamma u &= \frac{g}{2}\left(1+X^{-2n}\right)\cdot\bar{u}\Gamma_0 u\ ,\ \ \bar{v}\Gamma v = \frac{g}{2}\left(1+X^{-2n}\right)\cdot\bar{v}\Gamma_0 v\ ,\\
\bar{v}\Gamma u &= g\ X^{-n}\cdot\bar{v}\Gamma_0 u\ ,\ \ \bar{u}\Gamma v = g\ X^{-n}\cdot\bar{u}\Gamma_0 v\ ,
\end{aligned} \tag{2.5}$$

which means again pair-suppression for large X and $n >> 1$. We note that with $n = N-1$, and interpreting $x = 1/X < 1$ as the probability of $q\bar{q}$-pair creation, the vertex (2.4) models SU_c(N) pair-suppression for large N [15].

Then, one has that

$$\bar{\psi}_f\, \not p\, \psi_i \;\rightarrow\; -\widetilde{\psi}_f\, C^{-1} \not p\, C \widetilde{\bar{\psi}}_i = +\bar{\psi}_i\, \not p\, \psi_f \;,$$
$$\bar{\psi}_f\, \Gamma_0\, \not p\, \psi_i \;\rightarrow\; -\widetilde{\psi}_f\, C^{-1}\Gamma_0\, \not p\, C \widetilde{\bar{\psi}}_i = +\bar{\psi}_i\, \not p\, \left(\Gamma_0\right)_c \psi_f \;, \tag{2.9}$$

with $\left(\Gamma_0\right)_c = \widetilde{C^{-1}\Gamma_0 C}$ [5]. *From this it is clear that (2.3) gives C-invariance.*

For a proper representation in x-space we have to split the fermion field in a positive and negative frequency part, like in the normal-ordering products. We write

$$\psi(x) = \psi^{(+)}(x) + \psi^{(-)}(x) \;, \tag{2.10}$$

which splitting is obviously invariant under orthochronous Lorentz transformations $L_+^\uparrow$. Then,

$$\exp\left(\widetilde{a}\not p\right) \rightarrow \psi(x|a) \equiv \exp\Big[+ i\widetilde{a}\not \partial \Big]\psi^{(+)}(x) + \exp\Big[- i\widetilde{a}\not \partial \Big]\psi^{(-)}(x) \;. \tag{2.11}$$

So, with this prescription we can write down an effective interaction Lagrangian corresponding to the vertex (2.3). We write

$$\mathcal{L}_I(x) = \frac{g}{2}\left[\bar{\psi}(x|a)\Gamma_0\; \psi(x|a) + \bar{\psi}(x|-a)\Gamma_0\; \psi(x|-a)\right] \cdot e^{-a} \;. \tag{2.12}$$

Now, in perturbation theory the fields are free fields, "in"- or "out"-fields with mass M, so that for e.g. absolute pair-suppression, i.e. $a = \infty$,

$$\mathcal{L}_I(x) \Rightarrow \frac{g}{2}\left[\overline{\psi^{(+)}}(x)\Gamma_0\; \psi^{(+)}(x) + \overline{\psi^{(-)}}(x)\Gamma_0\; \psi^{(-)}(x)\right] \;, \tag{2.13}$$

where $\overline{\psi^{(\pm)}}(x) \equiv \psi^{(\pm)\dagger}(x)\gamma^0$.

Strong-pair suppression: For $a < \infty$, and large, we have 'strong-pair-suprresion'. For this we could try to develop the non-local theory described by e.g. the interaction Lagrangian (2.12), which however seems rather complicated. Instead, we can indroduce next and similar to (2.13) also a 'pair-production' Lagrangian $\mathcal{L}'_I$ by

$$\mathcal{L}'_I(x) \Rightarrow \frac{g'}{2}\left[\overline{\psi^{(+)}}(x)\Gamma_0\; \psi^{(-)}(x) + \overline{\psi^{(-)}}(x)\Gamma_0\; \psi^{(+)}(x)\right] \;, \tag{2.14}$$

where for example the pair-creation coupling $g' = \exp(-a)\; g$, with $a > 0$ and large.
For the rest of these notes we restrict ourselves in working only out the consequences for the interaction (2.12).

[5] Such relations also hold in momentum-space, which follows from the properties of the Dirac-spinors [22]

$$e^{i\phi(p,s)} v(p,s) = C\; \widetilde{\bar{u}}(p,s) \quad , \quad e^{i\phi(p,s)} u(p,s) = C\; \widetilde{\bar{v}}(p,s) \;,$$

where e.g. $C = i\gamma^2\gamma^0 = -C^{-1}$, in the Pauli-Dirac representation. and where the phase factors are unity.

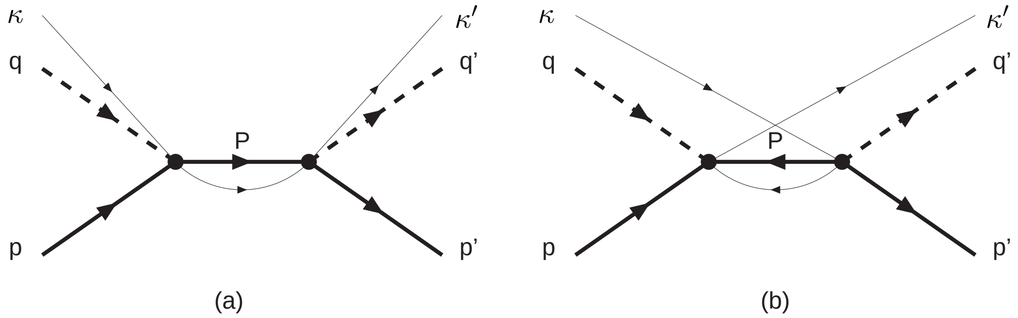

FIG. 2: *Baryon-resonance s-channel graphs.*

III. $J^P = \frac{1}{2}^+$- RESONANCE, PS-COUPLING

This section deals with the baryons like $N(940)$ and the P_{11}-resonance $N^*(1470)$, i.e. the Roper resonance. The two Kadyshevsky graphs are shown in figure 2. The ps-vertex means that $\Gamma_0 = \gamma_5$ [6]. Then, from the Kadyshevsky rules given in Appendix A one finds for graph (a) and graph (b) [16]

$$\begin{aligned}
M^{(a)}_{\kappa',\kappa} &= -\frac{1}{4} g^2 \left(1+e^{-2a}\right)^2\ \bar{u}(\mathbf{p}') \left[\left\{-\frac{1}{2}(M_f+M_i)+M_B\right\} - \not{Q} + \left\{\Delta_{E,s} - A_s\right\} \not{n}\right] u(\mathbf{p}) \cdot \\
&\times \frac{1}{2A_s} \frac{1}{\frac{1}{2}(\kappa'+\kappa)+\Delta_{E,s}-A_s+i\epsilon}\ , \\
M^{(b)}_{\kappa',\kappa} &= -g^2 e^{-2a}\ \bar{u}(\mathbf{p}') \left[\left\{-\frac{1}{2}(M_f+M_i)+M_B\right\} - \not{Q} + \left\{\Delta_{E,s} + A_s\right\} \not{n}\right] u(\mathbf{p}) \cdot \\
&\times \frac{1}{2A_s} \frac{1}{\frac{1}{2}(\kappa'+\kappa)-\Delta_{E,s}-A_s+i\epsilon}\ . \qquad (3.1)
\end{aligned}$$

Here,

$$\Delta_s = \frac{1}{2}\left[(p'+p)+(q'+q)\right] \quad , \quad \Delta_{E,s} = \Delta_s \cdot n\ . \qquad (3.2)$$

and

$$A_s = \left\{M_B^2 - \Delta_s^2 + (\Delta_s \cdot n)^2\right\}^{1/2} \qquad (3.3)$$

A. Absolute Pair-suppression, $a \to \infty$

In this case one has for the on-energy-shell amplitudes, i.e. $\kappa' = \kappa = 0$, $M^{(b)}_{0,0} = 0$, and $M^{(a)}_{0,0}$ becomes

$$M^{(a)}_{0,0} \Rightarrow -\frac{1}{4} g^2\ \bar{u}(\mathbf{p}') \left[\frac{(M_B - M) - \not{Q}}{2A_s\left(\Delta_s \cdot n - A_s\right)} + \frac{\not{n}}{2A_s}\right] u(\mathbf{p})\ . \qquad (3.4)$$

[6] This means that $g \equiv i g_{\pi N}$, otherwise the interaction Lagrangian (2.13) is not hermitean! So, these notes have to be updated in the final version!

In order to make this amplitude frame-independent, i.e. n_μ independent, we have to subtract from (3.4) the piece

$$\Delta M^{(a)} \;=\; -\frac{1}{4}g^2\;\bar{u}(\mathbf{p}')\left\{[(M_B-M)-\not{Q}\,]\,\frac{\Delta_s\cdot n}{2A_s\,(s-M_B^2)}+\frac{\not{n}}{2A_s}\right\}u(\mathbf{p})\;. \qquad (3.5)$$

This gives

$$\begin{aligned}\bar{M}^{(a)}_{0,0} \;&=\; M^{(a)}_{0,0}-\Delta M^{(a)} = -\frac{1}{4}g^2\;\bar{u}(\mathbf{p}')\,[(M_B-M)-\not{Q}]\,u(\mathbf{p})\,\cdot\\ &\quad\times\frac{1}{2A_s}\left\{\frac{1}{\Delta_s\cdot n-A_s}-\frac{\Delta_s\cdot n}{s-M_B^2}\right\}\\ &=\; -\frac{1}{8}g^2\;\bar{u}(\mathbf{p}')\,[(M_B-M)-\not{Q}\,]\,u(\mathbf{p})\;\frac{1}{s-M_B^2}\;. \end{aligned} \qquad (3.6)$$

Here we used the identity $s-M_B^2=(\Delta_s\cdot n)^2-A_s^2$.

One sees that by rescaling the coupling $g\rightarrow 2\sqrt{2}g$, one obtains the same amplitudes as usual !!?

In the next section, we derive the subtraction terms using the Gross-Jackiw scheme [18], by an adaption of the R-product in the Kadyshevsky field theory.

IV. GROSS-JACKIW R-PRODUCT ANALYSIS FOR PS-COUPLING

In reference [23] the perturbation formalism in the case of interaction Lagrangians with derivatives is discussed. The interaction (2.3) is such a case with derivatives of any order. Therefore, one may expect many contributions to $\Delta\mathcal{H}_I$, and it seems a very complicated affair to evaluate these. However, in the perturbation formalism the fields are 'free-fields', namely "in"- or "out"-fields with mass M.

In the situation with pair-suppression, the R-product formulation is the right starting point in perturbation theory. For example in case of 'absolute pair-suppression', the contractions in the Wick-expansion do not lead to $S_F(x,y)$-functions. This can be seen as follows. The Feynman green function is

$$S_F(x-y)_{\beta\alpha}=-i\theta(x^0-y^0)\langle 0|\psi_\beta(x)\bar{\psi}_\alpha(y)|0\rangle+i\theta(y^0-x^0)\langle 0|\bar{\psi}_\alpha(y)\psi_\beta(x)|0\rangle\;, \qquad (4.1)$$

where

$$\begin{aligned}\langle 0|\psi_\beta(x)\bar{\psi}_\alpha(y)|0\rangle &= \langle 0|\psi^{(+)}_\beta(x)\overline{\psi^{(+)}}_\alpha(y)|0\rangle\;,\\ \langle 0|\bar{\psi}_\alpha(y)\psi_\beta(x)|0\rangle &= \langle 0|\overline{\psi^{(-)}}_\alpha(y)\psi^{(-)}_\beta(x)|0\rangle\;. \end{aligned} \qquad (4.2)$$

But, the negative-frequency terms never occur in the Wick-expansion for 'absolute pair-suppression.

For the analysis of the frame-dependent terms in the Kadyshevsky formalism we consider, see [23] section III.B,

$$P^{\alpha\beta}\frac{\partial}{\partial n^\beta}\theta[n\cdot(x-y)]\mathcal{L}_I(x)\mathcal{L}_I(y)=P^{\alpha\beta}(x-y)_\beta\delta[n\cdot(x-y)]\mathcal{L}_I(x)\mathcal{L}_I(y) \qquad (4.3)$$

For πN we have that

$$\mathcal{L}_I(x) = \frac{g}{2}\left(\bar{\psi^{(+)}}(x)\Gamma_0\psi^{(+)}(x) + \bar{\psi^{(-)}}(x)\Gamma_0\psi^{(-)}(x)\right)\phi(x)$$
$$\sim \frac{g}{2}\left[\bar{\psi^{(+)}}\Gamma_0\psi^{(+)}(x)\right]\ \phi(x)\ . \tag{4.4}$$

Then, for the evaluation of (4.3) we get [7]

$$\delta[n\cdot(x-y)]\mathcal{L}_I(x)\mathcal{L}_I(y) \sim \delta(x^0-y^0)\ \overline{\psi^{(+)}}_\alpha(x)\psi^{(+)}_\beta(x)\cdot\overline{\psi^{(+)}}_\gamma(y)\psi^{(+)}_\delta(y) =$$
$$\delta(x^0-y^0)N\left[\ldots\right] + \delta(x^0-y^0)\overline{\psi^{(+)}}_\alpha(x)\ \left\{\psi^{(+)}_\beta(x),\overline{\psi^{(+)}}_\gamma(y)\right\}\ \psi^{(+)}_\delta(y) =$$
$$\delta(x^0-y^0)N\left[\ldots\right] + \delta(x^0-y^0)\overline{\psi^{(+)}}_\alpha(x)\cdot(+i\partial\!\!\!/_x + M)_{\beta\gamma}\,\Delta^{(+)}(x-y)\cdot\psi^{(+)}_\delta(y)\ , \tag{4.5}$$

where [24]

$$\Delta^{(+)}(x-y) = \frac{1}{2}\left[i\Delta(x-y) + \Delta^{(1)}(x-y)\right]\ , \tag{4.6}$$

with

$$\partial_0\Delta(x-y)\,|_{x^0=y^0} = -\delta(\mathbf{x}-\mathbf{y})\ \ ,\ \ \partial_0\Delta^{(1)}(x-y)\,|_{x^0=y^0} = 0\ . \tag{4.7}$$

Therefore,

$$\delta(x^0-y^0)\,(+i\partial\!\!\!/_x + M)_{\beta\gamma}\,\Delta^{(+)}(x-y) =$$
$$\frac{1}{2}\delta_{\beta\gamma}\delta^4(x-y) + \frac{1}{2}\delta(x^0-y^0)\,(+i\partial\!\!\!/_x + M)_{\beta\gamma}\,\Delta^{(1)}(x-y) \tag{4.8}$$

From this we get, for $\Gamma_0 = \gamma_5$, that

$$\int d^4x\ \int d^4y\ P^{\alpha\beta}(x-y)_\beta\delta[n\cdot(x-y)]\mathcal{L}_I(x)\mathcal{L}_I(y) \Rightarrow +\frac{g^2}{8}\int d^4x\ \int d^4y\ P^{\alpha\beta}(x-y)_\beta\cdot$$
$$\times\left\{\overline{\psi^{(+)}}(x)\cdot(-i\partial\!\!\!/_x + M)\,\Delta^{(1)}(x-y)\cdot\psi^{(+)}(y)\right\}\delta(x^0-y^0) =$$
$$P^{\alpha\beta}\frac{\partial}{\partial n_\beta}\left\{+\frac{g^2}{8}\int d^4x\ \int d^4y\ \theta[n\cdot(x-y)]\cdot\right.$$
$$\left.\times\ \overline{\psi^{(+)}}(x)\left[(-i\partial\!\!\!/_x + M)\,\Delta^{(1)}(x-y)\right]\cdot\psi^{(+)}(y)\right\}\ . \tag{4.9}$$

---

[7] The anti-commutator for free fields and the positive frequency components is given by

$$\left\{\psi^{(+)}_\alpha(x),\overline{\psi^{(+)}}_\beta(y)\right\} = \sum_s\int\frac{d^3p}{(2\pi)^3}\frac{M}{E_p}\ \left[u_\alpha(p,s)\bar{u}_\beta(p,s)\right]e^{-ip\cdot(x-y)} =$$
$$\int\frac{d^3p}{(2\pi)^3 2E_p}\,(p\!\!\!/ + M)_{\alpha\beta}\,e^{-ip\cdot(x-y)} = (+i\partial\!\!\!/ + M)_{\alpha\beta}\int\frac{d^3p}{(2\pi)^3 2E_p}e^{-ip\cdot(x-y)} =$$
$$(+i\partial\!\!\!/ + M)_{\alpha\beta}\,\Delta^{(+)}(x-y;M^2)\ .$$

Notice the appearance of the non-causal $\Delta^{(1)}(x-y)$-function. So, such terms have to be removed from the $M^{(2)}$-matrix. These non-causal terms are typical for non-local interactions as we analyze here in connection with pair-suppression.

Application to πN-scattering, and restricting ourselves to the s-channel resonance contribution, we have to evaluate the matrix element

$$
\begin{aligned}
& (2\pi)^4\delta\left(p'+q'-p-q\right)\ \Delta M^{(2)}_{0,0} = \langle|\Delta H_I|\rangle \equiv -\frac{i}{8}g^2\ \langle p'q'|\int d^4x\ \int d^4y\ \theta[n\cdot(x-y)]\cdot \\
& \times\left(\phi(x)\phi(y)\right)\cdot\overline{\psi^{(+)}}(x)\left[\left(-i\partial\!\!\!/_x+M\right)\Delta^{(1)}(x-y)\right]\cdot\psi^{(+)}(y)|pq\rangle \\
= & -\frac{i}{8}g^2\ \int d^4x\ \int d^4y\cdot-\frac{1}{2\pi i}\int d\kappa\ \frac{\exp\left[-i\kappa n\cdot(x-y)\right]}{\kappa+i\epsilon}\ e^{i(p'+q')\cdot x}\ e^{-i(p+q)\cdot y}\ . \\
& \times\bar{u}(p')\left[\int\frac{d^4P}{(2\pi)^3}\left(-P\!\!\!/+M\right)\delta\left(P^2-M^2\right)\ e^{-iP\cdot(x-y)}\right]\ u(p) \\
= & \ (2\pi)^4\delta\left(p'+q'-p-q\right)\cdot\frac{g^2}{8}\int\frac{d\kappa}{\kappa+i\epsilon}\left[\bar{u}(p')\left(-P\!\!\!/+M\right)u(p)\right]\ \delta\left(P^2-M^2\right)\ ,
\end{aligned}
\tag{4.10}
$$

where

$$
P=\Delta_s-\kappa n\ \ ,\ \ \Delta_s=\frac{1}{2}(p'+q'+p+q)\ . \tag{4.11}
$$

The solutions for κ are

$$
\kappa^{\pm}=-\Delta_s\cdot n\pm A_s\ \ ,\ \ P_0^{\pm}=\Delta_s-\kappa^{\pm}\ . \tag{4.12}
$$

This gives

$$
\delta\left(P^2-M^2\right)\ \Rightarrow\ \delta\left((\Delta_s-\kappa n)^2-M^2\right)=\frac{1}{2A_s}\left[\delta(\kappa-\kappa^+)+\delta(\kappa-\kappa^-)\right]\ . \tag{4.13}
$$

Performing now the κ-integral, we get for (4.10)

$$
\begin{aligned}
& \Delta M^{(2)}_{0,0}=+\frac{g^2}{8}\bar{u}(p')\Bigg[\\
& \frac{1}{2A_s}\left\{\frac{(-\Delta\!\!\!/_s+(\Delta_s\cdot n-A_s)n\!\!\!/+M)}{-\Delta_s\cdot n+A_s}+\frac{(-\Delta\!\!\!/_s+(\Delta_s\cdot n+A_s)n\!\!\!/+M)}{-\Delta_s\cdot n-A_s}\right\}\Bigg]\ u(p)= \\
& \frac{g^2}{16A_s}\bar{u}(p')\left\{\left[(M_B-M)-Q\!\!\!/\,\right]\left[\frac{1}{-\Delta_s\cdot n+A_s}-\frac{1}{\Delta_s\cdot n+A_s}\right]-2n\!\!\!/\right\}\ u(p)= \\
& \frac{g^2}{4}\ \bar{u}(p')\left\{\left[(M_B-M)-Q\!\!\!/\,\right]\frac{(\Delta_s\cdot n)}{2A_s\,(M_B^2-s)}-\frac{n\!\!\!/}{2A_s}\right\}\ u(p)\ .
\end{aligned}
\tag{4.14}
$$

This indeed explains the 'compensation terms' (3.5) we had to introduce in order to make the Kadyshevsky amplitude for 'absolute pair-suppression' Lorentz invariant!
We note that for baryon-exchange the analysis runs in a completely analogous way.

In view of the results so far in this section, we can now identify the effective extra Hamiltonian This $\Delta\mathcal{H}_I$ has to be such that $(2\pi)^4\delta(P_f-P_i)\Delta M^{(2)}_{0,0}=+\langle\ \ |\Delta\mathcal{H}_I|\ \ \rangle$. Then, from (4.10) we infer that the corrective interaction Hamiltonian has to be

$$
\begin{aligned}
\Delta H_I\ \equiv\ & -i\frac{g^2}{8}\ \int d^4x\ \int d^4y\ \theta[n\cdot(x-y)]\cdot\phi(x)\phi(y) \\
& \times\bar{\psi^{(+)}}(x)\left[\left(-i\partial\!\!\!/_x+M\right)\Delta^{(1)}(x-y)\right]\cdot\psi^{(+)}(y)\ .
\end{aligned}
\tag{4.15}
$$

Of course, a similar term has to be added for $\pi\bar{N}$-scattering.

We note that, although non-local as expected, the extra Hamiltonian exhibits again 'absolute pair-suppression'!

V. OFF-ENERGY-SHELL MATRIX ELEMENTS

The explicit form of ΔH_I in (4.15) enables us to analyze properly the off-energy-shell behavior of the corrections to the πN matrix elements. We will demonstrate that for the spin-1/2 baryon graphs, and subsequently extract the rule to derive the result by a rather simple substitution prescription for the 'covariantization' of the second-order amplitudes.

A. Off-energy-shell subtraction

As an example, we treat here again the spin-1/2 resonance amplitude with PS-coupling. Then, considering the off-energy-shell matrix πN matrix elements of ΔH_I, we get

$$\begin{aligned}
&(2\pi)^4\delta\left(p'+q'+\kappa' n-p-q-\kappa n\right)\ \Delta M^{(2)}_{\kappa',\kappa} \equiv \\
&-\frac{i}{8}g^2\ \langle p'q'\kappa'|\int d^4x\ \int d^4y\ \theta[n\cdot(x-y)]\cdot\phi(x)\phi(y) \\
&\times\bar{\psi^{(+)}}(x)\left[(-i\not{\partial}_x+M)\,\Delta^{(1)}(x-y);M_B^2\right]\cdot\psi^{(+)}(y)|pq\kappa\rangle = \\
&-\frac{i}{8}g^2\ \int d^4x\ \int d^4y\cdot-\frac{1}{2\pi i}\int d\kappa_1\ \frac{\exp\left[-i\kappa_1 n\cdot(x-y)\right]}{\kappa_1+i\epsilon}\ e^{i(p'+q'+\kappa' n)\cdot x}\ e^{-i(p+q+\kappa n)\cdot y}\ . \\
&\times\bar{u}(p')\left[\int\frac{d^4P}{(2\pi)^3}\left(-\not{P}+M_B\right)\delta\left(P^2-M_B^2\right)\ e^{-iP\cdot(x-y)}\right]\ u(p) = \\
&(2\pi)^4\delta\left(p'+q'+\kappa' n-p-q-\kappa n\right)\cdot\frac{g^2}{8}\int\frac{d\kappa_1}{\kappa_1+i\epsilon}\left[\bar{u}(p')\left(-\not{P}+M_B\right)u(p)\right]\ \delta\left(P^2-M_B^2\right)
\end{aligned} \tag{5.1}$$

where now

$$P=\frac{1}{2}(\kappa'+\kappa)+\Delta_s-\kappa_1 n\ , \tag{5.2}$$

and $\Delta_s=\frac{1}{2}(p'+q'+p+q)$ which is the same as for $\kappa'=\kappa=0$, and the solutions for κ_1 are

$$\kappa^{\pm}=\frac{1}{2}(\kappa'+\kappa)+\Delta_s\cdot n\pm A_s\ \ ,\ \ P(\kappa^{(\pm)})=\Delta_s-\kappa^{\pm}n=\Delta_s-\left[(\Delta_s\cdot n)\pm A_s\right]n\ . \tag{5.3}$$

Again using

$$\delta\left(P^2-M_B^2\right)\ \Rightarrow\ \delta\left((\Delta_s-\kappa_1 n)^2-M_B^2\right)=\frac{1}{2A_s}\left[\delta(\kappa_1-\kappa^+)+\delta(\kappa_1-\kappa^-)\right]\ , \tag{5.4}$$

we can perform the κ_1-integral. With the short-hand notation $\bar{\kappa} = (\kappa' + \kappa)/2$, we get for (5.1)

$$\Delta M^{(2)}_{\kappa',\kappa} = \frac{g^2}{8}\bar{u}(p')\Bigg[\frac{1}{2A_s}\left\{\frac{(-\not{\Delta}_s + (\Delta_s\cdot n + A_s)\not{n} + M_B)}{\frac{1}{2}(\kappa'+\kappa) + \Delta_s\cdot n + A_s} + \frac{(-\not{\Delta}_s + (\Delta_s\cdot n - A_s)\not{n} + M_B)}{\frac{1}{2}(\kappa'+\kappa) + \Delta_s\cdot n - A_s}\right\}\Bigg]\ u(p) =$$
$$\frac{g^2}{16A_s}\bar{u}(p')\left\{[(M_B - M) - \not{Q}\,]\left[\frac{1}{\bar{\kappa} + \Delta_s\cdot n + A_s} + \frac{1}{\bar{\kappa} + \Delta_s\cdot n - A_s}\right] + \left[\frac{(\Delta_s\cdot n + A_s)}{\bar{\kappa} + \Delta_s\cdot n + A_s} + \frac{(\Delta_s\cdot n - A_s)}{\bar{\kappa} + \Delta_s\cdot n - A_s}\right]\not{n}\right\}\ u(p) \tag{5.5}$$

Reminding that for $a = \infty$ the off-energy-shell matrix element is, see (3.1),

$$M^{(a)}_{\kappa',\kappa} = -\frac{1}{4}g^2\ \bar{u}(\mathbf{p}')\left\{[(M_B - M) - \not{Q}] + ((\Delta_s\cdot n) - A_s)\not{n}\right\}u(\mathbf{p})\cdot \times \frac{1}{2A_s}\frac{1}{\bar{\kappa} + (\Delta_s\cdot n) - A_s + i\epsilon}\ , \tag{5.6}$$

Then,

$$\bar{M}^{(2)}_{\kappa',\kappa} \equiv M^{(a)}_{\kappa',\kappa} + \Delta M^{(2)}_{\kappa',\kappa} = -\frac{1}{8}g^2\cdot\bar{u}(\mathbf{p}')\left\{[(M_B - M) - \not{Q}\,] - \bar{\kappa}\not{n}\right\}u(\mathbf{p})\cdot \times\frac{1}{(\bar{\kappa} + \Delta_s\cdot n)^2 - A_s^2}\ , \tag{5.7}$$

which in the limit $\kappa', \kappa \to 0$ agrees with (3.6).

This settles the off-energy-shell behavior of the amplitudes!

B. Prescription 'covariantization' second-order amplitudes

From the foregoing analysis we have found for 'absolute pair-suppression' the following recipe for the 'covariantization' of the Kadyshevsky amplitudes:

Subtraction Non-causal parts: Make in the amplitude from graph's of type (a) the substitution [8]

$$\Delta^{(+)}(x - y; M^2) \to \frac{i}{2}\Delta(x - y; M^2) \tag{5.8}$$

[8] We remind here the spectral representation of the invariant singular functions:

$$\Delta^{(+)}(x) = \int \frac{d^4P}{(2\pi)^3}\ \theta(P_0)\delta(P^2 - M^2)\ e^{-iP\cdot x}\ ,$$
$$\Delta^{(1)}(x) = \int \frac{d^4P}{(2\pi)^3}\ \delta(P^2 - M^2)\ e^{-iP\cdot x}\ ,$$
$$\Delta(x) = -i\int \frac{d^4P}{(2\pi)^3}\ \epsilon(P_0)\delta(P^2 - M^2)\ e^{-iP\cdot x}\ .$$

With the substitution (5.8) the amplitude $M^{(2)}_{\kappa',\kappa}$ contains the propagator factor

$$\theta[n\cdot(x-y)]\ \Delta(x-y;M^2)\ =\ -\Delta_{ret}(x-y;M^2)=\int\frac{d^4p}{(2\pi)^4}\frac{e^{-ip\cdot(x-y)}}{p^2-M^2+i0p_0}$$
$$=\int\frac{d^4p}{(2\pi)^4}\ \frac{1}{2E_p}\left[\frac{1}{p_0-E_p+i\epsilon}-\frac{1}{p_0+E_p+i\epsilon}\right]\ . \quad (5.9)$$

The same (retarded) factor emerges in the (symmetric) anti-nuclear matter of infinite density, described in an footnote above. See $S_F(p;p_F,\bar{p}_F)$ in (1.2), which for $n_F(p)=0$ and $\bar{n}_F(p)=1$, indeed corresponds with (5.9).
Sofar, we have seen that in the cases where the interaction L_I does not contain derivatives of the fermion fields the substitution (8.9) works for πN. Here, we have to subtract out the non-causal part to arrive at Lorentz invariant expressions for pair-suppression. When dealing with the $J^P=(3/2)^+$-resonance in section VIII, using the gauge-invariant coupling, this will not be the complete story. Because, in that case apart from the non-causal parts we also have to remove the non-invariant 'contact terms'.

VI. $J^P=\frac{1}{2}^+$- BARYON-EXCHANGE, PS-COUPLING

We note that the case of Baryon-exchange can be handled in a completely analogous fashion. We get

$$\bar{M}^{(2)}_{\kappa',\kappa}\equiv M^{(a)}_{\kappa',\kappa}+\Delta M^{(2)}_{\kappa',\kappa}\ =\ -\frac{1}{8}g^2\cdot\bar{u}(\mathbf{p}')\left\{[(M_B-M)+\not{Q}\,]-\bar{\kappa}\not{n}\right\}u(\mathbf{p})\cdot$$
$$\times\frac{1}{(\bar{\kappa}+\Delta_u\cdot n)^2-A_u^2}\ , \quad (6.1)$$

where

$$\Delta_u=\frac{1}{2}\left(p'+p-q'-q\right)\quad,\quad A_u=\sqrt{M_B^2-\Delta_u^2+(\Delta_u\cdot n)^2}\ . \quad (6.2)$$

Two remarks:

(i) Crossing symmetry: under the transformation

$$q'\to -q\ ,\ q\to -q' \quad (6.3)$$

the amplitude in (6.1) goes over into the amplitude (5.7), i.e. su-crossing symmetry,

---

So, one sees that although there are no positive and negative energy transitions, in the $\Delta(x)$-functions both positive and negative energies occur. However, in contrast to the Feynman function, the positive as well as the negative energies propagate forward in time. *Although 'internally' negative energies contribute to the propagator there will never be non-zero matrix elements between positive and negative energy states in case of 'absolute' pair suppression.*
Furthermore, we note that after the substitution (5.8) we have effectively in the Kadyshevsky graphs the 'retarded propagator' $\Delta_R=-\theta(x_0)\Delta(x)$.

(ii) The partial-wave expansion of (6.1) is simple!!

Off-energy-shell su-crossing symmetry is very convenient in checking explicit expressions for spin-1/2 and spin-3/2 resonce and exchange amplitudes!

VII. $J^P = \frac{1}{2}^+$- RESONANCE, PV-COUPLING

In this case the basic interaction, with no-pair-suppression, reads

$$\mathcal{L}_{PV} = \frac{f}{m_\pi}\bar{\psi}(x)\gamma_5\gamma_\mu\psi(x)\cdot\partial^\mu\phi(x) \ . \tag{7.1}$$

In this case we get, as is easily seen from the derivations above,

$$\begin{aligned}\Delta H_I^{(2)} \equiv & -\frac{f^2}{8m_\pi^2}\int d^4x \int d^4y\ \theta[n\cdot(x-y)]\cdot\partial^\mu\phi(x)\partial^\nu\phi(y) \\ & \times\overline{\psi^{(+)}}(x)\gamma_5\gamma_\mu\left[\left(-i\not{\partial}_x+M\right)\Delta^{(1)}(x-y)\right]\cdot\gamma_5\gamma_\nu\psi^{(+)}(y) \ .\end{aligned} \tag{7.2}$$

In [16] the amplitude for graph (a) is given:

$$M^{(a)}_{\kappa',\kappa} = -\frac{f^2}{4m_\pi^2}\int\frac{d\kappa_1}{\kappa_1+i\epsilon}\Delta^{(+)}(P_a)\cdot\bar{u}(p')\left[\not{q}'\left\{\not{P}_a+M_B\right\}\not{q}\right]\ u(p) \ . \tag{7.3}$$

Applying the prescription, given section V B, we make in (7.3) the substitution

$$\Delta^{(+)}(P)\Rightarrow\Delta(P) = -\frac{i}{2}\epsilon(P_0)\delta\left(P^2-M_B^2\right) \ . \tag{7.4}$$

The solutions for $\kappa_1^\pm = -\Delta_s\cdot n \pm A_s$, and $P_a \to P^\pm = \Delta_s - \kappa_1^\pm n$ have been given above, when dealing with the PS-coupling. Therefore, we find

$$M^{(2)}_{\kappa',\kappa} \Rightarrow -\frac{f^2}{8m_\pi^2}\bar{u}(p')\ \not{q}'\cdot\frac{1}{2A_s}\left[\frac{-\not{P}^+ + M_B}{\bar{\kappa}+\Delta_s\cdot n+A_s} - \frac{-\not{P}^- + M_B}{\bar{\kappa}+\Delta_s\cdot n-A_s}\right]\not{q}\ u(p) \ , \tag{7.5}$$

where as before $\bar{\kappa} = (\kappa'+\kappa)/2$. Working this out by combining the denominators, we arrive at

$$M^{(2)}_{\kappa',\kappa} \Rightarrow -\frac{f^2}{8m_\pi^2}\bar{u}(p')\ \not{q}'\cdot\left[\frac{\not{\Delta}_s - M_B + \bar{\kappa}\not{n}}{(\bar{\kappa}+\Delta_s\cdot n)^2 - A_s^2}\right]\not{q}\ u(p) \ , \tag{7.6}$$

which is clearly n_μ-independent on-enegy-shell ($\bar{\kappa}=0$).
Again, the case of Baryon-exchange can be handled in a completely analogous fashion, and the expression for this case can be derived by applying the crossing-symmetry transformation (6.3) to (7.6).

VIII. $J^P = \frac{3}{2}^+$- RESONANCE, GAUGE-INVARIANT COUPLING

For πN the gauge-invariant (GI) interaction with 'absolute pair-suppression', reads

$$\mathcal{L}_I \Rightarrow \frac{g}{2}\left[\left(\partial_\mu\overline{\Psi^{(+)}}_\nu\right)\Gamma_\alpha\psi^{(+)}\cdot\partial_\beta\phi + \overline{\psi^{(+)}}\Gamma_\alpha\left(\partial_\mu\Psi^{(+)}_\nu\right)\cdot\partial_\beta\phi\right] \ , \tag{8.1}$$

where $\Gamma_\alpha = \gamma_5\gamma_\alpha$. From [16] we have for the graph of type (a) the matrix element

$$M^{(a)}_{\kappa',\kappa} = -\frac{g^2}{4}\epsilon^{\mu'\nu'\alpha'\beta'}\epsilon^{\mu\nu\alpha\beta}\ q'_{\beta'}q_\beta \cdot \int_{-\infty}^{+\infty} \frac{d\kappa_1}{\kappa_1 + i\epsilon}\ P_{a,\mu'}P_{a,\mu} \cdot$$
$$\times\bar{u}(p',s')\left[\Gamma_{\alpha'}\left\{\theta(P^0_a)\delta(P^2_a - M^2)\,(\not{P}_a + M)\left(g_{\nu'\nu} - \frac{1}{3}\gamma_{\nu'}\gamma_\nu\right)\right\}\ \Gamma_\alpha\right]\ u(p,s)\ . \quad (8.2)$$

where we used the (simple) RS-propagator. The δ-function constraint in (8.2) gives

$$P_a = P(\kappa_1^+) = +\Delta_s - (\Delta_s\cdot n)n - A_s\ \ ,\ \ \Delta_s = \frac{1}{2}\left[(p'+p) + (q'+q)\right]\ . \quad (8.3)$$

A. Removal Non-causal Parts

Following the prescription, given before, we make in (8.2) the substitution

$$\Delta^{(+)}(P_a, M^2) = \theta(P^0_a)\delta^2(P^2_a - M^2) \Rightarrow \frac{i}{2}\Delta(P_a, M^2) = \frac{i}{2}\epsilon(P^0)\delta(P^2_a - M^2)\ , \quad (8.4)$$

so that now both the solutions $P^\pm$ for P_a contribute with opposite sign,

$$P^\pm = +\Delta_s - ((\Delta_s\cdot n)n \pm A_s)\ . \quad (8.5)$$

Then, the amplitude (8.2) becomes

$$M_{\kappa',\kappa} \Rightarrow -\frac{i}{8}g^2\ \epsilon^{\mu'\nu'\alpha'\beta'}\epsilon^{\mu\nu\alpha\beta}\ q'_{\beta'}q_\beta \cdot \left\{\bar{u}(p',s')\ \Gamma_{\alpha'}\cdot\right.$$
$$\times\frac{1}{2A_s}\left[P^+_{\mu'}P^+_\mu \frac{\not{P}^+ + M}{\bar{\kappa} + \Delta_s\cdot n + A_s} - P^-_{\mu'}P^-_\mu \frac{\not{P}^- + M}{\bar{\kappa} + \Delta_s\cdot n - A_s}\right]$$
$$\left.\times\left(g_{\nu'\nu} - \frac{1}{3}\gamma_{\nu'}\gamma_\nu\right)\right\}\ \Gamma_\alpha\ u(p,s)\ . \quad (8.6)$$

For the further evaluation, we note that

$$P^\pm_{\mu'}P^\pm_\mu\ =\ \Delta_{s,\mu'}\Delta_{s,\mu} - (\Delta_s\cdot n \pm A_s)\left(\Delta_{s,\mu'}n_\mu + \Delta_{s,mu}n_{\mu'}\right) + \left(\Delta_s\cdot n \pm A_s\right)^2 n_{\mu'}n_\mu\ .$$

Then, we get for the expression between the square-bracket in (8.6)

$$\frac{1}{2A_s}\Big[\ldots\Big]\ =\ \left\{-\left(\not{\Delta}_s + M + \bar{\kappa}\not{n}\right)\ \left(\Delta_{s,\mu'} + \bar{\kappa}n_{\mu'}\right)\left(\Delta_{s,\mu} + \bar{\kappa}n_\mu\right)\cdot\right.$$
$$\times\left(\ (\bar{\kappa} + \Delta_s\cdot n)^2 - A_s^2\right)^{-1}$$
$$\left.+\left(\not{\Delta}_s + M + \bar{\kappa}\not{n}\right)n_{\mu'}n_\mu + \not{n}\left(\Delta_{\mu'}n_\mu + \Delta_\mu n_{\mu'}\right) - 2(\Delta_s\cdot n)\not{n}\ n_{\mu'}n_\mu\right\} \quad (8.7)$$
$$\stackrel{\bar{\kappa}\to 0}{\Longrightarrow}\left\{-\left(\not{\Delta}_s + M\right)\ \Delta_{s,\mu'}\Delta_{s,\mu}\cdot\left(\Delta_s^2 - M^2\right)^{-1}\right.$$
$$\left.+\left(\not{\Delta}_s + M\right)n_{\mu'}n_\mu + \not{n}\left(\Delta_{\mu'}n_\mu + \Delta_\mu n_{\mu'}\right) - 2(\Delta_s\cdot n)\not{n}\ n_{\mu'}n_\mu\right\}\ \ ,\ \ s = \Delta_s^2 \quad (8.8)$$

From this expression it is clear that there remain 'contact-terms', which we eliminate by analyzing the equal time contributions from the time-derivatives of the $\Delta(x-y;M^2)$-function.

B. Removal Contact-terms

We now analyze the on-energy-shell contact terms, i.e. $\bar{\kappa} = 0$. From above we have for the 'contact-terms' in the square-bracket (8.6)

$$\frac{1}{2A_s}\left[\ldots\right]\Big|_{c.t.} = (\not{\Delta}_s + M)\ n_{\mu'}n_\mu + \Big[\Delta_{s,\mu'}n_\mu + \Delta_{s,mu}n_{\mu'} - 2(\Delta_s\cdot n)n_{\mu'}n_\mu\Big]\ \not{n}\ . \quad (8.9)$$

We noticed the following identities:

$$(i) \qquad P_{\omega\rho}\frac{\partial}{\partial_\rho}\left(n_{\mu'}n_\mu\right) = n_{\mu'}P_{\mu\omega} + n_\mu P_{\mu'\omega}\ . \quad (8.10)$$

This expression occurred in [23], Eq. (4.31), in the expression for $P_{\omega\rho}\partial\tau_1(x-y;n)/\partial_\rho$, and [9]

$$\begin{aligned}(ii) \qquad & P_{\omega\rho}\frac{\partial}{\partial n_\rho}\left\{\left[\left(\Delta_{s,\mu'}n_\mu + \Delta_{s,\mu}n_{\mu'} - 2(\Delta_s\cdot n)n_{\mu'}n_\mu\right)\not{n}\right] + \not{\Delta}_s\ n_{\mu'}n_\mu\right\} = \\ & \left\{\left(P_{\mu\omega}P_{\mu'\sigma} + P_{\mu\sigma}P_{\mu'\omega} - 2n_\mu n_{\mu'}P_{\omega\sigma}\right)\not{n}\right. \\ & \left. +\gamma^\tau\left(P_{\tau\omega}P_{\mu'\sigma} + P_{\tau\sigma}P_{\mu'\omega}\right)\ n_\mu + \gamma^\tau\left(P_{\tau\omega}P_{\mu\sigma} + P_{\tau\sigma}P_{\mu\omega}\right)\ n_{\mu'}\right\}\ \Delta_s^\sigma\ , \quad (8.11)\end{aligned}$$

the validity we checked by FORM [27]. This expression occurred in [23], Eq. (4.32), in the expression for $P_{\omega\rho}\partial\tau_2(x-y;n)/\partial_\rho$. For the effected interaction Lagrangian needed for the elemination of the n_μ-dependense, both 'contact'- and 'non-causal'-terms, we have

$$\begin{aligned}P_{\omega\rho}\frac{\partial}{\partial n_\rho}\Delta L_I \ &\sim\ P_{\omega\rho}\frac{\partial}{\partial n_\rho}\int d^x\int d^y\ \theta\left[n\cdot(x-y)\right]\ \mathcal{L}_I(x)\mathcal{L}_I(y) \\ &\Rightarrow\ \frac{g^2}{4}\epsilon^{\mu'\nu'\alpha'\beta'}\epsilon^{\mu\nu\alpha\beta}\int d^4x\int d^4y\ \left(\partial_{\beta'}\phi(x)\partial_\beta\phi(y)\right)\cdot \\ &\quad \times\overline{\psi^{(+)}}(x)\Gamma_{\alpha'}\left(g_{\nu\nu'} - \frac{1}{3}\gamma_{\nu'}\gamma_\nu\right)\cdot\left[P_{\omega\rho}\frac{\partial}{\partial n_\rho}\left\{\theta\left[n\cdot(x-y)\right]\cdot\right.\right. \\ &\quad \left.\left.\times\ \left(i\not{\partial}_x + M\right)\partial^x_{\mu'}\partial^x_\mu\Delta^{(+)}(x-y;M^2)\right\}\right]\cdot\Gamma_\alpha\psi^{(+)}(y)\ . \quad (8.12)\end{aligned}$$

Now, as already mentioned before $\Delta^{(+)}(x,M^2) = i\Delta(x,M^2)/2 + \Delta^{(1)}(x,M^2)/2$ which terms lead, when inserted in (8.12), to $\Delta L_{c.t.}$ and $\Delta L_{n.c.}$ respectively. Focussing here on the first one, we note that because

$$\frac{\partial}{\partial n_\rho}\theta\left[n\cdot(x-y)\right] = (x-y)^\rho\delta\left[n\cdot(x-y)\right]$$

[9] In terms of projection operators $P_{\mu\nu}$ there is also the relation

$$n_{\mu'}\partial_\mu + n_\mu\partial_{\mu'} - 2n_{\mu'}n_\mu(n\cdot\partial) = \left(n_{\mu'}P_{\mu\sigma} + n_\mu P_{\mu'\sigma}\right)\partial^\sigma\ .$$

we have in this case essentially '*equal-time derivatives*', and only terms with a derivative on the δ-function will survive. Therefore, in (8.12) the expression between curly-brackets becomes

$$\left\{\ldots\right\}_{x^0=y^0} \Rightarrow \frac{i}{2}\left\{ -i(\gamma\cdot n)\left(P_{\mu'\kappa}P_{\mu\sigma}-n_{\mu'}n_\mu P_{\kappa\sigma}\right)\partial^\kappa\partial^\sigma \right.$$
$$\left. -\left(i\gamma^\tau P_{\tau\kappa}\partial^\kappa+M\right)\left(n_{\mu'}P_{\mu\sigma}+n_\mu P_{\mu'\sigma}\right)\partial^\sigma\right\}\times\delta^3(\mathbf{x}-\mathbf{y})\ . \tag{8.13}$$

Then,

$$\begin{aligned}
&(x-y)^\rho\delta\left[n\cdot(x-y)\right]\left\{\ldots\right\}_{x^0=y^0} \stackrel{p.i.}{\rightarrow}\\
&-\frac{i}{2}\left[-i(\gamma\cdot n)\Big(P_{\mu'\rho}P_{\mu\sigma}+P_{\mu'\sigma}P_{\mu\rho}-2n_{\mu'}n_\mu P_{\rho\sigma}\Big)\right.\\
&\left.-i\gamma^\tau\Big(P_{\tau\sigma}\left(n_{\mu'}P_{\mu\rho}+n_\mu P_{\mu'\rho}\right)+P_{\tau\rho}\left(n_{\mu'}P_{\mu\rho}+n_\mu P_{\mu'\rho}\right)\right]\ \partial^\sigma\delta^4(x-y)\\
&+\frac{i}{2}M\left(n_{\mu'}P_{\mu\rho}+n_\mu P_{\mu'\rho}\right)\delta^4(x-y)=\\
&-\frac{1}{2}\gamma^\tau\left[n_\tau\Big(P_{\mu'\rho}P_{\mu\sigma}+P_{\mu'\sigma}P_{\mu\rho}-2n_{\mu'}n_\mu P_{\rho\sigma}\Big)\right.\\
&\left.+n_{\mu'}\Big(P_{\tau\sigma}P_{\mu\rho}+P_{\tau\rho}P_{\mu\sigma}\Big)+n_\mu\Big(P_{\tau\sigma}P_{\mu'\rho}+P_{\tau\rho}P_{\mu\sigma}\Big)\right]\ \partial^\sigma\delta^4(x-y)\\
&+\frac{i}{2}M\left(n_{\mu'}P_{\mu\rho}+n_\mu P_{\mu'\rho}\right)\delta^4(x-y)
\end{aligned} \tag{8.14}$$

Now, using $P^\rho_\omega P_{\rho\chi}=P_{\omega\chi}$ we obtain, applying the identities in (8.10) and (8.11),

$$\begin{aligned}
&P_{\omega\rho}(x-y)^\rho\delta\left[n\cdot(x-y)\right]\left\{\ldots\right\}_{x^0=y^0}\Rightarrow\\
&\left\{-\frac{1}{2}P_{\omega\rho}\frac{\partial}{\partial n_\rho}\left\{\left[n_{\mu'}\partial_\mu+n_\mu\partial_{\mu'}-2n_{\mu'}n_\mu(n\cdot\partial)\right]\ \not{n}+n_{\mu'}n_\mu\not{\partial}\right\}+\frac{i}{2}M\ P_{\omega\rho}\frac{\partial}{\partial n_\rho}n_{\mu'}n_\mu\right\}\ \delta^4(x-y)=\\
&P_{\omega\rho}\frac{\partial}{\partial n_\rho}\left\{-\frac{1}{2}\left[n_{\mu'}\partial_\mu+n_\mu\partial_{\mu'}-2n_{\mu'}n_\mu(n\cdot\partial)\right]\ \not{n}+\frac{i}{2}\left(i\not{\partial}+M\right)n_{\mu'}n_\mu\right\}\ \delta^4(x-y)\ .
\end{aligned} \tag{8.15}$$

From these results, we infer that

$$\begin{aligned}
-\Delta H_{c.t.}\sim\ &+\frac{g^2}{8}\epsilon^{\mu'\nu'\alpha'\beta'}\epsilon^{\mu\nu\alpha\beta}\int d^4x\int d^4y\ \left(\partial_{\beta'}\phi(x)\partial_\beta\phi(y)\right)\cdot\overline{\psi^{(+)}}(x)\Gamma_{\alpha'}\left(g_{\nu\nu'}-\frac{1}{3}\gamma_{\nu'}\gamma_\nu\right)\cdot\\
&\times\left\{\left[n_{\mu'}\partial^x_\mu+n_\mu\partial^x_{\mu'}-2n_{\mu'}n_\mu(n\cdot\partial^x)\right]\not{n}-i\left(i\not{\partial}^x+M\right)n_{\mu'}n_\mu\right\}\cdot\\
&\times\delta^4(x-y)\cdot\Gamma_\alpha\psi^{(+)}(y)\ .
\end{aligned} \tag{8.16}$$

Taking now the off-energy-shell πN matrix element of (8.16) we get

$$\langle p', q'; \kappa'| - i\Delta H_{c.t.}|p, q; \kappa\rangle = +i\frac{g^2}{8}\epsilon^{\mu'\nu'\alpha'\beta'}\epsilon^{\mu\nu\alpha\beta}(q'_{\beta'}q_\beta)\cdot\int\frac{d^4K}{(2\pi)^4}\Bigg[$$
$$\int d^4x\int d^4y\; e^{i(p'+q'+\kappa'n)\cdot x)}e^{-i(p+q+\kappa n)\cdot y)}e^{-iK\cdot(x-y)}\cdot\bar{u}(p',s')\;\Gamma_{\alpha'}\cdot$$
$$\times\left\{[n_{\mu'}K_\mu + n_\mu K_{\mu'} - 2n_{\mu'}n_\mu(n\cdot K)]\,\not{n} + (\not{K}+M)\,n_{\mu'}n_\mu\right\}\cdot$$
$$\times\left(g_{\nu\nu'} - \frac{1}{3}\gamma_{\nu'}\gamma_\nu\right)\cdot\Gamma_\alpha u(p,s)\Bigg]\;. \tag{8.17}$$

Writing

$$\langle p', q'; \kappa'| - i\Delta H_{c.t.}|p, q; \kappa\rangle = (2\pi)^4\delta\left(p'+q'+\kappa' n - p - q - \kappa n\right)\;\left(\Delta M_{\kappa',\kappa}\right)_{c.t.}\;, \tag{8.18}$$

we obtain from (8.17) the expression

$$\left(\Delta M_{\kappa',\kappa}\right)_{c.t.} = +i\frac{g^2}{8}\epsilon^{\mu'\nu'\alpha'\beta'}\epsilon^{\mu\nu\alpha\beta}(q'_{\beta'}q_\beta)\cdot\bar{u}(p',s')\Bigg[\Gamma_{\alpha'}$$
$$\times\;\left\{[n_{\mu'}K_\mu + n_\mu K_{\mu'} - 2n_{\mu'}n_\mu(n\cdot K)]\,\not{n} + (\not{K}+M)\,n_{\mu'}n_\mu\right\}\Bigg|_{K=\Delta_s+\bar{\kappa}n}\cdot$$
$$\times\left(g_{\nu\nu'} - \frac{1}{3}\gamma_{\nu'}\gamma_\nu\right)\cdot\Gamma_\alpha\Bigg]\,u(p,s)\;. \tag{8.19}$$

This matrix element cancels the c.t.-terms in (8.6) and (8.8)! (CHECK!!)

So, cancellation of the contact terms leads from (8.6) to the final (off-energy-shell) amplitude

$$M_{\kappa',\kappa}\;\Rightarrow\;+(?)\frac{g^2}{8}\;\epsilon^{\mu'\nu'\alpha'\beta'}\epsilon^{\mu\nu\alpha\beta}\;q'_{\beta'}q_\beta\cdot$$
$$\times\left(\Delta_{s,\mu'}+\bar{\kappa}n_{\mu'}\right)\left(\Delta_{s,\mu}+\bar{\kappa}n_\mu\right)\cdot$$
$$\times\bar{u}(p',s')\left\{\Gamma_{\alpha'}\cdot\left(\not{\Delta}_s + M + \bar{\kappa}\not{n}\right)\cdot\left(g_{\nu'\nu}-\frac{1}{3}\gamma_{\nu'}\gamma_\nu\right)\right\}\cdot\Gamma_\alpha\;u(p,s)\cdot$$
$$\times\left[\left(\Delta_s\cdot n + \bar{\kappa}\right)^2 - A_s^2\right]^{-1}\;. \tag{8.20}$$

One sees that for $\bar{\kappa}\to 0$ the amplitude becomes n_μ-independent, i.e. Lorentz-invariant, and the PW-expansion is similar to that for baryon-exchange, see above, and therefore simple!.

In passing we note that

$$\left[+\Delta_{s,\mu'}\Delta_{s,\mu} + \bar{\kappa}\left(\Delta_{s,\mu'}n_\mu + \Delta_{s,\mu}n_{\mu'}\right) + \bar{\kappa}^2 n_{\mu'}n_\mu\right] =$$
$$+\left(\Delta_{s,\mu'}+\bar{\kappa}n_{\mu'}\right)\left(\Delta_{s,\mu}+\bar{\kappa}n_\mu\right)\equiv +\bar{P}_{\mu'}\bar{P}_\mu\;, \tag{8.21}$$

from which one can see the resemblance with expressions on this matter in for example [16]. Using FORM [27], and the short-hand notation

$$K_\mu = \Delta_{s,\mu} + \bar{\kappa}\ n_\mu\ , \tag{8.22}$$

we get for the numerator in (8.20)

$$\begin{aligned}
M_{\kappa',\kappa} \Rightarrow\ & \frac{1}{3}\bar{u}(p',s')\Big[M_\Delta\Big\{-\Big(\not{q}\,\not{q}' + (q'\cdot q)\Big)\ K^2 + (q'\cdot K)\not{q}\,\not{K} + (q\cdot K)\not{K}\,\not{q}'\Big\} \\
& +\Big(-(q\cdot K)\ \not{q}'\ -(q'\cdot K)\ \not{q}\ + \not{q}\ \not{K}'\ \not{q} - (q'\cdot q)\ \not{K}\Big)\ K^2 \\
& +2(q'\cdot K)(q\cdot K)\ \not{K}\Big]\ u(p,s) \\
=\ & \frac{1}{3}\bar{u}(p',s')\Big[M_\Delta\Big\{\Big(+\frac{1}{2}\left(\not{q}'\,\not{q} - \not{q}\,\not{q}'\right) - 2(q'\cdot q)\Big)\ K^2 + \frac{1}{2}(q'\cdot K)\left(\not{q}\ \not{K} - \not{K}\ \not{q}\right) \\
& -\frac{1}{2}(q\cdot K)\left(\not{q}'\ \not{K} - \not{K}\,\not{q}'\right) + 2(q'\cdot K)(q\cdot K)\Big\} \\
& +\Big\{-\frac{1}{2}\Big(\not{q}'\ \not{K}\,\not{q} - \not{q}\ \not{K}\,\not{q}'\Big) - 2(q'\cdot q)\not{K}\Big\}\ K^2 + 2(q'\cdot K)(q\cdot K)\ \not{K}\Big]\ u(p,s)
\end{aligned} \tag{8,23}$$

where in the last expression the symmetry under the interchange $q' \leftrightarrow q$ is exhibited.
For later it will be usefull to exhibit the terms independent, linear, quadratic, and cubic in $\bar{\kappa}$. Writing

$$M_{\kappa',\kappa} = M^{(0)}_{\kappa',\kappa} + \bar{\kappa}\ M^{(1)}_{\kappa',\kappa} + \bar{\kappa}^2\ M^{(2)}_{\kappa',\kappa} + \bar{\kappa}^3\ M^{(3)}_{\kappa',\kappa}\ , \tag{8.24}$$

and using (8.22), we get from (8.23)

$$M^{(0)}_{\kappa',\kappa} \Rightarrow \frac{1}{3}\bar{u}(p',s')\left[M_\Delta\left\{-\left(\not{q}\,\not{q}' + (q'\cdot q)\right)\ \Delta_s^2 + (q'\cdot\Delta_s)\not{q}\ \not{\Delta}_s + (q\cdot\Delta_s)\not{\Delta}_s\ \not{q}'\right\}\right.$$
$$+\left(-(q\cdot\Delta_s)\ \not{q}'\ -(q'\cdot\Delta_s)\ \not{q}\ +\not{q}\ \not{\Delta}_s\ \not{q}' - (q'\cdot q)\ \not{\Delta}_s\right)\ \Delta_s^2$$
$$\left.+2(q'\cdot\Delta_s)(q\cdot\Delta_s)\ \not{\Delta}_s\right]\ u(p,s)\ , \tag{8.25}$$

$$M^{(1)}_{\kappa',\kappa} \Rightarrow \frac{1}{3}\bar{u}(p',s')\left[M_\Delta\left\{-2\left(\not{q}\,\not{q}' + (q'\cdot q)\right)\ (\Delta_s\cdot n) + (q'\cdot n)\not{q}\ \not{\Delta}_s + (q'\cdot\Delta_s)\not{q}\ \not{n}\right.\right.$$
$$\left.+ (q\cdot n)\not{\Delta}_s\not{q}' + (q\cdot\Delta_s)\not{n}\ \not{q}'\right\}$$
$$+\left(-(q'\cdot n)\ \not{q}\ -(q\cdot n)\ \not{q}'\ +\not{q}\ \not{n}\ \not{q}' - (q'\cdot q)\ \not{n}\right)\ \Delta_s^2$$
$$+2\left(-(q'\cdot\Delta_s)\ \not{q}\ -(q\cdot\Delta_s)\ \not{q}'\ +\not{q}x'\ \not{\Delta}_s\not{q}i' - (q'\cdot q)\ \not{\Delta}_s\right)\ (\Delta_s\cdot n)$$
$$\left.+2(q\cdot n)(q'\cdot\Delta_s)\ \not{\Delta}_s + 2(q\cdot\Delta_s)(q'\cdot n)\ \not{\Delta}_s + 2(q'\cdot\Delta_s)(q\cdot\Delta_s)\ \not{n}\right]\ u(p,s)\ , \tag{8.26}$$

$$M^{(2)}_{\kappa',\kappa} \Rightarrow \frac{1}{3}\bar{u}(p',s')\left[M_\Delta\left\{-\left(\not{q}\,\not{q}' + (q'\cdot q)\right)\ + (q'\cdot n)\not{q}\ \not{n} + (q\cdot n)\not{n}\ \not{q}'\right\}\right.$$
$$+2\left(-(q'\cdot n)\ \not{q}\ -(q\cdot n)\ \not{q}'\ +\not{q}\ \not{n}\,\not{q}' - (q'\cdot q)\ \not{n}\right)\ (\Delta_s\cdot n)$$
$$+\left(-(q'\cdot\Delta_s)\ \not{q}\ -(q\cdot\Delta_s)\ \not{q}'\ +\not{q}\ \not{\Delta}_s\ \not{q}' - (q'\cdot q)\ \not{\Delta}_s\right)$$
$$\left.+2(q'\cdot n)(q\cdot n)\ \not{\Delta}_s + 2(q\cdot n)(q'\cdot\Delta_s)\ \not{n} + 2(q\cdot\Delta_s)(q'\cdot n)\ \not{n}\right]\ u(p,s)\ , \tag{8.27}$$

$$M^{(3)}_{\kappa',\kappa} \Rightarrow \frac{1}{3}\bar{u}(p',s')\left[+2(q'\cdot n)(q\cdot n)\ \not{n}\right]\ u(p,s)\ . \tag{8.28}$$

IX. KADYSHEVSKY INTEGRAL EQUATION

In this section we give a proof that the on-energy-shell solution of the Kadyshevsky integral equation is frame indenpendent, see also [1].

We restrict ourselves to the meson-baryon states, e.g the pion-nucleon etc. states. The Kadyshevsky analog of the Bethe-Salpeter equation, see [16] sections IX and X, reads

$$\begin{aligned}
&M_{\kappa',\kappa}(q'_a,p'_b;q_a,p_b) = I_{\kappa',\kappa}(q'_a,p'_b;q_a,p_b) + \int d^4q''_a \int d^4p''_b \int d\kappa'' \cdot \\
&\times I_{\kappa',\kappa''}(q'_a,p'_b;q''_a,p''_b) G_{\kappa''}(q''_a,p''_b) M_{\kappa'',\kappa}(q''_a,p''_b;q_a,p_b)\cdot \\
&\times\delta(q''_a + p''_b + \kappa'' n - q_a - p_b - \kappa n)\ ,
\end{aligned} \tag{9.1}$$

where the propagation of the meson, baryon, and of the quasi-particle is described by

$$G(q_a,p_b,\kappa)\ =\ \frac{-1}{(2\pi)^2}\delta_+(q_a^2-m_a^2)\delta_+(p_b^2-M_b^2)\cdot G_0(\kappa)\ , \tag{9.2}$$

and the propagator $G_0(\kappa)$ for the quasi-particles is given by [8]

$$G_0(\kappa) = (1/2\pi)\,[1/(\kappa + i\epsilon)] \ . \tag{9.3}$$

With 'strong(absolute) pair-suppression' only positive energy nucleons are involved, both in the initial/final but also in the intermediate states.

Introducing the momenta

$$\begin{aligned} &P = p_a + p_b \ \ , \ \ P' = p'_a + p'_b \ \ , \ \ P'' = p''_a + p''_b \ , \\ &p = \frac{1}{2}(p_a - p_b) \ \ , \ \ p' = \frac{1}{2}(p'_a - p'_b) \ \ , \ \ p'' = \frac{1}{2}(p''_a - p''_b) \ , \end{aligned} \tag{9.4}$$

we bring Eq. (9.1) in the form

$$\begin{aligned} M_{\kappa',\kappa}(P',p';P,p) = I_{\kappa',\kappa}(P',p;P,p) + \int d^4P'' \int d^4p'' \int d\kappa'' \cdot \\ \times \left[I_{\kappa',\kappa''}(P',p;P'',p'') G_{\kappa''}(P'',p'') M_{\kappa'',\kappa}(P'',p'';P,p) \right] |_{P''=P+(\kappa-\kappa'')n} \ . \end{aligned} \tag{9.5}$$

This equation we write schematically, for the on-energy-shell case, i.e. $\kappa = \kappa' = 0$, as

$$M_{0,0} = I_{0,0} + \int d\kappa \ I_{0,\kappa} \ G_\kappa \ M_{\kappa,0} \ . \tag{9.6}$$

Then,

$$P^{\alpha\beta}\frac{\partial}{\partial^\beta}M_{0,0} \ = \ P^{\alpha\beta}\int d\kappa \left[\frac{\partial I_{0,\kappa}}{\partial n^\beta} \ G_\kappa \ M_{\kappa,0} + I_{0,\kappa} \ G_\kappa \ \frac{\partial M_{\kappa,0}}{\partial n^\beta} \right] \ . \tag{9.7}$$

Next we observe that

$$\frac{\partial I_{0,\kappa}}{\partial n^\beta} \propto \kappa \ \ , \ \ \frac{\partial M_{\kappa,0}}{\partial n^\beta} \propto \kappa \ . \tag{9.8}$$

For the kernel $I_{0,\kappa}$ we have seen examples in the previous sections, and for $M_{\kappa,0}$ one may invoke the Born-series based on the integral equation (9.6). Furthermore,

$$G_\kappa \propto \frac{1}{\kappa + i\epsilon} = P\frac{1}{\kappa} - i\pi\delta(\kappa) \ . \tag{9.9}$$

From (9.8), and the absence of a pole at $\kappa = 0$, it follows that the $\delta(\kappa)$-term gives zero when used in (9.7).

For the $P(1/\kappa)$-term, we note that the singularities in the complex κ-plane from the amplitude denominators

$$D = \left[\left(\bar{\kappa} + \Delta_s \cdot n \right)^2 - A_s^2 \right]^{-1} \ \ , \tag{9.10}$$

are, because of their retarded character, in the lower half-plane. Namely at

$$\kappa_\pm = -\Delta_s \cdot n \pm \sqrt{M^2 - \Delta_s^2 + (\Delta_s \cdot n)^2} - i\epsilon \ , \tag{9.11}$$

as can be seen from Eq. (5.9). Then, we can close the intergration contour in the the upper-half complex κ-plane, and the contribution of $P(1/\kappa) \propto -i\pi Res[\ldots]_{\kappa=0}$ and consequently vanishes also.

Important for the proof is the behavior at infinity in the complex κ-plane. To fulfill the necessary requirement in general a phenomenological cut-off is needed in κ-space. As an example we suggest a 'form-factor'

$$F(\kappa) = \left(\frac{\Lambda_\kappa^2}{\Lambda_\kappa^2 - \kappa^2 - i0\epsilon(\kappa)}\right)^{N_\kappa} , \tag{9.12}$$

for a large Λ_κ and some power N_κ. Note that the poles in the κ-plane have a retarded character and are below the real axis. Of course, above $\kappa \equiv \Delta\kappa = \kappa_f - \kappa_i$.
This finishes the proof that the on-energy-shell solution of the Kadyshevsky equation is frame-, i.e. n_μ-independent.

X. DISCUSSION AND OUTLOOK

A. Absolute and Strong Pair-suppression and Green-functions

The interaction (2.3) describes also for $a = \infty$ both πN and $\pi\bar{N}$ scattering. However, in working out the Feynman rules for the interaction Lagrangian with $a = \infty$ there does not appear the Feynman propagator between two interaction vertices. For the Feynman propagator to appear one needs ' in the product of two interaction Lagrangians terms like

$$\bar{\psi^{(+)}}(x)\Gamma\left\{\psi^{(-)}(x), \bar{\psi^{(-)}}(y)\right\}\Gamma\psi^{(-)}(y) , \tag{10.1}$$

which are lacking for absolute pair-suppression. Therefore, the analysis of Lagrangians with pair-suppression is most easily done in the Kadyshevsky formalism.
Now in reality there will be "strong pair-suppression, i.e. a large but finite. It is clear that then we have to take into account, in principle, also the pair-terms and the Kadyshevsky equation becomes a coupled integral equation. Although more complicated and more laborious, in essence there are no extra problems to deal with this situation as compared to 'absolute pair-suppression dealt with in these notes.

We stress that it was found that absolute pair-suppression leads to a relativistic theory where the transitions between positive and negative energy state matrix elements are absent. However, to make the theory frame-independent we had to introduce extra non-local interaction Hamiltonians ΔH_I, leading to extra effective vertices where internally the negative energy solutions of the free Dirac-equation are important. However, there are only non-zero matrix elements of ΔH_I for positive energy states. This justifies the statements in the Introduction about the suppression of the positive negative energy transitions in the matrix elements.

B. Motivation for the Kadyshevsky Formalism

The usefulness of the Kadyshevsky formalism for our purpuses is twofold:

(i) Particles, in particularly baryons, stay on-mass-shell. This makes it possible to control pair-suppression in arbitrary complicated graph's. This feature made it possible to construct a relativistic interaction theory where for matrix elements the positive-negative energy transitions are absolutely) suppressed.

(ii) This formalism leads to 3-dimensional integral equations which partly justifies and explains the succes of such equations, e.g. the Thompson-equation [37], in phenomenological low-energy hadron-scattering models [5, 6].

(iii) In this formalism we can treat the higher spin particles, both for mesons (J=1,2) and baryons (J=3/2).

C. Prospects and Problems with Non-local Field Theories

Recently non-local regularizations of gauge theories have been reviewed and studied [19]. Historically, one has tried to cure the ultraviolet divergences by constructing non-local cut-off's. As mentioned in the introduction, interest has been revived because of the non-locality of string theories. One was able to achieve a finite, unitary, and Lorentz invariant perturbative S-matrix. A price for these advantages: off-shell non-causality, as well as instability. Furthermore, there is the problem of how to formulate a canonical formalism for non-local interactions. However, it is claimed recently that a general procedure exists for the canonical formulation of any perturbatively localizable interaction.

In [19] the cut-off

$$\mathcal{E}_m = \exp\left[\frac{\partial^2 - m^2}{2\Lambda^2}\right] \tag{10.2}$$

has been studied. Note the similarity with our vertex factors, which contain the square root in the exponential instead. Also, cut-off's of the type (10.2) were used in the Nijmegen soft-core baryon-baryon potentials [28], momentum-space πN-potentials [6], and in the study of the renormalization group equations [30].

Finally, we mention the possible applications of the formalism developed in these notes:
(i) Analogous systems: First, of course, the extension to Kaon-Nucleon scattering, and the application to photo-production. (ii) Baryon-baryon systems: the derivation of new two-meson-exchange potentials. (ii) Application to Few-body systems: the suppression of the positive-negative energy transition matrix elements seems to cure problems with these in e.g. the Bethe-Salpeter and/or relativistic Faddeev formalisms.

APPENDIX A: KADYSHEVSKY-RULES IN MOMENTUM-SPACE

In this section the Kadyshevsky formalism is briefly introduced by using the S-matrix formula in quantum-field-theory as a startpoint, and going from there to the rules for the Kadyshevsky-diagrams [7–10]. We follow the set up of the appendices B in [22] where the rules for the Feynman-graphs are given. The differences will then come to the surface in a most transparant manner. Starting from the expression of the S-operator, one has [25, 26]

$$\begin{aligned} S &= 1+\sum_{n=1}^{\infty}\left(\frac{i}{\hbar}\right)^n \int_{-\infty}^{+\infty}\ldots\int_{-\infty}^{+\infty}\theta(x_n^0-x_{n-1}^0)\theta(x_{n-1}^0-x_{n-2}^0)\ldots\theta(x_2^0-x_1^0)\cdot \\ &\qquad \times\mathcal{L}_I(x_n)\ \mathcal{L}_I(x_{n-1})\ldots\mathcal{L}_I(x_1)\cdot d^4x_n\ldots d^4x_1 \\ &\equiv 1+\sum_{n=1}^{\infty}S_n\ , \end{aligned} \tag{A1}$$

we follow [7] and introduce the time-like vector n^μ with $n^2=n_0^2-\mathbf{n}^2=1, n_0>0$. Then (A1) can be brought into a completely 4-dimensional covariant form, although frame-dependent, by the replacement

$$\theta(x^0)\to\theta(x\cdot n)\quad ,\quad n\cdot x=n_0x^0-\mathbf{n}\cdot\mathbf{x}\ . \tag{A2}$$

This gives ($\hbar=1$)

$$\begin{aligned} S_n &= i^n\int_{-\infty}^{+\infty}\ldots\int_{-\infty}^{+\infty}\theta[n\cdot(x_n-x_{n-1})]\theta[n\cdot(x_{n-1}-x_{n-2})]\ldots\theta[n\cdot(x_2-x_1)]\cdot \\ &\qquad \times\mathcal{L}_I(x_n)\ \mathcal{L}_I(x_{n-1})\ldots\mathcal{L}_I(x_1)\cdot d^4x_n\ldots d^4x_1\ . \end{aligned} \tag{A3}$$

The equivalence of S_n in equations (A1) and (A3) can be seen as follows. Assuming that the S-matrix defined in (A1) is Lorentz-invariant, and realizing that (A1) and (A3) are identical in the frame where $n^\mu=(1,\mathbf{0})$, it follows that they are equivalent in all frames because the expression in (A3) is manifest Lorentz-invariant. Also, it follows that the S-matrix defined in (A3) is independent of the four-vector n^μ. A more explicit elaboration on this issue and others is given in appendix A of [4].

From the expression (A3) one can work out the rules for the Kadyshevsky graphs in a way which parallels the derivation of the Feynman rules. The differences come from the treatment of the θ-functions. In the case of the Feynman graphs one includes the θ-functions into the propagators by applying the Wick-expansion to the T-products of the field operators. In the case of the Kadyshevsky graphs one employs a four-dimensional form of the θ-functions, exploiting (A2),

$$\theta(n\cdot x)=-\frac{1}{2\pi i}\int_{-\infty}^{+\infty}d\kappa\frac{\exp\left[-i\kappa(n\cdot x)\right]}{\kappa+i\epsilon}\ , \tag{A4}$$

and one applies the Wick-expansion to the ordinary products of the field operators. Then, the propagators are given by

$$\begin{aligned}
\langle 0|\phi(x)\phi(y)|0\rangle &= \Delta^{(+)}(x-y;\mu^2) = \int \frac{d^4q}{(2\pi)^3}\theta(q_0)\delta(q^2-\mu^2)\ e^{-iq\cdot(x-y)} \\
\langle 0|A_\mu(x)A_\nu(y)|0\rangle &= D^{(+)}_{\mu\nu}(x-y) = -g_{\mu\nu}\int \frac{d^4q}{(2\pi)^3}\theta(q_0)\delta(q^2)\ e^{-iq\cdot(x-y)} \\
\langle 0|\psi(x)_\beta\bar{\psi}(y)_\alpha|0\rangle &= S^{(+)}_{\beta\alpha}(x-y) = \int \frac{d^4p}{(2\pi)^3}\theta(p_0)\,(\not{p}+m)_{\beta\alpha}\,\delta(p^2-m^2)\ e^{-ip\cdot(x-y)} \\
\langle 0|\bar{\psi}(x)_\beta\psi(y)_\alpha|0\rangle &= S^{(-)}_{\beta\alpha}(x-y) = \int \frac{d^4p}{(2\pi)^3}\theta(p_0)\,(\not{p}-m)_{\beta\alpha}\,\delta(p^2-m^2)\ e^{-ip\cdot(x-y)}\ ,
\end{aligned} \tag{A5}$$

which are the so called Wightman-functions for free-fields. For the massive vector field $V_\mu(x)$ we have

$$\langle 0|V_\mu(x)V_\nu(y)|0\rangle = \Delta^{(+)}_{\mu\nu}(x-y;m_V^2) = \int \frac{d^4q}{(2\pi)^3}\theta(q_0)\delta\left(q^2-m_V^2\right)\ e^{-iq\cdot(x-y)}\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{m_V^2}\right) \tag{A6}$$

In the Kadyshevsky-graph theory the considered Hilbert-space is enlarged by admitting states containing 'quasi-particles'. The latter carry only 4-momentum, and serve to have formally four-momentum conservation at each vertex. The quasi-particles refer to the κ-variables in the Fourier transforms (A4) of the θ-functions appearing in (A3). These quasi-particle states $|\kappa_1,\ldots\rangle$ are normalized by

$$\langle \kappa_1'\ldots|\kappa_1,\ldots\rangle = \delta(\kappa_1'-\kappa_1)\ldots \tag{A7}$$

The θ-functions in (A3) connect only internal points of the graphs. In order to handle integral equations, occurring in for example the Bethe-Salpeter- and Schwinger-Dyson-equations, one needs to consider amplitudes with external quasi-particles as well as internal quasi-particles. The external quasi-particle entering a vertex is included only into the four-momentum conservation rule of that vertex, including both the external and the internal quasi-particle 4-momentum.

After these preliminary remarks we now list the momentum-space rules for the computation of the $-M_{\kappa',\kappa}$-amplitudes, defined by

$$S_{\kappa',\kappa} = 1_{\kappa',\kappa} - (2\pi)^4 i\delta^4(P_f+\kappa' n - P_i - \kappa n)\ M_{\kappa',\kappa}\ . \tag{A8}$$

The invariant amplitude $-M_{\kappa',\kappa}$ is computed by drawing all connected Feynman graphs for the considered process. The amplitude

$$-(2\pi)^4\delta(\sum_i p_{i,out}+\kappa' n-\sum_i p_{i,in}-\kappa n)M_{\kappa',\kappa}(G)$$

corresponding to graph G is built up by associating factors with the elements of the graph, which we list below:

I. Those factors, independent of the specific details of the interactions, are given by the following rules:

1. Draw the Feynman graph G. Arbitrarily number its vertices and orient each internal particle line from the vertex with the smaller number to the vertex with the larger number, assigning to it a 4-momentum p.

2. Connect with dotted lines the first vertex with the second, the second with the third, etc. Orient them in the direction of increasing numbers and assign to them a 4-momentum $\kappa_s n$, where $s = 1, 2, \ldots, n-1$ is the number of the vertex which a given dotted line leaves. Attach to the first vertex an incoming external dotted line with 4-momentum $\kappa_i n$, and to the last vertex n an outgoing external dotted line with 4-momentum $\kappa_f n$.

3. For incoming (outgoing) boson and fermion lines: identical to the rules for Feynman graphs [22].

4. For each internal dotted line with momentum κn a factor

$$G_0(\kappa) = -\frac{1}{\kappa + i\epsilon} \ . \tag{A9}$$

5. For each internal boson line with momentum q a factor

$$\Delta^{(+)}(q) = \theta(q_0)\delta(q^2 - \mu^2) \ . \tag{A10}$$

6. For each internal fermion line with momentum p and *positive energy* a factor

$$S^{(+)}_{\beta\alpha}(p) = (\not{p} + m)_{\beta\alpha}\,\theta(p_0)\delta(p^2 - m^2) \ . \tag{A11}$$

For each internal fermion line with momentum p and *negative energy* a factor

$$S^{(-)}_{\beta\alpha}(p) = (\not{p} - m)_{\beta\alpha}\,\theta(p_0)\delta(p^2 - m^2) \ . \tag{A12}$$

7. For each internal photon line, using the Feynman gauge, a factor

$$D^{(+)}(q)_{\mu\nu} = -g_{\mu\nu}\theta(q_0)\delta(q^2) \ . \tag{A13}$$

8a. For each vertex, number s, a factor

$$(2\pi)^4\delta^4\left(\sum_i p_{i,out} + \kappa_{s+1} - \sum_i p_{i,in} - \kappa_s\right) \ , \tag{A14}$$

where $p_{i,out}$ and $p_{i,in}$ are the outgoing respectively the incoming momenta at the vertex with number s.

8b. Integrate over each internal particle line , momentum l: $\int d^4l/(2\pi)^3$.

9. Integrate over each internal quasi-particle (dotted) line with momentum $\kappa_s n$: $\int_{-\infty}^{+\infty} d\kappa_s/(2\pi)$.

10. *Not* a factor -1 for each closed loop.

11. A factor -1 between graphs which differ only by an interchange of two-external fermions. This not only for the interchange of identical fermions in the final state, but also the interchange of e.g. an initial fermion and a similar anti-fermion in the final state.

12. Repeat the operations (1)-(11) for all $n!$ numberings of the vertices of the given Feynman graph and sum.

II. Those factors coming from the structure and type of vertices are, given for each vertex by the matrix element $\langle \ldots |\mathcal{L}_I(0)| \ldots \rangle$. Therefore, they are, apart from a factor $(-i)$, identical to that given in [22], appendices B.

APPENDIX B: SECOND QUANTIZATION MOMENTUM QUASI-PARTICLES

For the inclusion of the $\theta[n \cdot (x_{i-1} - x_i)]$-factors appearing in (A3) one may proceed as follows. Introducing $\tau' = n \cdot x',\ \tau = n \cdot x$ and consider the π-problem

$$\left(i\frac{\partial}{\partial\tau} + i\epsilon\right)\chi_\kappa(\tau) = \kappa\ \chi_\kappa(\tau)\ . \tag{B1}$$

The ortho-normal solutions of equation (B1) are

$$\chi_\kappa(\tau) = \frac{1}{\sqrt{2\pi}}e^{-i(\kappa - i\epsilon)\tau}\ ,\ -\infty < \kappa < \infty\ . \tag{B2}$$

The corresponding Green function satisfies the equation

$$\left(i\frac{\partial}{\partial\tau'} + i\epsilon\right)G(\tau', \tau) = -\delta(\tau' - \tau)\ , \tag{B3}$$

which can be expressed as

$$G(\tau', \tau)\ =\ -\int_{-\infty}^{\infty} d\kappa\ \frac{\chi_\kappa(\tau')\chi^*_\kappa(\tau)}{\kappa + i\epsilon} = i\theta(\tau' - \tau)\ . \tag{B4}$$

This last expression follows from (B2) and the representation (A4). Notice that we can also write for G the expression

$$G(\tau', \tau)\ =\ -\int_{C_R} \frac{d\kappa}{\kappa}\ \chi_\kappa(\tau')\chi^*_\kappa(\tau)\ , \tag{B5}$$

where the contour C_R in the complex κ-plane is $C_R = \{-\infty < \Re\kappa < \infty, \Im\kappa = i\epsilon\}$.
For the second quantization formalism we introduce auxiliary fields, henceforth called Kadyshevsky fields, by the operators

$$\begin{aligned} \chi(\tau)\ &=\ \int \frac{d\kappa}{\kappa + i\epsilon}\ a(\kappa)\ \chi_\kappa(\tau)\ , \\ \bar{\chi}(\tau)\ &=\ \int \frac{d\kappa}{\kappa + i\epsilon}\ a^\dagger(\kappa)\ \chi^*_\kappa(\tau)\ . \end{aligned} \tag{B6}$$

In second quantization, we postulate the commutator

$$[\chi(\tau'), \bar{\chi}(\tau)] = -i\theta(\tau' - \tau) \equiv -i\theta[n \cdot (x' - x)]\ , \tag{B7}$$

which follows from the canonical commutation rules for the annihilation and creation operators for the quasi-particles

$$\left[a(\kappa'), a^\dagger(\kappa)\right] = \kappa\delta(\kappa' - \kappa)\ . \tag{B8}$$

We note that with these normalizations

$$|\kappa\rangle = a^\dagger(\kappa)|0\rangle\ \ ,\ \ \chi(\tau)|\kappa\rangle = \frac{1}{\sqrt{2\pi}}e^{-i(\kappa - i\epsilon)\tau}\ . \tag{B9}$$

Next we introduce the following addition to the free Lagrangian density

$$\mathcal{L}_K = i\bar{\chi}(\tau)\dot{\chi}(\tau) + i\epsilon\bar{\chi}(\tau)\chi(\tau) \ , \tag{B10}$$

where $\dot{\chi} := \partial\chi/\partial\tau$. To the interaction Lagrangians we add a factor $\chi^\dagger\chi$, for example for the pseudo-scalar pion-nucleon interaction

$$\mathcal{L}_{ps} = g\bar{\psi}(x)\gamma_5\psi(x)\ \phi(x) \ \rightarrow\ \bar{\mathcal{L}}_{ps} = g\left[\bar{\psi}(x)\gamma_5\psi(x)\ \phi(x)\right]\cdot\{\bar{\chi}(n\cdot x)\chi(n\cdot x)\} \ . \tag{B11}$$

This additional factor will produce in the contractions between the vertices of a graph the factor

$$\langle 0|\chi(n\cdot x')\bar{\chi}(n\cdot x)|0\rangle = -i\theta[n\cdot(x'-x)] \ . \tag{B12}$$

With these changes in the Lagrangian etc. one can formally incorporate the θ-functions appearing in (A3) in a second-quantization formalism as follows. First we write (A3) in the equivalent form

$$\begin{aligned} S_n \ =\ & \frac{i^n}{n!}\sum_P \int_{-\infty}^{+\infty}\dots\int_{-\infty}^{+\infty} \theta[n\cdot(x_{\pi_1}-x_{\pi_2})]\theta[n\cdot(x_{\pi_2}-x_{\pi_3})]\dots\theta[n\cdot(x_{\pi n-1}-x_{\pi_n})]\cdot \\ & \times\mathcal{L}_I(x_{\pi_1})\ \mathcal{L}_I(x_{\pi_2})\dots\mathcal{L}_I(x_{\pi_n})\cdot d^4x_1\dots d^4x_n \ , \end{aligned} \tag{B13}$$

where the sum P includes all permutation $\pi(1,2,\dots,n)$. Then, in the κ-space one next defines the S_n-operator by

$$\langle\kappa'|S_n|\kappa\rangle \ =\ \frac{i^n}{n!}\sum_P \int_{-\infty}^{+\infty}\dots\int_{-\infty}^{+\infty} \langle\kappa'|\ \bar{\mathcal{L}}_I(x_{\pi_1})\ \bar{\mathcal{L}}_I(x_{\pi_2})\dots\bar{\mathcal{L}}_I(x_{\pi_n})|\kappa\rangle\ \cdot d^4x_1\dots d^4x_n \tag{B14}$$

where the change $\mathcal{L}_I(x)\rightarrow\bar{\mathcal{L}}_I(x)$ symbolizes the change in the interaction Lagrangians similar to that in (B9). Taking matrix elements of the expression in (B12) generates all Kadyshevsky-graphs as defined by the rules in Appendix B.
The matrix elements of the full S-operator can now be expressed as

$$\langle\kappa'|S|\kappa\rangle \ =\ \mathcal{S}\exp\left\{\frac{i}{\hbar}\int_{-\infty}^{+\infty}\bar{\mathcal{L}}_I(x)d^4x\right\} \ , \tag{B15}$$

where $\mathcal{S}$ stands for the symmetrizer

$$\mathcal{S}\Big(\mathcal{L}_I(x_1)\ \mathcal{L}_I(x_2)\dots\mathcal{L}_I(x_n)\Big) = \sum_P \mathcal{L}_I(x_{\pi_1})\ \mathcal{L}_I(x_{\pi_2})\dots\mathcal{L}_I(x_{\pi_n}) \ . \tag{B16}$$

APPENDIX C: RELATIVISTIC INVARIANT AMPLITUDES

In this appendix the contribution from the Kadyshevsky diagrams to the relativistic amplitudes $A_{\kappa',\kappa}(s,t,u)$, $A'_{\kappa',\kappa}(s,t,u)$, $B_{\kappa',\kappa}(s,t,u)$, and $B'_{\kappa',\kappa}(s,t,u)$ are listed in the case of 'absolute pair-suppression'. We give the results for the general mass case, so that the results apply to the elastic and the inelatic meson-baryon reactions.

1. Momentum space baryon-exchange diagrams

(i) $J^P = \frac{1}{2}^+$ baryon-exchange

(a) pseudoscalar coupling:

$$\begin{aligned} A_{fi}(PS) &= -\frac{1}{8} g_{14}^{(ps)} g_{23}^{(ps)} \left\{ -\frac{1}{2}\left(M_f+M_i\right)+M_B \right\} D_u\left(\Delta_u, n, \bar{\kappa}\right) , \\ B_{fi}(PS) &= -\frac{1}{8} g_{14}^{(ps)} g_{23}^{(ps)}\, D_u\left(\Delta_u, n, \bar{\kappa}\right) \quad , \quad B'_{fi}(PS)=0 , \\ A'_{fi}(PS) &= +\frac{1}{8} g_{14}^{(ps)} g_{23}^{(ps)}\, D_u\left(\Delta_u, n, \bar{\kappa}\right) , \end{aligned} \tag{C1}$$

where the denominator function is

$$D_u\left(\Delta_u, n, \bar{\kappa}\right) = \left[\left(\bar{\kappa}+\Delta_u\cdot n\right)^2 - A_u^2\right]^{-1} \quad , \quad A_u = \sqrt{M_B^2-\Delta_u^2+(\Delta_u\cdot n)^2} . \tag{C2}$$

(b) pseudovector coupling:

$$\begin{aligned} A_{fi}(PV) &= +\frac{1}{8}\frac{f_{14}^{(pv)} f_{23}^{(pv)}}{m_\pi^2} \left[\left\{\frac{1}{2}\left(M_f+M_i\right)+M_B\right\} \left\{\frac{1}{4}\left(s_{p'q'}+s_{pq}-u_{p'q}-u_{pq'}\right)+\frac{1}{4}\left(t_{p'p}+t_{q'q}\right)\right.\right. \\ &\left. -\frac{1}{2}\left(m_f^2+m_i^2\right)-\frac{1}{2}(\kappa'-\kappa)(q'-q)\cdot n-\frac{1}{2}(\kappa'-\kappa)^2\right\} + \frac{1}{4}\left(M_i-M_f\right)\cdot \\ &\left.\times\left(M_f^2-M_i^2-u_{p'q}+u_{pq'}\right)-\bar{\kappa}\left(\frac{1}{2}(M_i-M_f)(q'-q)\cdot n\right)\right]\cdot D_u\left(\Delta_u, n, \bar{\kappa}\right) , \\ B_{fi}(PV) &= +\frac{1}{8}\frac{f_{14}^{(pv)} f_{23}^{(pv)}}{m_\pi^2} \left[-\left\{\frac{1}{2}\left(M_f+M_i\right)+M_B\right\}\left(M_i+M_f\right)+\frac{1}{2}\left(M_f^2+M_i^2-u_{p'q}-u_{pq'}\right)\right. \\ &\left. -2\bar{\kappa}\Delta_u\cdot n\right]\cdot D_u\left(\Delta_u, n, \bar{\kappa}\right) , \\ A'_{fi}(PV) &= +\frac{1}{8}\frac{f_{14}^{(pv)} f_{23}^{(pv)}}{m_\pi^2} \left[\bar{\kappa}\left\{\frac{1}{4}\left(s_{p'q'}+s_{pq}-u_{p'q}-u_{pq'}\right)+\frac{1}{4}\left(t_{p'p}+t_{q'q}\right)-\frac{1}{2}\left(m_f^2+m_i^2\right)\right.\right. \\ &\left.\left. -\frac{1}{4}(\kappa'-\kappa)^2\right\}-\frac{1}{4}\left(\kappa'-\kappa\right)\left(M_f^2-M_i^2-u_{p'q}+u_{pq'}\right)\right]\cdot D_u\left(\Delta_u, n, \bar{\kappa}\right) , \\ B'_{fi}(PV) &= +\frac{1}{8}\frac{f_{14}^{(pv)} f_{23}^{(pv)}}{m_\pi^2} \left[-\frac{1}{2}(\kappa'-\kappa)\left\{\frac{1}{2}\left(M_f+M_i\right)+M_B\right\}-\frac{1}{2}\bar{\kappa}\left(M_i-M_f\right)\right]\cdot D_u\left(\Delta_u, n, \bar{\kappa}\right) , \end{aligned} \tag{C3}$$

where $\bar{\kappa} = (\kappa'+\kappa)/2$.

(ii) $J^P = \frac{1}{2}^+$ baryon-resonance

Using su-crossing symmetry, the results for the baryon-resonance (s-channel) contribution can be obtained from those of baryon-exchange (u-channel) by the replacements (i) $q \to -q'$ and $q' \to -q$, which means the substitutions $\Delta_u \leftrightarrow \Delta_s, m_f \leftrightarrow m_i$, (ii) add a minus sign to the B- and B'-invariant amplitude, and (iii) in the coupling suffixes: (14) $\to$ (12),

(23) → (34). Furthermore $\Delta_s \leftrightarrow \Delta_u$, $s_{pq} \leftrightarrow u_{pq'}$, and $s_{p'q'} \leftrightarrow u_{p'q}$.

(a) pseudoscalar coupling:

$$
\begin{aligned}
A_{fi}(PS) &= -\frac{1}{8} g_{12}^{(ps)} g_{34}^{(ps)} \left\{ -\frac{1}{2}(M_f + M_i) + M_B \right\} D_s(\Delta_s, n, \bar{\kappa}) \;, \\
B_{fi}(PS) &= +\frac{1}{8} g_{12}^{(ps)} g_{34}^{(ps)} \; D_s(\Delta_s, n, \bar{\kappa}) \quad , \quad B'_{fi}(PS) = 0 \;, \\
A'_{fi}(PS) &= +\frac{1}{8} g_{12}^{(ps)} g_{34}^{(ps)} \; D_s(\Delta_s, n, \bar{\kappa}) \;,
\end{aligned}
\tag{C4}
$$

where now the denominator function is

$$
D_s(\Delta_s, n, \bar{\kappa}) = \left[(\bar{\kappa} + \Delta_s \cdot n)^2 - A_s^2 \right]^{-1} \quad , \quad A_s = \sqrt{M_B^2 - \Delta_s^2 + (\Delta_s \cdot n)^2} \;. \tag{C5}
$$

(b) pseudovector coupling:

$$
\begin{aligned}
A_{fi}(PV) &= +\frac{1}{8} \frac{f_{12}^{(pv)} f_{34}^{(pv)}}{m_\pi^2} \left[\left\{ \frac{1}{2}(M_f + M_i) + M_B \right\} \left\{ \frac{1}{4}(u_{p'q} + u_{pq'} - s_{p'q'} - s_{pq}) + \frac{1}{4}(t_{p'p} + t_{q'q}) \right. \right. \\
&\quad \left. -\frac{1}{2}\left(m_f^2 + m_i^2\right) + \frac{1}{2}(\kappa' - \kappa)(q' - q) \cdot n - \frac{1}{2}(\kappa' - \kappa)^2 \right\} + \frac{1}{4}(M_i - M_f) \cdot \\
&\quad \left. \times \left(M_f^2 - M_i^2 - s_{p'q'} + s_{pq}\right) + \bar{\kappa} \left(\frac{1}{2}(M_i - M_f)(q' - q) \cdot n \right) \right] \cdot D_s(\Delta_s, n, \bar{\kappa}) \;, \\
B_{fi}(PV) &= -\frac{1}{8} \frac{f_{12}^{(pv)} f_{34}^{(pv)}}{m_\pi^2} \left[-\left\{ \frac{1}{2}(M_f + M_i) + M_B \right\} (M_i + M_f) + \frac{1}{2}\left(M_f^2 + M_i^2 - s_{p'q'} - s_{pq}\right) \right. \\
&\quad \left. - 2\bar{\kappa}\Delta_s \cdot n \right] \cdot D_s(\Delta_s, n, \bar{\kappa}) \;, \\
A'_{fi}(PV) &= +\frac{1}{8} \frac{f_{12}^{(pv)} f_{34}^{(pv)}}{m_\pi^2} \left[\bar{\kappa} \left\{ \frac{1}{4}(u_{p'q} + u_{pq'} - s_{p'q'} - s_{pq}) + \frac{1}{4}(t_{p'p} + t_{q'q}) - \frac{1}{2}\left(m_f^2 + m_i^2\right) \right. \right. \\
&\quad \left. \left. -\frac{1}{4}(\kappa' - \kappa)^2 \right\} - \frac{1}{4}(\kappa' - \kappa) \left(M_f^2 - M_i^2 - s_{p'q'} + s_{pq} \right) \right] \cdot D_s(\Delta_s, n, \bar{\kappa}) \;, \\
B'_{fi}(PV) &= -\frac{1}{8} \frac{f_{12}^{(pv)} f_{34}^{(pv)}}{m_\pi^2} \left[-\frac{1}{2}(\kappa' - \kappa) \left\{ \frac{1}{2}(M_f + M_i) + M_B \right\} - \frac{1}{2}\bar{\kappa}(M_i - M_f) \right] \cdot D_s(\Delta_s, n, \bar{\kappa})
\end{aligned}
\tag{C6}
$$

Here again, su-crossing symmetry relates the resonance amplitudes to those of baryon-exchange.

We note that 'on-energy-shell', i.e. $\kappa = \kappa' = \bar{\kappa} = 0$, the scalar variables become

$$
s_{pq} = s_{p'q'} = s \quad , \quad u_{pq'} = u_{p'q} = u \quad , \quad \Delta_s^2 = s \quad , \quad \Delta_u^2 = u \;, \tag{C7}
$$

and the denominators have the limit

$$
D_s(\Delta_s, n, \bar{\kappa}) \rightarrow \left[s - M_B^2 \right]^{-1} \quad , \quad D_u(\Delta_u, n, \bar{\kappa}) \rightarrow \left[u - M_B^2 \right]^{-1} \;. \tag{C8}
$$

Then, in this limit one can compare, after the redefinition of the couplings $g \rightarrow 2\sqrt{2}g$, $f \rightarrow 2\sqrt{2}f$, the expressions for the invariant amplitudes here with those in [5, 6].

(iv) $J^P = \frac{3}{2}^+$ baryon-resonance

These are the invariant amplitudes with absolute pair-suppression, in the Kadyshevsky formalism, using the gauge-invariant (GI) coupling.

a. $\underline{M^{(0)}_{\kappa',\kappa}}$ **invariant amplitudes**:

$$\begin{aligned}
A^{(0)}_{fi}(GI) \;=\; & +(?)\frac{1}{24}\frac{g^{(*)}_{12}g^{(*)}_{34}}{m_\pi^4}\left[\left\{\frac{1}{2}\left(M_f+M_i\right)+M_\Delta\right\}\cdot\right. \\
& \times\left\{\frac{1}{2}\left(m_i^2+m_f^2-t_{q'q}\right)+\frac{1}{4}\left(s_{p'q'}+s_{pq}-u_{p'q}-u_{pq'}\right)\right\}\ \Delta_s^2 \\
& -M_\Delta\left\{\frac{1}{2}\left(m_i^2+m_f^2-t_{q'q}\right)-\frac{1}{4}\left(u_{p'q}+u_{pq'}-s_{p'q'}-s_{pq}\right)\right\}\cdot \\
& \times\left\{\frac{1}{2}\left(3m_i^2+3m_f^2-t_{q'q}\right)-\frac{1}{2}\left(s_{p'q'}+s_{pq}-u_{p'q}-u_{pq'}\right)\right\} \\
& -3\left\{\frac{1}{2}\left(M_f+M_i\right)+M_\Delta\right\}\ \left\{\left(m_f^2+m_i^2\right)-t_{q'q}\right\}\ \Delta_s^2 \\
& +\frac{M_\Delta}{2}\ \left\{\frac{1}{4}\left(u_{pq'}-s_{p'q'}-M_i^2+M_f^2\right)-\frac{1}{4}t_{q'q}+m_i^2+\frac{1}{2}m_f^2\right\}\cdot \\
& \times\left\{u_{p'q}-t_{q'q}-M_f^2+m_f^2+m_i^2+\left(M_i-M_f\right)^2\right\} \\
& +\frac{M_\Delta}{2}\ \left\{\frac{1}{4}\left(u_{p'q}-s_{pq}+M_i^2-M_f^2\right)-\frac{1}{4}t_{q'q}+m_f^2+\frac{1}{2}m_i^2\right\}\cdot \\
& \times\left\{u_{pq'}-t_{q'q}-M_i^2+m_f^2+m_i^2+\left(M_i-M_f\right)^2\right\} \\
& +\left(M_f+M_i\right)\ \left\{\frac{1}{4}\left(u_{p'q}-s_{pq}+M_i^2-M_f^2\right)-\frac{1}{4}t_{q'q}+m_f^2+\frac{1}{2}m_i^2\right\}\cdot \\
& \times\left\{\frac{1}{4}\left(u_{pq'}-s_{p'q'}-M_i^2+M_f^2\right)-\frac{1}{4}t_{q'q}+m_i^2+\frac{1}{2}m_f^2\right\} \\
& \left.+\frac{1}{8}\left(M_i-M_f\right)\ \left(u_{pq'}-u_{p'q}+s_{p'q'}-s_{pq}+6m_f^2-6m_i^2\right)\right]\cdot D_s\left(\Delta_s,n,\bar{\kappa}\right) \qquad \text{(C9)}
\end{aligned}$$

$$\begin{aligned}
B^{(0)}_{fi}(GI) \;=\; & +(?)\frac{1}{24}\frac{g^{(*)}_{12}g^{(*)}_{34}}{m_\pi^4}\left[\left\{\frac{1}{2}\left(M_f+M_i\right)+M_\Delta\right\}\ \left(M_f+M_i\right)\ \Delta_s^2\cdot\right. \\
& -M_\Delta M_f\left[\frac{1}{4}\left(u_{pq'}-s_{p'q'}-M_i^2+M_f^2\right)-\frac{1}{4}t_{q'q}+\frac{1}{2}m_f^2+m_i^2\right] \\
& -M_\Delta M_i\left[\frac{1}{4}\left(u_{p'q}-s_{pq}+M_i^2-M_f^2\right)-\frac{1}{4}t_{q'q}+m_f^2+\frac{1}{2}m_i^2\right] \\
& +\frac{1}{2}\left(m_f^2+m_i^2-t_{q'q}\right)\left(-\left(u_{p'q}+u_{pq'}-s_{p'q'}-s_{pq}\right)+\frac{1}{2}\left(m_f^2+m_i^2\right)\right. \\
& \left.-\frac{3}{4}\left(M_f^2+M_i^2\right)+\frac{1}{4}t_{p'p}\right)+\frac{1}{8}\left[u_{p'q}-M_f^2+M_i^2\right]\left[u_{pq'}+M_f^2-M_i^2\right] \\
& \left.+\left[-\frac{1}{2}\left(u_{p'q}+u_{pq'}\right)+s_{p'q'}+s_{pq}-\frac{1}{2}\left(M_f^2+M_i^2\right)+5\left(m_f^2+m_i^2\right)\right]\ \Delta_s^2\right]\cdot D_s\left(\Delta_s,n,\bar{\kappa}\right) \qquad \text{(C10)}
\end{aligned}$$

$$A'^{(0)}_{fi}(GI) \;=\; +(?)\frac{1}{24}\frac{g_{12}^{(*)}g_{34}^{(*)}}{m_\pi^4}\cdot\frac{1}{2}(\kappa'-\kappa)\Bigg[-\left(M_i-M_f\right)M_\Delta\Bigg\{-\frac{1}{2}\left(s_{p'q'}+s_{pq}-u_{p'q}-u_{pq'}\right)+\frac{3}{2}\left(m_f^2+\right.$$

$$-\frac{1}{2}t_{q'q}\Bigg\}+\Bigg\{-\frac{1}{4}\left(s_{p'q'}-s_{pq}-u_{p'q}+u_{pq'}\right)+6\left(m_i^2-m_f^2\right)\Bigg\}\Bigg]\cdot D_s\left(\Delta_s,n,\bar\kappa\right)\ ,$$

$$B'^{(0)}_{fi}(GI) \;=\; +(?)\frac{1}{24}\frac{g_{12}^{(*)}g_{34}^{(*)}}{m_\pi^4}\cdot\frac{1}{2}(\kappa'-\kappa)\Bigg[\Bigg\{M_\Delta+\frac{1}{2}\left(M_f+M_i\right)\Bigg\}\ \Delta_s^2$$

$$-M_\Delta\Bigg\{-\frac{1}{2}\left(s_{p'q'}+s_{pq}-u_{p'q}-u_{pq'}\right)+\frac{3}{2}\left(m_f^2+m_i^2\right)-\frac{1}{2}t_{q'q}\Bigg\}\Bigg]\cdot D_s\left(\Delta_s,n,\bar\kappa\right) \qquad \text{(C12)}$$

b. $\underline{M^{(1)}_{\kappa',\kappa}}$ **invariant amplitudes**: Next we list the invariant amplitudes linear in $\bar\kappa$, from $M^{(1)}_{\kappa',\kappa}$. We first give them in an intermediate form, that is with innerproducts of the involved four-vectors, q, q', Q, and Δ_s. For the following, we use the short-hands

$$A_0 \;=\; Q^2+(p'+p)\cdot Q-\frac{1}{4}t_{p'p}-\frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n$$

$$= \frac{1}{2}\left(m_f^2+m_i^2-t_{q'q}\right)+\frac{1}{4}\left(s_{p'q'}+s_{pq}-u_{p'q}-u_{pq'}\right)\ , \qquad \text{(C13a)}$$

$$A_1 \;=\; Q^2+(p'+p)\cdot Q-\frac{1}{4}t_{p'p}+\frac{1}{4}(\kappa'-\kappa)^2$$

$$= \frac{1}{4}\left(t_{p'p}+t_{q'q}\right)+\frac{1}{4}\left(s_{p'q'}+s_{pq}-u_{p'q}-u_{pq'}\right)+\frac{1}{4}(\kappa'-\kappa)^2\ . \qquad \text{(C13b)}$$

$$A^{(1)}_{fi}(GI) \;=\; +(?)\frac{1}{24}\frac{g_{12}^{(*)}g_{34}^{(*)}}{m_\pi^4}\Bigg[(\Delta_s\cdot n)\Bigg\{\left(M_\Delta+\frac{1}{2}\left(M_f+M_i\right)\right)\ \left(-6(q'\cdot q)+2A_0\right)-(q\cdot\Delta_s)-(q'\cdot\Delta_s)\Bigg\}$$

$$-(q'\cdot n)\Bigg\{-p'\cdot q+q'\cdot q+\frac{1}{2}m_f^2-\frac{1}{2}q\cdot\Delta_s+\frac{1}{2}q'\cdot\Delta_s\Bigg\}\cdot M_\Delta$$

$$-(q\cdot n)\Bigg\{-p\cdot q'+q'\cdot q+\frac{1}{2}m_i^2+\frac{1}{2}q\cdot\Delta_s-\frac{1}{2}q'\cdot\Delta_s\Bigg\}\cdot M_\Delta$$

$$+\left(M_f+M_i\right)\Big((q'\cdot\Delta_s)(q\cdot n)+(q\cdot\Delta_s)(q'\cdot n)\Big)+\frac{1}{2}A_0\ M_\Delta\Big((q\cdot n)+(q'\cdot n)\Big)$$

$$+\frac{1}{2}\left(M_i-M_f\right)\Bigg\{+2(\Delta_s\cdot n)\left(p'\cdot q-p\cdot q'+\frac{1}{2}m_i^2-\frac{1}{2}m_f^2+2q'\cdot\Delta_s-2q\cdot\Delta_s\right)$$

$$+2\Delta_s^2(q\cdot n)-2\Delta_s^2(q'\cdot n)\Bigg\}$$

$$-\frac{1}{4}\left(M_i-M_f\right)^2\Bigg\{M_\Delta(q\cdot n)+M_\Delta(q'\cdot n)\Bigg\}\Bigg]\cdot D_s\left(\Delta_s,n,\bar\kappa\right)\ .$$

$$B^{(1)}_{fi}(GI) \;=\; +(?)\frac{1}{24}\frac{g_{12}^{(*)}g_{34}^{(*)}}{m_\pi^4}\Bigg[(\Delta_s\cdot n)\Bigg\{2\left(M_\Delta+\frac{1}{2}\left(M_f+M_i\right)\right)\ \left(M_f+M_i\right)+4(Q\cdot\Delta_s)+2\Delta_s^2$$

$$+2\left(-p'\cdot q-p\cdot q'+\frac{1}{2}m_f^2+\frac{1}{2}m_i^2\right)\Bigg\}-(q'\cdot n)\left\{-M_\Delta M_f-2(q\cdot\Delta_s)-\Delta_s^2\right\}$$

$$-(q\cdot n)\left\{-M_\Delta M_i-2(q'\cdot\Delta_s)-\Delta_s^2\right\}\Bigg]\cdot D_s\left(\Delta_s,n,\bar\kappa\right)\ . \qquad \text{(C15)}$$

$$A_{fi}^{(1)\,\prime}(GI) \;=\; +(?)\frac{1}{24}\frac{g_{12}^{(*)}g_{34}^{(*)}}{m_\pi^4}\left[(\Delta_s\cdot n)\left\{A_1\Delta_s^2-3(q'\cdot q)\Delta_s^2+M_\Delta\left(M_f+M_i\right)\;(Q\cdot\Delta_s)\right.\right.$$
$$\left.+\,2(q'\cdot\Delta_s)(q\cdot\Delta_s)\right\}+\frac{1}{2}(\kappa'-\kappa)\left\{2(\Delta_s\cdot n)\left(q\cdot\Delta_s-q'\cdot\Delta_s-p'\cdot q+p\cdot q'\right.\right.$$
$$\left.\left.\left.+\,\frac{1}{2}m_f^2-\frac{1}{2}m_i^2\right)-(q\cdot n)\Delta_s^2+(q'\cdot n)\Delta_s^2\right\}\right]\cdot D_s\left(\Delta_s,n,\bar\kappa\right)\;. \qquad \text{(C16)}$$

$$B_{fi}^{(1)\,\prime}(GI) \;=\; +(?)\frac{1}{24}\frac{g_{12}^{(*)}g_{34}^{(*)}}{m_\pi^4}\left[\frac{1}{2}M_\Delta\left((q'\cdot\Delta_s-(q\cdot\Delta_s)\right)+\frac{1}{2}\left(M_i-M_f\right)+\frac{1}{2}(\kappa'-\kappa)\cdot\right.$$
$$\left.\times\left\{2\left(M_\Delta+\frac{1}{2}\left(M_f+M_i\right)\right)(\Delta_s\cdot n)+\frac{1}{2}M_\Delta\left((q\cdot n)+(q'\cdot n)\right)\right\}\right]\cdot D_s\left(\Delta_s,n,\bar\kappa\right) \qquad \text{(C17)}$$

c. <u>$M^{(2)}_{\kappa',\kappa}$ **invariant amplitudes**</u>:

$$A_{fi}^{(2)}(GI) \;=\; +(?)\frac{1}{24}\frac{g_{12}^{(*)}g_{34}^{(*)}}{m_\pi^4}\left[\left(M_\Delta+\frac{1}{2}\left(M_f+M_i\right)\right)\;(-3(q'\cdot q)+A_0)\right.$$
$$+2(q'\cdot n)(q\cdot n)\left(M_\Delta+\frac{1}{2}\left(M_f+M_i\right)\right)\;-2(Q\cdot n)^2M_\Delta-2(Q\cdot n)(\Delta_s\cdot n)\;M_\Delta$$
$$\left.+\frac{1}{2}\left(M_i-M_f\right)\left\{-p\cdot q'+p'\cdot q+\frac{1}{2}m_i^2-\frac{1}{2}m_f^2+(q'\cdot\Delta_s)-(q\cdot\Delta_s)\right\}\right]\cdot D_s\left(\Delta_s,n,\bar\kappa\right) \qquad \text{(C18)}$$

$$B_{fi}^{(2)}(GI) \;=\; +(?)\frac{1}{24}\frac{g_{12}^{(*)}g_{34}^{(*)}}{m_\pi^4}\left[\left(M_\Delta+\frac{1}{2}\left(M_f+M_i\right)\right)\;(M_f+M_i)-p'\cdot q-p\cdot q'+\frac{1}{2}m_i^2-\frac{1}{2}m_f^2\right.$$
$$\left.-(q\cdot\Delta_s)+2(Q\cdot\Delta_s)+4(\Delta_s\cdot n)^2+4(\Delta_s\cdot n)(Q\cdot n)\right]\cdot D_s\left(\Delta_s,n,\bar\kappa\right)\;. \qquad \text{(C19)}$$

$$A_{fi}^{(2)\,\prime}(GI) \;=\; +(?)\frac{1}{24}\frac{g_{12}^{(*)}g_{34}^{(*)}}{m_\pi^4}\left[2(\Delta_s\cdot n)\left(A_1-3q'\cdot q\right)+M_\Delta\left(M_f+M_i\right)(Q\cdot n)\right.$$
$$-2(q\cdot n)(\Delta_s\cdot n)-2(q'\cdot n)(\Delta_s\cdot n)-\frac{1}{2}(\kappa'-\kappa)\cdot$$
$$\times\left\{+\,2(\Delta_s\cdot n)(q\cdot n)-2(\Delta_s\cdot n)(q'\cdot n)-p\cdot q'+p'\cdot q+\frac{1}{2}m_i^2-\frac{1}{2}m_f^2\right.$$
$$\left.\left.+\,(q'\cdot\Delta_s)-(q\cdot\Delta_s)\right\}\right]\cdot D_s\left(\Delta_s,n,\bar\kappa\right)\;. \qquad \text{(C20)}$$

$$B_{fi}^{(2)\,\prime}(GI) \;=\; +(?)\frac{1}{24}\frac{g_{12}^{(*)}g_{34}^{(*)}}{m_\pi^4}\left[\left(M_i-M_f\right)(\Delta_s\cdot n)-\frac{1}{2}M_\Delta(q\cdot n)+\frac{1}{2}M_\Delta(q'\cdot n)\right]\cdot$$
$$\times D_s\left(\Delta_s,n,\bar\kappa\right)\;. \qquad \text{(C21)}$$

(iii) $J^P = \frac{3}{2}^+$ baryon-exchange

Application of su-crossing symmetry enables one to derive the invariant amplitudes for the s-channel, using the u-channel amplitudes.
We repeat the su-crossing rules: (i) $q \to -q'$ and $q' \to -q$, which means the substitutions $\Delta_u \leftrightarrow \Delta_s, m_f \leftrightarrow m_i$, (ii) add a minus sign to the B- and B'-invariant amplitude, and (iii) in the coupling suffixes: (14) $\to$ (12), (23) $\to$ (34). Furthermore $\Delta_s \leftrightarrow \Delta_u$, $s_{pq} \leftrightarrow u_{pq'}$, and $s_{p'q'} \leftrightarrow u_{p'q}$.
From the baryon-exchange formulas (C9)-(C21) we obtain

APPENDIX D: MISCELLANEOUS IDENTITIES

We consider general meson-baryon scattering, the topic of this paper, and list a number of useful identities which are valid when sandwiched between the Dirac-spinors $\bar{u}(p', s')$ and $u(p, s)$, and for particles on-mass-shell. Also, we assume 'quasi four momentum' conservation, i.e.

$$p + q + \kappa n = p' + q' + \kappa' n \ . \tag{D1}$$

$$\not{q} = -\frac{1}{2}(M_i - M_f) + \not{Q} + \frac{1}{2}(\kappa' - \kappa)\not{n} \ , \tag{D2a}$$

$$\not{q}' = +\frac{1}{2}(M_i - M_f) + \not{Q} - \frac{1}{2}(\kappa' - \kappa)\not{n} \ , \tag{D2b}$$

$$\begin{aligned}\not{q}\not{q}' = & \left[Q^2 - (p' + P)\cdot Q - \frac{1}{4}t_{p'p} - \frac{1}{2}(\kappa' - \kappa)\cdot n\right] \\ & + (M_f + M_i)\not{Q} + \frac{1}{2}(\kappa' - \kappa)\ [\not{n}, \not{Q}]_- \ ,\end{aligned} \tag{D2c}$$

$$\begin{aligned}\not{q}\not{n}\not{q}' = & -\frac{1}{2}(M_i - M_f)\ (q' - q)\cdot n + \left[2n\cdot Q - (p' + p)\cdot n\right]\not{Q} \\ & - \left[Q^2 - (p' + p)\cdot Q - \frac{1}{4}t_{p'p} + \frac{1}{4}(\kappa' - \kappa)^2\right]\not{n} \\ & - \frac{1}{2}(M_i - M_f)\ [\not{n}, \not{Q}]_- \ .\end{aligned} \tag{D2d}$$

$$\not{q}'\not{\Delta}_s = +\frac{1}{2}(M_i - M_f)\ \not{q}' - \frac{1}{2}\not{q}\ \not{q}' + p'\cdot q' + q'\cdot q + \frac{1}{2}m_f^2 \ , \tag{D3a}$$

$$\not{\Delta}_s'\not{q}_s = -\frac{1}{2}(M_i - M_f)\ \not{q} - \frac{1}{2}\not{q}\ \not{q}' + p'\cdot q + q'\cdot q + \frac{1}{2}m_i^2 \ . \tag{D3b}$$

Some kinematic relations are

$$s_{pq} = (p + q)^2 = M_i^2 + m_i^2 + 2p\cdot q \ , \tag{D4a}$$

$$s_{p'q'} = (p' + q')^2 = M_f^2 + m_f^2 + 2p'\cdot q' \ , \tag{D4b}$$

$$u_{pq'} = (p - q')^2 = M_i^2 + m_f^2 - 2p\cdot q' \ , \tag{D4c}$$

$$u_{p'q} = (p' - q)^2 = M_f^2 + m_i^2 - 2p'\cdot q \ , \tag{D4d}$$

$$t_{p'p} = (p' - p)^2 = M_f^2 + M_i^2 - 2p'\cdot p \ , \tag{D4e}$$

$$t_{q'q} = (q' - q)^2 = m_f^2 + m_i^2 - 2q'\cdot q \ . \tag{D4f}$$

and the inverse relations

$$p \cdot q = \frac{1}{2}\left[s_{pq} - M_i^2 - m_i^2\right] , \quad \text{(D5a)}$$

$$p' \cdot q' = \frac{1}{2}\left[s_{p'q'} - M_f^2 - m_f^2\right] , \quad \text{(D5b)}$$

$$p \cdot q' = \frac{1}{2}\left[M_i^2 + m_f^2 - u_{pq'}\right] , \quad \text{(D5c)}$$

$$p' \cdot q = \frac{1}{2}\left[M_f^2 + m_i^2 - u_{p'q}\right] , \quad \text{(D5d)}$$

$$p' \cdot p = \frac{1}{2}\left[M_f^2 + M_i^2 - t_{p'p}\right] , \quad \text{(D5e)}$$

$$q' \cdot q = \frac{1}{2}\left[m_f^2 + m_i^2 - t_{q'q}\right] . \quad \text{(D5f)}$$

Miscellaneous combinations

$$Q^2 - (p'+p)\cdot Q - \frac{1}{4}t_{p'p} = -\frac{1}{4}\left(s_{p'q'} + s_{pq} - u_{p'q} - u_{pq'}\right) - \frac{1}{4}\left(t_{p'p} + t_{q'q}\right) , \quad \text{(D6a)}$$

$$Q^2 - (p'+p)\cdot Q - \frac{1}{4}t_{p'p} - \frac{1}{2}(\kappa' - \kappa)(p'-p)\cdot n = q'\cdot q - \frac{1}{2}(p'+p)\cdot(q'+q) =$$
$$-\frac{1}{4}\left(s_{p'q'} + s_{pq} - u_{p'q} - u_{pq'}\right) + \frac{1}{2}\left(m_f^2 + m_i^2 - t_{q'q}\right) \equiv A_0 . \quad \text{(D6b)}$$

$$\Delta_s^2 = \frac{1}{4}(p'+p)^2 + \frac{1}{4}(q'+q)^2 + \frac{1}{2}(p'+p)\cdot(q'+q)$$
$$= \frac{1}{2}\left(M_f^2 + M_i^2 + m_f^2 + m_i^2\right) - \frac{1}{4}t_{p'p} - \frac{1}{4}t_{q'q} + \frac{1}{4}\left(s_{p'q'} + s_{pq} - u_{p'q} - u_{pq'}\right) \text{(D7a)}$$

$$q\cdot\Delta_s = \frac{1}{2}\left(p'\cdot q + p\cdot q + m_i^2 + q'\cdot q\right)$$
$$= \frac{1}{4}\left(s_{pq} - u_{p'q} - M_i^2 + M_f^2\right) - \frac{1}{4}t_{q'q} + m_i^2 + \frac{1}{2}m_f^2 , \quad \text{(D7b)}$$

$$q'\cdot\Delta_s = \frac{1}{2}\left(p'\cdot q' + p\cdot q' + m_f^2 + q'\cdot q\right)$$
$$= \frac{1}{4}\left(s_{p'q'} - u_{pq'} + M_i^2 - M_f^2\right) - \frac{1}{4}t_{q'q} + m_f^2 + \frac{1}{2}m_i^2 , \quad \text{(D7c)}$$

$$Q\cdot\Delta_s = \frac{1}{2}\left(s_{p'q'} + s_{pq} - u_{pq'} - u_{p'q}\right) + m_f^2 + m_i^2 + q'\cdot q . \quad \text{(D7d)}$$

APPENDIX E: TAKAHASHI-UMEZAWA THEORY INTERACTION REPRESENTATION

In this section we describe the method of Takahashi and Umezawa in the canonical treatment of interaction in local field theory [17, 31, 32]. In these notes, for pedagogical purposes, we deal with a set of independent local fields $\Phi_\alpha(x)$, so that the relation between the fields in the Heisenberg- and the (Tomanaga-Schwinger) Interaction-representation, henceforth refered to as H.R. and I.R. respectively, reads [10]

$$\mathbf{\Phi}_\alpha(x) = U^{-1}[\sigma]\ \Phi_\alpha(x)\ U[\sigma]\Big|_{x/\sigma} \quad , \tag{E1}$$

where $\sigma = \sigma(x)$ is a space-like surface and x/σ indicates that the point x lies on the surface σ. $U(\sigma)$ is the unitary transformation between the H.R. and I.R. in the Tomonaga-Schwiger [33, 34] generalization of the Schrödinger equation, see e.g. [26] section 13a.

Let the interaction Lagrangian in the Heisenberg representation be

$$\mathcal{L}_I(x) = \mathcal{L}'\left(\mathbf{\Phi}_\alpha(x), \partial_\mu \mathbf{\Phi}_\alpha(x), \ldots\right) \ , \tag{E2}$$

and the equations of motion

$$\Lambda_{\alpha\beta}(\partial)\ \mathbf{\Phi}_\beta(x) = \mathbf{J}_\alpha(x) \ , \tag{E3}$$

where

$$\mathbf{J}_\alpha(x) \ = \ \frac{\partial \mathcal{L}'}{\partial \mathbf{\Phi}_\alpha} - \partial_\mu \frac{\partial \mathcal{L}'}{\partial \mathbf{\Phi}_{\alpha;\mu}} + \ldots \equiv \sum_{n=0}^{\infty} D_{\mu_1 \ldots \mu_n}\ \mathbf{j}_{\alpha;\mu_1\ \ldots\ \mu_n}(x) \ , \tag{E4}$$

with

$$\begin{aligned} D_{\mu_1\ \ldots\ \mu_n} \ &\equiv \ (-1)^n \partial_{\mu_1}\ \ldots\ \partial_{\mu_n} \equiv D_a \ , \\ \mathbf{j}_{\alpha;\mu_1\ \ldots\ \mu_n}(x) \ &\equiv \ \frac{\partial \mathcal{L}'}{\partial \mathbf{\Phi}_{\alpha;\mu_1\ \ldots \mu_n}} \equiv \mathbf{j}_{\alpha;a}(x) \ . \end{aligned}$$

Furtheron, we will symbolically denote (E4) as

$$\mathbf{J}_\alpha(x) \equiv D_a \mathbf{j}_{\alpha;a}(x) \ , \tag{E5}$$

The fields in the Interaction-representation, *free field quantities, e.g. in-fields*, $\Phi_\alpha(x)$ are assumed to satisfy the field equations [11]

$$\Lambda_{\alpha\beta}(\partial)\ \Phi_\beta(x) = 0 \ , \tag{E6}$$

---

[10] *In the following, most equations are weak-equations in the LSZ-sense [35].*

[11] (i) For bosons: $\Phi_\alpha(x) = \phi(x)$, and

$$\Lambda_{\alpha\beta}(\partial) = \left(\Box - m^2\right) \delta_{\alpha\beta} \quad , \quad R_{\alpha\beta} = \delta_{\alpha\beta} \ .$$

(ii) For fermions: $\Phi_\alpha(x) = \psi(x)$, and

$$\Lambda_{\alpha\beta}(\partial) = (i\nabla\!\!\!/ - M)\,\delta_{\alpha\beta} \quad , \quad R_{\alpha\beta}(\partial) = (-i\nabla\!\!\!/ - M)\,\delta_{\alpha\beta} \ .$$

which can be reduced to satisfying the Klein-Gordon equations

$$\left(\Box - m_\alpha^2\right) \Phi_\alpha(x) = 0 \ . \tag{E7}$$

Using the operator $R_{\beta\gamma}(\partial)$ defined by

$$\Lambda_{\alpha\beta}(\partial)\ R_{\beta\gamma}(\partial)\ = \left(\Box - m_\alpha^2\right) \delta_{\alpha\gamma} \ , \tag{E8}$$

the commutation relations read

$$\left[\Phi_\alpha(x), \Phi_\beta(x')\right]_\pm = i\hbar R_{\alpha\beta}(\partial)\ \Delta(x - x') \ . \tag{E9}$$

In terms of the Green functions $\Delta_G(x - x')$ equation (E3) reads in integral form [36]

$$\mathbf{\Phi}_\beta(x)\ =\ \Phi_\alpha(x) + \int d^4x'\ R_{\alpha\beta}(\partial)\ D_a\ \Delta_G(x - x')\ \mathbf{j}_{\beta;a}(x') \ . \tag{E10}$$

Here, $\Delta_G(x)$ is linear combination of $\Delta, \Delta^{(1)}$ etc. satisfying

$$\Lambda_{\alpha\beta}(\partial) R_{\beta\gamma}(\partial) \Delta_G(x - x') = \left(\Box - m_\alpha^2\right) \Delta_G(x - x')\delta_{\alpha\gamma} = \delta(x - x')\delta_{\alpha\gamma} \ , \tag{E11}$$

and $\Phi_\alpha(x)$ is the free field solution for equation (E6).

1. Derivation of the Interaction Hamiltonian

In order to find the unitary transformation $U[\sigma]$ connecting equations (E3) and (E6) one introduces the following auxiliary (Heisenberg) field operator

$$\Phi_\alpha[x, \sigma] \equiv \Phi_\alpha(x) + a \int_{-\infty}^{\sigma} d^4x'\ R_{\alpha\beta}(\partial)\ D'_a\ \Delta(x - x')\ \mathbf{j}_{\beta;a}(x') \ , \tag{E12}$$

where x not necessarily lies on the surface σ, *i.e.* x *and* σ *are considered as independent variables.* The constant a in front of the integral in (E12) is determined from the requirement on the field commutator, see (E18) below, and it turns out that $a = 1$ (see Appendix F). Hence the constant a is omitted in the following.
As is easily seen, $\Phi_\alpha[x, \sigma]$ satisfies the free field equation (E6)

$$\Lambda_{\alpha\beta}(\partial)\ \Phi_\beta[x, \sigma] = 0 \ , \tag{E13}$$

because $\Phi_\alpha(x)$ etc. satisfy this equation.

This does not hold for the Heisenberg fields, because in the transformation x *and* σ *are not independent, instead one has*

$$\begin{aligned} \mathbf{\Phi}_\alpha(x)\ &=\ U^{-1}[\sigma]\Phi_\alpha(x)U[\sigma]\Big|_{x/\sigma} \ , \\ \mathbf{\Pi}_\alpha(x)\ &=\ U^{-1}[\sigma]\Pi_\alpha(x)U[\sigma]\Big|_{x/\sigma} \ , \end{aligned} \tag{E14}$$

for the set of canonically conjugated pair of variables $(\Phi_\alpha(x), \Pi_\alpha(x))$.

Also, from (E12) it is clear that by the choice of the lower limit in the integral in (E12)

$$\Phi_\alpha[x, -\infty] \equiv \Phi_\alpha(x) \tag{E15}$$

i.e. $\Phi[x, -\infty] = \Phi_{in}(x)$. This choice also implies that the Heisenberg field is given by

$$\mathbf{\Phi}_\alpha(x) = \Phi_\alpha(x) + \int d^4x' \ \{R_{\alpha\beta}(\partial) \ D_a \ \Delta_{ret}(x - x')\} \cdot \mathbf{j}_{\beta;a}(x') \ , \tag{E16}$$

with

$$\Delta_{ret}(x - x') \ = \ \theta(x_0 - x_0')\Delta(x - x') = \frac{1}{2}\left[1 + \epsilon(x - x')\right] \Delta(x - x') \ .$$

From (E12) and (E16) one obtains

$$\mathbf{\Phi}_\alpha(x) = \Phi_\alpha[x/\sigma] + \frac{1}{2}\int d^4x' \ \Big[R_{\alpha\beta}(\partial) \ D_a' \ , \ \epsilon(x - x')\Big] \ \Delta(x - x') \ \mathbf{j}_{\beta;a}(x') \ , \tag{E17}$$

Next one *requires* the commutation relation

$$\Big[\Phi_\alpha[x, \sigma] \ , \ \Phi_\beta[x', \sigma]\Big]_\pm = i\hbar \ R_{\alpha\beta}(\partial) \ \Delta(x - x') \ , \tag{E18}$$

since, after all, also $\Phi_\alpha(x)$ satisfy such commutation relations, and because *this would follow directly from the existence of* $U[\sigma]$.

This requirement can be satisfied only if a unitary transformation $U[\sigma]$ *exists for which*

$$\Phi_\alpha[x, \sigma] = U^{-1}[\sigma] \ \Phi_\alpha(x) \ U[\sigma] \ , \tag{E19}$$

with $U[\sigma] \equiv U(\sigma, -\infty)$, *connecting* $\Phi_\alpha[x, \sigma]$ *with (E12).* Here, we used the transformation between surfaces, i.e.

$$\Phi_\alpha(x, \sigma) = U^{-1}(\sigma; \sigma')\Phi_\alpha(x, \sigma') \ U(\sigma; \sigma') \ , \tag{E20}$$

If now, $U[\sigma]$ is determined by [12]

$$i\hbar\frac{\delta U(\sigma)}{\delta\sigma(x)} = \mathcal{H}_I(x : n)U[\sigma]\Big|_{x/\sigma} = U[\sigma] \ \mathcal{H}_I(x/\sigma; n) \ , \tag{E27}$$

[12] In fact, we can prove this for a broad class of $\mathcal{L}_I$'s as follows. First, from (E19) we derive that

$$U[\sigma]\frac{\delta\Phi_\alpha[x, \sigma]}{\delta\sigma(x')} \ U^{-1}[\sigma] = \left[\Phi_\alpha(x), \frac{\delta U[\sigma]}{\delta\sigma(x')} \ U^{-1}[\sigma]\right] \ , \tag{E21}$$

and second, from (E12) we find that

$$U[\sigma]\frac{\delta\Phi_\alpha[x, \sigma]}{\delta\sigma(x')} \ U^{-1}[\sigma] = \Delta(x - x') \ j(x') \ . \tag{E22}$$

Combining these two relations gives

$$\left[\Phi_\alpha(x), \frac{\delta U[\sigma]}{\delta\sigma(x')} \ U^{-1}[\sigma]\right] = \Delta(x - x') \ j(x') \ . \tag{E23}$$

where $\mathcal{H}_I$, which will in general depend on the vector $n_\mu(x)$ locally normal to the surface $\sigma(x)$, i.e. $n^\mu(x)d\sigma_\mu = 0$, is hermitean because of the unitarity of $U(\sigma)$. Then, from (E19) one gets that

$$i\hbar\frac{\delta\Phi_\alpha(x,\sigma)}{\delta\sigma(x')} = U^{-1}[\sigma]\left[\Phi_\alpha(x), H_I(x':n)\right]\ U[\sigma]\ . \tag{E28}$$

On the other hand, from (E12)

$$i\hbar\frac{\delta\Phi_\alpha[x,\sigma]}{\delta\sigma(x')} = i\hbar\ \mathbf{j}_{\beta;a}(x')\left\{D'_a\ R_{\alpha\beta}(\partial)\ \Delta(x-x')\right\}\ . \tag{E29}$$

Comparing (E28) and (E29) gives the relation

$$\left[\Phi_\alpha(x), H_I(x':n)\right] = i\hbar\ \left(U(\sigma)\mathbf{j}_{\beta;a}(x')U^{-1}(\sigma)\right)\ \left\{D'_a\ R_{\alpha\beta}(\partial)\ \Delta(x-x')\right\}\ . \tag{E30}$$

This is the fundamental equation by which the interaction Hamiltonian $H'(x:n)$ must be determined [31]. The existence of $H'(x:n)$ is necessary for the feasibility of the connection between (E12) and (E19).

The unitary transformation $U(\sigma)$ connecting the Heisenberg and Interaction representation is subsequently obtained by solving Eq. (E27).
However, see appendix F for comments on the question of the existence or non-existence of this connection!

2. Application Takahashi-Umezawa $\mathcal{H}_I[x:n]$-construction I

Via (E30) $H'[x:n]$ can be obtained, as a power series in the coupling constants, by rewriting the Heisenberg current $\mathbf{j}_{\beta;a}(x')$ as a function of the fields $\Phi[x,\sigma]$. Given any

---

Now, in the case that the interaction Lagrangian is proportional to some power p of the field $\Phi_\alpha(x)$, and consider the case with no derivatives, one has that

$$\left[\Phi_\alpha(x), \mathcal{H}_I(x')\right] = i\Delta(x-x')\ j(x')\ \ ,\ \ j(x') = -\frac{\partial\mathcal{H}_I(x')}{\partial\Phi_\alpha(x')}\ . \tag{E24}$$

Then, from (E23) it follows that

$$\frac{\delta U[\sigma]}{\delta\sigma(x')}\ U^{-1}[\sigma] = -i\mathcal{H}_I(x')\ \ ,\ \ \text{or}\ \ i\hbar\frac{\delta U[\sigma]}{\delta\sigma(x')} = \mathcal{H}_I(x')\ U[\sigma]\ . \tag{E25}$$

Q.E.D.
Corollary: the commutation relations for $\Phi_\alpha[x,\sigma]$ are identical to those for $\Phi_\alpha(x)$ (E9):

$$\begin{aligned}\left[\Phi_\alpha[x,\sigma]\ ,\ \Phi_\beta[x',\sigma]\right]_\pm &= U^{-1}(\sigma)\left[\Phi_\alpha(x)\ ,\ \Phi_\alpha(x')\right]_\pm U(\sigma)\\ &= i\hbar\ \Delta(x-x')\ ,\end{aligned} \tag{E26}$$

differential operator $M(\partial)$, this can be done, exployting (E17), by the formula

$$
\begin{aligned}
M(\partial)\boldsymbol{\Phi}_\alpha(x,\sigma) &= \left(M(\partial)\Phi_\alpha[x,\sigma]\right)_{x/\sigma} \\
&\quad + \frac{1}{2}\int d^4x'\ \mathbf{j}_{\beta;a}(x')\ \Big[M(\partial)R_{\alpha\beta}(\partial)\ D'_a\ ,\ \epsilon(x-x')\Big]\ \Delta(x-x')\ . \qquad \text{(E31)}
\end{aligned}
$$

(1) The pseudo-vector pion-nucleon interaction: Here the set of Heisenberg fields is $\boldsymbol{\Phi}_\alpha = \{\boldsymbol{\psi}, \boldsymbol{\Phi}\}$, and the interaction Lagrangian density

$$
\mathcal{L}_{pv}(x) = \frac{f}{m_\pi}\ \bar{\boldsymbol{\psi}}(x)\gamma_\mu\gamma_5\boldsymbol{\psi}(x)\cdot\partial^\mu\boldsymbol{\phi}(x)\ . \qquad \text{(E32)}
$$

Then, equation (E30) implies the relations

$$
\begin{aligned}
\Big[\phi(x)\ ,\ \mathcal{H}_I[x':n]\Big] &= -i(f/m_\pi)\ U[\sigma]\ \partial'_\mu\left(\bar{\boldsymbol{\psi}}(x')\gamma^\mu\gamma_5\ \boldsymbol{\psi}(x')\right)\ U^{-1}[\sigma]\ \Delta(x-x')\ , \\
\Big[\psi(x)\ ,\ \mathcal{H}_I[x':n]\Big] &= -(f/m_\pi)\ \gamma^\mu\gamma_5\ U[\sigma]\ \boldsymbol{\psi}(x')\cdot\partial'_\mu\boldsymbol{\phi}(x')\ U^{-1}[\sigma]\ S(x-x')\ , \qquad \text{(E33)}
\end{aligned}
$$

Then, from (E31) one obtains

$$
\begin{aligned}
\boldsymbol{\psi}(x) &= \psi[x/\sigma]\ , \qquad \text{(E34a)} \\
\boldsymbol{\phi}(x) &= \phi[x/\sigma] - \frac{f}{2m_\pi}\int d^4y\ \bar{\boldsymbol{\psi}}(y)\gamma^\nu\gamma_5\boldsymbol{\psi}(y)\ \Big[\partial_\nu\ ,\ \epsilon(x-y)\Big]\Delta(x-y) \\
&= \phi[x/\sigma]\ , \qquad \text{(E34b)} \\
\partial_\mu\boldsymbol{\phi}(x) &= \left(\partial_\mu\phi[x,\sigma]\right)_{x/\sigma} - \frac{f}{2m_\pi}\int d^4y\ \bar{\boldsymbol{\psi}}(y)\gamma^\nu\gamma_5\boldsymbol{\psi}(y)\ \Big[\partial_\mu\partial_\nu\ ,\ \epsilon(x-y)\Big]\Delta(x-y) \\
&= \left(\partial_\mu\phi[x,\sigma]\right)_{x/\sigma} + \frac{f}{m_\pi}\left(\bar{\psi}[x/\sigma]\gamma^\nu\gamma_5\psi[x/\sigma]\right)\ n_\mu(x)\ n_\nu(x)\ . \qquad \text{(E34c)}
\end{aligned}
$$

Here, we used the identities, see e.g. [17],

$$
\begin{aligned}
& F^{\mu\nu}(x)n_\mu(x)n_\nu(x) = -\frac{1}{2}\int d^4y\ F^{\mu\nu}(y)\ \Big(\partial_\mu\epsilon(x-y)\Big)\ \Big(\partial_\nu\Delta(x-y)\Big)\ , \qquad \text{(E35a)} \\
& \Big[\partial_\nu\ ,\ \epsilon(x-y)\Big]\ \Delta(x-y) = \partial_\nu\Big[\epsilon(x-y)\Delta(x-y)\Big] - \epsilon(x-y)\cdot\partial_\nu\Delta(x-y) = \\
& \Delta(x-y)\ \partial_\nu\epsilon(x-y) = 0\ , \qquad \text{(E35b)} \\
& \Big[\partial_\mu\partial_\nu\ ,\ \epsilon(x-y)\Big]\Delta(x-y) = -2n_\mu n_\nu\ \delta^4(x-y)\ . \qquad \text{(E35c)}
\end{aligned}
$$

Substituting the last results into (E33) gives

$$
\begin{aligned}
\Big[\phi(x)\ ,\ \mathcal{H}_I[x':n]\Big] &= -i(f/m_\pi)\ \partial'_\mu\left(\bar{\psi}(x')\gamma^\mu\gamma_5\ \psi(x')\right)\ \Delta(x-x')\ , \qquad \text{(E36a)} \\
\Big[\psi(x)\ ,\ \mathcal{H}_I[x':n]\Big] &= -(f/m_\pi)\ \gamma^\mu\gamma_5\ \psi(x')\cdot\partial'_\mu\phi(x')\cdot S(x-x') \\
&\quad - \left(\frac{f}{m_\pi}\right)^2\ \gamma_\mu\gamma_5\psi(x')\cdot\left(\bar{\psi}(x')\gamma_\nu\gamma_5\psi(x')\right)S(x-x')\ n^\mu(x')n^\nu(x')\ \text{(E36b)}
\end{aligned}
$$

From these results one infers that

$$\mathcal{H}_I[x:n] \;=\; -\frac{f}{m_\pi}\bar{\psi}(x)\gamma_\mu\gamma_5\psi(x)\cdot\partial^\mu\phi(x)+\frac{1}{2}\left(\frac{f}{m_\pi}\right)^2\left[\bar{\psi}(x)\gamma_\mu\gamma_5\psi(x)\cdot n^\mu(x)\right]^2 \;. \quad \text{(E37)}$$

3. Application Takahashi-Umezawa $\mathcal{H}_I[x:n]$-construction II

Here we consider the *positive frequency part* of the interaction Lagrangian with 'absolute pair-suppresion' for the pseudo-scalar coupling (2.13), written in the Heisenberg representation [13] :

$$\mathcal{L}_I(x) \Rightarrow \frac{g}{2}\left[\overline{\boldsymbol{\psi}^{(+)}}(x)\;\gamma_5\;\boldsymbol{\psi}^{(+)}(x)\right]\cdot\boldsymbol{\phi}(x)\;. \quad \text{(E38)}$$

Then, in order to apply the Umezawa-Takahashi formalism of the foregoing subsections, we have to adapt this formalism at several points. First, with 'absolute-pair suppression' we can split the fields into its posituive and negative enegy imply because they never mix. Then, in the formalism of this section we have $\Phi_\alpha(x)\to\Phi_\alpha^{(+)}(x)$ etc. So, the auxiliary fields are given by

$$\Phi_\alpha^{(+)}[x,\sigma]\equiv\Phi_\alpha^{(+)}(x)-i\int_{-\infty}^{\sigma}d^4x'\;R_{\alpha\beta}(\partial)\;D'_a\;\Delta^{(+)}(x-x')\;\mathbf{j}_{\beta;a}(x')\;. \quad \text{(E39)}$$

Here, the factor $-i$ in front of the integral is choosen so that when a similar formula is written down for $\Phi_\alpha^{(-)}[x,\sigma]$ the sum of the formulas leads to (E12). Now, for (E38) $D'_a=1$, i.e. no derivatives in the interaction Lagrangian, and for the ψ- and ϕ-field

$$\psi\;:\;R_{\alpha\beta}(\partial)=-\left(i\nabla\!\!\!/+M\right)\quad,\quad\phi\;:\;R_{\alpha\beta}=1\;, \quad \text{(E40)}$$

and

$$\psi\;:\;\mathbf{j}_{\beta;a}(x)\Rightarrow\frac{g}{2}\;\gamma_5\;\boldsymbol{\psi}^{(+)}(x)\;\boldsymbol{\phi}(x)\;, \quad \text{(E41a)}$$

$$\phi\;:\;\mathbf{j}_{\beta;a}(x)\Rightarrow\frac{g}{2}\;\overline{\boldsymbol{\psi}^{(+)}}(x)\;\gamma_5\;\boldsymbol{\psi}_\alpha^{(+)}(x)\;. \quad \text{(E41b)}$$

For the Heisenberg field we take again, see (E16)

$$\boldsymbol{\Phi}_\alpha^{(+)}(x)=\Phi_\alpha^{(+)}(x)+\int d^4x'\;\left\{R_{\alpha\beta}(\partial)\;D_a\;\Delta_{ret}(x-x')\right\}\cdot\mathbf{j}_{\beta;a}(x')\;, \quad \text{(E42)}$$

Subsequently, equation (E17) becomes

$$\begin{aligned}\boldsymbol{\Phi}_\alpha^{(+)}(x)\;&=\;\Phi_\alpha^{(+)}[x/\sigma]+\int d^4x'\;\left\{\left[\frac{1}{4}R_{\alpha\beta}(\partial)\;D'_a\;,\;\epsilon(x-x')\right]\;\Delta(x-x')\right.\\&\quad\left.-\frac{i}{2}\theta(x-x')\;R_{\alpha\beta}(\partial)\;D'_a\;\Delta^{(1)}(x-x')\right\}\cdot\mathbf{j}_{\beta;a}(x')\\&\equiv\;\Phi_\alpha^{(+)}[x/\sigma]+g_H(x;n)+g_H^{(1)}(x;n)\;,\end{aligned} \quad \text{(E43)}$$

[13] Like in (2.13) the coupling is imaginary, $g\equiv ig_{\pi N}$, in order that the interaction Lagrangian is hermitean. Of course, in the final version we should make the replacement $g\to ig$ everywhere.

where we used [24]

$$\Delta^{(+)}(x-y) = \frac{1}{2}\left[i\Delta(x-y) + \Delta^{(1)}(x-y)\right] ,$$
$$\Delta^{(-)}(x-y) = \frac{1}{2}\left[i\Delta(x-y) - \Delta^{(1)}(x-y)\right] , \tag{E44}$$

and

$$g_H(x;n) = \frac{1}{4}\int d^4x' \left\{\left[R_{\alpha\beta}(\partial)\ D'_a\ ,\ \epsilon(x-x')\right]\ \Delta(x-x')\right\} \mathbf{j}_{\beta;a}(x') , \tag{E45a}$$
$$g_H^{(1)}(x;n) = -\frac{i}{2}\int d^4x' \left\{\theta(x-x')\ R_{\alpha\beta}(\partial)\ D'_a\ \Delta^{(1)}(x-x')\right\}\cdot \mathbf{j}_{\beta;a}(x') . \tag{E45b}$$

Then, because of

$$[\partial_\mu, \epsilon(x-x')]\ \Delta(x-x') = 0\ \ ,\ \ [\partial_\mu\partial_\nu, \epsilon(x-x')]\,\Delta(x-x') = -2n_\mu n_\nu\ \delta^4(x-x') , \tag{E46}$$

and we get for the ψ-field

$$g_H(x;n) = -\frac{1}{4}\int d^4x' \left\{\left[i\nabla\!\!\!/ + M\ ,\ \epsilon(x-x')\right]\ \Delta(x-x')\right\} \mathbf{j}_{\beta;a}(x') = 0 , \tag{E47}$$

and

$$g_H^{(1)}(x;n) = \frac{ig}{4}\int d^4y\ \theta[n_x\cdot(x-y)]\cdot\left\{(i\nabla\!\!\!/_x + M)\ \Delta^{(1)}(x-y)\right\}\ \gamma_5\ \psi_H^{(+)}(y)\ \phi_H(y)$$
$$= -\frac{ig}{4}\int d^4y\ \theta[n_x\cdot(x-y)]\cdot S^{(1)}(x-y)\ \gamma_5\ \psi_H^{(+)}(y)\ \phi_H(y) , \tag{E48}$$

where we introduced the standard notation, see e.g. [22],

$$S^{(1)}(x-y) \equiv -(i\nabla\!\!\!/_x + M)\ \Delta^{(1)}(x-y) ,$$

and similarly for $S^{(+)}(x-y)$.
The determining equation (E30) becomes for the ψ-field

$$\left[\psi(x), H_I(x':n)\right] = \frac{1}{2}g\ S^{(+)}(x-x')\ \gamma_5\ U(\sigma_x)\left\{\psi_H^{(+)}(x')\ \phi_H(x')\right\}\ U^{-1}(\sigma_x) , \tag{E49}$$

and because $g_H(x;n) = 0$ we have for the Heisenberg field (E42)

$$U(\sigma_x)\ \psi_H^{(+)}(x')\ U^{-1}(\sigma_x) = U(\sigma,\sigma')\ \psi(x'/\sigma')U^{-1}(\sigma,\sigma') +$$
$$-\frac{ig}{4}\int d^4y\ \theta[n'_x\cdot(x'-y)]\ S^{(1)}(x-y)\ \gamma_5\ U(\sigma',\sigma'')\ \psi^{(+)}(y/\sigma'')\ \phi(y/\sigma'')\ U^{-1}(\sigma',\sigma'') , \tag{E50}$$

where we used the notations $\sigma' = \sigma_{x'}$ and $\sigma'' = \sigma_y$. From the Dyson-solution of (E27)

$$U(\sigma_1,\sigma_0) = 1 + \sum_{n=1}^{\infty}\frac{(-i)^n}{n!}\int_{\sigma_0}^{\sigma_1}\ \dots\ \int_{\sigma_0}^{\sigma_1} d^4y_1\ \dots\ d^4y_n\ \cdot$$
$$\times T\left[\mathcal{H}_I(y_1:n_1)\ \dots\ \mathcal{H}_I(y_n:n_n)\right] , \tag{E51}$$

and so, neglecting higher-order terms in the coupling g one has the approximation

$$U(\sigma_x)\ \psi_H^{(+)}(x')\ U^{-1}(\sigma_x) = \psi(x'/\sigma') + \\ +\frac{g}{4}\int d^4y\ \theta[n'_x\cdot(x'-y)]\ S^{(1)}(x-y)\ \gamma_5\ \psi^{(+)}(y/\sigma'')\ \phi(y/\sigma'')\ + O\left(g^2\right)\ , \quad \text{(E52)}$$

and then we get for (E49)

$$\Big[\psi(x), H_I(x':n)\Big] \approx \frac{1}{2}g\ S^{(+)}(x-x')\ \gamma_5\ \{\psi(+)(x')\ \phi(x')\} + \\ +\frac{g^2}{8}\int d^4y\ \theta[n'_x\cdot(x'-y)]\ S^{(1)}(x-y)\ \gamma_5\ \psi^{(+)}(y)\ \phi(y)\ . \quad \text{(E53)}$$

Then, using

$$\langle 0|\psi^{(+)}(x)\overline{\psi^{(+)}}(y)|0\rangle = \langle 0|\left[\psi^{(+)}(x),\overline{\psi^{(+)}}(y)\right]|0\rangle = -iS^{(+)}(x-y)\ , \quad \text{(E54)}$$

we infer from these results the interaction Hamiltonian

$$\mathcal{H}_I(x)\ \Rightarrow\ -\frac{g}{2}\left[\overline{\psi^{(+)}}(x)\gamma_5\ \psi^{(+)}(x)\right] - i\frac{g^2}{8}\int d^4y\ \theta[n\cdot(x-y)]\ \cdot \\ \times\overline{\psi^{(+)}}(x)\left[\gamma_5\ S^{(1)}(x-y)\ \gamma_5\right]\psi^{(+)}(y)\cdot(\phi(x)\phi(y))\ . \quad \text{(E55)}$$

Notice that the second term on the right hand side in (E54) agrees with ΔH_I in (4.15).

APPENDIX F: ADDITIONAL NOTES ON THE TAKAHASHI-UMEZAWA FORMALISM

In this appendix we collect some comments and additional background material on the Takahashi-Umezawa (TU) theory of interactions.

1. On the existence of the Interaction-representation

According to Haag's theorem [38] in general there does not exist a unitary transformation which relates the in(out)-fields $\Phi_\alpha(x)$ to the Heisenberg fields as conjectured in (E14)

$$\mathbf{\Phi}_\alpha(x) = U^{-1}[\sigma]\Phi_\alpha(x)U[\sigma]\Big|_{x/\sigma}\ , \\ \mathbf{\Pi}_\alpha(x) = U^{-1}[\sigma]\Pi_\alpha(x)U[\sigma]\Big|_{x/\sigma}\ . \quad \text{(F1)}$$

On the other hand there does exist a unitary $U[\sigma]$ such that

$$\Phi_\alpha[x,\sigma] = U^{-1}[\sigma]\ \Phi_\alpha(x)\ U[\sigma]\ , \quad \text{(F2)}$$

with $U[\sigma]\equiv U(\sigma,-\infty)$*, connecting* $\Phi_\alpha[x,\sigma]$ *with (E12).* This because both fields are free fields with equal mass. Since different surfaces σ and σ' are connected by

$$\Phi_\alpha(x,\sigma) = U^{-1}(\sigma;\sigma')\Phi_\alpha(x,\sigma')\ U(\sigma;\sigma')\ , \quad \text{(F3)}$$

the flat surfaces $\sigma = -\infty$ and $\sigma' = +\infty$ are connected, the transformation $U(+\infty, -\infty)$ connects the in- and out-fields, and hence provides the S-matrix. Therefore, we can derive the perturbation formulas without using the Heisenberg fields and its ETCR's.

Remark F.1 Conclusion: In the TU-theory the transformation between the Heisenberg and the asymptotic in-, or out-fields is actually never used in the derivation of the interaction hamiltonian, and therefore not sensitive to the Haag-theorem. *(See also section H on the BMP-theory.)*

However, in the LSZ-formalism the Heisenberg fields are essential for the Green-functions. So, we have to find the connection with the Green-functions:

2. Commutation relations for auxiliary fields $Q[x, \sigma]$

In this section we evaluate the commutation relation for the auxiliary fields up to and including the second order in the coupling, in the Takahashi-Umezawa scheme [17, 31, 32]. The auxiliary (Heisenberg) field is defined by (E12)

$$\Phi_\alpha[x, \sigma] \equiv \Phi_\alpha(x) + a \int_{-\infty}^{\sigma} d^4x' \; R_{\alpha\beta}(\partial) \; D'_a \; \Delta(x - x') \; \mathbf{j}_{\beta;a}(x') \; , \tag{F4}$$

where x not necessarily lies on the surface σ, *i.e. x and σ are considered as independent variables.*
In (F4) we introduced the parameter a, which we determine in this section by checking to second order in the coupling the requirement

$$\Big[\Phi_\alpha[x, \sigma] \; , \; \Phi_\beta[x', \sigma]\Big]_\pm = \Big[\Phi_\alpha(x) \; , \; \Phi_\beta(x')\Big]_\pm = iR_{\alpha\beta}(\partial) \; \Delta(x - x') \; . \tag{F5}$$

since, after all, also $\Phi_\alpha(x)$ satisfy such commutation relations, and because *this would follow directly from the existence of* $U[\sigma]$ (E19), which gives the relation between the Heisenberg-, denoted with the subscript H, and the Interaction-representation

$$\Phi_H(x) = U^{-1}[\sigma] \; \Phi(x) \; U[\sigma] \; . \tag{F6}$$

3. Example with trilinear interaction scalar fields

For this exercise, we choose

$$\mathcal{L}_I = \frac{1}{2} g \; \chi^2 \phi = -\mathcal{H}_I \; , \tag{F7}$$

where χ and χ are scalar fields with in principle unequal masses. The corresponding currents are

$$j_{\chi,H}(x) = -g\chi(x)\phi(x) \quad , \quad j_{\phi,H}(x) = -\frac{1}{2} g\chi^2(x) \; . \tag{F8}$$

Then,

$$U[\sigma] = U(\sigma,-\infty) = 1 - i\int_{-\infty}^{\sigma} d^4x\ \mathcal{H}_I(x)\ \ldots$$
$$= 1 + \frac{i}{2}g\int_{-\infty}^{\sigma} d^4x\ \chi^2(x)\phi(x) + O(g^2)\ . \tag{F9}$$

In the following we use the notation $j_H \equiv j_{\chi,H}$, and

$$[\chi(x),\chi(x')] = i\Delta(x-x')\ \ ,\ \ [\phi(x),\phi(x')] = i\Delta_\phi(x-x')\ . \tag{F10}$$

Then,

$$\chi[x,\sigma] = \chi(x) + a\int_{-\infty}^{\sigma} d^4x'\ \Delta(x-x')\ j_H(x')\ , \tag{F11a}$$
$$\chi[y,\sigma] = \chi(y) + a\int_{-\infty}^{\sigma} d^4y'\ \Delta(y-y')\ j_H(y')\ , \tag{F11b}$$

and

$$\begin{aligned}\Big[\chi[x,\sigma],\chi[y,\sigma]\Big] = &\Big[\chi(x),\chi(y)\Big] + \\ &+a\Big[\chi(x),\int_{-\infty}^{\sigma} d^4y'\ \Delta(y-y')\ j_H(y')\Big] \\ &+a\Big[\int_{-\infty}^{\sigma} d^4x'\ \Delta(x-x')\ j_H(x'),\chi(y)\Big] \\ &+a^2\int_{-\infty}^{\sigma} d^4x'\int_{-\infty}^{\sigma} d^4y'\ \Delta(x-x')\ \Delta(y-y')\cdot\Big[j_H(x'),j_H(y')\Big]\end{aligned} \tag{F12}$$

In the field-current commutator

$$\Big[\chi(x),j_H(y)\Big] = -g\Big[\chi(x),U^{-1}[\sigma]\chi(y)\phi(y)U[\sigma]\Big] \tag{F13}$$

1. *The first-order g-term*:

$$\begin{aligned}&+a\Big[\chi(x),\int_{-\infty}^{\sigma} d^4y'\ \Delta(y-y')\ j(y')\Big] \\ &-a\Big[\chi(y),\int_{-\infty}^{\sigma} d^4x'\ \Delta(x-x')\ j(x')\Big] \\ = &-iag\Big[\int_{-\infty}^{\sigma} d^4y'\ \Delta(y-y')\ \Delta(x-y')\ \phi(y') \\ &-\int_{-\infty}^{\sigma} d^4x'\ \Delta(x-x')\ \Delta(y-x')\ \phi(x')\Big] \\ = &\ 0\ .\end{aligned} \tag{F14}$$

2. <u>*The second-order g^2-term*</u>: The g^2-term in this commutator is given by

$$\left[\chi(x), \frac{i}{2}g^2 \int_{-\infty}^{\sigma_{x'}} \left[\chi^2(x')\phi(x'), \chi(y')\phi(y')\right]\right] =$$
$$\frac{i}{2}g^2 \int_{-\infty}^{\sigma_{y'}} d^4x' \left\{ -2\Delta(x-x')\Delta(x'-y')\phi(x')\phi(y') \right.$$
$$-2\Delta(x-x')\ \Delta_\phi(x'-y')\ \chi(y')\chi(x')$$
$$\left. -\ \Delta(x-y')\ \Delta_\phi(x'-y')\ \chi^2(x') \right\} . \tag{F15}$$

a) The g^2-terms linear in the parameter a are

$$-iag^2 \int_{-\infty}^{\sigma} d^4y' \int_{-\infty}^{\sigma_{y'}} d^4x'\ \Delta(y-y') \left\{ \right.$$
$$\Delta(x-x')\ \Delta(x'-y')\ \phi(x')\ \phi(y') + \Delta(x-x')\ \Delta_\phi(x'-y')\ \chi(y')\ \chi(x')$$
$$\left. + \frac{1}{2}\Delta(x-y')\ \Delta_\phi(x'-y')\ \chi^2(x') \right\}$$
$$+iag^2 \int_{-\infty}^{\sigma} d^4x' \int_{-\infty}^{\sigma_{x'}} d^4y'\ \Delta(x-x') \left\{ \right.$$
$$\Delta(y-y')\ \Delta(y'-x')\ \phi(y')\ \phi(x') + \Delta(y-y')\ \Delta_\phi(y'-x')\ \chi(x')\ \chi(y')$$
$$\left. + \frac{1}{2}\Delta(y-x')\ \Delta_\phi(y'-x')\ \chi^2(y') \right\} \tag{F16}$$

Here, for flat space-like surfaces $\sigma_x = x_0'$ and $\sigma_y = y_0'$. So, both are not equal to σ! Note that the terms with $\chi^2(x')$ and $\chi^2(y')$ cancel against each other.

Next, it is important to realize that

$$\int_{-\infty}^{\sigma} d^4y' \int_{-\infty}^{\sigma_{y'}} d^4x' = \int_{-\infty}^{\sigma} d^4y' \int_{-\infty}^{\sigma} d^4x'\ \theta(\sigma_{y'} - \sigma_{x'}) ,$$
$$\int_{-\infty}^{\sigma} d^4x' \int_{-\infty}^{\sigma_{x'}} d^4y' = \int_{-\infty}^{\sigma} d^4x' \int_{-\infty}^{\sigma} d^4y'\ \theta(\sigma_{x'} - \sigma_{y'}) , \tag{F17}$$

which gives for (F16) the result

$$= -iag^2 \int_{-\infty}^{\sigma} d^4x' \int_{-\infty}^{\sigma} d^4y'\ \Delta(x-x')\ \Delta(y-y') \left\{ \right.$$
$$\Delta(x'-y')\ \Big(\theta(\sigma_{y'}-\sigma_{x'})\ \phi(x')\ \phi(y') + \theta(\sigma_{x'}-\sigma_{y'})\ \chi(y')\ \chi(x')\Big)$$
$$\left. \Delta_\phi(x'-y')\ \Big(\theta(\sigma_{y'}-\sigma_{x'})\ \chi(y')\ \chi(x') + \theta(\sigma_{x'}-\sigma_{y'})\ \chi(x')\ \chi(y')\Big) \right\}$$
$$= -iag^2 \int_{-\infty}^{\sigma} d^4x' \int_{-\infty}^{\sigma} d^4y'\ \Delta(x-x')\ \Delta(y-y') \left\{ \right.$$
$$\Delta(x'-y')\ \Big[\phi(x')\ \phi(y') + i\theta(\sigma_{x'}-\sigma_{y'})\ \Delta_\phi(y'-x')\Big]$$
$$\left. +\Delta_\phi(x'-y')\ \Big[\chi(y')\ \chi(x') + i\theta(\sigma_{x'}-\sigma_{y'})\ \Delta(x'-y')\Big] \right\} . \tag{F18}$$

---

b) The g^2-terms quadratic in the parameter a are

$$ia^2g^2 \int_{-\infty}^{\sigma} d^4x' \int_{-\infty}^{\sigma} d^4y' \left\{ + \Delta(x-x')\Delta(y-y')\Delta(x'-y')\ \phi(x')\phi(y') \right.$$
$$\left. + \Delta(x-x')\Delta(y-y')\Delta_\phi(x'-y')\ \chi(y')\chi(x') \right\} . \tag{F19}$$

c) <u>Case $a = 1$</u>: Now for $a = 1$ one sees that the sum of the linear and quadratic terms in a from (F18) and (F19) cancel! *This demonstrates that the choice of the integral term in (E12) is fixed by the requirement of the comutation relation (E18*!

4. Scalar Fields with General Interaction Hamiltonian

We repeat the basic formulas with $a = 1$ for the starting point of the demonstration:

$$\chi[x,\sigma] = \chi(x) + \int_{-\infty}^{\sigma} d^4x'\ \Delta(x-x')\ j_H(x')\ , \tag{F20a}$$
$$\chi[y,\sigma] = \chi(y) + \int_{-\infty}^{\sigma} d^4y'\ \Delta(y-y')\ j_H(y')\ , \tag{F20b}$$

and

$$\begin{aligned}
\Big[\chi[x,\sigma],\chi[y,\sigma]\Big] &= \Big[\chi(x),\chi(y)\Big] + \\
&+ \Big[\chi(x), \int_{-\infty}^{\sigma} d^4y'\ \Delta(y-y')\ j_H(y')\Big] \\
&+ \Big[\int_{-\infty}^{\sigma} d^4x'\ \Delta(x-x')\ j_H(x'), \chi(y)\Big] \\
&+ \int_{-\infty}^{\sigma} d^4x' \int_{-\infty}^{\sigma} d^4y'\ \Delta(x-x')\ \Delta(y-y') \cdot \Big[j_H(x'), j_H(y')\Big]\ .(F21)
\end{aligned}$$

We check the second-order in the interaction. Then

$$j_H(y') = U^{-1}[\sigma_{y'}]\chi(y)\phi(y)U[\sigma_{y'}] \Rightarrow -i \int_{-\infty}^{\sigma_{y'}} d^4x'\ \Big[\mathcal{H}_I(x'), j(y')\Big] \tag{F22}$$

Furthermore, we need the commutators

$$\begin{aligned}
\Big[\chi(x), \mathcal{H}_I(x')j(y')\Big] &= \Big[\chi(x), \mathcal{H}_I(x')\Big]\ j(y') + \mathcal{H}_I(x')\ \Big[\chi(x), j(y')\Big] \\
&= -i\Delta(x-x')\ j(x')\ j(y') + \mathcal{H}_I(x')\ \Big[\chi(x), j(y')\Big]\ , \\
\Big[\chi(x), j(y')\ \mathcal{H}_I(x')\Big] &= \Big[\chi(x), j(y')\Big]\mathcal{H}_I(x') - i\Delta(x-x')\ j(y')\ j(x')\ . \qquad (F23a)
\end{aligned}$$

This gives

$$\Big[\chi(x), j_H(y')\Big] \Rightarrow -i \int_{-\infty}^{\sigma_{y'}} d^4x' \ \Big(-i\Delta(x-x') \Big[j(x'), j(y')\Big] + \Big[\mathcal{H}_I(x'), [\chi(x), j(y')]\Big]\Big), \tag{F24}$$

and leads to

$$\begin{aligned} \Big[\chi(x), \int_{-\infty}^{\sigma} d^4y' \ \Delta(y-y') \ j_H(y')\Big] &\Rightarrow -i \int_{-\infty}^{\sigma} d^4y' \ \int_{-\infty}^{\sigma_{y'}} d^4x' \ \Delta(y-y') \cdot \\ \times \Big(-i\Delta(x-x') \Big[j(x'), j(y')\Big] + \Big[\mathcal{H}_I(x'), [\chi(x), j(y')]\Big]\Big) &\equiv I_1 \ , \end{aligned} \tag{F25}$$

Similarly,

$$\begin{aligned} \Big[\chi(y), \int_{-\infty}^{\sigma} d^4x' \ \Delta(x-x') \ j_H(x')\Big] &\Rightarrow -i \int_{-\infty}^{\sigma} d^4x' \ \int_{-\infty}^{\sigma_{x'}} d^4y' \ \Delta(x-x') \cdot \\ \times \Big(-i\Delta(y-y') \Big[j(y'), j(x')\Big] + \Big[\mathcal{H}_I(y'), [\chi(y), j(x')]\Big]\Big) &\equiv I_2 \ , \end{aligned} \tag{F26}$$

Again, like in (F17), it is important to realize that

$$\begin{aligned} \int_{-\infty}^{\sigma} d^4y' \int_{-\infty}^{\sigma_{y'}} d^4x' \ &= \ \int_{-\infty}^{\sigma} d^4y' \int_{-\infty}^{\sigma} d^4x' \ \theta(\sigma_{y'} - \sigma_{x'}) \ , \\ \int_{-\infty}^{\sigma} d^4x' \int_{-\infty}^{\sigma_{x'}} d^4y' \ &= \ \int_{-\infty}^{\sigma} d^4x' \int_{-\infty}^{\sigma} d^4y' \ \theta(\sigma_{x'} - \sigma_{y'}) \ , \end{aligned} \tag{F27}$$

so that

$$\begin{aligned} I_1 + I_2 \ = \ - \int_{-\infty}^{\sigma} d^4x' \ \int_{-\infty}^{\sigma} d^4y' \Big\{ & \Delta(x-x') \ \Delta(y-y') \ \Big[j(x'), j(y')\Big] \\ & -i\theta(\sigma_{y'} - \sigma_{x'}) \ \Delta(y-y') \ \Big[\mathcal{H}_I(x'), \Big[\chi(x), j(y')\Big]\Big] \\ & +i\theta(\sigma_{x'} - \sigma_{y'}) \ \Delta(x-x') \ \Big[\mathcal{H}_I(y'), \Big[\chi(y), j(x')\Big]\Big]\Big\} \ . \end{aligned} \tag{F28}$$

Now, it is easy to see that

$$\Big[\chi(x), j(y')\Big] \ = \ i\Delta(x-y') \ f(y') \ , \tag{F29a}$$

$$\Big[\chi(y), j(x')\Big] \ = \ i\Delta(y-x') \ f(x') \ , \tag{F29b}$$

which leads to the cancellation of the 2nd and 3rd term in (F28), as can be seen by making the interchange $x' \leftrightarrow y'$ in one of these terms. Also, the 1st term in (F28) is seen to cancel the current comutator term in (F21). So, we demonstrated that

$$\Big[\chi[x,\sigma], \chi[y,\sigma]\Big] \ = \ \Big[\chi(x), \chi(y)\Big] \ , \tag{F30}$$

which we set out to prove.

APPENDIX G: MISCELLANEOUS COMMUTATION RELATIONS

$$1. \quad \left[\chi^2(x')\ \phi(x'), \chi(y')\ \phi(y')\right] = \left[\chi^2(x'), \chi(y')\right]\ \phi(x')\ \phi(y') + \chi(y')\ \chi^2(x')\ \left[\phi(x'), \phi(y')\right]$$
$$= 2i\ \Delta(x'-y')\ \chi(x')\ \phi(x')\ \phi(y') + i\ \Delta_\phi(x'-y')\ \chi(y')\ \chi^2(x')\ ,$$
$$2. \quad \left[\chi(x), \chi(y')\ \chi^2(x')\ \right] = \left[\chi(x), \chi(y')\right]\ \chi^2(x') + \chi(y')\ \left[\chi(x), \chi^2(x')\right]$$
$$= i\Delta(x-y')\ \chi^2(x') + 2i\Delta(x-x')\ \chi(y')\ \chi(x')\ . \tag{G1a}$$

APPENDIX H: S-MATRIX FORMULATION OF THE TAKAHASHI-UMEZAWA METHOD, BMP-THEORY

According to Haag's theorem [38] in general there does not exist a unitary transformation which relates the in(out)-fields $\phi_{as}(x)$ to the Heisenberg fields $\mathbf{\Phi}(x)$ as conjectured in (E14). On the other hand there is no objection against the existence of a unitary $U[\sigma]$ relating the TU-auxiliary fields and the asymptotic fields

$$\Phi[x,\sigma] = U^{-1}[\sigma]\ \phi_{as}(x)\ U[\sigma]\ , \tag{H1}$$

with $U[\sigma] \equiv U(\sigma, -\infty)$*, connecting* $\Phi[x,\sigma]$ *with (E12).* This because both fields are free fields with equal mass.
In this section we will establish these matters, derive the relation between the TU interaction Hamiltonian, the $U[\sigma]$, and a perturbative formula of the S-matrix in the context of the axiomatic S-matrix theory. We follow here the framework of Bogoliubov and collaborators [21, 39, 40], see also [26], section 18b. We refer in the following to this as the BMP-theory.

1. Asymptotic completeness, Yang-Feldman equations

In the context of axiomatic field theory, Lehmann, Symanzik, and Zimmermann (LSZ) [35] formulated an asymptotic condition utilizing the notion of weak convergence in the Hilbert space of state vectors. See e.g. [22] for an detailed exposition of the LSZ-formalism. Here, the Heisenberg field operator $\Phi(x)$ and the asymptotic fields ϕ_{as} with $as = -in, -out$ satisfy the equations [14]

$$(\Box + m^2)\Phi(x) = j(x)\quad ,\quad (\Box + m^2)\phi_{as} = 0\ , \tag{H2}$$

where $j(x)$ is the Heisenberg current, and the asymtotic fields are *free fields.* Then, the relation between these fields is given by the Yang-Feldman (YF) [36] equations

$$\Phi(x) = \phi_{in}(x) + \int \Delta_{ret}(x-y)\ j(y)\ d^4y$$
$$= \phi_{out}(x) + \int \Delta_{adv}(x-y)\ j(y)\ d^4y\ . \tag{H3a}$$

[14] In the following paragraphs we use for the Heisenberg current the notation: $\mathbf{j}(x) \equiv j(x)$.

Upon subtraction we get the relation

$$\phi_{out}(x) \ = \ \phi_{in}(x) - \int \Delta(x-y)\ j(y)\ d^4y\ . \tag{H4}$$

Next one assumes the completeness of both the asymtotic fields, i.e. for the corresponding Hilbert spaces and vacuum states one has

$$\mathcal{H}_{in} = \mathcal{H}_{out} = \mathcal{H} \ \ , \ \ |0\rangle_{in} = |0\rangle_{out} = |0\rangle\ . \tag{H5}$$

It can be shown that on a certain dense set of $\mathcal{H}_{as}$ the LSZ asymptotic consditions and the YF-equations are valid, cmfr. [40], chapter 14.

2. Scattering Matrix, Unitarity, Microcausality, Commutators

In order to bypass the use of a unitary operator U as a mediator between the asymptotic and the (interacting) Heisenberg fields, we follow here the method of Bogoliubov and collaborators, see [39, 40].

Assume that the S-operator is a functional of the asymptotic fields $\chi_\rho(x)$. In the following we use in-fields, i.e. $\chi_\rho(x) = \phi_{in,\rho}(x)$. Then, the assumption of asymptotic completeness implies the existence of the expansion

$$\begin{aligned} S \ = \ 1 + \sum_{n=1}^{\infty} \int d^4x_1 \ldots d^4x_n\ S_n(x_1\alpha_1, \ldots, x_n\alpha_n) \cdot \\ \times : \chi_{\alpha_1}(x_1) \ldots \chi_{\alpha_n}(x_n) :\ . \end{aligned} \tag{H6}$$

This is no restriction, because any e.g. ordinary product of fields can be expanded in normal-ordered ones by Wick's theorem. Also, the use of the normal-ordered products here ensures the stability of the vacuum, i.e. $\langle 0|S|0\rangle = 1$

1. Then, functional derivatives of the S-operator have the form

$$\begin{aligned} \frac{\delta S}{\delta \chi_\alpha(x)} \ = \ \sum_n \sum_j \delta_{\alpha\alpha_j} \int d^4x_1 \ldots \widehat{d^4x_j} \ldots d^4x_n\ S_n(x_1\alpha_1, \ldots, x_j = x\alpha_j = \alpha, \ldots x_n\alpha_n) \cdot \\ \times : \chi_{\alpha_1}(x_1) \ldots \widehat{\chi_{\alpha_j}(x_j)} \ldots \chi_{\alpha_n}(x_n) :\ . \end{aligned} \tag{H7}$$

2. Unitarity $S^\dagger\ S = 1$ gives upon functional differentiation

$$\frac{\delta S^\dagger}{\delta \chi_\rho(x)} S = -S^\dagger\ \frac{\delta S}{\delta \chi_\rho(x)}\ , \tag{H8}$$

and a similar relation starting from $S\ S^\dagger = 1$.

3. The *Heisenberg current* is defined as [15]

$$j_\rho(x) = iS^\dagger \ \frac{\delta S}{\delta \chi_\rho(x)} \tag{H9}$$

4. Then *microcausality* takes the form, see [39], chapter 17 [16],

$$\frac{\delta j_\rho(x)}{\delta \chi_\lambda(y)} = 0 \quad , \quad \text{for} \quad x \leq y \ . \tag{H10}$$

Theorem H.1 *Let $H(x)$ be a (local) function of the asymptotic fields $\chi_\alpha(x)$, which is defined even when $\chi_\alpha(x)$ do not satisfy free field equations. Let the S-operator be defined as the time-ordered exponential*

$$S = T\left[\exp\left\{-i\int d^4x \ H_I(x)\right\}\right] \ . \tag{H11}$$

Then, the microcausility condition

$$\frac{\delta}{\delta \chi_\beta(y)}\left\{S^\dagger \ \frac{\delta S}{\delta \chi_\alpha(x)}\right\} = 0 \ \ \text{for} \ \ x \leq y \ . \tag{H12}$$

This illustrates that the intuitive notion of causality is reflected in the expression of the S-matrix as the time-ordered exponential. See [39] for the details on this point of view. Furthermore, it follows from (H9) and (H11) that

$$j_\rho(x) = \frac{\partial H_I(x)}{\partial \chi_\rho(x)} \ . \tag{H13}$$

For example in QED $H_I(x) = e\bar{\psi}(x)\gamma_\mu\psi(x)A^\mu(x)$, and we get that the electromagnetic current operator is $j_\mu(x) = e\bar{\psi}(x)\gamma_\mu\psi(x)$, which is correct [22].

5. It can be shown that with the current (H9) the asymptotic fields $\chi_{in/out,\rho}(x)$ satisfy the Yang-Feldman equation

$$\chi_{out,\rho}(x) = \chi_{in,\rho}(x) - \int d^4y \ \Delta(x-y) \ j_\rho(y) \ . \tag{H14}$$

For the demonstration see section I.

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[15] Note that in [40] the out-field is used. Then

$$j_\rho(x) = i\frac{\delta S}{\delta \chi_\rho(x)}S^\dagger \ .$$

[16] Here $x \leq y$: or $(x-y)^2 \geq 0, x^0 < y^0$, or $(x-y)^2 < 0$. So, the point x is in the past of or is spacelike separated from the point y.

3. Correspondence with the LSZ Theory

The correspondence is obtained by the identification

$$j_\rho(x) = iS^\dagger \ \frac{\delta S}{\delta\chi_\rho(x)} \equiv \left(\Box + m^2\right) \ \boldsymbol{\chi}_\rho(x) \ , \tag{H15}$$

where $\boldsymbol{\chi}_\rho(x)$ denotes the (interacting) Heisenberg field. We note that for a hermitean field $\chi_\rho(x)$ the current is also hermitean, due to the relation (H8)

$$j^\dagger_\rho(x) = -i\frac{\delta S^\dagger}{\delta\chi_\rho(x)} S = j_\rho(x) \ . \tag{H16}$$

Functionally differentiating the current gives the equations

$$\begin{aligned}
\frac{\delta j_\rho(x)}{\delta\chi_\sigma(y)} \ &= \ iS^\dagger \ \frac{\delta^2 S}{\delta\chi_\sigma(y)\delta\chi_\rho(x)} + i\frac{\delta S^\dagger}{\delta\chi_\sigma(y)} SS^\dagger \frac{\delta S}{\delta\chi_\rho(x)} \\
&= \ iS^\dagger \ \frac{\delta^2 S}{\delta\chi_\sigma(y)\delta\chi_\rho(x)} + ij_\sigma(y) \ j_\rho(x) \ , &\text{(H17a)} \\
\frac{\delta j_\sigma(y)}{\delta\chi_\rho(x)} \ &= \ iS^\dagger \ \frac{\delta^2 S}{\delta\chi_\rho(x)\delta\chi_\sigma(y)} + ij_\rho(x) \ j_\sigma(y) \ . &\text{(H17b)}
\end{aligned}$$

Subtraction gives

$$\frac{\delta j_\rho(x)}{\delta\chi_\sigma(y)} - \frac{\delta j_\sigma(y)}{\delta\chi_\rho(x)} \ = \ -i\left[j_\rho(x) \ , j_\sigma(y)\right] \ . \tag{H18}$$

Note that for spacelike separations, i.e. $(x-y)^2 < 0$, causality and (H18) imply that the current commutators vanish. Moreover, the application of the causality condition (H12) to equations (H17b) for $x \neq y$ gives the following important relation

$$H_2(x\rho, y\sigma) \ \equiv \ S^\dagger \ \frac{\delta^2 S}{\delta\chi_\rho(x)\delta\chi_\sigma(y)} = -T\left[j_\rho(x)j_\sigma(y)\right] \ . \tag{H19}$$

It follows that for all x and y

$$H_2(x\rho, y\sigma) = -T\left[j_\rho(x)j_\sigma(y)\right] - i\Lambda_{\rho\sigma}(x,y) \ , \tag{H20}$$

where $\Lambda_{\rho\sigma}$ is a *quasilocal operator*

$$\Lambda_{\rho\sigma}(x,y) = \Lambda_{\sigma\rho}(y,x) = 0 \ \text{ if } \ \mathrm{x} \neq \mathrm{y} \ . \tag{H21}$$

Theorem H.2 *If $\chi_\rho(x)$ is hermitean, then the quasilocal operator $\Lambda_{\rho\sigma}$ is hermitean.*

This can be proven by differentiating functionally (H8) w.r.t. $\chi_\sigma(y)$, using the definition of the currents (H9), and comparing the result with (H20) and its hermitean conjugate.

Substitution of (H19) into equation (H17b) gives

$$\frac{\delta j_\rho(x)}{\delta\chi_\sigma(y)} \ = \ -i\theta(x^0-y^0)\left[j_\rho(x), j_\sigma(y)\right] + \Lambda_{\rho\sigma}(x,y) \ . \tag{H22}$$

Now we are prepared to derive the locality properties assumed in the LSZ theory. Above, the local commutivity of the currents has been shown to follow from microcausality. Using the YF-equations (H3a) one can show the following theorem

Theorem H.3 *For spacelike separations the Heisenberg fields commute with the currents and among themselves*

$$\Big[\mathbf{\Phi}_\rho(x), j_\sigma(y)\Big] = 0 \quad \text{for} \quad (\mathrm{x}-\mathrm{y})^2 < 0 \ , \tag{H23a}$$

$$\Big[\mathbf{\Phi}_\rho(x), \mathbf{\Phi}_\sigma(y)\Big] = 0 \quad \text{for} \quad (\mathrm{x}-\mathrm{y})^2 < 0 \ . \tag{H23b}$$

This theorem can be shown as follows: From the expansion of $j(x)$ in asymptotic fields, like that for the S-matrix in (H6), and the commutation relations for asymtotic fields $\chi_\rho(x)$ one has [17]

$$\Big[\chi_\rho(x), j_\sigma(y)\Big] = +i\int d^4x'\ \Delta(x-x')\frac{\delta j_\sigma(y)}{\delta\chi_\rho(x')} \ . \tag{H24}$$

Using (H22), (H24) and the relation

$$\Delta(x) = \Delta_{ret}(x) - \Delta_{adv}(x) \ , \tag{H25}$$

one gets with the Yang-Feldman equation (H3a) that (CHECK!!)

$$\Big[\mathbf{\Phi}_\rho(x), j_\sigma(y)\Big] = +i\int d^4x'\ \Delta(x-x')\Lambda_{\rho\sigma}(x',y)$$
$$-\int\Big\{\Delta_{ret}(x-x')\theta_{x'y} - \Delta_{adv}(x-x')\theta_{yx'}\Big\}\Big[j_\rho(x'), j_\sigma(y)\Big] \Rightarrow 0 \ . \tag{H26}$$

Here $\theta_{xy} \equiv \theta(x^0-y^0)$. The vanishing of the first term on the r.h.s. is due to the quasi-local character of $\Lambda_{\rho\sigma}$ and the properties of $\Delta(x-x')$. As for the term with $\Delta_{ret}(x-x')$ we note that for $(x-y)^2 < 0$ the points x and y are outside each others lightcones. Now because of $\theta_{x'y}$ the point x' can not be in the lightcone of both x and y, and therefore there is no contribution to the integral. In the same way one can reason that the same is the case for the temr with $\Delta_{adv}(x-x')$. This concludes the proof.

Similarly one can prove the second commutator of this theorem.

In [40] the following is shown: Let be given (i) the locality condition (H26), (ii) the current $j_\rho(x)$ defined by the operation of the Klein-Gordon operator on the Heisenberg field $\mathbf{\Phi}_\rho(x)$, (iii) the asymptotic condition in the form of the Yang-Feldman equations, and (iv) the current expressed as a functional derivative of the S-matrix, which is given as functional series similar to (H6). Then, under these hypotheses it is possible to exploit the arbitrariness in the extrapolation of the S-matrix off-mass-shell so as to fulfill the microcausality condition (H10).

---

[17] From $\chi_{out} = S^{-1}\ \chi_{in}\ S$ follows

$$\chi_{out}(x) = \chi_{in}(x) + S^{-1}\ \Big[\chi_{in}(x), S\Big] \ ,$$

which means for any operator $Q(y)$, so also for $\chi_{out}(y)$, expressible as a series similar to (H6) in χ_{in} that

$$\Big[\chi_{in}(x), O(y)\Big] = i\int d^4x'\ \Delta(x-x')\frac{\delta O(y)}{\delta\chi(x')} \ .$$

Here, we used $\delta\chi_{in}(y)/\delta\chi_{in}(x') = \delta(x'-y)$.

4. Perturbation series, Interaction Hamiltonian, U-transformation

From equation (H20), using (H9) for the current, one derives a defining equation for the S-operator in terms of $\Lambda_{\rho\sigma}(x,y)$, namely

$$\frac{\delta^2 S}{\delta\chi_\rho(x)\delta\chi_\sigma(y)} = ig\Lambda(\rho x, \sigma y) + \\ +\theta_{xy}\frac{\delta S}{\delta\chi_\rho(x)}S^\dagger\frac{\delta S}{\delta\chi_\sigma(y)} + \theta_{yx}\frac{\delta S}{\delta\chi_\sigma(y)}S^\dagger\frac{\delta S}{\delta\chi_\rho(x)} \ , \tag{H27}$$

where

$$ig\Lambda(\rho x, \sigma y) \equiv -S\ \Lambda_{\rho\sigma}(x,y) \ . \tag{H28}$$

A solution as a power series in the coupling g

$$S = 1 + \sum_{n=1}^{\infty}(ig)^n\ S_n \ , \tag{H29}$$

leads to a system of *recursive relations* for the coefficients S_n. The first order suggests the introduction of the ineraction Hamiltonian by

$$\frac{\delta^2 S}{\delta\chi_\rho(x)\delta\chi_\sigma(y)} = \Lambda(\rho x, \sigma y \equiv -\frac{\delta^2 H_I'}{\delta\chi_\rho(x)\delta\chi_\sigma(y)} \ . \tag{H30}$$

Namely, assuming for this Hamiltonian the expansion in asymptotic fields of the form

$$H_I \equiv gH_I' = \sum_{n,\rho_i}\int H_{\rho_1\ldots\rho_n}\ :\chi_{\rho_1}(x)\ldots\chi_{\rho_n}(x):\ d^4x_1\ldots d^4x_n \ , \tag{H31}$$

and utilizing

$$\delta\chi_\rho(x)/\delta\chi_\sigma(y) = \delta_{\rho\sigma}\delta(x-y) \ , \tag{H32}$$

indeed leads to

$$S_1 = -iH_I = -i\int d^4x\ \mathcal{H}_I(x) \ . \tag{H33}$$

Conjecture H.1 *The time ordered exponential*

$$S = T\left\{\exp\left[-i\int d^4x\ \mathcal{H}_I(x)\right]\right\} \tag{H34}$$

satisfies the functional differential equations (H27), together with the auxiliary conditions

$$\langle 0|S|0\rangle = 1 \ \ , \ \ \langle 1|S|0\rangle = 0 \ , \tag{H35}$$

if $\Lambda_{\rho\sigma}(x,y)$ has the form (H30).
The operator (H34) is a solution of equation (H27) which coincides with the solution from the recursion relations.

One notices that the S-matrix is determined by the interaction Hamiltonian. To determine H_I we follow Takahashi-Umezawa [17, 31]. Using their auxiliary field

$$\chi[x,\sigma] \equiv \chi(x) + \int_{-\infty}^{\sigma} d^4x'\ \Delta(x-x')\ j(x')\ , \tag{H36}$$

which like $\chi(x) \equiv \chi_{in}(x)$ satisfies the free KG-equation with mass m.
First, we prove the following theorem

Conjecture H.2

$$\Big[\chi[x,\sigma],\chi[y,\sigma]\Big] = \Big[\chi(x),\chi(y)\Big] = i\Delta(x-y;m)\ . \tag{H37}$$

Proof: H.1 *Using (H36) gives*

$$\Big[\chi[x,\sigma],\chi[y,\sigma]\Big] - \Big[\chi(x),\chi(y)\Big] = +\int_{-\infty}^{\sigma} d^4y'\ \Delta(y-y')\ \Big[\chi(x),j(y')\Big]$$
$$-\int_{-\infty}^{\sigma} d^4x'\ \Delta(x-x')\ \Big[\chi(y),j(x')\Big]$$
$$+\int_{-\infty}^{\sigma}\int_{-\infty}^{\sigma} d^4x'd^4y'\ \Delta(x-x')\Delta(y-y')\ \Big[j(x'),j(y')\Big]\ .$$

Now, we use (H18) and (H24) to rewrite the above expression:

$$\Big[\chi[x,\sigma],\chi[y,\sigma]\Big] - \Big[\chi(x),\chi(y)\Big] = -i\int_{-\infty}^{\sigma} d^4y' \int_{-\infty}^{\infty} d^4x'\Delta(x-x')\Delta(y-y')\frac{\delta j(y')}{\delta j(x')}$$
$$+i\int_{-\infty}^{\sigma} d^4x' \int_{-\infty}^{\infty} d^4y'\ \Delta(x-x')\Delta(y-y')\frac{\delta j(x')}{\delta j(y')}$$
$$-i\int_{-\infty}^{\sigma} d^4x' \int_{-\infty}^{\sigma} d^4y'\ \Delta(x-x')\Delta(y-y')\left(\frac{\delta j(x')}{\delta j(y')} - \frac{\delta j(y')}{\delta j(x')}\right)$$
$$\Rightarrow 0\ ,$$

which follows from the microcausality condition that annihilates the possibly contributions from the difference in boundary values in the integrals. (Q.E.D.)

This justifies the conjecture of the existence of a unitary operator $U[\sigma]$ such that

$$\chi[x,\sigma] = U^{-1}[\sigma]\ \chi(x)\ U[\sigma]\ . \tag{H38}$$

Functional differentiation of (H36) and (H38) gives

$$\frac{\delta\chi[x,\sigma]}{\delta\sigma(x')} = U^{-1}[\sigma]\left[\chi(x),\frac{\delta U[\sigma]}{\delta\sigma(x')}U^{-1}[\sigma]\right]\ U[\sigma] = \Delta(x-x')\ j(x')\ . \tag{H39}$$

defining the interaction hamiltonian density by the equation

$$i\frac{\delta U[\sigma]}{\delta\sigma(x')} = \mathcal{H}_I(x,\sigma)\ U[\sigma]\ , \tag{H40}$$

which has as solution the time ordered exponential

$$U[\sigma] = T\left\{\exp\left[-i\int_{-\infty}^{\sigma} d^4x\ \mathcal{H}_I(x,n)\right]\right\}\ . \tag{H41}$$

With these results we arrive at the *fundamental equation* of Takahashi-Umezawa: for the determination of the interaction hamiltonian:

$$\left[\chi(x), \mathcal{H}_I(x,n)\right] = i\Delta(x-x')\ U[\sigma]j(x)U^{-1}[\sigma]\ . \tag{H42}$$

Finally, we notice that the field $\chi[x,\sigma]$ provides a *free field interpolation* between $\chi_{in}(x)$ and $\chi_{out}(x)$: see (H3a) and (H36). Therefore

$$\chi_{out} = \chi[x,\infty] = U^{-1}[\infty]\ \chi_{in}\ U[\infty]\ , \tag{H43}$$

with $\chi_{in}(x) \equiv \chi(x)$. One sees that

$$S = U[+\infty] = U[+\infty,-\infty]\quad ,\quad U[-\infty] = 1\ , \tag{H44}$$

which brings (H43) into the familiar form

$$\chi_{out}(x) = S^{-1}\ \chi_{in}(x)\ S\ . \tag{H45}$$

This establishes the relation of the U-matrix, and ipso facto the interaction hamiltonian, to the S-matrix.

We conclude that we have established the Takahashi-Unezawa-method in the framework of the axiomatic S-matrix theory, but without the defect indicated by the Haag-theorem.

5. Application TU-scheme in BMP-Theory

It is clear from above that in the BMP-theory the TU-equations for the Heisenberg fields (E17), and the determining equation for the interaction Hamiltonian (E30) are valid. Next we give two examples of application of the formalism described above.

1. *PV-Coupling Nucleons*: Then, starting with the interaction Lagrangian

$$\mathcal{L}_{pv}(x) = \frac{f}{m_\pi}\ \bar{\boldsymbol{\psi}}(x)\gamma_\mu\gamma_5\boldsymbol{\psi}(x)\cdot\partial^\mu\boldsymbol{\phi}(x)\ . \tag{H46}$$

Then, like in equation (E33) one gets that

$$\begin{aligned}\left[\phi(x)\ ,\ \mathcal{H}_I[x':n]\right] &= -i(f/m_\pi)\ U[\sigma]\ \partial'_\mu\left(\bar{\boldsymbol{\psi}}(x')\gamma^\mu\gamma_5\ \boldsymbol{\psi}(x')\right)\ U^{-1}[\sigma]\ \Delta(x-x')\ ,\\ \left[\psi(x)\ ,\ \mathcal{H}_I[x':n]\right] &= -(f/m_\pi)\ \gamma^\mu\gamma_5\ U[\sigma]\ \boldsymbol{\psi}(x')\cdot\partial'_\mu\boldsymbol{\phi}(x')\ U^{-1}[\sigma]\ S(x-x')\ ,\end{aligned} \tag{H47}$$

and similarly to (E34c)

$$\boldsymbol{\psi}(x) = \psi[x/\sigma]\quad ,\quad \boldsymbol{\phi}(x) = \phi[x/\sigma]\ , \tag{H48a}$$

$$\partial_\mu\boldsymbol{\phi}(x) = \left(\partial_\mu\phi[x,\sigma]\right)_{x/\sigma} + \frac{f}{m_\pi}\left(\bar{\psi}[x/\sigma]\gamma^\nu\gamma_5\psi[x/\sigma)\right)\ n_\mu(x)\ n_\nu(x)\ . \tag{H48b}$$

So, in this case there was no problem with the handling of the operators involving the Heisenberg fields $\boldsymbol{\psi}$ and $\boldsymbol{\phi}$, which occur in

$$U[\sigma]\,\mathbf{j}(x')\,U^{-1}[\sigma]\ .$$

This, because of the identity of the $U[\sigma]$-transformed free fields $\psi[x/\sigma]$ and $\phi[x/\sigma]$ as expressed in the first two identities in (H48b). Therefore, the construction of the interaction Hamiltonian can be done and is the same as that found previously in this case: expression (E37).

2. *Vector-Coupling Nucleons*:

6. BPM-theory and Haag-theorem

There still seems to be a problem: considering the PV-coupling example above, the equality $\boldsymbol{\psi}(x) = \psi[x/\sigma]$ in (H48b) implies for the ETCR

$$\Big[\psi[x/\sigma],\psi[y/\sigma]\Big]_+ = U[\sigma]\ \Big[\psi[x/\sigma],\psi[y/\sigma]\Big]_+ U^{-1}[\sigma] = iS(x-y;M)\ \ ,$$

which is identical to that of the free field $\psi(x)$. How to reconcile this result with Haag's theorem [38]?

First we remark that the Heisenberg fields are not *kinematically independent fields* [41]. This because $\partial_\mu\boldsymbol{\phi}(x)$ contains $\boldsymbol{\psi}(x)$, and so does $\boldsymbol{\pi}_\phi(y)$. Therefore one has that

$$\Big[\boldsymbol{\pi}_\phi(x),\boldsymbol{\psi}(y)\Big] \neq 0\ . \tag{H49}$$

So, not all of the ETCR's are those of the corresponding free fields, and therefore the Haag theorem does not apply, i.e. the theory is *non-trivial.* Moreover, the ETCR's between the Heisenberg fields will in general not be of the most simple type, but may have Schwinger terms etc. on the rh.s. [41]. Therefore, the operator ring of the Heisenberg fields $\mathcal{P}\left\{\boldsymbol{\chi}_\rho\right\}$ is not isomorf with the operator ring of the asymptotic (free) fields $\mathcal{P}\left\{\chi_\rho\right\}$.
When in a field model for all fields one has the identity $\boldsymbol{\chi}_\rho = \chi_\rho[x/\sigma]$ it is clear that because of Haag's theorem the Heisenberg fields describe a free field and the S-matrix is the unit operator in Hilbert-space.
The same applies to the Vector-coupling model, see e.g. [17]. Calling theories equivalent to free field theories *trivial* [42] we come to the following conclusions [18]:

Conjecture H.3 *The following interaction models are* **non-trivial***:*

(i) Pseudo-vector pseudoscalar interactions (e.g. non-linear chiral-models),

[18] These conclusions do not mean that the *trivial* theories/couplings are not usefull as *effective theories/couplings.* It only means that they are unsuitable as *fundamental theories.* Noteworthy though is that the *non-trivial theories/couplings* are those that are the most succesful experimentally.

(ii) Gauge-theories (QED, QCD, Standard-model),

(iii) Interactions with Pair-suppression.

Conjecture H.4 *The following interaction models are* **trivial***:*

(i) Pseudo-scalar pseudoscalar interactions,

(ii) Scalar interactions without Pair-suppression,

(iii) $\phi^4, \phi^6, \ldots$*-theories.*

This concludes our description of a TU-theory within the framework of the axiomatic BMP-theory, which does not require the existence of a unitary transformation between the Heisenberg- and the asymptotic-fields. (In retrospect, the same holds for the TU-method, see section F 1.)

APPENDIX I: RECONSTRUCTION FIELDS FROM S-MATRIX

As for the literature and more references, we refer here to [41], chapter 7, where this topic is discussed. One makes the following assumptions [19]:

(i) There exist asymptotic fields ϕ^{as} with $as = in, out$, which operators form an *irreducible operator ring* (in the absence of bound states), and obey

$$\left(\Box + m^2\right) \phi^{as} = 0 \ , \tag{I1}$$

and

$$[\phi^{as}(x), \phi^{as}(y)] = -i\Delta(x-y) \ . \tag{I2}$$

(ii) There exists a Poincaré invariant and unitary S-matrix defined by

$$\phi^{out}(x) = S^{-1}\phi^{in}(x) \ S \ . \tag{I3}$$

The program of this section is the deduction of the scalar field theory from these two assumptions.

Starting point: because of the irreducibility of the operator ring $\phi^{in}(x)$, one can express S in the form

$$S = 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \int dx_1 \ldots \int d_n x \ c_n(x_1, \ldots, x_n) \ N\left(\phi^{in}(x_1)\ldots\phi^{in}(x_n)\right) \ . \tag{I4}$$

Of course, this serves merely as a definition of the c-number coefficients c_n. The normal product is chosen in order to have the stability of the vacuum, i.e. $\langle 0|S|0\rangle = 1$. Since

$$N_n \equiv N\left(\phi^{in}(x_1)\ldots\phi^{in}(x_n)\right) \tag{I5}$$

is symmetric in the x_i, one may take also the c_n coefficients as symmetric in the x_i.

[19] As for the literature and more references, we refer here to [41], chapter 7, where this topic is discussed.

1. Source operator and Yang-Feldman equation

First, one defines the source from the S-matrix, see also our exposition of the BMP-theory, as follows

$$j(y) \equiv iS^{-1}\frac{\delta S}{\delta\phi^{in}(y)} \ . \tag{I6}$$

Then,

Theorem I.1 *The Yang-Feldman equation holds*

$$\phi^{out}(x) = \phi^{in}(x) - \int \ \Delta(x-y)j(y)d^4y \ . \tag{I7}$$

Proof: I.1 *Introducing the abbreviation*

$$N_{n,k} \equiv N\left(\phi^{in}(x_1)\dots\phi^{in}(x_{k-1})\phi^{in}(x_{k+1})\dots\phi^{in}(x_n)\right) , \tag{I8}$$

it follows by induction and elementary properties of the normal products that

$$\left[\phi^{in}(x), N_n\right] = \sum_{k=1}^{n}\left[\phi^{in}(x), \phi^{in}(x_k)\right] \ N_{n,k} = -i\sum_{k=1}^{n}\Delta(x-x_k) \ N_{n,k} \ . \tag{I9}$$

Then, from (I4) and (I9)

$$\left[\phi^{in}(x), S\right] \ = \ \sum_{n=1}^{\infty}\frac{-i}{n!}\int\sum_{k=1}^{n}\Delta(x-x_k) \ c_n(x_1,\dots,x_k,\dots,x_n) \ N_{n,k} \ dx_1\dots dx_n \ . \tag{I10}$$

Now, using the symmetry of c_n and N_n, and denoting x_k by y, equation (I10) can be rewritten as

$$\begin{aligned}\left[\phi^{in}(x), S\right] \ &= \ \sum_{n=2}^{\infty}\frac{-i}{(n-1)!}\int dy\Delta(x-y)\int dx_2\dots dx_n \cdot \\ &\quad \times c_n(y,x_2,\dots,x_n) \ N\left(\phi^{in}(x_2)\dots\phi^{in}(x_n)\right) \\ &= \ \sum_{m=1}^{\infty}\frac{-i}{m!}\int dy\Delta(x-y)\int dx_1\dots dx_m \cdot \\ &\quad \times c_{m+1}(y,x_1,\dots,x_m) \ N\left(\phi^{in}(x_1)\dots\phi^{in}(x_m)\right) \ .\end{aligned} \tag{I11}$$

Because of

$$\frac{\delta\phi^{in}(x_k)}{\delta\phi^{in}(y)} = \delta(x_k - y) \ , \tag{I12}$$

one can write

$$\begin{aligned}&\sum_{m=1}^{\infty}\frac{1}{m!}\int dx_1\dots dx_m \ c_{m+1}(y,x_1,\dots,x_m) \ N\left(\phi^{in}(x_1)\dots\phi^{in}(x_m)\right) = \\ &\frac{\delta}{\delta\phi^{in}(y)}\sum_{m=1}^{\infty}\frac{1}{(m+1)!}\int dx_1dx_2\dots dx_{m+1} \ c_{m+1}(x_1,x_2,\dots,x_{m+1}) \\ &\qquad N\left(\phi^{in}(x_1)\phi^{in}(x_2)\dots\phi^{in}(x_{m+1})\right) \ .\end{aligned}$$

Comparison of the r.h.s. with (I4) shows that

$$\sum_{m=1}^{\infty} \frac{1}{m!} \int dx_1 \dots dx_m \; c_{m+1}(y, x_1, \dots, x_m) \; N\left(\phi^{in}(x_1) \dots \phi^{in}(x_m)\right) = \frac{\delta S}{\delta \phi^{in}(y)} \; . \quad \text{(I13)}$$

Hence, equation (I11) assumes the the form

$$\left[\phi^{in}(x), S\right] = -i \int dy \Delta(x-y) \frac{\delta S}{\delta \phi^{in}(y)} \; . \quad \text{(I14)}$$

Furthermore, the definition(I3) of S can be rewritten as

$$\phi^{out}(x) = \phi^{in}(x) + S^{-1}\left[\phi^{in}(x), S\right] \; , \quad \text{(I15)}$$

so that using (I14) yields

$$\phi^{out}(x) = \phi^{in}(x) - \int dy \Delta(x-y) \; \left\{ i S^{-1} \frac{\delta S}{\delta \phi^{in}(y)} \right\} \; . \quad \text{(I16)}$$

In view of the definition of the source (I6) the Yang-Feldman equation (I7) follows, Q.E.D.

2. Derivation *interpolating field* from the S-matrix

For details, we refer again to [41]. One introduces the definition

Definition I.1 *The interpolating field is defined by the Yang-Feldman equation*

$$\phi(x) = \phi^{in}(x) + \int dy \; \Delta_R(x-y) \; j(y) \; , \quad \text{(I17)}$$

with $j(y)$ given by (I6).

Application of the K.G. operator shows immediately that

$$\left(\Box + m^2\right) \phi(x) = j(x) \; , \quad \text{(I18)}$$

which is the usual property of the interacting Heisenberg field.

Now one show that $\phi(x)$ (i) is a scalar field, and (ii) that it satifies the LSZ asymptotic condition [35] rigorously. One introduces first the smeared-out fields

$$\phi^{\alpha}(t) = i \int_{x^0=t} f^*_{\alpha}(x) \; \overleftrightarrow{\partial}_0 \; \phi(x) d^3\mathbf{x} \; , \quad \text{(I19)}$$

and the one-particle annihilation operator is given by

$$a^{in}_{\alpha} = i \int_{x^0=t} f^*_{\alpha}(x) \; \overleftrightarrow{\partial}_0 \; \phi^{in}(x) d^3\mathbf{x} \; , \quad \text{(I20)}$$

Then, it can be shown, using the Riemann-Lebesque lemma on Fourier transforms that

$$\lim_{t \to -\infty} \langle A|\phi^{\alpha}(t)|B\rangle = \langle A|a^{in}_{\alpha}|B\rangle \; . \quad \text{(I21)}$$

For questions concerning the *uniqueness* of $\phi(x)$ we refer again to the discussion given in [41], chapter 7.

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First, we use Dirac spinors with the normalization $u^\dagger(\mathbf{p})\ u(\mathbf{p}) = 2E(\mathbf{p})$, which makes the normalization factors for the mesons and fermions very similar. Secondly, we have a $(-)$-sign in the definition of the M-matrix in relation to the S-matrix,

$$\langle f|S|i\rangle = \delta_{fi} - (2\pi)^4\delta(P_f - P_i)\ \langle f|M|i\rangle\ .$$

This means in Born-Approximation

$$\langle f|M|i\rangle\Big|_{B.A.} = +\langle f|\mathcal{H}_I|i\rangle\ ,$$

and

$$(2\pi)^4\delta(P_f - P_i)\langle f|M|i\rangle = \langle f|\Delta H_I|i\rangle\ .$$

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