Derivative Scalar- and Miyazawa-Fujita-pair Exchange Nucleon-nucleon Potential.

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Abstract

To complete the Extended-Soft-Core (ESC) model, we derive in this note the nucleon-nucleon and hyperon-nucleon potentials due to meson-pairs with $J^{PC} = 0^{++}$, with derivative couplings. In effective chiral field theory models this represents the so called c_3-term. In this note we assume that it is dominated by the Δ_{33}-resonance. This enables the SU(3) generalization to all baryon-baryon channels where the baryons belong to the {8}-irrep, containing N, Λ, Σ, and Ξ, i.e. the $J^{PC} = \frac{1}{2}^+$-states. The potentials are worked out explicitly, and the SU(3)-matrix elements are constructed for all two-baryon channels.

Important applications are: (i) Construction of a new ESC-model, (ii) The SU(3)-generalization of the Fugita-Myazawa threebody forces.

I. INTRODUCTION

To complete the Extended-Soft-Core (ESC) model, we derive in this note the nucleon-nucleon potential due to meson-pairs with $J^{PC} = 0^{++}$, with derivative couplings. The philosophy and technical devices used in this derivation are described in detail in [1, 2]. In connection with the ESC-model we mention the following points:

- In the Nijmegen soft-core models, the form factor is taken to be Gaussian. This means that the form factors do not contain the two-meson contributions accurately, but at best in some mean sense.
- Many unstable boson-exchanges H_j contain in principle also effects from their decay channels. It is especially important to include the long and intermediate range parts in nucleon-nucleon potentials designed for the interactions where $T_{lab} \leq 400$ MeV.
- Meson-baryon resonances R_i, notably the $\Delta_{33}(1236)$-resonance, in low energy BB-channels can be approximated as non-propagating and the corresponding box graphs lead to long range potentials.

As pointed out in [1] all three points are met by the inclusion of the meson-pair vertices. In particularly 2π-exchange effects can be included by the meson-pair potentials, see Fig. 1. Also, when the two-meson contributions are taken care off by the pair interactions, the Gaussian form factors more truly represent the effects of the quark composition composition of the nucleons.

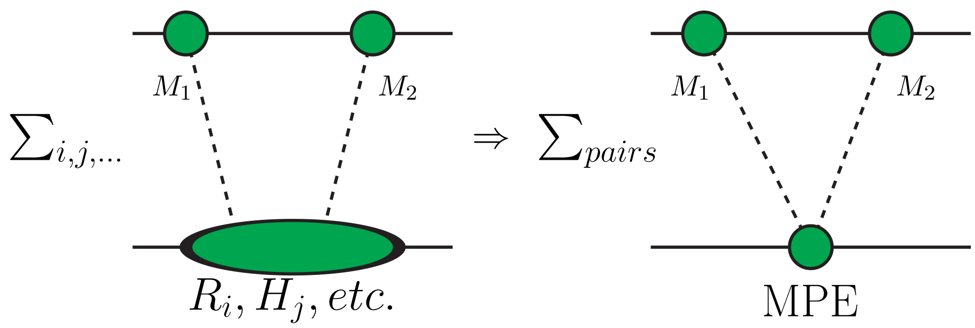

FIG. 1: *Meson-pair description and low-energy approximation.*

As pointed out by Ko and Rudaz [3] besides the most simple Lagrangian $\mathcal{L}^{(0)}_{\sigma\pi\pi} = g_{\sigma\pi\pi}\sigma\boldsymbol{\pi}\cdot\boldsymbol{\pi}$ also the coupling with two derivatives appears in the linear σ-model Lagrangian, which is useful in keeping the scalar meson width's within reasonable bounds as the scalar mass increases. Also, such couplings and the corresponding contribution to the BB-potentials were considered in the context of an $SU_f(3)$ generalization in [4]. This Lagrangian reads $\mathcal{L}^{(1)}_{\sigma\pi\pi} = g'_{\sigma\pi\pi}\sigma(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi})$ Also, such a coupling of the scalar mesons can give an account for the c_3-term in the ($NN2\pi$ effective field-theory interaction Lagrangian [5, 6]

$$\mathcal{L}^{(1)} = -\bar{\psi}\left[8c_1 D^{-1} m_\pi^2 \frac{\boldsymbol{\pi}^2}{F_\pi^2} + 4c_3 \mathbf{D}_\mu \cdot \mathbf{D}^\mu + 2c_4 \sigma_{\mu\nu} \boldsymbol{\tau}\cdot\mathbf{D}^\mu \times \mathbf{D}^\nu\right]\psi\ , \qquad (1.1)$$

where $D = 1 + \boldsymbol{\pi}^2/F_\pi^2$ and $\mathbf{D}_\mu = D^{-1}\partial_\mu\boldsymbol{\pi}/F_\pi$, with $F_\pi = 2f_\pi = 185$ MeV. The c_3-term has been determined in e.g. nucleon-nucleon [9]. Notice that because we use the conventions of [7] there is a minus sign in the c_3-term. Since we have shown elsewhere [8] that tensor-meson exchange can account only 20% of the c_3-coefficient, we assume that the remaining part comes from scalar-meson exchange. This is the motivation for the derivation given in this note of the nucleon-nucleon pair-potentials due to the derivative coupling of a scalar $(\pi\pi)_0$-pair coupling.

Application of the c_3-term NN-potential, with a gaussian cut-off $\Lambda \approx 1$ Gev/c^2, to a fit in nucleon-nucleon, using the ESC-model, reveals that it is impossible to reach a sensible description of the NN-phases when we fix the value at $c_3 = -5\ \text{GeV}^{-1}$, obtained in [9]. This is caused by the large oscillations of this potential below 1 fm. Only by making Λ much smaller it should be possible to use such a potential in the ESC-model. In view of this fact, we analyze an interpretation of this interaction in terms of scalar and diffractive exchange, using an expansion of the πN-amplitude valid for low t-values. It turns out that the c_3-term can be ascribed to a form factor effect in the NN-system. As such this interaction is largely already contained implicitly in the ESC-model.
However, this is inadequate for the description of the three-body NNN force (TBF) using the ESC-model pair-interactions. Namely, it appears not possible to reproduce the Fujita-Miyazawa three-nucleon interaction [10] . Also, for YN- and YNN-forces the SU(3)-structure of the decuplet resonances $\Delta_{33}(1236), \Sigma^*(1385)$, etc. is not yet represented in the ESC-modeling. Therefore, it will be a natural next step to incorporate the c_3-pair interaction, interpreted as coming from the decuplet resonances, in the ESC-model BB-potentials.

In section II we give the $\pi\pi NN$-vertex. In section II-V the consequences for the nucleon-nucleon potential are worked out up the $1/M$-corrections. This is similar to the techniques employed for the other pair potentials, see [1]. In Appendices A and B some useful definitions and results are collected that were not given in the references. In Appendix C we give the small t interpretation of the c_3 interaction, and discuss how it is accomodated by the ESC-model. In Appendix VI the SU(3) structure of the c_3-term in (1.1) is derived. In Appendix VII useful SU(2) Glebsch-Gordon coefficients are listed. In Appexdix E the gauge-invariant $\pi N\Delta_{33}$-coupling is applied to the computation of the Δ_{33}-resonance contribution to the πN-amplitude. The corresponding $c_{1,3}$-coefficients are computed, and the relation with the FM-interaction is given.

II. DERIVATIVE SCALAR-PAIR NN-INTERACTION HAMILTONIANS

We consider the $\pi N \to \pi N$ amplitude and denote the initial and final pion momenta respectively as q and q'. Similarly, the initial and final nucleon momenta are denoted by p and p'. As usual, we introduce the variables

$$P = \frac{1}{2}(p'+p) \quad Q = \frac{1}{2}(q'+q) \ \ , \ \ \Delta = q'-q = p'-p\ . \tag{2.1}$$

The interaction Hamiltonian for the derivative scalar type $(\pi\pi)$-coupling to nucleons reads as follows

$$\mathcal{H}_I^{S2} \ = \ +\frac{g_{S2}}{m_\pi^3}\left(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}\right)\left(\bar{\psi}\psi\right)\ . \tag{2.2}$$

Comparison of (2.2) and (1.1) yields $g_{S2} = +4(c_3 m_\pi)\,(m_\pi/F_\pi)^2 \approx -1.60$.
Remark: check conventions and signs!

For the interaction Hamiltonian (2.2) we get the 'pionic' vertex factor

$$(q'|\partial_\mu\pi\cdot\partial^\mu\pi|q) \Rightarrow 2(q'\cdot q) = 2(Q^2-\frac{1}{4}\Delta^2)\ . \tag{2.3}$$

In the CM-system, we use the customary combinations of nucleon momenta:

$$\mathbf{q} = \frac{1}{2}\left(\mathbf{p}'+\mathbf{p}\right)\ ,\quad \mathbf{k} = (\mathbf{p}'-\mathbf{p}) = \mathbf{k}_1+\mathbf{k_2}\ . \tag{2.4}$$

The nucleon-spinor vertex factor is

$$\begin{aligned}\Gamma(p',p) &= (p'|\left(\bar{\psi}\psi\right)|p) = \bar{u}(p')u(p)\\ &\Rightarrow \sqrt{\frac{(E'+M)(E+M)}{4M^2}}\left[1-\frac{\boldsymbol{\sigma}\cdot\mathbf{p}'\boldsymbol{\sigma}\cdot\mathbf{p}}{(E'+M)(E+M)}\right]\\ &\approx 1-\frac{1}{4M^2}\left[\left(\mathbf{q}^2-\frac{1}{4}\mathbf{k}^2\right)+i\boldsymbol{\sigma}\cdot\mathbf{q}\times\mathbf{k}\right]\approx 1+O\left(\frac{1}{M^2}\right)\ .\end{aligned} \tag{2.5}$$

Here, the reduction to the Pauli-spinors is carried through in the CM-system. In the following we evaluate the potentials up to and including the $1/M$-corrections.

III. DERIVATIVE-SCALAR-PAIR EXCHANGE: 1-PAIR-EXCHANGE

The general formulas for the fourth-order meson-pair exchange kernels are given in [1], equations (2.1) and (2.2). Here, we use the same conventions w.r.t. to the exchanged momenta $k^\mu_{1,2} = (\omega_{1,2},\mathbf{k}_{1,2})$, and the nomenclature of the contributing momentum-space graphs. The connection with the momenta q, q' introduced in section 2 will be given below in this section.
The pair-potentials can be written as

$$\begin{aligned}V^{(n)}_{pair}(\alpha\beta) &= C^{(n)}(\alpha\beta)\ g^{(n)}(\alpha\beta)\ \int\frac{d^3k_1d^3k_2}{(2\pi)^6}\cdot e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{r}}\ F_\alpha(\mathbf{k}_1^2)F_\beta(\mathbf{k}_2^2)\cdot\\ &\quad\times\sum_p\widetilde{O}^{(n)}_{\alpha\beta,p}\left(\mathbf{q};\mathbf{k}_1,\omega_1;\mathbf{k}_2,\omega_2\right)\ D^{(n)}_p(\omega_1,\omega_2)\ .\end{aligned} \tag{3.1}$$

Here, the index n distinguishes the 1-pair ($n=1$) and two-pair ($n=2$) meson-pair exchange, and the index p distinguishes the different time-ordered graphs. The labels $(\alpha\beta)$ refer to the particular pion-pair, or more generally pseudo-scalar meson-pair, that is being exchanged. So, in this note always $\alpha=\pi,\beta=\pi$. For the couplings we have $g^{(1)} = g^{S2}_{(\pi\pi)_0}\ (f_P/m_\pi)^2$ and $g^{(2)} = \left(g^{S2}_{(\pi\pi)_0}\right)^2$, with powers of m_π, depending on the definitions of the Hamiltonians. Note that in this paper the isospin factor $C^{(n)}=6$, see [1], Table I. In the following of these notes we suppress this isospin factor $C^{(n)}$ in the formulas for the potentials.

In the adiabatic approximation, i.e. $E(\mathbf{p}) \approx M$, the energy denominators of the various time-ordered graphs for 1-pair exchange are

$$\begin{aligned} D_a^{(1)}(\omega_1,\omega_2) &= \frac{1}{2\omega_1\omega_2}\frac{1}{\omega_2(\omega_1+\omega_2)} , \\ D_b^{(1)}(\omega_1,\omega_2) &= \frac{1}{2\omega_1^2\omega_2^2} , \\ D_c^{(1)}(\omega_1,\omega_2) &= \frac{1}{2\omega_1\omega_2}\frac{1}{\omega_1(\omega_1+\omega_2)} . \end{aligned} \tag{3.2}$$

A. MTPE, One-Pair-Exchange with Vertex I

The $(NN\pi\pi)$-vertex for $\mathcal{H}_I$ we write as, cfrm. (2.5),

$$\begin{aligned} \bar{u}(p')\Gamma_{S2}^{(2)}u(p) &= \frac{g_{S2}}{m_\pi^3}\, 2g^{\mu\nu}\left(Q_\mu Q_\nu - \frac{1}{4}\Delta_\mu\Delta_\nu\right)\cdot[\bar{u}(p')u(p)] \\ &= 2\frac{g_{S2}}{m_\pi^3}\left(Q^2-\frac{1}{4}\Delta^2\right)\cdot\Gamma(p',p) . \end{aligned} \tag{3.3}$$

The reduction to Pauli-spinors in the CM-system is given above.

To evaluate the contribution from the graphs (a), (b), and (c), we need the Q^μ-vector and the Δ^μ-vector. One has

$$Q^\mu = \frac{1}{2}(q'+q)^\mu = \begin{cases} (a) \ : \ Q^\mu = (-\omega_1+\omega_2, +\mathbf{k}_1-\mathbf{k}_2)/2 \\ (b) \ : \ Q^\mu = (+\omega_1+\omega_2, +\mathbf{k}_1-\mathbf{k}_2)/2 \\ (c) \ : \ Q^\mu = (+\omega_1-\omega_2, +\mathbf{k}_1-\mathbf{k}_2)/2 \end{cases} \tag{3.4}$$

$$\Delta^\mu = (q'-q)^\mu = \begin{cases} (a) \ : \ \Delta^\mu = (+\omega_1+\omega_2, -\mathbf{k}_1-\mathbf{k}_2) \\ (b) \ : \ \Delta^\mu = (-\omega_1+\omega_2, -\mathbf{k}_1-\mathbf{k}_2) \\ (c) \ : \ \Delta^\mu = (-\omega_1-\omega_2, -\mathbf{k}_1-\mathbf{k}_2) \end{cases} \tag{3.5}$$

From (3.4) and (3.5) one deduces

$$q'\cdot q = Q^2 - \frac{1}{4}\Delta^2 = \begin{cases} (a) \ : \ -\omega_1\omega_2+\mathbf{k}_1\cdot\mathbf{k}_2 \\ (b) \ : \ +\omega_1\omega_2+\mathbf{k}_1\cdot\mathbf{k}_2 \\ (c) \ : \ -\omega_1\omega_2+\mathbf{k}_1\cdot\mathbf{k}_2 \end{cases} \tag{3.6}$$

We write the operators in (3.3) factorized as a product of the operator for the 1-pair vertex and an operator for the two single pion couplings, i.e.

$$\begin{aligned} &\widetilde{O}^{(n)}_{\alpha\beta,p}(\mathbf{q};\mathbf{k}_1,\omega_1;\mathbf{k}_2,\omega_2) = \widetilde{O}^{(S2)}_{\alpha\beta,p}\,\widetilde{O}^{(2PS)}_{\alpha\beta} , \quad \widetilde{O}^{(2PS)}_{\alpha\beta} = -\left(\frac{f_P}{m\pi}\right)^2(\mathbf{k}_1\cdot\mathbf{k}_2 - i\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\times\mathbf{k}_2) , \\ &\widetilde{O}^{(S2)}_{\alpha\beta,p} = +2\frac{g_{S2}}{m_\pi^3}(q'\cdot q) = +2\frac{g_{S2}}{m_\pi^3}(\pm\omega_1\omega_2+\mathbf{k}_1\cdot\mathbf{k}_2) . \end{aligned} \tag{3.7}$$

Next, we evaluate the contribution to the pair-vertex summed over the graphs:

$$\sum_{p=a,b,c}[q'(p)\cdot q(p)]\ D_p^{(1)}(\omega_1,\omega_2) \tag{3.8}$$

The different contributions are:

$$1)\qquad \sum_{p=a,b,c} [q_0'(p) q_0(p)] \ \ D_p^{(1)}(\omega_1,\omega_2) = 0 \ ,$$

$$2)\qquad \sum_{p=a,b,c} [\mathbf{q}'(p)\cdot\mathbf{q}(p)] \ \ D_p^{(1)}(\omega_1,\omega_2) = \frac{\mathbf{k}_1\cdot\mathbf{k}_2}{\omega_1^2\omega_2^2} \tag{3.9}$$

The mirror graphs give similar contributions and are included by making the replacement

$$\boldsymbol{\sigma}_1 \rightarrow \frac{1}{2}\left(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2\right)\ , \tag{3.10}$$

everywhere in the matrix elements. Collecting now all results, and selecting the symmetric, and taking into account that the pair vertex can be left and right in the 1-pair graphs, one finally obtains the contribution

$$\sum_p \widetilde{O}_{\pi\pi}^{(S2,1)}(\mathbf{k}_1,\omega_1;\mathbf{k}_2,\omega_2) D_p^{(1)}(\omega_1,\omega_2) =$$

$$-2\left(\frac{g_{S2}}{m_\pi^3}\right)\left(\frac{f_P}{m_\pi}\right)^2 (\mathbf{k}_1\cdot\mathbf{k}_2)\left\{\mathbf{k}_1\cdot\mathbf{k}_2-\frac{i}{2}\left(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2\right)\cdot\mathbf{k}_1\times\mathbf{k}_2\right\}\cdot\frac{1}{\omega_1^2\omega_2^2}\ . \tag{3.11}$$

The Fourier transformation leads to the potential

$$\begin{aligned} V_{ad}^{(S2,1)}(r) &= -2\left(\frac{g_{S2}}{m_\pi^3}\right)\left(\frac{f_P}{m_\pi}\right)^2 \int \frac{d^3k_1 d^3k_2}{(2\pi)^6}\ e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{r}}\ F_1(\mathbf{k}_1^2)F_2(\mathbf{k}_2^2)\cdot(\mathbf{k}_1\cdot\mathbf{k}_2)^2\cdot\frac{1}{\omega_1^2\omega_2^2} \\ &= -2\left(\frac{g_{S2}}{m_\pi^3}\right)\left(\frac{f_P}{m_\pi}\right)^2 \lim_{\mathbf{r}_1,\mathbf{r}_2\rightarrow\mathbf{r}} (\boldsymbol{\nabla}_1\cdot\boldsymbol{\nabla}_2)^2 I_{2,\pi}(m_\pi,r_1) I_{2,\pi}(m_\pi,r_2) \\ &= -2\left(\frac{g_{S2}}{m_\pi^3}\right)\left(\frac{f_P}{m_\pi}\right)^2 \left[\frac{2}{r^2} I_{2,\pi}'(m_\pi,r_1) I_{2,\pi}'(m_\pi,r_2) + I_{2,\pi}''(m_\pi,r_1) I_{2,\pi}''(m_\pi,r_2)\right] \end{aligned} \tag{3.12}$$

Notice that the term in the integrand odd under the interchange $\mathbf{k}_1 \leftrightarrow \mathbf{k}_2$ vanishes upon integration. For a definition of the function $I_{2,\pi}(r)$, and the lateron used $I_{2,\pi}(r)$ and $F_\pi(r)$, we refer to [2, 11].
The non-local central potential, coming from the $-(\mathbf{q}^2+\mathbf{k}^2/4)/4M^2$-piece in (2.5) is $\varphi(r) = -(2M_{red}/(4M^2)\ V_{ad}^{(S_2)}(r)$, where $\varphi(r)$ is defined as in Eq. (35) of [12]. We expect that $\varphi(r) < 0.05$. (The non-local spin-orbit is negected in the Nijmegen work.) The $\mathbf{k}^2/8M^2$-piece (2.5) leads to corrections via the extra derivatives $[\boldsymbol{\nabla}_1^2 + 2\boldsymbol{\nabla}_1\cdot\boldsymbol{\nabla}_2 + \boldsymbol{\nabla}_2^2]$. This leads to contributions from the third- and fourth-order derivatives of the $\phi_C^0(m_\pi,r)$-functions and are (presumably) small in view of the $m_\pi^2/8M^2$-coefficient.

IV. $1/M$ CORRECTIONS

A. Non-adiabatic contributions

The nonadiabatic corrections from the $1/M$ expansion of the energy denominators is explained in Ref. [2] and also used in [1], section IV. The expansion of the denominators gives an extra momentum dependent factor in the numerator, which can be rewritten as

$$\left[\mathbf{k}_1\cdot\mathbf{k}_2 - \mathbf{q}\cdot(\mathbf{k}_1-\mathbf{k}_2)\right]/2M\ . \tag{4.1}$$

The one-pair energy denominators become

$$\begin{aligned} D_a^{na}(\omega_1,\omega_2) &= \frac{1}{2\omega_1\omega_2}\frac{1}{\omega_2^2(\omega_1+\omega_2)} , \\ D_b^{na}(\omega_1,\omega_2) &= \frac{1}{2\omega_1\omega_2}\left(\frac{1}{\omega_1^2\omega_2}+\frac{1}{\omega_1\omega_2^2}\right), \\ D_c^{na}(\omega_1,\omega_2) &= \frac{1}{2\omega_1\omega_2}\frac{1}{\omega_1^2(\omega_1+\omega_2)} . \end{aligned} \tag{4.2}$$

Notice that all $D_p^{na}(\omega_1,\omega_2)$ are symmetric in $\mathbf{k}_1$ and $\mathbf{k}_2$. Therefore, only the terms symmetric in the exchanged momenta will survive the k_1, k_2 integrations.
The numerator is a product of (i) the nonadiabatic momentum dependent factor, alluded to above, (ii) the factor in (3.7), and (iii) the $2q'\cdot q$-factor worked out in (3.6). Taking all this into account, we arrive at the nonadiabatic correction

$$\begin{aligned} V_{na}^{(S2,1)}(r) &= -2\frac{g_{S2}}{m_\pi^3}\left(\frac{f_P}{m_\pi}\right)^2 \cdot \frac{1}{2M}\cdot \int \frac{d^3k_1 d^3k_2}{(2\pi)^6}\ e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{r}}\ F_1(\mathbf{k}_1^2)F_2(\mathbf{k}_2^2)\cdot \\ &\times\left[(\mathbf{k}_1\cdot\mathbf{k}_2)^2+\frac{i}{2}\left(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2\right)\cdot\mathbf{k}_1\times\mathbf{k}_2\ \mathbf{q}\cdot(\mathbf{k}_1-\mathbf{k}_2)\right]\cdot \\ &\times\sum_p \widetilde{O}_{\pi\pi,p}^{(S2,1)}\ D_p^{na}(\omega_1,\omega_2) , \end{aligned} \tag{4.3}$$

where

$$\widetilde{O}_{\pi\pi,p}^{(S2,1)} = \pm\omega_1\omega_2+\mathbf{k}_1\cdot\mathbf{k}_2 . \tag{4.4}$$

Here for $p=a,c$ the $(-)$-sign and for $p=b$ the $(+)$-sign applies. Using (4.2) and (4.4) the sum over the graphs in (4.3) is easily performed and yields

$$\sum_p \widetilde{O}_{\pi\pi,p}^{(S2,1)}\ D_p^{na}(\omega_1,\omega_2) = \frac{1}{\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}+\frac{1}{\omega_1^2\omega_2^2}\left[\frac{1}{\omega_1}+\frac{1}{\omega_2}-\frac{1}{\omega_1+\omega_2}\right]\ (\mathbf{k}_1\cdot\mathbf{k}_2) . \tag{4.5}$$

The potential (4.3) can now be worked our further, giving for $\alpha=\pi, \beta=\pi$

$$\begin{aligned} V_{na,C}^{(S2,1)}(r) &= -\frac{g_{S2}}{m_\pi^3}\left(\frac{f_P}{m_\pi}\right)^2\cdot\frac{1}{M}\cdot\lim_{r_1,r_2\to r}\left[\left(\frac{2}{r^2}\frac{\partial^2}{\partial r_1\partial r_2}+\frac{\partial^2}{\partial r_1^2}\frac{\partial^2}{\partial r_2^2}\right)B_{1,1}(r_1,r_2)\right. \\ &\left.-\left\{\frac{6}{r^2}\left(\frac{\partial^2}{\partial r_1^2}-\frac{1}{r_1}\frac{\partial}{\partial r_1}\right)\left(\frac{\partial^2}{\partial r_2^2}-\frac{1}{r_2}\frac{\partial}{\partial r_2}\right)+\frac{\partial^3}{\partial r_1^3}\frac{\partial^3}{\partial r_2^3}\right\}B_{\pi\pi}^{na}(r_1,r_2)\right] \\ V_{na,SO}^{(S2,1)}(r) &= -\frac{g_{S2}}{m_\pi^3}\left(\frac{f_P}{m_\pi}\right)^2\cdot\frac{1}{M}\cdot\frac{1}{r^2}\lim_{r_1,r_2\to r}\left[2\frac{\partial^2 B_{1,1}}{\partial r_1\partial r_2}(r_1,r_2)\right. \\ &\left.-\left(2\frac{\partial^2}{\partial r_1^2}\frac{\partial^2}{\partial r_2^2}-\frac{1}{r_1}\frac{\partial}{\partial r_1}\frac{\partial^2}{\partial r_2^2}-\frac{1}{r_2}\frac{\partial}{\partial r_2}\frac{\partial^2}{\partial r_1^2}\right)B_{\pi\pi}^{na}(r_1,r_2)\right] . \end{aligned} \tag{4.6}$$

The functions $B_{m,n}$ and $B_{\alpha\beta}^{na}$ are defined in Appendix A.

B. Pseudo-vector vertex contributions

The pseudovector vertex gives $1/M$-terms as can be seen from

$$\bar{u}(\mathbf{p}')\Gamma_P^{(1)}u(\mathbf{p}) = -i\frac{f_P}{m_\pi}\left[\boldsymbol{\sigma}\cdot(\mathbf{p}'-\mathbf{p})\pm\frac{\omega}{2M}\boldsymbol{\sigma}\cdot(\mathbf{p}'+\mathbf{p})\right] , \tag{4.7}$$

where upper (lower) sign applies for creation (absorption) of the pion at the vertex. For graph (a) the operator for the nucleon line on the right is readily seen to be

$$-\left(\frac{f_P}{m_\pi}\right)^2 \frac{1}{2M}\left[\left(\omega_1\mathbf{k}_2^2-\omega_2\mathbf{k}_1^2\right)-2\mathbf{q}\cdot(\omega_1\mathbf{k}_2+\omega_2\mathbf{k}_1)+2i\boldsymbol{\sigma}_2\cdot\mathbf{q}\times(\omega_1\mathbf{k}_2-\omega_2\mathbf{k}_1)\right] \tag{4.8}$$

The same expression for gragh (b) is obviously obtained from (4.7) by making the the substitution $\omega_1 \to -\omega_1$, and for graph (c) the substitution $\omega_{1,2} \to -\omega_{1,2}$. The mirror graphs are included by making again the replacement given in (3.10). Combining all this with the denominators $D_i^1(\omega_1,\omega_2)$ in (3.2) gives the following $1/M$-corrections

$$\begin{aligned} V_{pv}^{(S2,1)}(r) &= -\frac{g_{S2}}{m_\pi^3}\left(\frac{f_P}{m_\pi}\right)^2\cdot\frac{1}{M}\cdot\int\frac{d^3k_1d^3k_2}{(2\pi)^6}F_1(\mathbf{k}_1^2)F_2(\mathbf{k}_2^2)\cdot e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{r}}\ \cdot \\ &\times\frac{1}{\omega_1\omega_2\,(\omega_1+\omega_2)}\left[\left\{m_\pi^2(\omega_1^2+\omega_2^2)-2\omega_1^2\omega_2^2\right\}-(\mathbf{k}_1^2+\mathbf{k}_2^2)(\mathbf{k}_1\cdot\mathbf{k}_2)\right. \\ &\left.-\,i\,(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{q}\times\mathbf{k}\left(\omega_1^2+\omega_2^2+\mathbf{k}_1\cdot\mathbf{k}_2\right)\right]\ . \end{aligned} \tag{4.9}$$

Using the Fourier transforms given in A, one obtains

$$\begin{aligned} V_{pv,C}^{(S2,1)}(r) &= -\frac{g_{S2}}{m_\pi^3}\left(\frac{f_P}{m_\pi}\right)^2\cdot\frac{1}{M}\cdot\lim_{r_1,r_2\to r}\left[\left\{m_\pi^2\,(B_{-1,1}+B_{1,-1})-2B_{-1,-1}\right\}(r_1,r_2)+\right. \\ &\left.+\frac{\partial^2}{\partial r_1\partial r_2}\left\{B_{-1,1}+B_{1,-1}-2m_\pi^2B_{1,1}\right\}\right]\ , \\ V_{pv,SO}^{(S2,1)}(r) &= +\frac{g_{S2}}{m_\pi^3}\left(\frac{f_P}{m_\pi}\right)^2\cdot\frac{1}{M}\cdot\frac{2}{r}\frac{d}{dr}\lim_{r_1,r_2\to r}\left[(B_{-1,1}+B_{1,-1})-\frac{\partial^2}{\partial r_1\partial r_2}B_{1,1}\right](r_1,r_2) \end{aligned} \tag{4.10}$$

V. DERIVATIVE SCALAR MESON-PAIR EXCHANGE: 2-PAIR-EXCHANGE

There are 2 two-pair exchange diagrams, cfrm. [1], and we designate the left nucleon line by the suffix a and the right nucleon line by the suffix b. Then, for the two-pair graph with the lower vertex on line (a) and the upper vertex on line (b).

$$(q'\cdot q)_a=(q'\cdot q)_b=-\omega_1\omega_2+\mathbf{k}_1\cdot\mathbf{k}_2\ . \tag{5.1}$$

The energy denominators, summed over the 2 graphs give

$$D^{(2)}(\omega_1,\omega_2)\ =\ -\frac{1}{2\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}\ . \tag{5.2}$$

$$\begin{aligned} V_{ad,C}^{(S2,2)}(r) &= -\left(\frac{g_{S2}}{m_\pi^3}\right)^2\cdot\int\frac{d^3k_1d^3k_2}{(2\pi)^6}\ e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{r}}\ F_1(\mathbf{k}_1^2)F_2(\mathbf{k}_2^2)\cdot(-\omega_1\omega_2+\mathbf{k}_1\cdot\mathbf{k}_2)^2\ D^{(2)}(\omega_1,\omega_2) \\ &= -\frac{1}{2}\left(\frac{g_{S2}}{m_\pi^3}\right)^2\cdot\lim_{r_1,r_2\to r}\left\{B_{-1,-1}(r_1,r_2)+2(\boldsymbol{\nabla}_1\cdot\boldsymbol{\nabla}_2)B_{0,0}(r_1,r_2)+(\boldsymbol{\nabla}_1\cdot\boldsymbol{\nabla}_2)^2B_{1,1}(r_1,r_2)\right\} \end{aligned} \tag{5.3}$$

Here, the functions $B_{n,m}(r_1,r_2)$ and the $(\boldsymbol{\nabla}_1\cdot\boldsymbol{\nabla}_2)$-operations are given in Appendix B.

There are no $1/M$-corrections to the 2-pair contribution to the potentials.

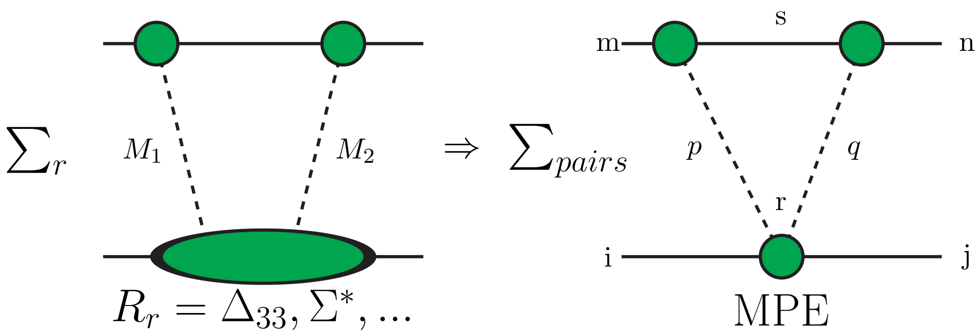

FIG. 2: *Meson-pair description Δ_{33}-induced BB-interaction*

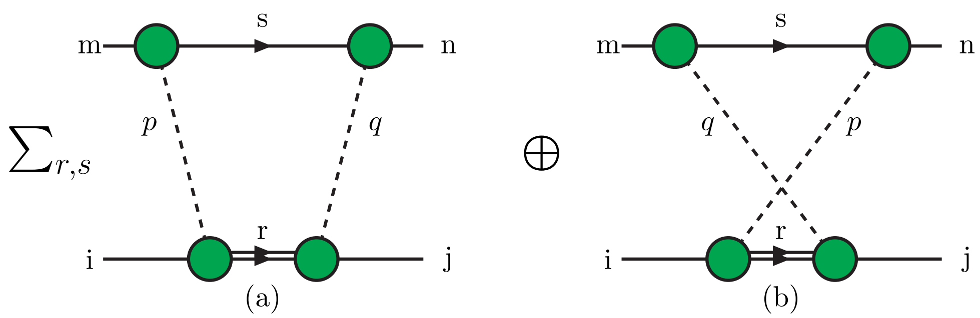

FIG. 3: *One-pair SU(3) matrix elements $M_{1-pair}(j,n;i,m) = M^{(a)}(j,n;i,m) + M^{(b)}(j,n;i,m)$*

VI. SU(3) STRUCTURE Δ_{33} INTERPRETATION c_3-TERM

In this appendix we work out the SU(3)-structure coefficients for the one and two pair-approximation to the Δ_{33}-induced BB-interaction, see Fig. 3. The interaction Lagrangian for the coupling of the SU(3)-decuplet $\{\kappa_1\} = \{10^*\}$ to the SU(3)-octets for the baryons $\{\kappa_2\} = \{8\}$ and the mesons $\{\mu\} = \{8\}$ is [15]

$$\mathcal{L}_{int}(x) = \sum_{\nu_1\nu_2\nu_3\gamma} g_\gamma \begin{pmatrix} \kappa_1 & \kappa_2^* & \mu & \\ & & & \gamma \\ \nu_1 & \nu_2 & -\nu_3 & \end{pmatrix} \cdot \begin{pmatrix} \mu^* & \mu & 1 \\ & & \\ -\nu_3 & \nu_3 & 0 \end{pmatrix} \cdot \bar{\chi}^{(\kappa_1)}_{-\nu_1}(x)\ \psi^{(\kappa_2)}_{\nu_2}(x) \phi^{(\mu)}_{\nu_3}(x). \quad (6.1)$$

Here, $\kappa_1 = \{10\}$, $\kappa_2 = \{8\}$, and $\mu = \{8\}$. Furthermore,

$$\begin{pmatrix} \mu & \mu^* & 1 \\ & & \\ \nu & -\nu & 0 \end{pmatrix} = (-)^{I_3+I_H+(Y+Y_H)/2}/\sqrt{N_\mu} \Rightarrow -\frac{1}{2\sqrt{2}}(-)^{Q(\nu_3)}. \quad (6.2)$$

So,

$$\mathcal{L}_{int}(x) \;\Rightarrow\; \frac{1}{2\sqrt{2}} \sum_{\nu_1\nu_2\nu_3\gamma} g_\gamma \begin{pmatrix} \kappa_1 & \kappa_2^* & \mu & \\ & & & \gamma \\ \nu_1 & \nu_2 & -\nu_3 & \end{pmatrix} \cdot \begin{pmatrix} \mu & \mu^* & 1 \\ & & \\ \nu_3 & -\nu_3 & 0 \end{pmatrix} \cdot$$

$$\times(-)^{Q(\nu_1)}(-)^{Q(-\nu_3)}\ \bar{\chi}^{(\kappa_1)}_{-\nu_1}(x)\ \psi^{(\kappa_2)}_{\nu_2}(x)\phi^{(\mu)}_{\nu_3}(x). \tag{6.3}$$

This Lagrangian is invariant under transformations $g \in SU(3)$ i.e.

$$U[g]\ \mathcal{L}_{int}\ U^{-1}[g] = \mathcal{L}_{int}. \tag{6.4}$$

Within an SU(3)-irrep each state can be tranformed into any other state within the irreps. Therefore, (6.4) means for matrix elements

$$(U[g]f'|\mathcal{L}_{int}|U[g]f) = (f'|\mathcal{L}_{int}|f)\,. \tag{6.5}$$

Below, we evaluate the matrix elements for the coupling of the decuplet $\Delta_{33}, \Sigma^*, ...$ $\{10^*\}$-states to the meson-baryon $\{8 \times 8\}$-states.

The coefficients for the vertices of the meson-baryon coupling to the SU(3)-decuplet are given by [16]

$$d^*(r; i, p) \equiv (r|\mathcal{L}_{int}(0)|i, p)\,, \tag{6.6}$$

Here, r $(1, .., 10)$ denotes the decuplet states, i, p $(1, ..., 8)$ denote the baryon and meson octet states. The 10 decuplet states and their coupling to the $\{8\} \otimes \{8\}$-states are given in Ref. [19], Table 3.4. This makes the construction of the ten 8x8-mtrices $d^*(r; i, p)$ in principle straightforward. However, this is rather tricky because of convention-sensitivity, and therefore we prefer here to use the octet-model representation, *i.e.* SU(3)/Z(3) representation, where the $\{10^*\} \oplus \{10\}$-states can be represented by a symmetric tensor $S_{kl}, k, l = 1, 8$ [17]. Here, including a factor $1/\sqrt{2}$ to avoid double counting, the decuplet $\{10\}$ couplings to $\{8\} \otimes \{8\}$-states we describe by

$$\Gamma^*(k, l; i, p) \equiv (k, l|\mathcal{L}_{int}(0)|i, p)\,/\sqrt{2}. \tag{6.7}$$

Then, the SU(3) matrix elements for the 1-pair graph in Fig. 2 is given by the sum of the graph (a) and (b) in Fig. 2, where the index s runs over the 8 octet baryon states. (Notice that the meson lines have no direction due to the fact that the meson fields are hermitean, i.e. $\phi_i^\dagger = \phi_i$.) Similarly for the 2-pair matrix element, where now the line with index s in Fig. 3 runs over the 10 decuplet irrep $\{10\}$-states. So,

$$M_{1-pair}(j, n; i, m) \;=\; \sum_{p,q,s=1}^{8} \sum_{k,l=1}^{8} \left[O(s; m, p)\ O(n; s, q) + O(s; m, q)\ O(n; s, p)\right] \cdot$$

$$\times \left[\bar{\Gamma}^*(kl; j, q)\right]\ \left[\Gamma^*(kl; i, p)\right], \tag{6.8a}$$

$$M_{2-pair}(j, n; i, m) \;=\; \sum_{p,q}^{8} \sum_{kl,rt=1}^{8} \left[\bar{\Gamma}^*(kl; j, q)\right] \left[\Gamma^*(kl; i, p)\right] \cdot \left[\bar{\Gamma}^*(rt; n, q)\right] \left[\Gamma^*(rt; m, p)\right] \tag{6.8b}$$

Here, $\bar{\Gamma}^*$ are the complex conjugates, and for pseudoscalars

$$O(s; m, p) \;=\; -i\alpha_P\ f_{s,m,p} + (1 - \alpha_P)\ d_{s,m,p}, \tag{6.9}$$

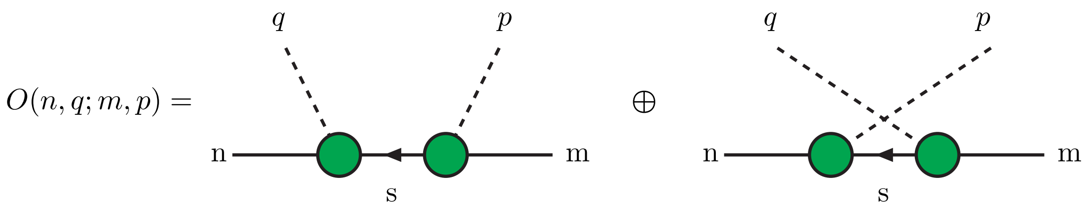

FIG. 4: *Meson-baryon SU(3) matrix elements* $O(n,q;m,p)$

where $f_{s,m,p}$ and $d_{s,m,p}$ are the su(3)-algebra structure constants.
Using (6.9) and some identities for the structure constants [17, 18] we can work out the 1-pair matrix element further. Denoting $a = \alpha_P$ and $b = 1 - \alpha_P$ we have, $\sum_s$ is understood,

$$
\begin{aligned}
\mathcal{O}(n,q;m,p) &\equiv O(s;m,p)O(n;s,q) + O(s;m,q)O(n;s,p) \\
&= \left[-ia\ f_{smp} + b\ d_{smp}\right]\left[-ia\ f_{nsq} + b\ d_{nsq}\right] + \\
&\quad \left[-ia\ f_{smq} + b\ d_{smq}\right]\left[-ia\ f_{nsp} + b\ d_{nsp}\right] \\
&= -a^2\left(f_{smp}\ f_{nsq} + f_{smq}\ f_{nsp}\right) + b^2\left(d_{smp}\ d_{nsq} + d_{smq}\ d_{nsp}\right) \\
&\quad -iab\ \left(f_{smp}\ d_{nsq} + d_{smp}\ f_{nsq} + f_{smq}\ d_{nsp} + d_{smq}\ f_{nsp}\right)
\end{aligned}
$$

1. ad ab-term: Using identity [Ditt15] [18] we have

$$
\begin{aligned}
\Big(\ \ldots\ \Big) &= -\left(f_{mps}\ d_{nqs} + f_{mqs}\ d_{pms}\right) = +f_{mns}\ d_{pqs}, \\
&\quad -\left(f_{nqs}\ d_{mps} + f_{nps}\ d_{qns}\right) = r -_{mns}\ \ d_{pqs},
\end{aligned}
$$

which means that the ab-term vanishes.

2. ad a^2-term: Using identity [Ditt24] [18]

$$
\begin{aligned}
f_{mps}\ f_{nqs} &= \frac{2}{3}\left(\delta_{mn}\delta_{pq} - \delta_{mq}\delta_{np}\right) + d_{mns}\ d_{pqs} - d_{mqs}\ d_{nps}, \\
f_{mqs}\ f_{nps} &= \frac{2}{3}\left(\delta_{mn}\delta_{pq} - \delta_{mp}\delta_{nq}\right) + d_{mns}\ d_{pqs} - d_{mps}\ d_{nqs},
\end{aligned}
$$

Summing these terms we get for the a^2-term

$$
\begin{aligned}
-\Big(\ \ldots\ \Big) &= \frac{2}{3}\left[2\delta_{mn}\delta_{pq} - \delta_{mq}\delta_{np} - \delta_{mp}\delta_{nq}\right] \\
&\quad +2d_{mns}\ d_{pqs} - \left\{d_{mqs}\ d_{nps} + d_{mps}\ d_{nqs}\right\}
\end{aligned}
$$

The identity [Ditt23] [18] gives

$$
\left\{d_{mqs}\ d_{nps} + d_{mps}\ d_{nqs}\right\} = \frac{1}{3}\left(\delta_{mq}\delta_{np} + \delta_{mn}\delta_{pq} + \delta_{mp}\delta_{nq}\right) - d_{mns}\ d_{pqs},
$$

which gives upon substitution in the a^2-term

$$a^2 \ : \ [\delta_{mn}\delta_{pq} - \delta_{mq}\delta_{np} - \delta_{mp}\delta_{nq}) + 3d_{mns}\ d_{pqs},$$

3. ad b^2-term: Using identity [Ditt23] [18]

$$+\Big(\ldots\Big) \ = \ \frac{1}{3}\left[\delta_{mp}\delta_{nq} + \delta_{mn}\delta_{pq} + \delta_{mq}\delta_{np}\right] - d_{mns}\ d_{pqs}.$$

Collecting the results we obtain

$$\begin{aligned}
\mathcal{O}(n,q;m,p) &= a^2\left\{[\delta_{mn}\delta_{pq} - \delta_{mq}\delta_{np} - \delta_{mp}\delta_{nq}] + 3d_{mns}\ d_{pqs}\right\} \\
&+\frac{1}{3}b^2\left\{[\delta_{mn}\delta_{pq} + \delta_{mq}\delta_{np} + \delta_{mp}\delta_{nq}] - 3d_{mns}\ d_{pqs}\right\} = \\
&\left(a^2+\frac{1}{3}b^2\right)\ \delta_{mn}\delta_{pq} - \left(a^2-\frac{1}{3}b^2\right)\ \left\{(\delta_{mp}\delta_{nq} + \delta_{mq}\delta_{np}) - 3d_{mns}\ d_{pqs}\right\}
\end{aligned} \tag{6.10}$$

For $\alpha_P = 2/5$ we have

$$\begin{aligned}
a^2 + \frac{1}{3}b^2 &= \frac{1}{3}\left(4\alpha_P^2 - 2\alpha_P + 1\right) \to \frac{7}{25}, \\
a^2 - \frac{1}{3}b^2 &= \frac{1}{3}\left(2\alpha_P^2 + 2\alpha_P - 1\right) \to \frac{1}{25}.
\end{aligned}$$

A. Interpretation: t-channel exchange SU(3)-irreps

From the symmetry in the pseudoscalar labels m and n, and the baryon labels p and q, it is clear that from the t-channel exchange viewpoint only the SU(3)-irreps $\{1\}, \{8_s\}$, and $\{27\}$ are involved. Following [17] we multiply with the octet vectors a_p and a_q which gives:

a) $(a^2+b^2/3)$-term: $\delta_{mn}[\sum_p a_p a_p]$ which is a $\{1\}$-irrep.
b) $(a^2-b^2/3)$-term:

$$\begin{aligned}
&3(a^2-b^2/3)\left\{\frac{2}{3}a_m a_n - d_{mns}(a*a)_s\right\} = \\
&2(a^2-b^2/3)\left\{\left[a_m a_n - \frac{3}{5}d_{mns}(a*a)_s - \frac{1}{8}\delta_{mn}(\sum_p a_p a_p)\right]\right. \\
&\left.-\frac{9}{10}d_{mns}(a*a)_s + \frac{1}{8}\delta_{mn}(\sum_p a_p a_p)\right\} \Rightarrow \\
&2(a^2-b^2/3)\left\{[M_{27}]_{mn} - \frac{9}{10}\,[M_{8_s}]_{mn} + \frac{1}{8}\,[M_1]_{mn}\right\}.
\end{aligned} \tag{6.11}$$

Conclusion: *the SU(3) generalization of the c_3-term in the πN interaction Lagrangian, using Δ_{33}-dominance, leads to the $\{1\}_t-, \{8_s\}_t-$, and $\{27\}_t$-irrep scalar derivative pair-interaction.*

VII. SU(3) STRUCTURE MPE FROM OCTET BARYON RESONANCES

In this appendix we work out the SU(3)-structure coefficients for the one and two pair-approximation to the baryon resonance induced BB-interactions, similarly to the graphs of Fig. 2. Here we evaluate the matrix elements for the coupling of the baryon-resonance octet states to the meson-baryon $\{8 \times 8\}$-states.

The coefficients for the vertices of the meson-baryon coupling to the SU(3)-octet are given by [16]

$$\Gamma^*(k,l;i,p) \equiv (r|\mathcal{L}_{int}(0)|i,p)\,/\sqrt{2}. \tag{7.1}$$

Here, k,l $(1,..,8)$ give the decuplet states, and i,p $(1,...,8)$ denote the baryon and meson octet states. Then, the SU(3) matrix elements for the 1-pair graph in Fig. 2 is given by the sum of the graph (a) and (b) in Fig. 2, where the index s runs over the 8 octet baryon states. (Notice that the meson lines have no direction due to the fact that the meson fields are hermitean, i.e. $\phi_i^\dagger = \phi_i$.) Similarly for the 2-pair matrix element, where now the line with index s in Fig. 2 runs over the 8 octet states. So,

$$\begin{aligned} M_{1-pair}(j,n;i,m) \;&=\; \sum_{p,q,s=1}^{8}\sum_{k,l=1}^{8} \left[O(s;m,p)\ O(n;s,q) + O(s;m,q)\ O(n;s,p)\right]\cdot \\ &\qquad\qquad \times\left[\bar{\Gamma}^*(k,l;j,q)\right]\ \left[\Gamma^*(k,l;i,p)\right], &(7.2a)\\ M_{2-pair}(j,n;i,m) \;&=\; \sum_{p,q}^{8}\sum_{k,l,s,t=1}^{8} \left[\bar{\Gamma}^*(k,l;j,q)\right]\left[\Gamma^*(k,l;i,p)\right]\cdot \\ &\qquad\qquad \times\left[\bar{\Gamma}^*(s,t;n,q)\right]\left[\Gamma^*(s,t;m,p)\right]. &(7.2b)\end{aligned}$$

Here, $\bar{\Gamma}^*$ are the complex conjugates, and again for pseudoscalars

$$O(s;m,p) \;=\; -i\alpha_P\ f_{s,m,p} + (1-\alpha_P)\ d_{s,m,p}, \tag{7.3}$$

where $f_{s,m,p}$ and $d_{s,m,p}$ are the su(3)-algebra structure constants.
The rest of the construction is completely the same as in Appendix VI.

A. SU(3)/Z$_3$ Octet-model states

The baryon and meson states are given in Table I, see [19]. From Table I one readily finds the physical states in terms of the octet-base states B_i, P_i. We have

$$\Sigma^+ = (\psi_1 - i\psi_2)/\sqrt{2}\ ,\ \pi^+ = (\phi_1 - i\phi_2)/\sqrt{2}, \tag{7.4a}$$
$$\Sigma^- = (\psi_1 + i\psi_2)/\sqrt{2}\ ,\ \pi^- = (\phi_1 + i\phi_2)/\sqrt{2}, \tag{7.4b}$$
$$\Sigma^0 = \psi_3\ ,\ \pi^0 = \phi_3, \tag{7.4c}$$
$$p = (\psi_4 - i\psi_5)/\sqrt{2}\ ,\ K^+ = (\phi_4 - i\phi_5)/\sqrt{2}, \tag{7.4d}$$
$$\Xi^- = -(\psi_4 + i\psi_5)/\sqrt{2}\ ,\ K^- = (\phi_4 + i\phi_5)/\sqrt{2}, \tag{7.4e}$$
$$n = (\psi_6 - i\psi_7)/\sqrt{2}\ ,\ K^0 = (\phi_6 - i\phi_7)/\sqrt{2}, \tag{7.4f}$$
$$\Xi^0 = (\psi_6 + i\psi_7)/\sqrt{2}\ ,\ \bar{K}^0 = (\phi_6 + i\phi_7)/\sqrt{2}, \tag{7.4g}$$
$$\Lambda = \psi_8\ ,\ \eta_8 = \phi_8. \tag{7.4h}$$

TABLE I: Wave functions of the Baryon and meson octet representations in terms of physical components.

states	$\underline{\psi(i=1-8)}$	state	$\underline{\phi(i=1-8)}$
$B_1=\psi_1$	$(\Sigma^+ + \Sigma^-)/\sqrt{2}$	$P_1=\phi_1$	$(\pi^+ + \pi^-)/\sqrt{2}$
$B_2=\psi_2$	$i(\Sigma^+ - \Sigma^-)/\sqrt{2}$	$P_2=\phi_2$	$i(\pi^+ - \pi^-)/\sqrt{2}$
$B_3=\psi_3$	Σ^0	$P_3=\phi_3$	π^0
$B_4=\psi_4$	$(p-\Xi^-)/\sqrt{2}$	$P_4=\phi_4$	$(K^+ + K^-)/\sqrt{2}$
$B_5=\psi_5$	$i(p+\Xi^-)/\sqrt{2}$	$P_5=\phi_5$	$i(K^+ - K^-)/\sqrt{2}$
$B_6=\psi_6$	$(n+\Xi^0)/\sqrt{2}$	$P_6=\phi_6$	$(K^0+\bar{K}^0)/\sqrt{2}$
$B_7=\psi_7$	$i(n-\Xi^0)/\sqrt{2}$	$P_7=\phi_7$	$i(K^0-\bar{K}^0)/\sqrt{2}$
$B_8=\psi_8$	Λ	$P_8=\phi_8$	η_8

For the isospin states in terms of the particle states we use the Condon-Shortley phase convention. This implies for the proper isospinor states of the K-on and anti-K-on spinors

$$K = \left(\begin{array}{c} K^+ \\ K^0 \end{array}\right), \quad \bar{K} = i\tau_2 K^\dagger = \left(\begin{array}{c} \bar{K}^0 \\ -K^- \end{array}\right), \tag{7.5}$$

B. Computation $\{8\}\otimes\{8\}-\{10\}$-coupling Vertices in SU(3)/$\mathbf{Z}_3$

Representing the octet meson and baryon states by 8-dimensional vectors with components a_m and b_i respectively, we split the meson-baryon wave function components as follows in a symmetric and antisymmetric part

$$a_m b_i = \frac{1}{2}\left(a_m b_i + a_i b_m\right) + \frac{1}{2}\left(a_m b_i - a_i b_m\right) \equiv S_{mi} + A_{mi} \tag{7.6}$$

Then

$$\begin{aligned} S_{ij} &= \left[\frac{1}{8}\delta_{ij}\delta_{kl} + \frac{3}{5}d_{ijm}d_{mkl} + \frac{1}{2}\left\{(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})\right.\right. \\ &\left.\left. -\frac{6}{5}d_{ijp}d_{pmn}\left(\delta_{mk}\delta_{nl}+\delta_{ml}\delta_{nk}\right) - \frac{1}{4}\delta_{ij}\delta_{kl}\right\}\right]\ S_{kl}, \qquad (7.7a) \\ A_{ij} &= \left[\frac{1}{3}f_{ijm}f_{mkl} + \frac{1}{2}\left\{(\delta_{ik}\delta_{jl}-\delta_{il}\delta_{jk}) - \frac{2}{3}f_{ijp}f_{pmn}\left(\delta_{mk}\delta_{nl}-\delta_{ml}\delta_{nk}\right)\right\}\right] A_{kl}. \qquad (7.7b) \end{aligned}$$

Here, A_{ij} is splitted into the irreps $\{8_a\}$ and $\{10\}\oplus\{10^*\}$, and similarly S_{ij} into the irreps $\{1\}$, $\{8_s\}$, and $\{27\}$.
For the vertex $\Gamma^*(k,l;i,m)$ connecting the meson-baryon state $|i,m\rangle$ to the $\{10\}$-state

$|10;k,l\rangle$ one has

$$\Gamma^*[k,l;i,m] \;=\; \frac{1}{2}\left(\delta_{ki}\delta_{lm}-\delta_{km}\delta_{li}\right)-\frac{1}{6}\sum_{r=1}^{8} f_{klr}f_{rst}\left(\delta_{si}\delta_{tm}-\delta_{sm}\delta_{ti}\right) \tag{7.8}$$

Then, the s-channel Δ_{33}-resonance contribution to the meson-baryon matrix element for $|i,m\rangle \to |j,n\rangle$ is given by

$$M\left(j,n;i,m\right) \;=\; \sum_{k,l=1}^{8} \Gamma^*[k,l;j,n]\;\Gamma^*[k,l;i,m] \tag{7.9}$$

The evaluation of the 4 terms in this product is straightforward:

1. $\frac{1}{4}\sum_{k,l=1}^{8}\left(\delta_{kj}\delta_{ln}-\delta_{kn}\delta_{lj}\right)\left(\delta_{ki}\delta_{lm}-\delta_{km}\delta_{li}\right)=\frac{1}{2}\left[\delta_{ij}\delta_{mn}-\delta_{in}\delta_{jm}\right],$
2. $-\frac{1}{12}\sum_{k,l=1}^{8}\left(\delta_{kj}\delta_{ln}-\delta_{kn}\delta_{lj}\right)f_{klr'}f_{r's't'}\left(\delta_{s'i}\delta_{t'm}-\delta_{s'm}\delta_{t'i}\right)=-\frac{1}{3}\sum_{r=1}^{8}f_{imr}f_{jnr},$
3. $-\frac{1}{12}\sum_{k,l=1}^{8}f_{klr}f_{rst}\left(\delta_{sj}\delta_{tn}-\delta_{sn}\delta_{tj}\right)\left(\delta_{ki}\delta_{lm}-\delta_{km}\delta_{li}\right)=-\frac{1}{3}\sum_{r=1}^{8}f_{imr}f_{jnr},$
4. $+\frac{1}{36}\sum_{k,l=1}^{8}f_{klr}f_{rst}\left(\delta_{sj}\delta_{tn}-\delta_{sn}\delta_{tj}\right)f_{klr'}f_{r's't'}\left(\delta_{s'i}\delta_{t'm}-\delta_{s'm}\delta_{t'i}\right)=+\frac{1}{3}\sum_{r=1}^{8}f_{imr}f_{jnr},$

which gives

$$M\left(j,n;i,m\right) \;=\; \frac{1}{2}\left[\delta_{ij}\delta_{mn}-\delta_{in}\delta_{jm}\right]-\frac{1}{3}\sum_{r=1}^{8}f_{imr}f_{jnr}, \tag{7.10}$$

and

$$\langle M_f B_f|M|B_i M_m\rangle = \sum_{j,n=1}^{8}\sum_{i,m=1}^{8} a_n^* b_j^* M\left(j,n;i,m\right)\; a_m b_i. \tag{7.11}$$

The three-body force due to the Δ_{33}-resonance is depicted in the right panel (b) of Fig. 5. In the "effective two-body potential" the "third nucleon" is summed and integrated over. This has the consequence that for the "third-nucleon" the pseudoscalars do not couple. Therefore, in matter the nucleon line, or in general the baryon line, with the Δ_{33}-resonance is integrated out for a non-zero contribution. Then, for symmetric baryonic matter, the relevant pion-nucleon operator in panel (b) is

$$\sum_{i,i'=1}^{8} M\left(i',n;i,m\right)\;\delta_{i'i} \;=\; \frac{5}{2}\;\delta_{mn}. \tag{7.12}$$

This is very useful for making SU(3) checks on the "effective two-body" NN, YN, and YY potentials.

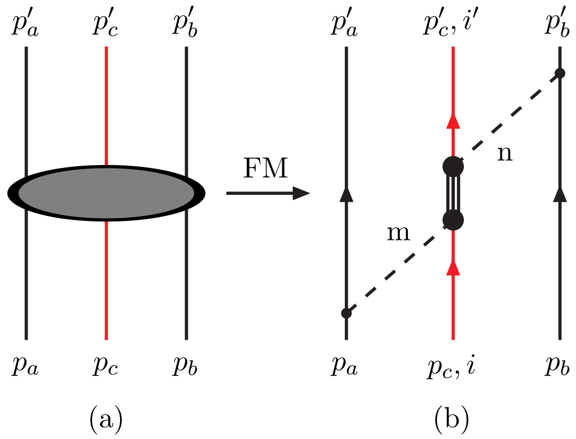

FIG. 5: *FM Three-particle amplitude from Δ_{33}-resonance.*

VIII. FUJITA-MIYAZAWA AMPLITUDE

The expressions in (E14) should be compared to those in Fugita-Miyazawa [10] Δ_{33} amplitude for πN, and with the c_3-pair term in the Lagrangian.

$$\mathcal{L}_{FM} = +\bar{\psi}\left[\left\{\left((A+B)\boldsymbol{\nabla}_1\cdot\boldsymbol{\nabla}_2 + D\right)\delta_{ij} - (A-B)\,\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}_1\times\boldsymbol{\nabla}_2\;\epsilon_{ijk}\tau_k\right\}\cdot \right.$$
$$\left.\times\,\pi_{1,i}(x)\pi_{2,j}(x)\right]\;\psi, \qquad (8.1)$$

where

$$A = \frac{5}{18\pi^2}\int \frac{\sigma_{33}}{\omega_p^2}dp, \quad B = \frac{3}{5}A, \quad D = \frac{2\pi}{3}(a_1+2a_3), \qquad (8.2)$$

with the numerical values $J \equiv \int dp\;\sigma_{33}/\omega_p^2 = 3.7m_\pi^{-3}$, and $a_1+2a_3 = -0.06m_\pi^{-1}$. This implies that $A,B>0$ and $D<0$ and for the ratio $(D/A)_{FM} \approx -0.4m_\pi^2$.

Remark: *The D-term represents s-wave πN scattering at low energies. This is not given by the Δ_{33}-resonance, but by the the nucleon s- and u- exchange diagrams, $\pi\pi$-pair diagrams, plus others. In the Miyazawa paper the D-term comes from the subtraction term in the πN dispersion relations due to Oehme.*
So, the D-term could be omitted when calculating the $\pi\pi$-pair interaction for the two-body NN-, YN-, and YY-potentials. Inclusion of the D-term implies merely a shift in the $\pi\pi$-pair coupling constants. We include the D-term also in the two-body potential because it is convenient to refer always to the complete FM-interaction!
Furthermore, the SU3-generalization does not apply to the subtraction terms!

Comparing the isoscalar part with the pair-interaction

$$\mathcal{H}_I^{(S_2)} = +\frac{g_{S_2}}{m_\pi^3}\;\bar{\psi}\left[\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}\right]\;\psi$$

we have $g_{S_2} = m_\pi^3\,(A+B) \approx 0.167$, and $g_{S_2} = -m_\pi D \approx 0.126$.

To cover **completely** *the δ_{ij}-term in (8.1) we need the additional pair interaction*

$$\mathcal{H}_I^{(S_1)} = +\frac{\Delta g_{S_1}}{m_\pi} \left[\bar{\psi}\psi\right] \cdot (\boldsymbol{\pi} \cdot \boldsymbol{\pi})$$

with $\Delta g_{S_1} = g_{S_2} + m_\pi D \approx 0.04$.

In momentum space the FM Lagrangian gives, $(M_{FM})_{ij} = -\langle p', q'|\mathcal{L}_{FM}(0)|p, q\rangle$,

$$\begin{aligned}(M_{FM})_{ij} &= -\bar{u}(p', s') \left[\left((A+B)\ \mathbf{q}' \cdot \mathbf{q} - D\right)\ \delta_{ij}\right. \\ &\qquad \left. - (A-B)\ (\mathbf{q} \times \mathbf{q}' \cdot \boldsymbol{\sigma})\ \epsilon_{ijk}\tau_k\right]\ u(p, s). \end{aligned} \tag{8.3}$$

Comparison with (E14) gives for case I, i.e. $s = M^2$,

$$A + B = \frac{3\bar{g}_{GI}^2}{m_\pi^2 \Delta M} \left(1 + \frac{3m}{2M} - \frac{3m^2}{4M^2}\right), \tag{8.4a}$$

$$D = -\frac{3\bar{g}_{GI}^2}{m_\pi^2 \Delta M} \left(\frac{3m^2}{8M^2}\right) m^2. \tag{8.4b}$$

For case II, i.e. $s = (M+m)^2$,

$$A + B = \frac{2\bar{g}_{GI}^2}{3m_\pi^2 \Delta M} \left(1 + \frac{7m}{4M} - \frac{m^2}{2M^2}\right), \tag{8.5a}$$

$$D = -\frac{2\bar{g}_{GI}^2}{3m_\pi^2 \Delta M} \left(1 - \frac{7m}{4M} + \frac{5m^2}{2M^2}\right) m^2. \tag{8.5b}$$

We note that the ratio $(D/A)_I \ll (D/A)_{FM}$ and $(D/A)_{II} \sim (D/A)_{FM}$ showing that case II is the more appropriate approximation.

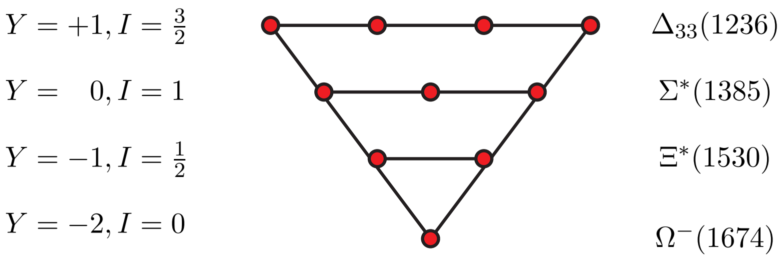

FIG. 6: *Contents SU(3) $\{10\}$-irrep.*

A. SU(3)-generalization Fujita-Miyazawa amplitude

The SU(3)-structure of the three-body amplitude in Fig. 7 is

$$M(m,l,n;i,k,j) = \sum_{p,q=1}^{8} O(m:k,p)\ O(n;l,q)\ \left[\sum_{r=1}^{10} \bar{d}^*(r;l,q)\ d^*(r;k,p)\right] \tag{8.6}$$

$$= \sum_{p,q=1}^{8} O(m:k,p)\ O(n;l,q)\ \left[\sum_{k,l=1}^{8} \bar{\Gamma}^*(k,l;l,q)\ \Gamma^*(k,l;k,p)\right] \tag{8.7}$$

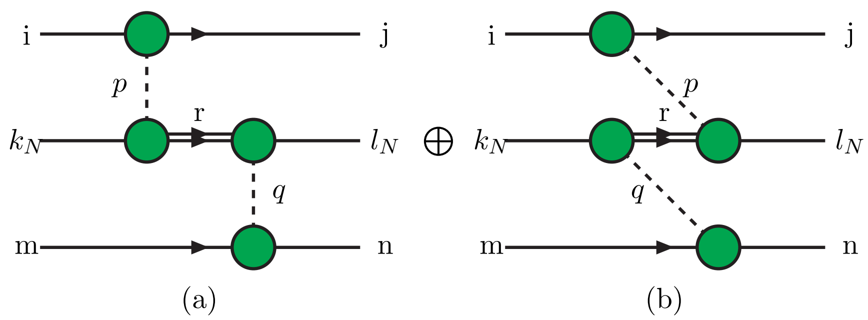

FIG. 7: *Three-body FM-graphs*

B. BB effective FM-potential in Nuclear Matter

The effective BB FM-potential in nuclear matter is obtained by integrating out the "third-nucleon". The resulting two-body potential we obtain by imposing charge and spin conservation for that nucleon is that for *symmetric matter*. This implies that only the Δ_{33}-excitation on the "third nucleon" will contribute because of the pseudoscalar character of the exchanged mesons. Using the notation $k_N = k$ and $l_N = l$ to denote the SU(3)-indices of the "third nucleon". We have for the effective two-body SU(3) matrix elements for protons and neutrons respectively

$$\bar{M}_p(p,q) = \sum_{k_N,l_N=4}^{5} \sum_{r,s=1}^{8} \bar{\Gamma}^*(r,s;l_N,q)\ \Gamma^*(r,s;k_N,p), \tag{8.8a}$$

$$\bar{M}_n(p,q) = \sum_{k_N,l_N=6}^{7} \sum_{r,s=1}^{8} \bar{\Gamma}^*(r,s;l_N,q)\ \Gamma^*(r,s;k_N,p), \tag{8.8b}$$

which gives, for N=p or N=n,

$$O_N(j,n;i,m) = O(j;i,p)\ O(n;m,q)\ \bar{M}_N(p,q), \tag{8.9a}$$

$$O(m;i,p) = 2\left[i\alpha_P\ f_{mip} + (1-\alpha_P)\ d_{mip}\right]. \tag{8.9b}$$

For nuclear matter we have

$$O_{NM} = x_p O_p + x_n O_n. \tag{8.10}$$

In case of symmetric matter $x_p = x_n = 1/2$, and for neutron matter $x_p = 0, x_n = 1$.

Footnote: Note that the π, η and K couplings to the "third nucleon" do not contribute in a nuclear medium becuase of the sum over the spins of this nucleon.

The matrix element between two BB-states is given by

$${}_f\langle BB|V_{FM}|BB\rangle_i \ = \ \frac{1}{4}\sum_{jn}\sum_{im} Z(\hat{j},\hat{\imath})\ Z(\hat{n},\hat{m})\ O_N(j,n;i,m)\ V_{FM}(r). \tag{8.11}$$

where the wave-function factors Z(...) are described in [28], and $\hat{\imath} = i - i_0$ etc are defined such that $\hat{\imath} = 1, 2$.
To illustrate the computation of the SU(3) matrix elements we give as an example the $\Sigma^+ n$ matrix element. Using the states defined in (7.4) we get

$$\begin{aligned}
\langle\Sigma^+ n|M_{FM}|\Sigma^+ n\rangle &= \frac{1}{4}\sum_{i,j,m,n=1}^{8}\sum_{p=1}^{8}\langle\psi_1 - i\psi_2|\psi_j\rangle\langle\psi_6 - i\psi_7|\psi_n\rangle\cdot \\
&\quad\times\langle\psi_j\psi_n|M_{FM}|\psi_i\psi_m\rangle\langle\psi_i|\psi_1 - i\psi_2\rangle\langle\psi_m|\psi_1 - i\psi_2\rangle \\
&= \frac{1}{4}\sum_{i,j,m,n=1}^{8}\sum_{p=1}^{8}(\delta_{1j} + i\delta_{2j})(\delta_{6n} + i\delta_{7n})(\delta_{1j} - i\delta_{2j})(\delta_{6n} - i\delta_{7n})\langle\psi_j\psi_n|M_{FM}|\psi_i\psi_m\rangle \\
&= \frac{1}{4}\sum_{i,j=1}^{2}\sum_{m,n=6}^{7}\sum_{p=1}^{8}\ Z(j,i)Z(n-5,m-5)\langle\psi_j\psi_n|M_{FM}|\psi_i\psi_m\rangle,
\end{aligned} \tag{8.12}$$

where Z denotes the 2x2 matrix

$$Z = \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}. \tag{8.13}$$

Acknowledgements

Discussions with Y. Yamamoto, M.M. Nagels, H.-J. Schulze, and S. Ishikawa are gratefully aknowledged.

APPENDIX A: INTEGRAL REPRESENTATIONS

We employ throughout these notes the integral representations introduced in [2] and used in [1, 11]. Special cases needed and not covered so far we give in this appendix.
From the inegral identity

$$\frac{1}{\omega_1 + \omega_2} = \frac{2}{\pi}\int_0^\infty d\lambda\ \frac{\lambda^2}{(\omega_1^2 + \lambda^2)(\omega_2^2 + \lambda^2)} \tag{A1}$$

Supplying the Gaussian form factors, the Fourier transform $B_{0,0}$ of (A1) is, see e.g. [2] for the complete procedure,

$$B_{0,0}(r_1, r_2) = \frac{2}{\pi} \int_0^\infty \lambda^2 d\lambda\ F_\alpha(\lambda, m_1, r_1) F_\beta(\lambda, m_2, r_2)\ . \tag{A2}$$

Using (A1) one easily derives

$$\frac{1}{\omega_1^2} \frac{1}{\omega_1 + \omega_2} = \frac{2}{\pi} \int_0^\infty d\lambda\ \left[\frac{1}{\omega_1^2} - \frac{1}{\omega_1^2 + \lambda^2}\right] \frac{1}{\omega_2^2 + \lambda^2} \tag{A3}$$

Again, adding the Gaussian form factors, the Fourier transformation yields

$$\Rightarrow \frac{2}{\pi} \int_0^\infty d\lambda\ \Big[I_2(m_1, r_1) - F_\alpha(\lambda, m_1, r_1)\Big] \cdot F_\beta(\lambda, m_2, r_2)\ . \tag{A4}$$

Here, the functions $I_2(m, r)$ and $F_\alpha(\lambda, m, r)$ have been defined in e.g. [11], eq. (4.5). $I_2(m, r) = (m/4\pi)\phi_C^0(mr)$ the basic generalized Gauss-Yukawa function for meson exchange with a gaussian form factor. Using (A3)-(A4) one easily derives the Fourier transform

$$\begin{aligned}\left(\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}\right) \frac{1}{\omega_1 + \omega_2}\ \Rightarrow\ \frac{2}{\pi} \int_0^\infty d\lambda\ \Big[& I_{2,\alpha}(m_1, r_1) F_\beta(\lambda, m_2, r_2) + F_\alpha(\lambda, m_1, r_1) I_{2,\beta}(m_2, r_2) \\ & - 2F_\alpha(\lambda, m_1, r_1)\ F_\beta(\lambda, m_2, r_2)\Big]\ .\end{aligned} \tag{A5}$$

The integral representation

$$\frac{\omega_1^2}{\omega_2^2} \frac{1}{\omega_1 + \omega_2} = \frac{2}{\pi} \int_0^\infty d\lambda\ \left[1 - \frac{\lambda^2}{\omega_1^2 + \lambda^2}\right] \left[\frac{1}{\omega_2^2} - \frac{1}{\omega_2^2 + \lambda^2}\right] \tag{A6}$$

From the identity (A6) one derives the Fourier transform for the left hand side as being given by the expression

$$\begin{aligned}\Rightarrow\ & \frac{2}{\pi} \int_0^\infty d\lambda\ \Big\{ \left[I_{0,\alpha}(m_1, r_1) - \lambda^2 F_\alpha(\lambda, m_1, r_1)\right] \cdot \left[I_{2,\beta}(m_2, r_2) - F_\beta(\lambda, m_2, r_2)\right] \Big\} \\ \equiv\ & B_{-2,2}(r_1, r_2)\end{aligned} \tag{A7}$$

Similarly,

$$\frac{\omega_2^2}{\omega_1^2} \frac{1}{\omega_1 + \omega_2} \quad \text{F.T.}\ \Rightarrow\ B_{2,-2}(r_1, r_2) \tag{A8}$$

Notice that we have introduced the (non-local) Fourier transforms

$$\frac{1}{\omega_1^m \omega_2^n} \frac{1}{\omega_1 + \omega_2} \quad \text{F.T.}\ \Rightarrow B_{m,n}(r_1, r_2)\ . \tag{A9}$$

In particular, quite often appears

$$B_{1,1}(r_1, r_2) = \frac{2}{\pi} \int_0^\infty d\lambda\ F_\alpha(\lambda, m_1, r_1)\ F_\beta(\lambda, m_2, r_2)\ . \tag{A10}$$

In the non-adiabatic $1/M$-correction computation, we encountered the denominator

$$D^{na} = \frac{1}{\omega_1^2\omega_2^2}\left[\frac{1}{\omega_1}+\frac{1}{\omega_2}-\frac{1}{\omega_1+\omega_2}\right] \ , \tag{A11}$$

which gives upon F.T. the coordinate space function

$$B^{na}_{\alpha\beta}(r_1,r_2) \ = \ \frac{2}{\pi}\int_0^\infty \frac{d\lambda}{\lambda^2}\left[I_2(m_\alpha,r_1)I_2(m_\beta,r_2) - F_\alpha(\lambda,r_1)F_\beta(\lambda,r_2)\right] \ . \tag{A12}$$

APPENDIX B: DIFFERENTIATION DICTIONARY

We employ throughout these notes the integral representations introduced in [2] and used in [1, 11]. Special cases needed and not covered so far we give in this appendix.

$$\begin{aligned}
&(i) \quad \lim_{r_1,r_2\to r}\frac{1}{2}\left(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2\right)\cdot\mathbf{q}\times\left(\boldsymbol{\nabla}_1+\boldsymbol{\nabla}_2\right)\ F(r_1)G(r_2) \Rightarrow -\frac{1}{r}\frac{d}{dr}\left[F(r)G(r)\right]\ \mathbf{L}\cdot\mathbf{S},\\
&(ii) \quad \lim_{r_1,r_2\to r}\frac{1}{2}\left(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2\right)\cdot\mathbf{q}\times\left(\boldsymbol{\nabla}_1+\boldsymbol{\nabla}_2\right)\ \left(\boldsymbol{\nabla}_1\cdot\boldsymbol{\nabla}_2\right)\ F(r_1)G(r_2) \Rightarrow -\frac{1}{r}\frac{d}{dr}\left[F'(r)G'(r)\right]\ \mathbf{L}\cdot\mathbf{S},\\
&(iii) \quad \lim_{r_1,r_2\to r}\frac{1}{2}\left(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2\right)\cdot\boldsymbol{\nabla}_1\times\boldsymbol{\nabla}_2\ \mathbf{q}\cdot\left(\boldsymbol{\nabla}_1-\boldsymbol{\nabla}_2\right)\ F(r_1)G(r_2) \Rightarrow -\frac{2}{r^2}F'G'\ \mathbf{L}\cdot\mathbf{S},\\
&(iv) \quad \lim_{r_1,r_2\to r}\frac{1}{2}\left(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2\right)\cdot\boldsymbol{\nabla}_1\times\boldsymbol{\nabla}_2\ \mathbf{q}\cdot\left(\boldsymbol{\nabla}_1-\boldsymbol{\nabla}_2\right)\ \left(\boldsymbol{\nabla}_1\cdot\boldsymbol{\nabla}_2\right)\ F(r_1)G(r_2) \Rightarrow\\
&\quad -\left[\frac{1}{r}\left(F_1'G_1+F_1G_1'\right)+2F_1'G_1'\right]\ \mathbf{L}\cdot\mathbf{S} = -\frac{1}{r^2}\left[\left(F''-\frac{1}{r}F'\right)\ G''+F''\ \left(G''-\frac{1}{r}G'\right)\ \right]\ \mathbf{L}\cdot\mathbf{S}
\end{aligned} \tag{B1}$$

Here $F_1 = F(r_1)/r_1$, $F_1' = \left(F''-F'/r\right)/r$, and similarly for G_1, G_1'.
The $B_{m,n}$-functions have the generic form

$$B_{m,n}(r_1,r_2) = \frac{2}{\pi}\int_0^\infty d\lambda\ \sum_k F_\alpha^{(k)}(\lambda,r_1)\ G_\beta^{(k)}(\lambda,r_2) \ , \tag{B2}$$

where F and G are I_n or $F_\alpha(\lambda,r)$ functions. From this it is clear that taking the partial derivatives and the limit $\mathbf{r}_1,\mathbf{r}_2\to\mathbf{r}$ give functions of the form

$$\frac{\partial^{p+q}}{\partial r_1^p\partial r_2^q}B_{m,n}(r_1,r_2) \ \Rightarrow\ \frac{2}{\pi}\int_0^\infty d\lambda\ \sum_k\frac{d^pF_\alpha^{(k)}}{dr^p}(\lambda,r)\frac{d^qG_\beta^{(k)}}{dr^q}(\lambda,r) \ . \tag{B3}$$

One can verify easily that

$$\begin{aligned}
&(i) \quad \lim_{r_1,r_2\to r}\left(\boldsymbol{\nabla}_1\cdot\boldsymbol{\nabla}_2\right)B_{m,n}(r_1,r_2) = \frac{2}{\pi}\int_0^\infty d\lambda\ \sum_k\left[\frac{dF_\alpha^{(k)}}{dr}\frac{dG_\beta^{(k)}}{dr}\right](\lambda,r) \ ,\\
&(ii) \quad \lim_{r_1,r_2\to r}\left(\boldsymbol{\nabla}_1\cdot\boldsymbol{\nabla}_2\right)^2B_{m,n}(r_1,r_2) = \frac{2}{\pi}\int_0^\infty d\lambda\ \sum_k\left[\frac{2}{r^2}\frac{dF_\alpha^{(k)}}{dr}\frac{dG_\beta^{(k)}}{dr}+\frac{d^2F_\alpha^{(k)}}{dr^2}\frac{d^2G_\beta^{(k)}}{dr^2}\right](\lambda,r)
\end{aligned} \tag{B4}$$

APPENDIX C: INTERPRETATION c_1 AND c_3-INTERACTION

1. The πN-amplitude from the c_1- and c_3-term: The pair-interaction Hamiltonians corresponding to c_1 and c_3 in (1.1) are of the form

$$\mathcal{H}_I^{S1} = +\frac{g_{S1}}{m_\pi}\left(\boldsymbol{\pi}\cdot\boldsymbol{\pi}\right)\left(\bar{\psi}\psi\right) \quad , \quad \mathcal{H}_I^{S2} = +\frac{g_{S2}}{m_\pi^3}\left(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}\right)\left(\bar{\psi}\psi\right) \ . \tag{C1}$$

The lowest order contribution to the πN-amplitude is

$$\mathcal{M} = \frac{2}{m_\pi}\left[g_{S_1} + g_{S_2}\frac{q'\cdot q}{m_\pi^2}\right] = \frac{2}{m_\pi}\left(g_{S_1}+g_{S_2}\right) - \frac{g_{S_2}}{m_\pi}\frac{t}{m_\pi^2} \ , \tag{C2}$$

where q and q' are respectively the initial and final pion momenta, and $t = (q'-q)^2$.

2. The πN-amplitude from σ and P-exchange: The $\pi\pi$-coupling of the σ and pomeron are defined by

$$\mathcal{L}_{\sigma\pi\pi} = \frac{1}{2}g_{\sigma\pi\pi}m_\pi\left(\sigma\boldsymbol{\pi}\cdot\boldsymbol{\pi}\right) \quad , \quad \mathcal{L}_{P\pi\pi} = \frac{1}{2}g_{P\pi\pi}m_\pi\left(P\boldsymbol{\pi}\cdot\boldsymbol{\pi}\right) \ . \tag{C3}$$

Then, in terms of the width $g_{\sigma\pi\pi}$ is given by the formula $g^2_{\sigma\pi\pi}/4\pi = 2(m_\sigma^2/m_\pi^2)(\Gamma_\sigma/p)$, where $p = \sqrt{m_\sigma^2 - 4m_\pi^2}/2$. In Born-Approximation one has

$$\mathcal{M} = m_\pi\left[\frac{g_{\sigma\pi\pi}g_{\sigma NN}}{t-m_\sigma^2}e^{a_\sigma t} + \frac{g_{P\pi\pi}g_{PNN}}{\mathcal{M}^2}e^{a_P t}\right] \ , \tag{C4}$$

From $c_1 \approx 0$, one has

$$-\frac{g_{\sigma\pi\pi}g_{\sigma NN}}{m_\sigma^2} + \frac{g_{P\pi\pi}g_{PNN}}{\mathcal{M}^2} \approx 0 \ . \tag{C5}$$

Introducing the expansion

$$\mathcal{M} = \mathcal{M}^{(0)} + \mathcal{M}^{(1)}t + \dots \ , \tag{C6}$$

for low t gives, using (C4) and (C5), that the term linear in t of the πN-amplitude is given by

$$\begin{aligned}\mathcal{M}^{(1)} &\approx m_\pi\frac{g_{\sigma\pi\pi}g_{\sigma NN}}{m_\sigma^2}\left(a_P - a_\sigma - \frac{1}{m_\sigma^2}\right) \\ &= m_\pi\frac{g_{\sigma\pi\pi}g_{\sigma NN}}{m_\sigma^2}\left(\frac{1}{4m_P^2} - \frac{1}{\Lambda_\sigma^2} - \frac{1}{m_\sigma^2}\right) \\ &\approx -m_\pi\frac{g_{\sigma\pi\pi}g_{\sigma NN}}{m_\sigma^2}\frac{1}{\Lambda_\sigma^2} \ .\end{aligned} \tag{C7}$$

Here, we used $m_P \approx 310$ MeV and $m_\sigma \approx 2m_P$.

3. Conjecture: Assuming now that $\mathcal{M}^{(1)}$ corresponds to the c_3-term in (1.1), and so also to the pair-interaction (2.2), one has the relation

$$\frac{g_{S_2}}{4\pi} \approx +\frac{m_\pi^4}{m_\sigma^2\Lambda_\sigma^2}\frac{g_{\sigma\pi\pi}g_{\sigma NN}}{4\pi} \ . \tag{C8}$$

In order to answer the question whether equation (C8) is a reasonable one, we use the value for g_{S_2} given in section II, and from [13] the estimates

$$m_\sigma^2 \approx 30 m_\pi^2, \quad \frac{g_{\sigma\pi\pi} g_{\sigma NN}}{4\pi} \approx 25 \ . \tag{C9}$$

Then, we obtain $\Lambda_\sigma \approx 2.56 m_\pi = 360$ MeV. Now we must realize that

$$\frac{1}{\Lambda_\sigma^2} = \frac{1}{\Lambda_{\sigma\pi\pi}^2} + \frac{1}{\Lambda_{\sigma NN}^2} \tag{C10}$$

Assuming that the cut-off's for the $\sigma\pi\pi$- and σNN-vertex are approximately equal, we obtain from (C10)

$$\Lambda_{\sigma\pi\pi} \approx \Lambda_{\sigma NN} \approx \sqrt{2}\Lambda_\sigma = 510 \text{ MeV} \ . \tag{C11}$$

This is a perfectly acceptable value for these cut-off parameters. Therefore, we conclude that the c_3-term in the Lagrangian (1.1) finds a natural explanation in σ and P-exchange. It can be considered as due to the first term in the low t expansion of the form factor.

The upshot of this analysis is that in dynamical models for low energy πN the c_3-term must not be included. Inclusion of this term with the strength as estimated in [9] would mean 'double counting'.

4. Note on c_1: From C1 and 1.1 one easily finds that

$$g_{S_1} = 8(c_1 m_\pi)\left(\frac{m_\pi}{F_\pi}\right)^2 \tag{C12}$$

Using the result of Ref. [9] $c_1 = -0.76 \pm 0.07$, one gets $g_{S_1} = -0.43 \pm 0.04$. This would imply a very large $(\pi\pi)_0$-pair interaction in the ESC-model, much larger as found in fitting the data using the ESC-model. Also, a consequence in ΛN would be that the attraction would mainly come from the region beyond $r = 1$ fm, which is in conflict with the studies of Λ-hypernuclei [14] The natural explanation is also here that indeed the effective c_1 is mainly produced by σ and P exchange.

5. Miscelaneous relations Indroducing the dimensionless parameters $\widetilde{c}_1 =: c_1/\mathcal{M}$ and $\widetilde{c}_3 =: c_1/\mathcal{M}$, where $\mathcal{M} = 1$ GeV/c^2, one readily finds in Born-approximation for the contribution to the πN-amplitude

$$\mathcal{M} = \frac{1}{m_\pi}\left(\frac{m_\pi}{F_\pi}\right)^2 \left\{8\left(2\tilde{c}_1 - \tilde{c}_3\right) + 4\tilde{c}_3 \frac{t}{m_\pi^2}\right\} \tag{C13}$$

The low t-expansion of (C4), see also (C7), gives upon comparison with (C13) the relations

$$\begin{aligned} 2\tilde{c}_1 - \tilde{c}_3 &= \frac{m_\pi^2}{8}\frac{M}{m_\pi}\left(\frac{F_\pi}{m_\pi}\right)^2 \left[-\frac{g_{\sigma\pi\pi} g_{\sigma NN}}{m_\sigma^2} + \frac{g_{P\pi\pi} g_{PNN}}{\mathcal{M}^2}\right] \\ \tilde{c}_3 &= \frac{m_\pi^2}{4}\frac{M}{m_\pi}\left(\frac{F_\pi}{m_\pi}\right)^2 \left[-\frac{g_{\sigma\pi\pi} g_{\sigma NN}}{m_\sigma^2}\left(\frac{m_\pi^2}{m_\sigma^2} + \frac{m_\pi^2}{\Lambda_\sigma^2}\right) + \frac{g_{P\pi\pi} g_{PNN}}{\mathcal{M}^2}\frac{m_\pi^2}{4m_P^2}\right] \ . \end{aligned} \tag{C14}$$

Using these formulas one can compute the contribution to $c_{1,3}$ for the $f_0(760)$- and $f_0(980)$-states.

6. Application to Nucleon-nucleon: The $(\pi\pi)_0$- and $(\pi\pi)_1$-pair interactions in nucleon-nucleon (NN) represent in the diagrams with one pair-vertex the effects of the 2π-cut in

the scalar and the vector meson form factor. The πN-diagrams, discussed above, are subgraphs of the NN-graphs containing these 2π-cuts. Therefore, taking into account these one-pair terms is covering the interactions contained in the Lagrangian (1.1). The NN-graphs with two pair vetices describe the decays of the scalar and vector mesons in two pions. They are contained in the 'broad meson' description of scalar $\sigma = \epsilon(760)$ etc. and ($\rho(760)$ etc exchange.

In the case of other pairs one must decide in each case whether the inclusion of the two-pair graphs in essence is double counting. For example in the case of the $(\pi\rho)_1$-pair, which has the same quantum numbers as the $a_1(1270)$, the two-pair term is included because a_1-exchange is not included in the OBE-set. The same is true for $(\pi\pi)$ with the quantum numbers of the tensor mesons.

Another class of graphs is the following. Consider the exchange of σ and P, coupling to the nucleon lines via their 2π-decay on both nucleon lines. For $t = 0$ these graphs look like a graph with an internal $(\boldsymbol{\pi} \cdot \boldsymbol{\pi})^4$-vertex. In view of the cancelation (C4) we conclude that this vertex is very weak, and justifies the neglect of these graphs.

In conclusion, the pair terms in ESC cover the effects of the terms generated by the effective field theories [5]. It has the advantage not being limited to the long range region, but leads to potentials for all distances. Of course, the included physics is restricted to low and intermediate momentum transfers ($|t| \leq 1$ GeV).

APPENDIX D: PHENOMENOLOGICAL $\pi N\Delta_{33}$ COUPLING

In this section we use the following local (N*N P) interaction

$$\mathcal{H}_{Y^*NP} = -i\left(i\frac{f^*_{Y^*NP}}{m_{\pi^+}}\right)\ \bar{\psi}_N \psi_{Y^*,\mu} \cdot \partial^\mu \phi_P, \tag{D1}$$

where $\psi_{Y^*,\mu}$ denotes the Rarita-Schwinger spinor. It now well known that in this form the 33-resonance does not couple to the right spin-3/2 degrees of freedom, hence we call it "phenomenological". In momentum space the vertex is given by

$$\begin{aligned}\langle P|\int d^4x\ \mathcal{H}_{Y^*NP}(x)|p,q\rangle &= (2\pi)^4\delta^4(P-p-q)\cdot \frac{f_{Y^*NP}}{m_{\pi^+}}\cdot \bar{u}^*_\mu(\mathbf{P},s')\ u(\mathbf{p},s)\cdot q^\mu \\ &\equiv (2\pi)^4\delta^4(P-p-q)\left[\bar{u}^*_\mu(\mathbf{P},s')\ \Gamma^\mu_{Y^*NP}(P;p,q)u(\mathbf{p},s)\right],\end{aligned} \tag{D2}$$

which gives the vertex

$$\Gamma^\mu_{Y^*NP}(P;p,q) = \frac{f^*_{Y^*NP}}{m_{\pi^+}}. \tag{D3}$$

The $J^P = \frac{3}{2}^+$-resonance is described by the Rarita-Schwinger Dirac spinors [20] satisfying the equations

$$(\not{P} - M_Y)\ u^\mu_Y(P) = 0\ ,\ \gamma_\mu u^\mu_Y(P) = P_\mu u^\mu_Y(P) = 0, \tag{D4}$$

and the resonance propagator is

$$P_{\mu\nu}(p) = \frac{U_{\mu\nu}(p)}{p^2 - M^2_Y + i\epsilon}, \tag{D5}$$

where the spin projector is [20]

$$
\begin{aligned}
U_{\mu\nu}(p) &= \sum_{\sigma=-3/2}^{+3/2} u_\mu(p,\sigma)\bar{u}_\nu(p) = (\not p + M_Y)\cdot \\
&\cdot\left(-g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3M_Y}\left(\gamma_\mu p_\nu - \gamma_\nu p_\mu\right) + \frac{2}{3M_Y^2}p_\mu p_\nu\right). \qquad \text{(D6)}
\end{aligned}
$$

The $J^P = \frac{3}{2}^+$-pole S-matrix element is

$$
\begin{aligned}
S_{fi} &= \frac{f_{12}^* f_{34}^*}{m_{\pi^+}^2}(-i)^2 \int \frac{d^4P}{(2\pi)^4}\bar{u}(\mathbf{p}_f, s_f)\left[(-q^\mu)\frac{iU_{\mu\nu}(P_\mu)}{P^2 - M_Y^2 + i\epsilon}(-q'^\nu)\right]\ u(\mathbf{p}_i, s_i) \\
&\times (2\pi)^4\delta^4(P-p-q)(2\pi)^4\delta^4(p'-q'-P) \\
&= -(2\pi)^4 i\delta^4(p'+q'-p-q)\left(f_{12}^* f_{34}^*/m_{\pi^+}^2\right) \\
&\times \bar{u}(\mathbf{p}_f, s_f)\left[\frac{q^\mu U_{\mu\nu}(P)q'^\nu}{s - M_Y^2 + i\epsilon}\right]\ u(\mathbf{p}_i, s_i), \qquad \text{(D7)}
\end{aligned}
$$

with $s = (p+q)^2 = (p'+q')^2$. The corresponding invariant amplitudes are [21]

$$
\begin{aligned}
A_{fi} &= +\frac{f_{12}^* f_{34}^*}{m_{\pi^+}^2}\cdot\frac{1}{s - M_Y^2 + i\epsilon}\cdot\left\{\frac{1}{2}(t - 2m^2)(M + M_Y)\right. \\
&+ \frac{M}{6M_Y^2}\left[M^2 - m^2 - s\right]^2 + \frac{1}{3}M_Y\left(s - M^2\right] \\
&\left. + \frac{M}{3}\left[s - M^2\right] + \frac{m^2}{3M_Y}\left[M^2 - m^2 - s\right]\right\}, \qquad \text{(D8a)} \\
B_{fi} &= -\frac{f_{12}^* f_{34}^*}{m_{\pi^+}^2}\cdot\frac{1}{s - M_Y^2 + i\epsilon}\cdot\left\{-\frac{1}{2}(t - 2m^2) - \frac{1}{6M_Y^2}\left[s - M^2 + m^2\right]^2\right. \\
&\left. + \frac{2}{3}M\ M_Y - \frac{1}{3}\left(m^2 - 2M^2\right) + \frac{M}{3M_Y}\left(M^2 - m^2 - s\right)\right\}. \qquad \text{(D8b)}
\end{aligned}
$$

Here, m and M denote the pion and nucleon mass respectively.

π N-**threshold**: With $s \approx M^2 + 2mM$, $t - 2m^2 = -2q'\cdot q$, $s + m^2 - M^2 = m(2M+m) \approx 2Mm$, we get

$$
\begin{aligned}
A_{fi} &\approx -\frac{f_{12}^* f_{34}^*}{m_{\pi^+}^2}\cdot\left(M_Y - M - \frac{2Mm}{M + M_Y}\right)^{-1}\left[-q'\cdot q + \frac{2}{3}Mm\left\{1 + \frac{m(M^2 - mM_Y)}{M_Y^2(M + M_Y)}\right\}\right], \\
B_{fi} &\approx +\frac{f_{12}^* f_{34}^*}{m_{\pi^+}^2}\cdot\left(M_Y - M - \frac{2Mm}{M + M_Y}\right)^{-1}\left[+q'\cdot q + \frac{2}{3}M^2\left\{1 - \frac{m}{M_Y} + \frac{M_Y}{M}\right.\right. \\
&\left.\left. -(2M^2 + M_Y^2)\frac{m^2}{2M^2 M_Y^2}\right\}\right](M + M_Y)^{-1}. \qquad \text{(D9)}
\end{aligned}
$$

The $M_{33} = A_{fi} - m_\pi B_{fi}$ amplitude becomes

$$
\begin{aligned}
M_{33} &\approx -\frac{f^*_{12}f^*_{34}}{m^2_{\pi^+}} \cdot \left(M_Y - M - \frac{2Mm}{M+M_Y}\right)^{-1} \bar{u}(p',s')\Bigg[-\left(1+\frac{m}{M+M_Y}\right)(q'\cdot q) \\
&\quad + \frac{2}{3}Mm\Bigg\{1+\frac{m(M^2-mM_Y)}{M_Y^2(M+M_Y)} - \frac{M}{M+M_Y}\left(1-\frac{m}{M_Y}+\frac{M_Y}{M}-\frac{2M^2+M_Y^2}{2M^2M_Y^2}m^2\right)\Bigg\}\Bigg]\ u(p,s) \\
&\approx -\frac{f^*_{12}f^*_{34}}{m^2_{\pi^+}} \cdot \left(M_Y - M - \frac{2Mm}{M+M_Y}\right)^{-1} \bar{u}(p',s')\Bigg[-(q'\cdot q) - \frac{2}{3}Mm\cdot \\
&\quad \times\Bigg\{1-\frac{m}{M}\left(1+\frac{\Delta M}{2M}\right)/\left(1+\frac{5\Delta M}{2M}\right)\Bigg\}\Bigg]\ u(p,s) \\
&\approx -2\frac{f^*_{12}f^*_{34}}{m^2_{\pi^+}\Delta M}\cdot\left(1-\frac{m}{M}\right)\ \bar{u}(p',s')\Bigg[-(q'\cdot q)-\frac{2}{3}Mm\cdot \\
&\quad \times\Bigg\{1-\frac{m}{M}\left(1-2\frac{\Delta M}{M}\right)\Bigg\}\Bigg]\ u(p,s). \qquad \text{(D10)}
\end{aligned}
$$

In [21] the fitted Δ_{33}-coupling is $f^2_{N\Delta\pi}/4\pi = 0.478$, and a cut-off $\Lambda = 603.22$ MeV. Comparing the with the coefficient of the $q'\cdot q$-term we have for the corresponding c_3-pair term coupling

$$
g_{S_2}/4\pi \approx (f^2_{N\Delta\pi}/4\pi)(m_\pi/\Delta M) = 0.216. \qquad \text{(D11)}
$$

Here, $\Delta M = M_\Delta - M_N$.

APPENDIX E: GAUGE-INVARIANT $\pi N\Delta_{33}$ COUPLING

The so-called 'gauge-invariant coupling of the Δ_{33}-resonance which is a spin-3/2 particle, restricting to the positive energy states, reads [24–26]

$$
\mathcal{L}_{GI} \ = \ g_{GI}\ \epsilon^{\mu\nu\alpha\beta}\left[\left(\partial_\mu\overline{\Psi^{(+)}_\nu}\right)\gamma_5\gamma_\alpha\psi^{(+)} + \overline{\psi^{(+)}}\gamma_5\gamma_\alpha\left(\partial_\mu\Psi^{(+)\nu}\right)\right]\ (\partial_\beta\phi) \qquad \text{(E1)}
$$

The Ψ_μ field contains besides the spin-3/2 also spin-1/2 components. Using the interaction (E1) it is assured that only the spin-3/2 components couple to the πN-channel. This is not the case for the coupling in (D1).

In [25], section **5.4**, the interaction Hamiltonian in the Takahashi-Umezawa formalism [27] is derived, which leads to the s- and u-channel πN amplitudes, [25] eqn. (5.43),

$$
\begin{aligned}
M_{\kappa',\kappa}(s) &= -\frac{1}{2}g^2_{GI}\int\frac{d\kappa_1}{\kappa_1+i\epsilon}\ \epsilon^{\mu\nu\alpha\beta}\bar{u}(p',s')\gamma_\alpha\gamma_5\ q'_\beta(\bar{P}_s)_\mu(\bar{P}_s)_{\mu'}\left(\bar{\not{P}}_s+M_\Delta\right)\cdot \\
&\quad \times\left(g_{\nu\nu'}-\frac{1}{3}\gamma_\nu\gamma_{\nu'}\right)\Delta(P_s)\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_{\alpha'}\gamma_5 q_{\beta'}\ u(p,s), \qquad \text{(E2a)} \\
M_{\kappa',\kappa}(u) &= -\frac{1}{2}g^2_{GI}\int\frac{d\kappa_1}{\kappa_1+i\epsilon}\ \epsilon^{\mu\nu\alpha\beta}\bar{u}(p',s')\gamma_\alpha\gamma_5\ q_\beta(\bar{P}_s)_\mu(\bar{P}_u)_{\mu'}\left(\bar{\not{P}}_u+M_\Delta\right)\cdot \\
&\quad \times\left(g_{\nu\nu'}-\frac{1}{3}\gamma_\nu\gamma_{\nu'}\right)\Delta(P_u)\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_{\alpha'}\gamma_5 q'_{\beta'}\ u(p,s), \qquad \text{(E2b)}
\end{aligned}
$$

where $\bar{P}_i = P_i + n\kappa_1$, i=s,u. Here, $P_i = \Delta_i + n\bar{\kappa} - n\kappa_1$ and $\Delta(P_i) = \epsilon(P_i^0)\delta(P_i^2 - M_\Delta^2)$ $(i = s, u)$. The Δ_i are

$$\Delta_s = \frac{1}{2}(p' + p + q' + q), \quad \Delta_u = \frac{1}{2}(p' + p - q' - q). \tag{E3}$$

Using the identity

$$\begin{aligned} \delta(P_i^2 - M_\Delta^2) &= \frac{1}{|\kappa_1^+ - \kappa_1^-|}\left(\delta(\kappa_1 - \kappa_1^+) + \delta(\kappa_1 - \kappa_1^-)\right], \\ \kappa_1^{pm} &= \Delta_i \cdot n + \bar{\kappa} \pm A_i, \end{aligned} \tag{E4}$$

where $A_i = \sqrt{(n \cdot \Delta_i)^2 - \Delta_i^2 + M_\Delta^2}$. The $\epsilon(P_i^0)$ selects both solutions with a relative minus sign, and since $\bar{P}_i$ are κ_1-independent, the κ_1-integral applies only to the quasi scalar propagator $1/(\kappa_1 + i\epsilon)$, and gives the factor

$$\begin{aligned} &\frac{1}{2A_i}\left[\frac{1}{\Delta_i \cdot n + \bar{\kappa} - A_i + i\epsilon} - \frac{1}{\Delta_i \cdot n + \bar{\kappa} + A_i + i\epsilon}\right] = \\ &\frac{1}{(\Delta_i \cdot n + \bar{\kappa})^2 - A_i^2 + i\epsilon}. \equiv D_i(\Delta_s, n, \bar{\kappa}). \end{aligned}$$

Contracting all indices the amplitudes become

$$\begin{aligned} M_{\kappa',\kappa}(s) &= -\frac{1}{2}g_{GI}^2\ \bar{u}(p', s')\left[\left(\bar{\not{P}}_s + M_\Delta\right)\left(\bar{P}_s^2(q' \cdot q) - \frac{1}{3}\bar{P}_s^2\not{q}'\not{q} - \frac{1}{3}\bar{\not{P}}_s\not{q}'(\bar{P}_s \cdot q)\right.\right. \\ &\left.\left. + \frac{1}{3}\bar{\not{P}}_s\not{q}(\bar{P}_s \cdot q') - \frac{2}{3}(\bar{P}_s \cdot q')(\bar{P}_s \cdot q)\right)\right]\ u(p, s) \cdot D(\Delta_s, n, \bar{\kappa}), \end{aligned} \tag{E5a}$$

$$\begin{aligned} M_{\kappa',\kappa}(u) &= -\frac{1}{2}g_{GI}^2\ \bar{u}(p', s')\left[\left(\bar{\not{P}}_u + M_\Delta\right)\left(\bar{P}_u^2(q' \cdot q) - \frac{1}{3}\bar{P}_u^2\not{q}\not{q}' - \frac{1}{3}\bar{\not{P}}_u\not{q}(\bar{P}_u \cdot q')\right.\right. \\ &\left.\left. + \frac{1}{3}\bar{\not{P}}_u\not{q}'(\bar{P}_u \cdot q) - \frac{2}{3}(\bar{P}_u \cdot q')(\bar{P}_u \cdot q)\right)\right]\ u(p, s) \cdot D(\Delta_u, n, \bar{\kappa}). \end{aligned} \tag{E5b}$$

These expressions are worked out in detail in Ref. [25, 26]. Here, for our discussion we use only the amplitudes for $\kappa = \kappa' = 0$. Using the standard decomposition

$$M_{0,0} = \bar{u}(p', s')\left[A_{0,0} + B_{0,0}\ \not{Q}\right]\ u(p, s), \ Q = \frac{1}{2}(q' + q), \tag{E6}$$

one obtains [25, 26],

$$
\begin{aligned}
A_{0,0}^{(s)} &= -\frac{1}{2} g_{GI}^2 \left[+s(M+M_\Delta)\ q' \cdot q - \frac{s}{3}(s-M^2)(M+M_\Delta) - \frac{1}{6} m^2 M_\Delta (s-M^2+m^2) \right. \\
&\left. + \frac{1}{6} M_\Delta (s-M^2)(s-M^2+m^2) - \frac{1}{6}(M+M_\Delta)(s-M^2+m^2)^2 \right] \cdot D(s, M_\Delta^2), && \text{(E7a)} \\
B_{0,0}^{(s)} &= -\frac{1}{2} g_{GI}^2 \left[+s\ (q' \cdot q) - \frac{s}{3}(-2M(M+M_\Delta)+m^2) - \frac{1}{6}(s+MM_\Delta)(s-M^2+m^2) \right. \\
&\left. + \frac{1}{6}(s-MM_\Delta)(s-M^2+m^2) - \frac{1}{6}(s-M^2+m^2)^2 \right] \cdot D(s, M_\Delta^2), && \text{(E7b)} \\
A_{0,0}^{(u)} &= -\frac{1}{2} g_{GI}^2 \left[+u(M+M_\Delta)\ q' \cdot q - \frac{u}{3}(u-M^2)(M+M_\Delta) - \frac{1}{6} m^2 M_\Delta (u-M^2+m^2) \right. \\
&\left. + \frac{1}{6} M_\Delta (u-M^2)(u-M^2+m^2) - \frac{1}{6}(M+M_\Delta)(u-M^2+m^2)^2 \right] \cdot D(s, M_\Delta^2), && \text{(E7c)} \\
B_{0,0}^{(u)} &= -\frac{1}{2} g_{GI}^2 \left[-u\ (q' \cdot q) - \frac{u}{3}(2M(M+M_\Delta)-m^2) + \frac{1}{6}(u+MM_\Delta)(u-M^2+m^2) \right. \\
&\left. - \frac{1}{6}(u-MM_\Delta)(u-M^2+m^2) + \frac{1}{6}(u-M^2+m^2)^2 \right] \cdot D(u, M_\Delta^2). && \text{(E7d)}
\end{aligned}
$$

Defining

$$
A_{0,0} = A_{0,0}^{(s)} + A_{0,0}^{(u)}, \quad B_{0,0} = B_{0,0}^{(s)} + B_{0,0}^{(u)}, \tag{E8}
$$

and using in the denominators the approximation $s = u = M^2$, we have

$$
\begin{aligned}
A_{0,0} &= -\frac{1}{2} g_{GI}^2 \left[+(s+u)(M+M_\Delta)\ q' \cdot q - \frac{1}{3}(M+M_\Delta)\left\{ s(s-M^2) + u(u-M^2) \right\} \right. \\
&- \frac{1}{6} m^4 M_\Delta + \frac{1}{6} M_\Delta \left\{ (s-M^2)(s-M^2) + (u-M^2)(u-M^2) \right\} \\
&\left. - \frac{1}{6}(M+M_\Delta)\left\{ (s-M^2+m^2)^2 + (u-M^2+m^2)^2 \right\} \right] \cdot D(M^2, M_\Delta^2), && \text{(E9a)} \\
B_{0,0} &= -\frac{1}{2} g_{GI}^2\ (s-u) \left[+(q' \cdot q) + \frac{1}{3}\left\{ (2M(M+M_\Delta) - m^2 \right\}) - \frac{1}{3} MM_\Delta \right. \\
&\left. - \frac{1}{6}(s+u-2M^2+2m^2) \right] \cdot D(M^2, M_\Delta^2). && \text{(E9b)}
\end{aligned}
$$

$$
\begin{aligned}
A_{0,0} &= -\frac{1}{2}g_{GI}^2 \left[+(s+u)(M+M_\Delta)\; q'\cdot q - \frac{1}{3}(M+M_\Delta)(M^2+m^2)(s+u)\right. \\
&+\frac{1}{6}(M+M_\Delta)(2M^2+3m^2)(2M^2-m^2) - \frac{1}{6}(M_\Delta - M)m^4 \\
&\left.-\frac{1}{12}\left\{5(M+M_\Delta)-(M_\Delta-M)\right\}\left\{(s-M^2)^2+(u-M^2)^2\right\}\right]\cdot \\
&\times D(M^2,M_\Delta^2), \qquad \text{(E10a)} \\
B_{0,0} &= -\frac{1}{2}g_{GI}^2\;(s-u)\;\left[+(q'\cdot q)+\frac{1}{2}(M(M+M_\Delta)-\frac{1}{6}M(M_\Delta-M)-\frac{1}{3}m^2\right. \\
&\left.-\frac{1}{6}(s+u-2M^2+2m^2)\right]\cdot D(M^2,M_\Delta^2). \qquad \text{(E10b)}
\end{aligned}
$$

Next we note that $D(M^2,M_\Delta^2) = (M^2-M_\Delta^2)^{-1} = -[(M+M_\Delta)(M_\Delta-M)]^{-1}$, and keeping only the dominant terms, i.e. terms in the numerator proportional to $(M+M_\Delta)$, we get the low energy approximation

$$
\begin{aligned}
A_{0,0} &= +\frac{g_{GI}^2}{2\Delta M}\;\left[+(s+u)\;q'\cdot q - \frac{1}{3}(M^2+m^2)(s+u)\right. \\
&\left.+\frac{1}{6}(2M^2+3m^2)(2M^2-m^2) - \frac{5}{12}\left\{(s-M^2)^2+(u-M^2)^2\right\}\right], \qquad \text{(E11a)} \\
B_{0,0} &= +\frac{g_{GI}^2}{4\Delta M}\;(s-u)\;M, \qquad \text{(E11b)}
\end{aligned}
$$

with $\Delta M = M_\Delta - M$.

I. **nucleon-pole** $s = M^2$: From $s+t+u = 2M^2+2m^2$ we have $u - M^2 = -(s-M^2) + (2m^2-t) \Rightarrow 2(q'\cdot q)$ for $s \to M^2$. This gives

$$
s+u = 2M^2 + 2q'\cdot q, \quad s-u = -2q'\cdot q. \qquad \text{(E12)}
$$

and

$$
(s-M^2)^2 + (u-M^2)^2 \to 4(q'\cdot q)^2 \to 0.
$$

All this leads to the low energy approximation

$$
\begin{aligned}
A_{0,0} &= +\frac{g_{GI}^2}{2\Delta M}\;\left[\frac{2}{3}(2M^2-m^2)\;(q'\cdot q) - \frac{1}{2}m^4\right], \\
&\approx -\frac{g_{GI}^2}{2\Delta M}\;\left[\frac{2}{3}(2M^2-m^2)\;(\mathbf{q}'\cdot\mathbf{q}) - \frac{1}{6}\left(8M^2-7m^2\right)m^2\right], \\
B_{0,0} &= -\frac{g_{GI}^2}{2\Delta M}\;(q'\cdot q)\;M \approx +\frac{g_{GI}^2}{2\Delta M}\;M\;\left(\mathbf{q}'\cdot\mathbf{q} - m^2\right). \qquad \text{(E13)}
\end{aligned}
$$

For the amplitude this leads to the approximation

$$
\begin{aligned}
M_{33} &= -\frac{g_{GI}^2}{2\Delta M}\;\bar{u}(p',s')\left[\frac{2}{3}(2M^2-\frac{3}{2}Mm-m^2)\;(\mathbf{q}'\cdot\mathbf{q}) - \frac{1}{6}\left(8M^2-6Mm-7m^2\right)m^2\right]\;u(p,s) \\
&= -\frac{3\bar{g}_{GI}^2}{m_\pi^2\Delta M}\;\bar{u}(p',s')\left[(1-\frac{3m}{4M}-\frac{m^2}{2M^2})\;(\mathbf{q}'\cdot\mathbf{q}) - \left(1-\frac{3m}{4M}-\frac{7m^2}{8M^2}\right)m^2\right]\;u(p,s) \\
&\Rightarrow +\frac{3\bar{g}_{GI}^2}{m_\pi^2\Delta M}\;\bar{u}(p',s')\left[(1-\frac{3m}{4M}-\frac{m^2}{2M^2})\;(q'\cdot q) - \left(\frac{3m^2}{8M^2}\right)m^2\right]\;u(p,s). \qquad \text{(E14)}
\end{aligned}
$$

Note that we introduced in the last two lines the dimensionless coupling $\bar{g}_{GI} = g_{GI}/Mm_\pi$. The last line is obtained by the substitution $\mathbf{q}' \cdot \mathbf{q} \approx -(q' \cdot q - m^2)$ facilitating the relation with the coefficients $c_{1,3}$ in (1.1). We obtain

$$-4\frac{m_\pi^2}{F_\pi^2}\, c_3 = \frac{3}{\Delta M}\bar{g}_{GI}^2, \quad -8\frac{m_\pi^2}{F_\pi^2}\, c_1 = +\frac{3}{\Delta M}\frac{m^2}{m_\pi^2}\left(\frac{3m^2}{8M^2}\right). \tag{E15}$$

II. πN**-threshold** $s \approx M^2 + 2Mm$: From $s + t + u = 2M^2 + 2m^2$ we have $u - M^2 = -(s - M^2) + (2m^2 - t) \Rightarrow 2(q' \cdot q) - 2Mm$. This case gives

$$s + u = 2M^2 + 2q' \cdot q, \quad s - u = 4Mm - 2q' \cdot q. \tag{E16}$$

and

$$(s - M^2)^2 + (u - M^2)^2 \to 4(q' \cdot q)^2 \to 8Mm\,[Mm - q' \cdot q]\,.$$

The corresponding low energy approximation reads

$$\begin{aligned} A_{0,0} &\approx -\frac{g_{GI}^2}{2\Delta M}\left[\frac{4}{3}M^2\left(1 + \frac{5m}{2M} - \frac{m^2}{2M^2}\right)\,(\mathbf{q}' \cdot \mathbf{q}) + \frac{2}{3}M^2\left(3 - \frac{5m}{M} + \frac{7m^2}{4M^2}\right)m^2\right], \\ B_{0,0} &= +\frac{g_{GI}^2}{2\Delta M}\,[2Mm - q' \cdot q]\;M \approx +\frac{g_{GI}^2}{2\Delta M}\,M\,\left[\mathbf{q}' \cdot \mathbf{q} + (2Mm - m^2)\right]. \end{aligned} \tag{E17}$$

For the amplitude (E17) leads to the approximation

$$\begin{aligned} M_{33} &= -\frac{g_{GI}^2}{2\Delta M}\,\bar{u}(p', s')\left[\frac{2}{3}(2M^2 + \frac{7}{2}Mm - m^2)\,(\mathbf{q}' \cdot \mathbf{q}) - \frac{1}{6}\left(14Mm - 7m^2\right)m^2\right]\,u(p, s) \\ &= -\frac{2\bar{g}_{GI}^2}{3m_\pi^2\Delta M}\,\bar{u}(p', s')\left[(1 + \frac{7m}{4M} - \frac{m^2}{2M^2})\,(\mathbf{q}' \cdot \mathbf{q}) - \left(\frac{7m}{4M} - \frac{7m^2}{8M^2}\right)m^2\right]\,u(p, s) \\ &\Rightarrow +\frac{2\bar{g}_{GI}^2}{3m_\pi^2\Delta M}\,\bar{u}(p', s')\left[(1 + \frac{7m}{4M} - \frac{m^2}{2M^2})\,(q' \cdot q) - \left(1 - \frac{3m^2}{8M^2}\right)m^2\right]\,u(p, s). \end{aligned} \tag{E18}$$

APPENDIX F: NUCLEON-RESONANCE PAIR-INTERACTIONS

In this section we derive the effective πN-interaction generated by the s-channel Nucleon resonance states $P_{11}(1440)$ and $S_{11}(1535)$, and their SU(3) octet companions. We derive forms with coefficients A,B,C and D like in the FM-Lagrangian. The πN amplitude reads

$$M = \bar{u}(\mathbf{p}_f, s_f)\left[A(s,t,u) + \frac{1}{2}(\not{q}' + \not{q})\;B(s,t,u)\right]\,u(\mathbf{p}_i, s_i) \tag{F1a}$$

$$= \chi^\dagger(s_f)\left[F(\mathbf{p}_f, \mathbf{p}_i) + \boldsymbol{\sigma} \cdot \hat{\mathbf{q}}_f\;G(\mathbf{p}_f, \mathbf{p}_i)\;\boldsymbol{\sigma} \cdot \hat{\mathbf{q}}\right]\,\chi(s_i). \tag{F1b}$$

The relation between these the first (relativistic) presentation and the second (non-relativistic) presentation, using [7] Dirac-spinors, is

$$F = \sqrt{\frac{(E' + M')(E + M)}{4M'M}}\left\{A + \frac{1}{2}\left[(\sqrt{s'} - M') + (\sqrt{s} - M)\right]\;B\right\}, \tag{F2a}$$

$$G = \sqrt{\frac{(E' - M')(E - M)}{4M'M}}\left\{-A + \frac{1}{2}\left[(\sqrt{s'} + M') + (\sqrt{s} + M)\right]\;B\right\}. \tag{F2b}$$

We develop the amplitudes around the πN threshold, and use the approximations

$$
\begin{aligned}
&E(\mathbf{p}) \approx M + \frac{\mathbf{p}^2}{2M}\ ,\ E'(\mathbf{p}') \approx M' + \frac{\mathbf{p}'^2}{2M'} \\
&\sqrt{s} - M \approx m\ ,\ \sqrt{s} + M \approx 2M + m, \\
&\sqrt{s'} - M' \approx m'\ ,\ \sqrt{s'} + M' \approx 2M' + m', \\
&s - M_R^2 = (\sqrt{s} - M_R)(\sqrt{s} + M_R) \approx \frac{1}{2}(m' + m)\left[M_R + \frac{1}{2}(M' + M)\right], \\
&\sqrt{s} = \sqrt{s'} \approx \frac{1}{2}\left[(M' + M) + (m' + m)\right],
\end{aligned}
$$

where $M' = M_f, m' = m_f$. The last approximation is based on the on-energy-shell assumption.

We define the effective Lagrangian such that in first-order it reproduces the πN-amplitude similarly to [10], see expression (8.1), i.e.

$$
\begin{aligned}
\mathcal{L}_{eff} \ \equiv\ &+\bar{\psi}_R\left[\left\{\left((A+B)\boldsymbol{\nabla}_1\cdot\boldsymbol{\nabla}_2 + D\right)\delta_{ij} - (A-B)\ \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}_1\times\boldsymbol{\nabla}_2\ \epsilon_{ijk}\tau_k\right\}\cdot\right. \\
&\left.\times\ \pi_{1,i}(x)\pi_{2,j}(x)\right]\ \psi_N + h.c. \qquad \text{(F3)}
\end{aligned}
$$

1. $J^P = \frac{1}{2}^-$ baryon-resonance

For the P_{11} meson-baryon resonance, e.g. the Roper resonance $P_{11}(1440)$, the local (N'N P) interaction

$$
\mathcal{L}_{N_RNP} = f_{RNP}\ [\bar{\psi}_R\boldsymbol{\tau}\psi_N]\cdot\boldsymbol{\phi}_P + h.c. \qquad \text{(F4)}
$$

The amplitudes A and B are, see [21] Appendix C,

$$
A(s,t,u)\ =\ +\frac{f_{RNP}^2}{s - M_R^2 + i\epsilon}\left[\frac{1}{2}(M' + M) + M_R\right]\ ,\ B(s,t,u) = +\frac{f_{RNP}^2}{s - M_R^2 + i\epsilon} \qquad \text{(F5)}
$$

where the average baryon and meson masses are $\bar{M} = (M' + M)/2$ and $\bar{m} = (m' + m)/2$. Also, the last expressions for A and B refer to the threshold approximation. The Pauli-spinor threshold amplitudes become

$$
F\ \approx\ \frac{f'^2}{\bar{M} + \bar{m} - M_R + i\epsilon}\ ,\ G \approx \frac{p'p}{4M'M}\frac{f'^2}{\bar{M} + \bar{m} + M_R}, \qquad \text{(F6)}
$$

where the average baryon and meson masses are $\bar{M} = (M' + M)/2$ and $\bar{m} = (m' + m)/2$. Taking into account the resonance character we should use the resonance propagator

$$
\frac{1}{s - M_R^2 + i\epsilon} \Rightarrow \frac{1}{s - M_R^2 + iM_R\Gamma_R} = \frac{s - M_R^2 - iM_R\Gamma_R}{(s - M_R^2)^2 + M_R^2\Gamma_R^2} \qquad \text{(F7)}
$$

Restricting ourselves to the Real-part implies the multiplication of the F and G amplitudes above by

$$
\frac{s - M_R^2 + i\epsilon}{s - M_R^2 + iM_R\Gamma_R} \Rightarrow \frac{(s - M_R)^2}{(s - M_R^2)^2 + M_R^2\Gamma_R^2}
$$

which leads to

$$F \approx f'^2 \frac{(\bar{M}+\bar{m})^2 - M_R^2}{(\bar{s}-M_R^2)^2 + M_R^2\Gamma_R^2} \left(\bar{M}+\bar{m}+M_R\right), \tag{F8a}$$

$$G \approx f'^2 \frac{p'p}{4M'M} \frac{(\bar{M}+\bar{m})^2 - M_R^2}{(\bar{s}-M_R^2)^2 + M_R^2\Gamma_R^2} \left(\bar{M}+\bar{m}-M_R\right), \tag{F8b}$$

where $\bar{s} = (\bar{M}+\bar{m})^2$. In the M-amplitude becomes in the CM-system, i.e. $\mathbf{p}' = -\mathbf{q}$ and $\mathbf{p} = -\mathbf{q}$,

$$\begin{aligned} M \approx f'^2 \frac{(\bar{M}+\bar{m})^2 - M_R^2}{(\bar{s}-M_R^2)^2 + M_R^2\Gamma_R^2} & \left\{ \left(\bar{M}+\bar{m}+M_R\right) \right. \\ & \left. + \left(\bar{M}+\bar{m}-M_R\right) \frac{\boldsymbol{\sigma}\cdot\mathbf{q}'\,\boldsymbol{\sigma}\cdot\mathbf{q}}{4M'M} \right\}. \end{aligned} \tag{F9}$$

Then, the effective Lagrangian which reproduces the M-amplitude in first order reads

$$\begin{aligned} \mathcal{L}_{eff} = f'^2 \frac{(\bar{M}+\bar{m})^2 - M_R^2}{(\bar{s}-M_R^2)^2 + M_R^2\Gamma_R^2} \bar{\psi}_R & \left[\left(\left(\bar{M}+\bar{m}+M_R\right) + \frac{(\bar{M}+\bar{m}-M_R)}{4M'M} \cdot \right.\right. \\ \times (\boldsymbol{\nabla}_1\cdot\boldsymbol{\nabla}_2) \Big) \delta_{ij} & \left. + i\frac{(\bar{M}+\bar{m}-M_R)}{4M'M} \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}_1\times\boldsymbol{\nabla}_2 \epsilon_{ijk}\tau_k \right] \psi_N\ \phi_{1,i}\phi_{2,j}. \end{aligned} \tag{F10}$$

Here, we inserted the isospin operators, because the effective Lagrangian must be hermitean. In case of the isospin zero η-mesons the spin-orbit term vanishes.

Compared with the parameters in the effective Lagrangian for the FM-interaction we have

$$A + B = +f'^2 \frac{(\bar{M}+\bar{m})^2 - M_R^2}{(\bar{s}-M_R^2)^2 + M_R^2\Gamma_R^2} \frac{(\bar{M}+\bar{m}-M_R)}{4M'M}, \tag{F11a}$$

$$A - B = +f'^2 \frac{(\bar{M}+\bar{m})^2 - M_R^2}{(\bar{s}-M_R^2)^2 + M_R^2\Gamma_R^2} \frac{(\bar{M}+\bar{m}-M_R)}{4M'M}, \tag{F11b}$$

$$D = +f'^2 \frac{(\bar{M}+\bar{m})^2 + M_R^2}{(\bar{s}-M_R^2)^2 + M_R^2\Gamma_R^2} \left(\bar{M}+\bar{m}+M_R\right). \tag{F11c}$$

2. $J^P = \frac{1}{2}^+$ baryon-resonance

For the $S_{11}(1535)$ meson-baryon resonance the local (N'N P) interaction

$$\mathcal{L}_{N_R NP} = i f_{RNP}\ [\bar{\psi}_R\gamma_5\boldsymbol{\tau}\psi_N]\cdot\boldsymbol{\phi}_P + h.c. \tag{F12}$$

The amplitudes A and B are, see [21] Appendix C,

$$A(s,t,u) = +\frac{f_{RNP}^2}{s-M_R^2+i\epsilon} \left[\frac{1}{2}(M'+M) - M_R\right], \tag{F13a}$$

$$B(s,t,u) = +\frac{f_{RNP}^2}{s-M_R^2+i\epsilon} \tag{F13b}$$

The Pauli-spinor amplitudes around the threshold become

$$F \approx \frac{f'^2}{\bar{M}+\bar{m}+M_R+i\epsilon}, \tag{F14a}$$

$$G \approx \frac{p'p}{4M'M}\frac{f'^2}{\bar{M}+\bar{m}-M_R}. \tag{F14b}$$

Taking into account the resonance character, like in the previous subsection, we obtain

$$F \approx f'^2 \frac{(\bar{M}+\bar{m})^2-M_R^2}{s-M_R^2)^2+M_R^2\Gamma_R^2}\left(\bar{M}+\bar{m}-M_R\right), \tag{F15a}$$

$$G \approx f'^2 \frac{p'p}{4M'M}\frac{(\bar{M}+\bar{m})^2-M_R^2}{s-M_R^2)^2+M_R^2\Gamma_R^2}\left(\bar{M}+\bar{m}+M_R\right), \tag{F15b}$$

and the M-amplitude becomes in the CM-system, i.e. $\mathbf{p}'=-\mathbf{q}$ and $\mathbf{p}=-\mathbf{q}$,

$$\begin{aligned} M \approx\ & f'^2 \frac{(\bar{M}+\bar{m})^2-M_R^2}{s-M_R^2)^2+M_R^2\Gamma_R^2}\left\{\left(\bar{M}+\bar{m}-M_R\right)\right. \\ & \left. +\left(\bar{M}+\bar{m}+M_R\right)\frac{\boldsymbol{\sigma}\cdot\mathbf{q}'\ \boldsymbol{\sigma}\cdot\mathbf{q}}{4M'M}\right\}. \end{aligned} \tag{F16}$$

Then, the effective Lagrangian which reproduces the M-amplitude in first order reads

$$\begin{aligned} \mathcal{L}_{eff} =\ & f'^2\ \frac{(\bar{M}+\bar{m})^2-M_R^2}{s-M_R^2)^2+M_R^2\Gamma_R^2}\ \bar{\psi}_R\left[\left(\left(\bar{M}+\bar{m}-M_R\right)+\frac{(\bar{M}+\bar{m}+M_R)}{4M'M}\cdot\right.\right. \\ & \left.\left. \times(\boldsymbol{\nabla}_1\cdot\boldsymbol{\nabla}_2)\right)\ \delta_{ij}+i\frac{(\bar{M}+\bar{m})+M_R}{4M'M}\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}_1\times\boldsymbol{\nabla}_2\epsilon_{ijk}\tau_k\right]\ \psi_N\ \phi_{1,i}\phi_{2,j}. \end{aligned} \tag{F17}$$

Here, we inserted the isospin operators, because the effective Lagrangian must be hermitean. In case of the η-mesons the spin-orbit term vanishes.

Compared with the parameters in the effective Lagrangian for the FM-interaction we have

$$A+B = +f'^2\frac{(\bar{M}+\bar{m})^2+M_R^2}{(\bar{s}+M_R^2)^2+M_R^2\Gamma_R^2}\ \frac{(\bar{M}+\bar{m}+M_R)}{4M'M}, \tag{F18a}$$

$$A-B = +f'^2\frac{(\bar{M}+\bar{m})^2+M_R^2}{(\bar{s}+M_R^2)^2+M_R^2\Gamma_R^2}\ \frac{(\bar{M}+\bar{m}+M_R)}{4M'M}, \tag{F18b}$$

$$D = +f'^2\frac{(\bar{M}+\bar{m})^2+M_R^2}{(\bar{s}+M_R^2)^2+M_R^2\Gamma_R^2}\ \left(\bar{M}+\bar{m}-M_R\right). \tag{F18c}$$

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[1] Th.A. Rijken, and V.G.J. Stoks, Phys. Rev. C**54**, 2869 (1996).

[2] Th.A. Rijken, Ann. Phys. (NY) **208**, 253 (1991).

[3] P. Ko and S. Rudaz, Phys. Rev. **D 50** (1994) 6877.

[4] V.G.J. Stoks and Th.A. Rijken, Nucl. Phys. A **613** (1996) 311.

[5] C. Ordóñez and U. van Kolck, Phys. Lett. B **291** (1992) 459; C. Ordóñez, L. Ray, and U. van Kolck, Phys. Rev. Lett. **72** (1994) 1982, Phys. Rev. C **53** (1996) 2086.

[6] M.C.M. Rentmeester, *Applied Nucleon-Nucleon Partial Wave Analyses*, PHD-thesis Katholieke Universiteit, Nijmegen 2001, chapter 3.

[7] We follow the conventions of J.D. Bjorken and S.D. Drell, *Relativistic Quantum Mechanics* and *Relativistic Quantum Fields* (McGraw-Hill Inc., New York, 1965). This except for the defintion of the M-matrix. We define the M-matrix by

$$S_{fi} = \delta_{fi} - (2\pi)^4 i\delta(P_f - P_i)\ M_{fi}\ .$$

[8] Th.A. Rijken, 'Tensor-exchange and the c_3-term', preprint University of Nijmegen, January 2001 (unpublished).

[9] M.C.M. Rentmeester, R.G.E. Timmermans, J.L. Friar, and J.J. de Swart, Phys. Rev. Lett. **82** (1999) 4992. .

[10] H. Miyazawa, Phys. Rev. **104**, 1741 (1956); J.-I. Fujita and H. Miyazawa, Progr. Theor. Phys. **17**, 360 (1957).

[11] Th.A. Rijken, and V.G.J. Stoks, Phys. Rev. C**54**, 2851 (1996).

[12] M.M. Nagels, Th.A. Rijken, and J.J. deSwart, Phys. Rev. **D17**, 768 (1978).

[13] M.M. Nagels et al, Nucl. Phys. **B 147** (1979) 189-276.

[14] Y. Yamamoto, private communication.

[15] P.A. Verhoeven, *Off-shell Baryon-Baryon Scattering*, Ph.D. thesis University of Nijmegen (1966), section VII.4.

[16] This notation for the coupling $\{8\} \times \{8\} \to \{10^*\}$ is analogous to the coefficients f_{ijk} and d_{ijk} for the couplings $\{8\} \times \{8\} \to \{8\}$.

[17] A.J. Macfarlane, A. Sudbury and P.H. Weisz, Comm. math. Phys. **11**, 77-90 (1968).

[18] P. Dittner, Comm. math. Phys. **22**, 238-252 (1971).

[19] P.A. Carruthers, *Introduction to Unitary Symmetry*, Interscience Publishers 1966.

[20] P.A. Carruthers, in *Spin and Isospin in Particle Physics*, Gordon and breach Science Publishers, Inc., 1971.

[21] H. Polinder, "Strong Meson-Baryon Interactions", Ph.D.-thesis, Katholieke Universiteit Nijmegen, 2004.

[22] The symmetry relation for SU(2) Clebsch-Gordan coefficients is

$$C\begin{pmatrix} i_1 & i_2 & I \\ m_1 & m_2 & M \end{pmatrix} = (-)^{I-i_1-i_2} C\begin{pmatrix} i_2 & i_1 & I \\ m_2 & m_1 & M \end{pmatrix}.$$

Application of the CGC-tables in PDG [23] for $1 \times 1/2 \to 1/2$ gives a (-)-sign.

[23] Particle Data Group, "Review of Particle Properties", Phys. Lett. **B239** (1990) 1.

[24] V. Pascalutsa, Phys. Rev. **D 58**, 096002 (1998).

[25] J. W. Wagenaar, *Pion-Nucleon Scattering in Kadyshevsky Formalism and Higher Spin Field Quantization*, Ph.D. thesis University of Nijmegen, July 2009, e-Print: arXiv:0904.1398[nucl-th].

[26] J. W. Wagenaar and T.A. Rijken, *Pion-Nucleon Scattering in Kadyshevsky Formalism: II Baryon Exchange Sector*, e-Print: arXiv:0905.1408[nucl-th].

[27] Y. Takahashi and H. Umezawa, Progr. Theor. Phys. **9**, 14 (1953); *ibid* **9**, 501 (1953).

[28] Th.A. Rijken, and Y. Yamamoto, Phys. Rev. C**73**, 044008 (2006).