Constituent Quark model and NN-potentials

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In these notes, while focussing on the meson-nucleon vertices, we give a derivation of the nucleon-nucleon (NN) potentials from meson-exchange between the quarks. To establish such a relation the quark-quark-meson (QQM) interactions are properly defined. Hitherto, the coefficients in the Pauli-spinor expansion of the meson-nucleon-nucleon (NNM) vertices are equated with those of the QQM-vertices. In these notes we employ the description of the nucleon with Dirac-spinors in the SU(6) semi-relativistic "constituent" quark-model (CQM) as formulated by LeYouanc, *et al.* It appears that the "constituent" quark model, *i.e.* $m_Q = M_N/3$, is able to produce the same ratio's for the central-, spin-spin-, tensor-, spin-orbit-, and quadratic-spin-orbit Pauli-invariants as in the phenomenological NNM-vertices. In order to achieve this, the scalar-, magnetic-vector, and axial-vector interactions require, besides the standard ones, an extra coupling to the quarks without the introduction of new parameters. In the case of the axial-vector mesons an extra coupling to the quarks is necessary, which is related to the quark orbital-angular momentum contribution to the nucleon spin. Furthermore, a momentum correlation between the quark interacting with the meson and the remaining quark pair, and a gaussian QQM form factor, are necessary, to avoid "spurious" terms.

From these results we have a formulation of the QQ-interactions which are directly related to the nucleon-nucleon extended-soft-core (ESC) interactions. This could be utilized in e.g. a study of quark matter.

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I. INTRODUCTION

The main motivation to work out QQM-coupling in the context of the constituent quark-model (CQM) is that in the extended-softcore (ESC) baryon-baryon interactions, see *i.e.* [1–3], the quark-pair creation (QPC) model is very succesful to explain the meson-baryon-baryon (MBB) coupling constants.

A major succes of the non-relativistic (additive) quark model (CQM) has been the description of the magnetic moments of the baryons with $m_Q = M_N/3$.

Also in these notes the description of the nucleon with Dirac-spinors in the SU(6) semi-relativistic "constituent" quark-model (CQM) as formulated by LeYouanc, *et al* [4] is employed. In Fig. 1 the QPC-mechanism for NNM-coupling is illustrated. From the subfigure (a) it is clear that the basis is the assumption that the mesons couple in first instance to the quarks. Then, with folding this leads to the NNM-coupling illustrated in subfigure (b). In this paper we show that the quark-quark-meson QQM) interaction can be choosen such that in the folding with the 3-quark nucleon wave function the correct $1/\sqrt{M'M}$ expansion of the NN-potentials can be obtained.

In QCD two non-perturbative effects occur: confinement and chiral symmetry breaking. The $SU(3)_L$x$SU(3)_R$ chiral symmetry is spontaneously broken to an $SU(3)_v$ symmetry at some scale $\Lambda_{\chi SB} \approx 1$ GeV [5–7]. Below this scale there is an octet of pseudoscalar Nambu-Goldstone-bosons: (π, K, η). The confinement scale $\Lambda_{QCD} \approx 100 - 330$ MeV. The complex QCD-vacuum structure can be described as an BPST instanton/anti-instanton liquid giving the valence quarks a dynamical or constituent effective mass $\approx M_N/3$ [8, 9]. This corresponds to the CQM [7], and explains the success of the program proposed in this paper.

In these notes we consider the nucleon-nucleon (NN) potential from meson-exchange between the (single) quarks in impulse-approximation, and folding these with the nucleon quark wave functions. (In the CQM the 3-quark model wave functions for the SU(3) octet baryons are, with respect to flavor and color, properly antisymmetrized gaussian quark wave functions reflecting the ground state of an effective harmonic oscillator binding force.)

We employ the description of the nucleon with Dirac-spinors in the SU(6)-version of the CQM, see [4]. In this study we evaluate the NN-meson vertices and analyze whether the expansion of these vertices in Pauli-invariants is in accordance with the similar expansion used in NN-models using meson exchange at the nucleon level.

For elastic scattering with the (external) nucleons on the mass shell, Lorentz invariance and parity conservation imply that there are 6 independent amplitudes [10], i.e. the NN-amplitude can be expressed in terms of the free nucleon Dirac-spinors as follows

$$\mathcal{M} = \sum_{i=1}^{6} M_i(s,t)\ \left[\bar{u}_{N_1'}(p_1')\bar{u}_{N_2'}(p_2')\ O_i\ u_{N_1}(p_1)\ u_{N_2}(p_2)\right]$$

where a complete set independent (t-channel) Lorentz-invariants can be choosen as

$$\begin{array}{ll} O_1 = 1 \otimes 1 & O_2 = \gamma_5 \otimes \gamma_5 \\ O_3 = \gamma_\mu \otimes \gamma^\mu & O_4 = \gamma_5 \gamma_\mu \otimes \gamma_5 \gamma^\mu \\ O_5 = \sigma_{\mu\nu} \otimes \sigma^{\mu\nu} & O_6 = i \left\{ \gamma_\mu K^\mu \otimes 1 - 1 \otimes \gamma_\mu P^\mu \right\}, \end{array}$$

where $P = p_1 + p_1', K = p_2 + p_2'$. We note that $O_i = \Gamma_{1,i} \otimes \Gamma_{2,i}$ and that in the meson-exchange contribution to the NN-amplitude the NNM-vertex is of the form $\bar{u}(p', s')\Gamma u(p, s)$. The Lorentz structure of the NN-amplitude and NNM-vertices given above is general and independent of the internal structure of the nucleon. Therefore, the QQM-exchange vertices folded with the nucleon quark wave functions has to reproduce at the nucleon level the same structure. This observation is the key to the procedure followed in these notes to define the QQM-vertices.

Conjecture : The phenomenological expansion of the vertices in powers of $1/\sqrt{M'M}$ should not depend on the internal structure of the nucleons. So, the ratio's of the central-, spin-spin-, tensor-, and spin-orbit operators should be independent internal structure of the nucleon.

At the nucleon level, in Pauli-spinor space, the vertices have the general structure:

$$\begin{aligned} \bar{u}(p', s')\Gamma\, u(p, s) &= \chi_{s'}^{\prime\dagger} \left\{ \Gamma_{bb} + \Gamma_{bs} \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E + M} - \frac{\boldsymbol{\sigma}\cdot\mathbf{p}'}{E' + M'} \Gamma_{sb} - \frac{\boldsymbol{\sigma}\cdot\mathbf{p}'}{E' + M'} \Gamma_{ss} \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E + M} \right\} \chi_s \\ &\approx \chi_{s'}^{\prime\dagger} \left\{ \Gamma_{bb} + \Gamma_{bs} \frac{(\boldsymbol{\sigma}\cdot\mathbf{p})}{2\sqrt{M'M}} - \frac{(\boldsymbol{\sigma}\cdot\mathbf{p}')}{2\sqrt{M'M}} \Gamma_{sb} - \frac{(\boldsymbol{\sigma}\cdot\mathbf{p}')\, \Gamma_{ss}\, (\boldsymbol{\sigma}\cdot\mathbf{p})}{4M'M} \right\} \chi_s \\ &\equiv \sum_l c_{NN}^{(l)}\, O_l(\mathbf{p}', \mathbf{p}, \boldsymbol{\sigma})\, (\sqrt{M'M})^{\alpha_l} \quad (l = bb, bs, sb, ss), \end{aligned}$$

where $O_l(\mathbf{p}', \mathbf{p}, \boldsymbol{\sigma})$ denotes the set of operators 1, $\boldsymbol{\sigma}$, $\mathbf{p}$, $\mathbf{p}'$, $\boldsymbol{\sigma}\cdot\mathbf{p}$, $\boldsymbol{\sigma}\cdot\mathbf{p}'$, $\boldsymbol{\sigma}\cdot\mathbf{p}'\times\mathbf{p}$, etc.
The question is how this structure is reproduced using the coupling of the mesons to the quarks directly, *i.e.* whether for the constants $c_{CQM}^{(l)} = c_{NN}^{(l)}$. In fact, we want to demonstrate that for the CQM, i.e. $m_Q = \sqrt{M'M}/3$, the ratio's $c_{CQM}^{(l)}/c_{NN}^{(l)}$ are constant for each type of meson. Then, by scaling the expansion coffients can be made equal.

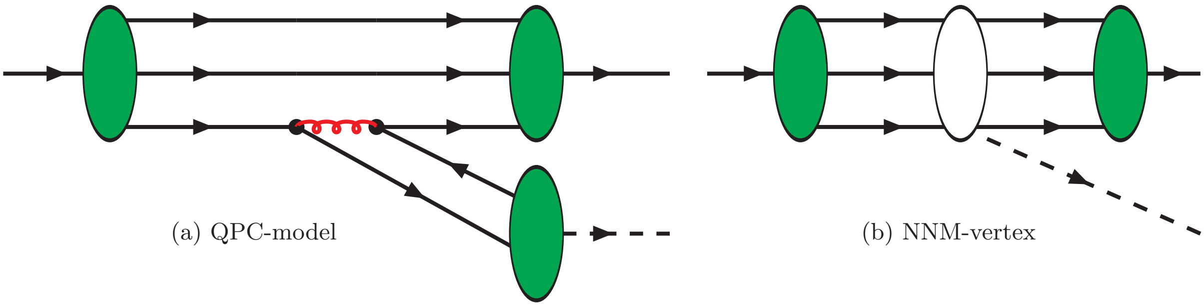

FIG. 1: Meson-nucleon-nucleon coupling.

Therefore, we expect these ratio's are essentially the same as for the expansion of the NNM-vertices with Pauli-spinor invariants.
We found this to be possible for most of the terms, up to order $1/M'M$, in the CQM where $m_Q = M_N/3$, for all couplings: pseudo-scalar (P), scalar (S), vector (V) and axial-vector (A) coupling.

In the scalar, vector, and axial-vector vertex there appear "spurious" terms $\propto 1/R_N^2$. This is only the case for the central and spin term of the scalar/vector and axial-vector respectively. In view of the "conjecture", these terms should not be present, and must be eliminated. We demonstrate that such "spurious" terms can be eliminated by introducing a momentum exchange between the "active" quark, i.e. the quark line with the meson vertex, and the two "spectator" quarks. (In the simplest model without such a momentum exchange, this amounts to the introduction of a gaussian momentum distribtion at the QQM-vertex.) .

In this study we evaluate the QQM vertices and analyze whether the expansion of these vertices in Pauli-invariants matches with the similar expansion used in NN-models using meson exchange at the nucleon level. To accomplish this we add the following vertices at the quark level: (i) for the vector-mesons a zero in the scalar derivative part, and (ii) in the case of the axial-vector coupling an additional pseudoscalar derivative interaction. To work out these ideas concretely, we use the description of the nucleon with Dirac-spinors in the SU(6)-version of the CQM, see [4]. In the CQM the rational for this is that since $M_N = 3m_Q$ the quark kinetic and potential energies cancel each other, which means that for the quark energies $E_i \approx m_i$.

As a final note: The QQM- and NNM-vertices are for potentials $\mathcal{V}$ in the Lippmann-Schwinger equation. For the relation with the (kinematically relativistic) Thompson, Kadyshevsky etc. equations, see Ref. [11].

The content of these notes is as follows. In section II the QCD basis of the CQM based on the instanton-model of the QCD-vacuum is briefly reviewed. In section III we review the quark wave functions and the overlap integrals. In section IV-VII we treat scalar-exchange, pseudo-scalar-, vector-, and axial-vector-meson exchange. In section IV C a method is given to remove "spurious" terms from the NN-vertices Γ_{CQM}. To complete this it is necessary to use a (gaussian) QQM cut-off. In section VIII we formulate our conclusions. In Appendix A the overlap integral for meson exchange is worked out. Similarly in Appendix B, where a momentum correlation is included between the quark with the meson-vertex and the remaining quark-pair, henceforth referred to as the "active" quark and "spectator" quarks respectively. It is shown that with such arrangement the "spurious" terms are eliminated, and can explain the procedure introduced in section IV C. In Appendix C we discuss the quark summation. In Appendix E we list the Pauli-spinor invariants for the nucleon-nucleon potentials. In Appendix F the extended-soft-core (ESC) quark-quark (QQ) OBE-interactions in momentum and configuration space are given for reference of the vertex structures with Pauli-invariants. In Appendix G the lower vertex for the scalar-meson QQ-coupling is worked out for comparison with the upper vertex. In Appendix H tensor-meson exchange is analyzed and compared with scalar- and vector-meson exchange.

II. CONSTITUENT QUARKS AND INSTANTONS

The spectra of the nucleons, Δ resonances and the hyperons Λ, Σ, Ξ are descibed in detail by the Glozman-Riska model [12]. This is a modern version of the constituent quark model (CQM) [13] based on the Nambu-Goldstone spontaneous chiral-symmetry breaking (SCSB) with quarks interacting by the exchange of the $SU(3)_F$ octet of pseudoscalar mesons [12]. The pseudoscalar octet are the Goldstone bosons associated with the hidden (approximate) chiral symmetry of QCD. The confining potential is choosen to be harmonic, as is rather common in constituent quark models. This is in line with the harmonic wave functions we used in the derivation of the connection between the meson-baryon and meson-quark couplings [14]. The η', which is dominantly an SU(3) singlet, decouples from the original pseudoscalar nonet because of the $U_A(1)$ anomaly [15, 16]. According to the two-scale picture of Manohar and Georgi [7] the effective degrees for the 3-flavor QCD at distances beyond that of SCSB ($\Lambda_{\chi SB}^{-1} \approx 0.2 - 0.3$ fm), but within that of the confinement scale $\Lambda_{QCD}^{-1} \approx 1$ fm, should be the constituent quarks and chiral meson fields. The two non-perturbative effects in QCD are confinement and chiral symmetry breaking. The $SU(3)_L \otimes SU(3)_R$ chiral symmetry is sponteneously broken to an $SU(3)_v$ symmetry at a scale $\Lambda_{\chi SB} \approx 1$ GeV. The confinement scale is $\Lambda_{QCD} \approx 100 - 300$ MeV, which roughly corresponds to the baryon radius ≈ 1 fm. Due to the complex structure of the QCD vacuum, which can be understood as a liquid of BPST instantons and anti-instantons [8, 9, 17, 18], the valence quarks acquire a dynamical or constituent mass [7, 9, 15, 18, 19]. The interaction between the instanton and the anti-instanton is a dipole-interaction [20], similar to ordinary molecules: weak attraction at large distances and strong repulsion at small ones. With the empirical value of the gluon condensate [21] as input the instanton density and radius become [20] $n_c = 8 \cdot 10^{-4}$ GeV^{-4}, and $\rho_c = (600 \text{ MeV})^{-1} \approx 0.3$ fm respectively. Also, with these parameters the non-perturbative vacuum expectation value for the quark fields is $\langle vac|\bar{\psi}\psi|vac\rangle \approx -10^{-2}$ GeV^3 and the quark effective mass ≈ 200 MeV, which is much larger than the almost massless (u,d) "current quarks". In the calculation of light quarks in the instanton vacuum [9] the effective quark mass $m_Q(p = 0) = 345$ MeV was calculated, which is remarkably close to the constituent mass $M_N/3$.

Very notable is the role of the instantons for the light meson spectrum. They give a non-perturbative gluonic interaction between quarks in QCD. For example the instanton-induced interaction, as proposed by 't Hooft [16], generates at low momenta the constituent quark mass [9], i.e. breaks chiral symmetry. This interaction supplies a strong attractive attraction in the pseudoscalar-isovector quark-antiquark system - pions -, which makes them anomalously light, with zero mass in the chiral limit. This is the mechanism by which the pions, being quark-antiquark bound states, appear as Nambu-Goldstone bosons of the SCSB symmetry. This strongly attractive interaction is absent in vector mesons [22, 23], making the masses of the vector mesons $\approx 2m_Q$ in accordance with $m_\rho \approx m_\omega \approx 2m_Q$. Since $\alpha_s \approx 0.3$ the one-gluon-exchage (OGE) is weak, and therefore the $\pi - \rho$ mass splitting is not due to the perturbative color-magnetic spin-spin interaction between the quark and antiquark [23]. Besides explaining the $\pi - \rho$ mass difference, the 't Hooft interaction also in a natural way solves the $U_A(1)$ problem, and gives the reason why the η' is heavy.

The 't Hooft four-fermion instanton mediated interaction for the light flavor doublet $\psi = (u, d)$, in the form of a generalized Nambu-Jona-Lasinio Lagrangian [6], is

$$\mathcal{L}_I = g_I \left[(\bar\psi\psi)^2 - (\bar\psi\gamma_5\boldsymbol{\tau}\psi)^2 - (\bar\psi\boldsymbol{\tau}\psi)^2 + (\bar\psi\gamma_5\psi)^2 \right] . \tag{2.1}$$

Here, the strength of the interaction g_I and the ultra-violet cut-off scale $1/r_0$ are related in the instanton liquid model [24]. In [25] Glozman and Varga show that the t-channel iteration of the instanton interaction (2.1) leads to isoscalar and isovector pseudoscalar and scalar exchange quark-quark potentials. Since the latter potentials are already included in our model, the four-fermion instanton interaction does not lead to extra pseudoscalar- and scalar-meson exchange potentials. So, only the instanton-exchange potential is new in our model.

In this paper we extend the meson-exchange between quarks by proposing to include, besides the pseudoscalar, all meson nonets: vector, axial-vector, scalar etc. *Since all these meson nonets can be considered as quark-antiquark bound states, there is no reason to exclude any of these mesons from the quark-quark interactions. Furthermore, our preferred value for the constituent quark mass has a solid basis in the instanton-liquid model of the QCD vacuum.*

III. QUARK WAVE FUNCTIONS OF THE NUCLEONS

A. Kinematics and Dirac spinors

We consider a nuclon having a momentum P and label the 3 quarks by a, b, c. The quark momenta are denoted by p_a, p_b, p_c.

The spatial part of the composite nucleon wave funtion is taken to be [4]

$$\psi(p_a, p_b, p_c) \;=\; \psi(p_1, p_2, p_3) = \left(\frac{\sqrt{3}R_N^2}{\pi}\right)^{3/2} \;\exp\left[-\frac{R^2}{6}\sum_{i<j}\left(\mathbf{p}_i - \mathbf{p}_j\right)^2 \right] \tag{3.1}$$

The normalization constant in (3.1) we denote by $\mathcal{N} \equiv \left(\sqrt{3}R_N^2/\pi\right)^{3/2}$.
In the constituent quark model (CQM) the nucleon (baryon) mass is given by the sum of quark masses, *i.e.* $M_N = 3m_Q$, the quark energies saisfy $E_Q = m_Q + T_Q + U_Q$, the kinetic(T_Q) and potential (U_Q) energies cancel approximately $T_Q + U_Q \approx 0$. Therefore, the constituent quark spinors are [4]

$$u_i^{(0)}(\mathbf{p}_i) = \sqrt{\frac{E_i + m_i}{2m_i}} \left[\begin{array}{c} 1 \\ \frac{\boldsymbol{\sigma}_i\cdot\mathbf{p}_i}{E_i + m_i} \end{array} \right] \otimes\ \chi_i \approx \left[\begin{array}{c} 1 \\ \frac{\boldsymbol{\sigma}_i\cdot\mathbf{p}_i}{2m_i} \end{array} \right] \otimes\ \chi_i, \tag{3.2}$$

where $\mathbf{p_i}_i$ denotes the three-momentum of the quarks in e.g. the CM-system.

B. Overlap Integrals, Vertex functions

We consider the nucleon-nucleon graph for meson-exchange between the consituent quarks of the two nucleons. In the following we will use, instead of indices a,b,c, the indices i=1,2,3.

In Fig. 2 we have given the momenta for the initial and final nucleons, and the assigned momenta of the quarks. From momentum conservation we have

$$\begin{aligned} p_1 &= k_1 + k_2 + k_3 \quad , \quad p_2 = q_1 + q_2 + q_3 \ , \\ p_1' &= k_1' + k_2' + k_3' \quad , \quad p_2' = q_1' + q_2' + q_3' \ , \end{aligned} \tag{3.3}$$

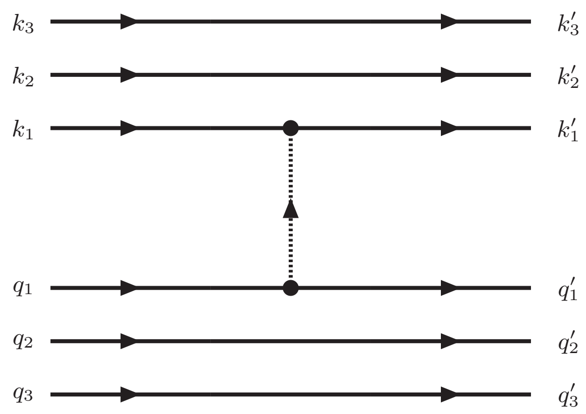

FIG. 2: External and internal momenta for meson-exchange

For meson-exchange with $p_1 - p'_1 = p'_2 - p_2 \equiv k$, we have for the matrix-element of the potential

$$\begin{aligned}
\langle p'_1 p'_2 | V | p_1 p_2 \rangle &= \int \prod_{i=1,3} d^3k_i \; \delta\left(\mathbf{p}_1 - \sum_i \mathbf{k}_i\right) \cdot \int \prod_{j=1,3} d^3k'_j \; \delta\left(\mathbf{p}'_1 - \sum_j \mathbf{k}'_j\right) \cdot \\
&\times \int \prod_{i=1,3} d^3q_i \; \delta\left(\mathbf{p}_2 - \sum_i \mathbf{q}_i\right) \cdot \int \prod_{j=1,3} d^3q'_j \; \delta\left(\mathbf{p}'_2 - \sum_j \mathbf{q}'_j\right) \cdot \\
&\times \; \tilde{\psi}^*_{p'_1}\left(\mathbf{k}'_1, \mathbf{k}'_2, \mathbf{k}'_3\right) \; \tilde{\psi}^*_{p'_2}\left(\mathbf{q}'_1, \mathbf{q}'_2, \mathbf{q}'_3\right) \cdot \tilde{\psi}_{p_1}\left(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3\right) \; \tilde{\psi}_{p_2}\left(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3\right) \; \cdot \\
&\times \; \delta^3\left(\mathbf{k}'_2 - \mathbf{k}_2\right) \; \delta^3\left(\mathbf{k}'_3 - \mathbf{k}_3\right) \; \delta^3\left(\mathbf{q}'_2 - \mathbf{q}_2\right) \; \delta^3\left(\mathbf{q}'_3 - \mathbf{q}_3\right) \; \cdot \\
&\times \; \frac{\gamma\left(k; k'_1, k_1\right) \; \gamma\left(k; q'_1, q_1\right)}{\mathbf{k}^2 + m_M^2} \cdot \delta^3\left(\mathbf{k} - \mathbf{k}'_1 + \mathbf{k}_1\right) \; \delta^3\left(\mathbf{k} + \mathbf{q}'_1 - \mathbf{q}_1\right) \; . \qquad (3.4)
\end{aligned}$$

In (3.4) the γ's denote the vertex functions. Using the gaussian wave function of equation (3.1), the overlap integral in Eq. (3.4) can be evaluated in a straightforward manner. For details see Appendix A.

For doing later integrals with explicit terms for the QQ-potential, it is useful to write the expression (A12) with separated vertex factors:

$$\begin{aligned}
\langle p'_1 p'_2 | V | p_1 p_2 \rangle &= \left(\frac{1}{8}\right)^3 \left(\frac{2\pi}{R_N^2}\right)^3 \mathcal{N}^4 \; \exp\left[-\frac{R_N^2}{3}\left(\mathbf{q}^2 + \mathbf{k}^2\right)\right] \; \cdot \\
&\times \int d^3Q \; \exp\left[-\frac{R_N^2}{6}\left\{\frac{9}{4}\left(\mathbf{Q}^2 - \frac{4}{3}\mathbf{q}\cdot\mathbf{Q}\right)\right\}\right] \cdot \\
&\times \int d^3S \; \exp\left[-\frac{R_N^2}{6}\left\{\frac{9}{4}\left(\mathbf{S}^2 + \frac{4}{3}\mathbf{q}\cdot\mathbf{S}\right)\right\}\right] \cdot \\
&\times V_{QQ}(\mathbf{Q}, \mathbf{S}; \mathbf{k}, \mathbf{q}) \; \times \delta\left(\mathbf{k}'_1 + \mathbf{q}'_1 - \mathbf{k}_1 - \mathbf{q}_1\right), \qquad (3.5)
\end{aligned}$$

where the QQ-potential is

$$V_{QQ}(\mathbf{Q}, \mathbf{S}; \mathbf{k}, \mathbf{q}) = \frac{\gamma\left(k; k'_1, k_1\right) \; \gamma\left(k; q'_1, q_1\right)}{\mathbf{k}^2 + m_M^2} \qquad (3.6)$$

with the momenta, see Appendix A, defined as

$$\mathbf{k}_1' = \frac{1}{2}(\mathbf{Q}+\mathbf{k}) \ , \ \mathbf{k}_1 = \frac{1}{2}(\mathbf{Q}-\mathbf{k}), \tag{3.7a}$$

$$\mathbf{q}_1' = \frac{1}{2}(\mathbf{S}-\mathbf{k}) \ , \ \mathbf{q}_1 = \frac{1}{2}(\mathbf{S}+\mathbf{k}). \tag{3.7b}$$

More explicitly for spin-J mesons (m,n =1, ..., 2J+1) the numerator in (3.6 stands for

$$\gamma(k;k_1',k_1)\ \gamma(k;q_1',q_1) \to \gamma^{\{\mu_m\}}(k;k_1',k_1)\ \mathcal{P}_{\{\mu_m\},\{\nu_n\}}(k)\gamma^{\{\nu_n\}}(k;q_1',q_1). \tag{3.8}$$

The basic d^3Q and d^3S integrals are

$$I_0(\mathbf{q}) = \int d^3Q\ \exp\left[-\frac{R_N^2}{6}\left\{\frac{9}{4}\left(\mathbf{Q}^2-\frac{4}{3}\mathbf{q}\cdot\mathbf{Q}\right)\right\}\right] = \left(\frac{8\pi}{3R_N^2}\right)^{3/2}\ \exp\left[\frac{1}{6}\mathbf{q}^2\ R_N^2\right], \tag{3.9a}$$

$$J_0(\mathbf{q}) = \int d^3S\ \exp\left[-\frac{R_N^2}{6}\left\{\frac{9}{4}\left(\mathbf{S}^2+\frac{4}{3}\mathbf{q}\cdot\mathbf{Q}\right)\right\}\right] = \left(\frac{8\pi}{3R_N^2}\right)^{3/2}\ \exp\left[\frac{1}{6}\mathbf{q}^2\ R_N^2\right]. \tag{3.9b}$$

Then,

$$I_i(\mathbf{q}) = \int d^3Q\ Q_i\ \exp\left[-\frac{R_N^2}{6}\left\{\frac{9}{4}\left(\mathbf{Q}^2-\frac{4}{3}\mathbf{q}\cdot\mathbf{Q}\right)\right\}\right] = \frac{2}{R_N^2}\boldsymbol{\nabla}_i\ I_0(\mathbf{q}) = +\frac{2}{3}\ q_i\ I_0(\mathbf{q}), \tag{3.10a}$$

$$J_i(\mathbf{q}) = \int d^3S\ S_i\ \exp\left[-\frac{R_N^2}{6}\left\{\frac{9}{4}\left(\mathbf{S}^2+\frac{4}{3}\mathbf{q}\cdot\mathbf{S}\right)\right\}\right] = -\frac{2}{R_N^2}\boldsymbol{\nabla}_i\ I_0(\mathbf{q}) = -\frac{2}{3}\ q_i\ I_0(\mathbf{q}), \tag{3.10b}$$

and

$$I_{i,j}(\mathbf{q}) = \int d^3Q\ Q_iQ_j\ \exp\left[-\frac{R_N^2}{6}\left\{\frac{9}{4}\left(\mathbf{Q}^2-\frac{4}{3}\mathbf{q}\cdot\mathbf{Q}\right)\right\}\right] = \left(\frac{2}{R_N^2}\right)^2\ \boldsymbol{\nabla}_i\boldsymbol{\nabla}_j\ I_0(\mathbf{q})$$
$$= \left[\frac{4}{3R_N^2}\delta_{i,j} + \frac{4}{9}\ q_iq_j\right] I_0(\mathbf{q}), \tag{3.11a}$$

$$J_{i,j}(\mathbf{q}) = \int d^3S\ S_iS_j\ \exp\left[-\frac{R_N^2}{6}\left\{\frac{9}{4}\left(\mathbf{S}^2+\frac{4}{3}\mathbf{q}\cdot\mathbf{S}\right)\right\}\right] = \left(\frac{2}{R_N^2}\right)^2\ \boldsymbol{\nabla}_i\boldsymbol{\nabla}_j\ I_0(\mathbf{q})$$
$$= \left[\frac{4}{3R_N^2}\delta_{i,j} + \frac{4}{9}\ q_iq_j\right] I_0(\mathbf{q}). \tag{3.11b}$$

The meson-nucleon vertex Γ is, analogously with V, given by

$$\langle p_1'|\Gamma|p_1\rangle = \mathcal{N}^2\left(\frac{1}{8}\right)^{3/2}\left(\frac{2\pi}{R_N^2}\right)^{3/2}\ \exp\left[-\frac{R_N^2}{6}(\mathbf{q}^2+\mathbf{k}^2)\right]\cdot$$
$$\times\int d^3Q\ \exp\left[-\frac{R_N^2}{6}\left\{\frac{9}{4}\left(\mathbf{Q}^2-\frac{4}{3}\mathbf{q}\cdot\mathbf{Q}\right)\right\}\right]\cdot\gamma(\mathbf{Q};\mathbf{k},\mathbf{q})\ \delta^{(3)}(\mathbf{p}_1'-\mathbf{p}_1-\mathbf{k})$$
$$\sim \mathcal{N}^2\left(\frac{2}{3}\right)^{3/2}\left(\frac{\pi}{R_N^2}\right)^3\cdot\exp\left[-\frac{1}{6}R_N^2\mathbf{k}^2\right]\ \bar{\Gamma}(\mathbf{q},\mathbf{k})\ \delta^{(3)}(\mathbf{p}_1'-\mathbf{p}_1-\mathbf{k}). \tag{3.12}$$

Here the last expression shows that the vertex has a Gaussian local form factor.

IV. SCALAR-EXCHANGE

The coupling of the scalar meson to the quarks we assume to be of the form

$$\mathcal{H}_S = \left[g_1\bar{\psi}\psi - g_2\Box(\bar{\psi}\psi)/(2\mu^2)\right]\ \sigma. \tag{4.1}$$

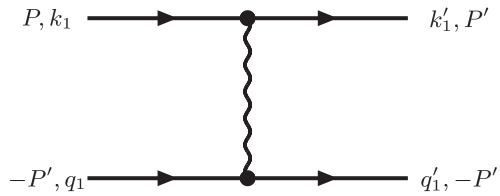

FIG. 3: V_{QQ} Scalar-exchange in the CM-frame

The corresponding vertex is

$$\Gamma_{QQ} \;=\; \bar{u}_Q(p')\left[g_1 + g_2\left(M^2 - p'\cdot p\right)/\mu^2\right]\; u_Q(p) \tag{4.2}$$

Now,

$$M^2 - p'\cdot p = M^2 - E'E + \mathbf{p}'\cdot\mathbf{p} \approx -\mathbf{k}^2/2. \tag{4.3}$$

Taking a common form factor for the two couplings, (4.3) implies a **zero in the potential**

$$0 = \left(g_1 - \frac{\mathbf{k}^2}{2\mu^2}g_2\right)^2 \approx g_1^2 - \frac{\mathbf{k}^2}{\mu^2}g_1 g_2,\;\; \mathbf{k}^2(0) = \mu^2\,\frac{g_1}{g_2}, \tag{4.4}$$

which marks an approximate simple zero. In ESC-models $g_1 = g_2$ and $\mathbf{k}^2(0) = m_\sigma^2$, so $\mu = m_\sigma \approx 2m_Q$.
On the quark level, the inclusion of the zero implies a change in the coefficients of the $\mathbf{k}^2$*-term which are of order* $1/M^2$.

A. Folding Amplitude Scalar-exchange

The Dirac-spinor part of the scalar-meson QQ-vertex is

$$\begin{aligned}
[\bar{u}_i(\mathbf{k}'_i)u_i(\mathbf{k}_i)] \;&=\; \sqrt{\frac{E'_i+m'_i}{2m'_i}\;\frac{E_i+m_i}{2m_i}}\cdot\chi_i'^{\dagger}\cdot\left[1 - \frac{\boldsymbol{\sigma}_i\cdot\mathbf{k}'_i}{E'_i+m_i}\frac{\boldsymbol{\sigma}_i\cdot\mathbf{k}_i}{E_i+m_i}\right] \\
&\approx\; \chi_i'^{\dagger}\left[1 - \frac{\mathbf{k}'_i\cdot\mathbf{k}_i}{4m_i^2} - \frac{i}{4m_i^2}\boldsymbol{\sigma}_i\cdot\mathbf{k}'_i\times\mathbf{k}_i\right]\;\chi_i \\
&=\; \chi_i'^{\dagger}\left[1 - \frac{\mathbf{Q}_i^2-\mathbf{k}^2}{16m_i^2} + \frac{i}{8m_i^2}\boldsymbol{\sigma}_i\cdot\mathbf{Q}_i\times\mathbf{k}\right]\;\chi_i.
\end{aligned} \tag{4.5}$$

Here is used that for the CQM $E_i \approx m_i$. The performance of the $\mathbf{Q}$-integral in (4.5) gives

$$[\bar{u}_i(\mathbf{k}'_i)u_i(\mathbf{k}_i)] \;\Rightarrow\; \chi_i'^{\dagger}\left[1 - \left(\frac{1}{4m_i^2R_N^2} + \frac{\mathbf{q}^2}{36m_i^2}\right) + \frac{\mathbf{k}^2}{16m_i^2} + \frac{i}{12m_i^2}\boldsymbol{\sigma}_i\cdot\mathbf{q}\times\mathbf{k}\right]\;\chi_i \tag{4.6}$$

Summing over the quarks leads to the vertex

$$\Gamma_{CQM} \;=\; \sum_{i=1-3}[\bar{u}_i(\mathbf{k}'_i)u_i(\mathbf{k}_i)] \Rightarrow 3\left[1 - \left(\frac{1}{4m_Q^2R_N^2} + \frac{\mathbf{q}^2}{36m_Q^2}\right) + \frac{\mathbf{k}^2}{16m_Q^2} + \frac{i}{36m_Q^2}\sum_i\boldsymbol{\sigma}_i\cdot\mathbf{q}\times\mathbf{k}\right] \tag{4.7}$$

The CQM replacement $m_Q \approx \sqrt{M'M}/3$ leads to

$$\Gamma_{CQM} \;=\; 3\left[\left(1 - \frac{1}{4m_Q^2R_N^2}\right) - \frac{\mathbf{q}^2}{4M'M} + \frac{9\mathbf{k}^2}{16M'M} + \frac{i}{4M'M}\sum_i\boldsymbol{\sigma}_N\cdot\mathbf{q}\times\mathbf{k}\right], \tag{4.8}$$

where we used $\sum_i \boldsymbol{\sigma}_i = \boldsymbol{\sigma}_N$. This assumes that the spin of the nucleon is given by the total spin of the quarks [27] This result should be compared with the $\big[\ldots\big]$-part of the vertex computed at the nucleon-level, $\boldsymbol{\Delta}^2 = (\mathbf{P}'-\mathbf{P})^2 = \mathbf{k}^2$,

$$\begin{aligned}
\Gamma_{NN} &\equiv \bar{u}(\mathbf{P}')\, u(\mathbf{P}) = \sqrt{\frac{E'+M'}{2M'}}\,\sqrt{\frac{E+M}{2M}}. \\
&\quad \times \chi'^{\,\dagger}\left[1 - \frac{\mathbf{p}'\cdot\mathbf{p}}{(E'+M')(E+M)} - \frac{i\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}}{(E'+M')(E+M)}\right]\chi \\
&\approx \sqrt{\frac{E'+M'}{2M'}}\,\sqrt{\frac{E+M}{2M}}\,\chi'^{\,\dagger}\left[1 - \frac{\mathbf{q}^2}{4M'M} + \frac{\boldsymbol{\Delta}^2}{16M'M} + \frac{i}{4M'M}\mathbf{q}\times\boldsymbol{\Delta}\cdot\boldsymbol{\sigma}\right]\chi \qquad (4.9a) \\
&\Rightarrow \left[1 - \frac{\mathbf{q}^2}{4M'M} + \frac{\boldsymbol{\Delta}^2}{16M'M} + \frac{i}{4M'M}\mathbf{q}\times\boldsymbol{\Delta}\cdot\boldsymbol{\sigma}\right]. \qquad (4.9b)
\end{aligned}$$

The last expression for Γ_{NN} is the correspondence of Γ_{CQM} in (4.8). This because in the transition from the potential V to the Lippmann-Schwinger potential $\mathcal{V}$ there occurs a factor $(E'+E)/(M'+M)$ [11]. Now,

$$\frac{E'+E}{M'+M} \approx 1 + \frac{\mathbf{q}^2+\boldsymbol{\Delta}^2/4}{2M'M},\ \ \frac{(E'+M')(E+M)}{4M'M} \approx 1 - \frac{\mathbf{q}^2+\boldsymbol{\Delta}^2/4}{2M'M}, \qquad (4.10)$$

showing that the product is $1 + O((M'M)^{-2} \approx 1$.

To bring Γ_{CQM} and Γ_{NN} in agreement the following:

a. The factor 3 is accounted for by scaling the quark-meson coupling, *i.e.* $g_Q^{(S)} = g^{(S)}/3$.

b. The "spurious" term $1/(4m_Q^2/R_N^2) = 9/(4M'MR_N^2)$ can be removed by introducing a gausslike distribution for $\mathbf{K}$, see subsection IV C.

c. Compared to Γ_{NN} the quark vertex Γ_{CQM} has an extra $8\mathbf{k}^2/(16M'M)$-term. This term can be cancelled by tuning the g_2-coupling. For that purpose we set

$$-g_2\frac{9}{16M'M} = g_1\frac{8}{16M'M},\ \ g_2/g_1 = -8/9 \approx -g_1. \qquad (4.11)$$

With these remarks it is shown that, although not identical, the QQ- and NN-vertex are (approximately) equivalent as far as the NN-potential is concerned. It also shows that the combination of scalar and vector exchange are necessary to bring this about.

This is consistent with the remarks after Eq. (4.4), i.e. $g_2 = g_1$ and $\mu = 2m_i$.

B. Scalar Form-factor Zero in QQ- and NN-vertex

Furthermore we remark that in the ESC-models we use a simple (first-order) zero for the scalar-meson exchange potential. Taking a zero at the vertex, as suggested by the analysis here, would imply a double zero. To match the practice in the ESC-models one can use the zero at the vertex partly for the proper generation of the $\mathbf{k}^2$*-term and partly for the simple zero in the potential, i.e. we expand*

$$\left(1 - \frac{\mathbf{k}^2}{2U^2}\right)^2 = \left(1 - \frac{\mathbf{k}^2}{U^2}\right) + O(\mathbf{k}^4/U^4).$$

Including the ESC-zero, the scalar-vertex becomes

$$\begin{aligned}
\Gamma_S &= \bar{u}(p')\left[g_1\left(1 - \frac{\mathbf{k}^2}{2U_S^2}\right) - g_2\frac{\mathbf{k}^2}{8m_Q^2}\right]\, u(p) \\
&\approx g_1\bar{u}(p')\left[1 - \left(\frac{1}{2U_S^2} + \frac{1}{8m_Q^2}\right)\,\mathbf{k}^2\right]\, u(p), \qquad (4.12)
\end{aligned}$$

which implies a zero in the scalar-quark coupling at

$$\mathbf{k}^2 \; = \; \frac{4m_Q^2 U_S^2}{U_S^2 + 4m_Q^2} \equiv 2U_Q^2.$$

With $U_S = 3M_N/3$ and $m_Q = M_N/3$ we get $U_Q \approx \sqrt{\frac{7}{32}}U_S \approx U_S/2$.
Considering scalar-exchange between a quark-line and a nucleon-line we have, up to terms of order $\mathbf{k}^4$,

$$\begin{aligned}\Gamma_Q \cdot \Gamma_N \; &\approx \; 1 - \left(\frac{1}{2U_Q^2} + \frac{1}{2U_S^2}\right)\mathbf{k}^2 = 1 - \left(\frac{U_Q^2 + U_S^2}{2U_Q^2 U_S^2}\right)\mathbf{k}^2 \\ &\equiv \; 1 - \frac{\mathbf{k}^2}{U_{Q+S}^2}, \quad U_{Q+S}^2 = 2U_S^2\left[1 + \frac{U_S^2}{U_Q^2}\right]^{-1}. \end{aligned} \tag{4.13}$$

We have

$$\frac{U_S^2}{U_Q^2} \; = \; 1 + U_S^2/(4m_Q^2) \approx 32/7,$$

where we used that $U_S = 750$ MeV $\approx (3/4)M_N$, and $m_Q = M_N/3$. This means that $U_{Q+S} \approx U_S/\sqrt{3}$.

Although this method can be chosen for the scalar- and and axial-meson coupling, it is not available for the vector-mesons. Therefore, the use of an extra coupling to match the $\mathbf{k}^2$ terms is preferable.

C. Removal spurious central term

Instead of the $\delta^3(\mathbf{K} - \mathbf{k})$-function we introduce a distribution of the momentum $\mathbf{K}$ exchange. Such a distribution might be caused by momentum exchange between the quark-line with the meson-vertex and the other two quarks in the nucleon, see Appendix B for an explicit demonstration. To produce a Γ_{CQM} without a "spurious" term, we consider the integrals

$$K_1(\mathbf{k}^2) \; = \; N_1 \int d^3K \; \exp\left[-\alpha\mathbf{K}^2 + \beta\mathbf{K}\cdot\mathbf{k}\right] \; e^{-\gamma\mathbf{k}^2} = N_1 \left(\frac{\pi}{\alpha}\right)^{3/2} \exp\left[-\left(\gamma - \frac{\beta^2}{4\alpha}\right)\mathbf{k}^2\right], \tag{4.14}$$

$$K_2(\mathbf{k}^2) \; = \; N_1 \int d^3K \; \mathbf{K}^2 \; \exp\left[-\alpha\mathbf{K}^2 + \beta\mathbf{K}\cdot\mathbf{k}\right] \; e^{-\gamma\mathbf{k}^2} = \left[\frac{3}{2}\alpha^{-1} + \frac{\beta^2}{4\alpha^2}\mathbf{k}^2\right] \; K_1(\mathbf{k}^2), \tag{4.15}$$

$$K_{3,i}(\mathbf{k}^2) \; = \; N_1 \int d^3K \; K_i \; \exp\left[-\alpha\mathbf{K}^2 + \beta\mathbf{K}\cdot\mathbf{k}\right] \; e^{-\gamma\mathbf{k}^2} = (\beta/2\alpha) \; \mathbf{k}_i \; K_1(\mathbf{k}^2), \tag{4.16}$$

and require

$$(i) \; K_1(\mathbf{k}^2) = \exp\left(-\frac{1}{6}R_N^2\mathbf{k}^2\right) \; , \; (ii) \; K_2(\mathbf{k}^2) = \left(\frac{4}{R_N^2} + \frac{\beta^2}{2\alpha^2}\mathbf{k}^2\right) \; K_1\mathbf{k}^2) \; , \; K_{3,i}(\mathbf{k}^2) = \mathbf{k}_i \; K_1(\mathbf{k}^2). \tag{4.17}$$

These conditions give $N_1 = (\pi/\alpha)^{-3/2}$ and the equations

$$a) \; \gamma - \frac{\beta^2}{4\alpha} = \frac{1}{6}R_N^2 \; , \; b) \; \frac{3}{2}\alpha^{-1} = \frac{1}{4}R_N^2 \; , \; c) \; \frac{\beta}{2\alpha} = 1. \tag{4.18}$$

It follows that $\alpha = (3/8)R_N^2$, $\beta = (3/4)R_N^2$, and $\gamma = (13/24)R_N^2$, and

$$K_1(\mathbf{k}^2) \; = \; N_1 \int d^3K \; \exp\left[-\frac{R_N^2}{6}\left\{\frac{9}{4}\mathbf{K}^2 - \frac{9}{2}\mathbf{K}\cdot\mathbf{k} + \frac{13}{4}\mathbf{k}^2\right\}\right]. \tag{4.19}$$

Using this expression the meson-nucleon vertex (3.12) becomes [1]

$$\begin{aligned}
\langle p_1'|\Gamma|p_1\rangle \; &= \; \mathcal{N}^2 \left(\frac{1}{8}\right)^{3/2} \left(\frac{2\pi}{R_N^2}\right)^{3/2} \; \left(\frac{3R_N^2}{8\pi}\right)^{3/2} \; \exp\left[-\frac{R_N^2}{6}\left(\mathbf{q}^2+\mathbf{k}^2\right)\right] \; \cdot \\
&\times \int d^3Q \; \exp\left[-\frac{R_N^2}{6}\left\{\frac{9}{4}\left(\mathbf{Q}^2 - \frac{4}{3}\mathbf{q}\cdot\mathbf{Q}\right)\right\}\right] \cdot \\
&\times \int d^3K \; \exp\left[-\frac{R_N^2}{6}\left\{\frac{9}{4}\left(\mathbf{K}^2 - 2\mathbf{k}\cdot\mathbf{K}+\mathbf{k}^2\right)\right\}\right] \cdot \\
&\times \gamma(\mathbf{Q},\mathbf{K};\mathbf{k},\mathbf{q}) \; \delta^{(3)}\left(\mathbf{p}_1' - \mathbf{p}_1 - \mathbf{k}\right). \qquad (4.20)
\end{aligned}$$

With this result we obtain

$$\Gamma_{CQM} \; = \; 3\left[1 - \frac{\mathbf{q}^2}{36m_Q^2} + \frac{\mathbf{k}^2}{16m_Q^2} + \frac{i}{36m_Q^2}\sum_i \boldsymbol{\sigma}_i\cdot\mathbf{q}\times\mathbf{k}\right]. \qquad (4.21)$$

With this method we reproduce the central and spin-orbit term in Eqn. (4.7) without the $1/(4m_i^2R_N^2)$ *term!*

V. PSEUDOSCALAR-EXCHANGE

We determine the QQ ps-exchange amplitude. Below, again i=1 is understood.
For the upper vertex in Fig. 3, line '1', we evaluate following spinor matrix-element

$$\begin{aligned}
[\bar{u}_i(\mathbf{k'}_i)\gamma_5 u_i(\mathbf{k}_i)] \; &= \; \sqrt{\frac{E_i'+m_i'}{2m_i'}\;\frac{E_i+m_i}{2m_i}}\cdot\chi_i'^{\dagger}\cdot\left[\frac{\boldsymbol{\sigma}_i\cdot\mathbf{k}_i}{E_i+m_i} - \frac{\boldsymbol{\sigma}_i\cdot\mathbf{k'}_i}{E_i'+m_i}\right]\;\chi_i \\
&\approx \; -\chi_i'^{\dagger}\left[\frac{\boldsymbol{\sigma}_i\cdot\mathbf{k}}{2m_i}\right]\;\chi_i, \qquad (5.1)
\end{aligned}$$

again because in the CQM $E_i \approx m_i$. Summing over the quarks gives

$$\widetilde{\Gamma}_{CQM,5} \; = \; \sum_{i=1-3}\,[\bar{u}_i(\mathbf{k'}_i)\gamma_5 u_i(\mathbf{k}_i)] = -3\chi_N'^{\dagger}\left[\frac{\boldsymbol{\sigma}\cdot\mathbf{k}}{2m_i}\right]\;\chi_N \qquad (5.2)$$

It is clear that this vertex is proportional to that for the OBE-coupling of the pseudoscalar meson. So, also for $m_i = \sqrt{M'M}/3$, i.e. the so-called "constituent" quarks, $\boldsymbol{\Gamma}_{QQ}$ in equivalent with $\boldsymbol{\Gamma}_{NN}$.
From $g_p = (2m_q/m_\pi)f_{pv}$, $g_P = (2M_N/m_\pi)f_{PV}$, and $g_p = g_P/3$ we find

$$f_{pv} = \frac{1}{3}\frac{M_N}{m_q}\;f_{PV},$$

which for $m_q = M_N/3$ the relation $f_{pv} = f_{PV}$.

---

[1] Comparing with (3.12) shows the $\mathbf{K}$-distribution change $\delta^{(3)}(\mathbf{K}-\mathbf{k}) \to (\pi\epsilon)^{-3/2}\exp\left[-\left(\mathbf{K}-\mathbf{k}\right)^2/\epsilon\right]$, $\epsilon = 8/(3R_N^2)$.

VI. FOLDING VECTOR-EXCHANGE VERTEX

The coupling of the vector mesons ($J^{PC} = 1^{--}$) to the quarks is given by the interaction Hamiltonian

$$\begin{aligned}\mathcal{H}_{VQQ} &= g_v \left(\bar{\psi}\gamma^\mu\psi\right)\ V_\mu + \frac{f_v}{4\mathcal{M}}\left(\bar{\psi}\ \sigma^{\mu\nu}\ \psi\right)\ (\partial_\mu V_\nu - \partial_\nu V_\mu) \\ &= \left[(\bar{\psi}\gamma^\mu\psi)F_{1,v} + \frac{i}{2}\left(\bar{\psi}\ \overleftrightarrow{\partial^\mu}\ \psi\right)\ F_{2,v}\right]\cdot V_\mu, \end{aligned} \tag{6.1}$$

where $a\ \overleftrightarrow{\partial^\mu}\ b = a\cdot\partial^\mu b - \partial^\mu a\cdot b$. The relation between the different coupling constants is $F_{1,v} = g_v + \frac{m_Q}{\mathcal{M}} f_v$, $F_{2,v} = -\frac{f_v}{\mathcal{M}}$, and reversely $g_v = F_{1,v} + m_Q F_{2,v}, f_v = -\mathcal{M}\ F_{2,e}$.

A. Direct-coupling

1. $\Gamma^0_{1,CQM}$-vertex: The QQ-meson vertices are

$$\begin{aligned}\left[\bar{u}_i(\mathbf{k'}_i)\gamma^0 u_i(\mathbf{k}_i)\right] &= \sqrt{\frac{E'_i+m'_i}{2m'_i}\ \frac{E_i+m_i}{2m_i}}\cdot\chi'^\dagger_i\cdot\left[1+\frac{\boldsymbol{\sigma}_i\cdot\mathbf{k'}_i}{E'_i+m_i}\frac{\boldsymbol{\sigma}_i\cdot\mathbf{k}_i}{E_i+m_i}\right] \\ &\approx \chi'^\dagger_i\left[1+\frac{\mathbf{k}'_i\cdot\mathbf{k}_i}{4m_i^2}+\frac{i}{4m_i^2}\boldsymbol{\sigma}_i\cdot\mathbf{k}'_i\times\mathbf{k}_i\right]\ \chi_i \\ &= \chi'^\dagger_i\left[1+\frac{\mathbf{Q}_i^2-\mathbf{k}^2}{16m_i^2}-\frac{i}{8m_i^2}\boldsymbol{\sigma}_i\cdot\mathbf{Q}_i\times\mathbf{k}\right]\ \chi_i. \end{aligned} \tag{6.2}$$

Notice that the $1/m_i^2$ terms are the same as for scalar-exchange apart from the sign. Therefore, from the expression (4.8) we now have

$$\Gamma^0_{1,CQM} = 3\left[\left(1+\frac{1}{4m_Q^2R_N^2}\right)+\frac{\mathbf{q}^2}{4M'M}-\frac{9\mathbf{k}^2}{16M'M}-\frac{i}{4M'M}\sum_i\boldsymbol{\sigma}_N\cdot\mathbf{q}\times\mathbf{k}\right], \tag{6.3}$$

The direct coupling to the nucleons gives

$$\begin{aligned}\Gamma^0_{1,NN} &= \left[\bar{u}_N(\mathbf{p}'\gamma^0u_N(\mathbf{p})\right] = \sqrt{\frac{E'+M'}{2M'}\ \frac{E+M}{2M}}\cdot\chi'^\dagger_N\cdot\left[1+\frac{\boldsymbol{\sigma}\cdot\mathbf{p}'}{E'+M'}\frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+M}\right] \\ &\approx \sqrt{\frac{E'+M'}{2M'}\ \frac{E+M}{2M}}\cdot\chi'^\dagger_N\left[1+\frac{\mathbf{p}'\cdot\mathbf{p}}{4M'M}+\frac{i}{4M'M}\boldsymbol{\sigma}\cdot\mathbf{p}'\times\mathbf{p}\right]\ \chi_N \\ &= \sqrt{\frac{E'+M'}{2M'}\ \frac{E+M}{2M}}\cdot\chi'^\dagger_N\left[1+\frac{\mathbf{q}^2-\mathbf{k}^2/4}{4M'M}-\frac{i}{4M'M}\boldsymbol{\sigma}\cdot\mathbf{q}\times\mathbf{k}\right]\ \chi_N. \\ &\Rightarrow \left[1+\frac{\mathbf{q}^2-\boldsymbol{\Delta}^2/4}{4M'M}-\frac{i}{4M'M}\boldsymbol{\sigma}\cdot\mathbf{q}\times\mathbf{k}\right]. \end{aligned} \tag{6.4}$$

To bring $\Gamma^0_{1,CQM}$ and $\Gamma^0_{1,NN}$ in agreement the following:

a. The factor 3 is accounted for by scaling the quark-meson coupling, *i.e.* $g_Q^{(V)} = g^{(V)}/3$.

b. The term $1/(4m_Q^2/R_N^2) = 9/(4M'MR_N^2) \approx 0.1$ for $R_N \approx 1$ fm, giving a 10% amplified of the central term..

c. Compared to Γ^0_{NN} the quark vertex Γ^0_{QQ} has an extra $-8\mathbf{k}^2/(16M'M)$-term. This term can be cancelled by introducing an extra QQV-interaction, similar to (4.1),

$$\Delta\mathcal{H}_V^{1)} = f'_{1,v}\left[\Box(\bar{\psi}\gamma^\mu\psi)(2\mu^2)\right]\ V_\mu, \tag{6.5}$$

and determine for $\mu = 0$ the coupling from the condition

$$f'_{1,v}\frac{9}{8M'M} = F_{1,v}\frac{8}{16M'M},\ f'_{1,v}/F_{1,v} = 4/9. \tag{6.6}$$

Terms for $\mu = m$ are of order $\sim 1/M^3$ which we neglect. With these remarks it is shown that, as in the scalar case, the QQ- and NN-vertex are (approximately) equivalent as far as the NN-potential is concerned. It also shows that the combination of scalar and vector exchange are necessary to bring this about. With the inclusion of the g_2'-contribution, the QQV-vertex becomes

$$\Gamma^0_{1,CQM} = 3\left[1+\left(\frac{1}{4m_Q^2 R_N^2}+\frac{\mathbf{q}^2-\mathbf{k}^2/4}{4M'M}\right)-\frac{i}{4M'M}\sum_i \boldsymbol{\sigma}_N\cdot\mathbf{q}\times\mathbf{k}\right], \tag{6.7}$$

So, the $\mathbf{q}^2$-, the $\mathbf{k}^2$-, and spin-orbit term are the same as for the coupling of the vector meson on the nucleon level. *The central term in $\Gamma^{(0)}_{QQ}$ has an extra $9/[4M'MR_N^2]$-term, which is a slight violation of the Idea/conjecture as formulated in the Introduction, similar to the scalar-meson case. As demonstrated in subsection IV C such "spurious" terms can be eliminated by introducing a $\mathbf{K}$-distribution. Henceforth, we omit such terms.*

2. $\boldsymbol{\Gamma}_{1,QQ}$-vertex: The QQ-meson vertices are

$$\begin{aligned}
[\bar{u}_i(\mathbf{k}'_i)\boldsymbol{\gamma}_i u_i(\mathbf{k}_i)] &= \sqrt{\frac{E_i'+m_i'}{2m_i'}\;\frac{E_i+m_i}{2m_i}}\cdot\chi_i'^{\dagger}\cdot\left[\frac{\boldsymbol{\sigma}_i\boldsymbol{\sigma}_i\cdot\mathbf{k}_i}{E_i+m_i}+\frac{\boldsymbol{\sigma}_i\cdot\mathbf{k}_i'\,\boldsymbol{\sigma}_i}{E_i'+m_i}\right]\chi_i \\
&\approx \chi_i'^{\dagger}\cdot\left[\frac{\mathbf{Q}_i}{2m_i}+\frac{i}{2m_i}(\boldsymbol{\sigma}_i\times\mathbf{k})\right]\chi_i \\
&\Rightarrow \chi_i'^{\dagger}\cdot\left[\frac{\mathbf{q}}{3m_i}+\frac{i}{2m_i}(\boldsymbol{\sigma}_i\times\mathbf{k})\right]\chi_i
\end{aligned} \tag{6.8}$$

Summing over the quarks leads to

$$\boldsymbol{\Gamma}_{1,CQM} = \sum_{i=1-3}[\bar{u}_i(\mathbf{k}'_i)\boldsymbol{\gamma}_i u_i(\bar{\mathbf{k}}_i)] = \chi_i'^{\dagger}\cdot\left[\frac{\mathbf{q}}{m_i}+\frac{i}{2m_i}(\boldsymbol{\sigma}_N\times\mathbf{k})\right]\chi_i \tag{6.9a}$$

$$\Rightarrow 3\left[\frac{\mathbf{q}}{M}+\frac{i}{2M}(\boldsymbol{\sigma}_N\times\mathbf{k})\right]. \tag{6.9b}$$

The direct coupling to the nucleons gives

$$\begin{aligned}
\boldsymbol{\Gamma}_{1,NN} &= [\bar{u}_N(\mathbf{p}')\boldsymbol{\gamma}u_N(\mathbf{p})] = \sqrt{\frac{E'+M'}{2M'}\;\frac{E+M}{2M}}\cdot\chi_N'^{\dagger}\cdot\left[\frac{\boldsymbol{\sigma}\boldsymbol{\sigma}\cdot\mathbf{p}}{E+M}+\frac{\boldsymbol{\sigma}\cdot\mathbf{p}'\,\boldsymbol{\sigma}}{E'+M'}\right]\chi_N \\
&\approx \sqrt{\frac{E'+M'}{2M'}\;\frac{E+M}{2M}}\cdot\chi_N'^{\dagger}\cdot\left[\frac{\mathbf{q}}{M}+\frac{i}{2M}(\boldsymbol{\sigma}\times\mathbf{k})\right]\chi_N \\
&\Rightarrow \chi_N'^{\dagger}\cdot\left[\frac{\mathbf{q}}{M}+\frac{i}{2M}(\boldsymbol{\sigma}\times\mathbf{k})\right]\chi_N
\end{aligned} \tag{6.10}$$

Again, we see that for $m_i = \sqrt{M'M}/3$, i.e. the so-called "constituent" quarks, $\boldsymbol{\Gamma}_{QQ}$ matches with $\boldsymbol{\Gamma}_{NN}$.

B. Derivative-coupling via Gordon-decomposition

It remains to established the relation of the vertices

$$\Gamma^\mu_{2,NN} = (p'+p)^\mu\ [\bar{u}(p'u(p] \quad\text{and}\quad \widetilde{\Gamma}^\mu_{2,QQ} = \sum_{i=1-3}(k'^\mu+k^\mu)\,[\bar{u}(\mathbf{k}_i')u(\mathbf{k}_i)]\,. \tag{6.11}$$

1. Γ^0_2-vertex: From the analysis of the scalar coupling, see Eqns. (4.5)-(4.8), the QQ-meson vertices are

$$\sum_{i=1-3}(k'_{i,0}+k_{i,0})\ [\bar{u}_i(\mathbf{k}'_i)\ u_i(\mathbf{k}_i)] \Rightarrow 6m_Q\ \left[1-\left(\frac{1}{4m_Q^2R_N^2}+\frac{\mathbf{q}^2}{36m_Q^2}\right)+\frac{\mathbf{k}^2}{16m_Q^2}+\frac{i}{36m_Q^2}\sum_i\boldsymbol{\sigma}_i\cdot\mathbf{q}\times\mathbf{k}\right] \tag{6.12}$$

The CQM replacement $m_Q \approx \sqrt{M'M}/3, (M'+M)/6$ leads to

$$\Gamma^0_{2,CQM} \approx (M'+M)\left[1-\left(\frac{1}{4m_Q^2R_N^2}+\frac{\mathbf{q}^2}{4M'M}\right)+\frac{9\mathbf{k}^2}{16M'M}+\frac{i}{4M'M}\sum_i \boldsymbol{\sigma}_N\cdot\mathbf{q}\times\mathbf{k}\right], \tag{6.13}$$

This vertex has to be compared with that at the NN-level:

$$\Gamma^0_{2,NN} \approx (M'+M)\left[1-\frac{\mathbf{q}^2}{4M'M}+\frac{\mathbf{k}^2}{16M'M}+\frac{i}{4M'M}\mathbf{q}\times\mathbf{k}\cdot\boldsymbol{\sigma}\right]. \tag{6.14}$$

So, the situation is again similar to the scalar case. The remedy to obtain agreement for the $\mathbf{k}^2$-term is the as in that case by introducing a zero in the coupling, or by adding an extra QQV-interaction, similar to (4.1) and (6.5),

$$\Delta\mathcal{H}_V^{(2)} = f'_{2,v}\left[\Box(i\bar{\psi}\overset{\leftrightarrow}{\partial}_\mu\psi)(2\mu^2)\right]\ V^\mu, \tag{6.15}$$

and determine for $\mu=0$ the coupling from the condition CHECK

$$f'_{2,v}\frac{9}{8M'M} = F_{2,v}\frac{8}{16M'M},\ f'_{2,v}/F_{2,v}=4/9. \tag{6.16}$$

Terms for $\mu=m$ are of order $\sim 1/M^3$ which again we neglect.

2. $\boldsymbol{\Gamma}_2$-vertex: For this term we neglect the $1/m_Q^2 \sim 1/M'M$-terms as in the NN-potential derivation, and therefore we get

$$\begin{aligned}\boldsymbol{\Gamma}_{2,CQM} &= \sum_{i=1-3}(\mathbf{k}'_i+\mathbf{k}_i)\ [\bar{u}_i(\mathbf{k'}_i)\ u_i(\mathbf{k}_i)] \\ &\Rightarrow\ 2\sum_i \mathbf{Q}_i\,[\bar{u}_i(\mathbf{k'}_i)\ u_i(\mathbf{k}_i)] \Rightarrow 2\mathbf{q},\end{aligned} \tag{6.17}$$

showing that without scaling, as in the case of $\Gamma^0_{2,QQ}$, the NN-vertex is produced.

So, with the results of the scalar and vector couplings, i.e. $\Gamma = 1, \gamma^\mu$, utilizing the Gordon-decomposition, the relation between QQM- and NNM-derivative couplings is most easily demonstrated.

C. Full quark-vector coupling

NOGTEDOEN:
At the quark-level the additional interaction is

$$\mathcal{H}_V^{(2)} = -h_v\left[\frac{\Box}{4m_Q^2}\left(i\bar{q}(x)\overset{\leftrightarrow}{\partial}_\mu q(x)\right)\right]\cdot\phi_V^\mu. \tag{6.18}$$

Since adaption is necessary in the direct and derivative term, we get for the full correction $h_v = g'_v + f'_v = (4/3)(g_V + f_V)/\mathcal{M}$, with $\mathcal{M} = m_Q/3$. Here, the $(\mathbf{p}'+\mathbf{p})$ term in (6.18) is
Since the adaption is in the direct and derivative term, we get for the full vector vertex

$$\bar{u}(p')\Gamma_v^\mu\ u(p) = \bar{u}(p')\left[G_{m,v}\gamma^\mu + \frac{1}{\mathcal{M}}G_{e,v}\ (p'+p)^\mu\right]\ u(p), \tag{6.19}$$

with

$$G_{m,v} = g_v + f_v\ ,\ G_{e,v} = -f_v....? = tedoen = -f_v\left[1+\frac{\kappa'}{\kappa}\frac{k^2}{8m_Q^2}\right], \tag{6.20}$$

where $f_v = \kappa_v\ g_v$. Now $k^2 \approx -\mathbf{k}^2$ so that $G_{e,v}$ exhibits a *zero* at $\mathbf{k}^2 = 8m_Q^2(\kappa_v/\kappa'_v)$.

VII. FOLDING AXIAL-VECTOR-EXCHANGE VERTEX

The coupling of the axial-vector mesons ($J^{PC}=1^{++}$, 1^{st} kind) to the quarks is given by the interaction Hamiltonian

$$\mathcal{H}_A = g_A[\bar{\psi}\gamma_\mu\gamma_5\psi]\phi^\mu_A + \frac{if_A}{\mathcal{M}}[\bar{\psi}\gamma_5\psi]\ \partial_\mu\phi^\mu_A. \tag{7.1}$$

1. Γ^0_5-vertex: The QQ-meson vertices are

$$\begin{aligned}[\bar{u}_i(\mathbf{k'}_i)\gamma^0_i\gamma_5 u_i(\mathbf{k}_i)] &= \sqrt{\frac{E'_i+m'_i}{2m'_i}\ \frac{E_i+m_i}{2m_i}}\cdot\chi'^\dagger_i\cdot\left[\frac{\boldsymbol{\sigma}_i\cdot\mathbf{k}_i}{E_i+m_i}+\frac{\boldsymbol{\sigma}_i\cdot\mathbf{k'}_i}{E'_i+m_i}\right]\ \chi_i \\ &\approx \chi'^\dagger_i\left[\frac{\boldsymbol{\sigma}_i\cdot\mathbf{Q}_i}{2m_i}\right]\ \chi_i \Rightarrow \chi'^\dagger_i\left[\frac{\boldsymbol{\sigma}_i\cdot\mathbf{q}}{3m_i}\right]\ \chi_i \end{aligned} \tag{7.2}$$

Summing over the quarks gives

$$\Gamma^0_{5,CQM} = \sum_{i=1-3}[\bar{u}_i(\mathbf{k'}_i)\gamma^0_i\gamma_5 u_i(\mathbf{k}_i)] = \chi'^\dagger_N\left[\frac{\boldsymbol{\sigma}_N\cdot\mathbf{q}}{3m_i}\right]\ \chi_N \Rightarrow \left[\frac{\boldsymbol{\sigma}_N\cdot\mathbf{q}}{\sqrt{M'M}}\right]. \tag{7.3}$$

It is clear that this vertex is proportional to that for the OBE-coupling of the axial-vector meson.

2. $\boldsymbol{\Gamma}_5$-vertex: The QQ-meson vertices are

$$\begin{aligned}[\bar{u}_i(\mathbf{k'}_i)\boldsymbol{\gamma}_i\gamma_5 u_i(\mathbf{k}_i)] &= \sqrt{\frac{E'_i+m'_i}{2m'_i}\ \frac{E_i+m_i}{2m_i}}\cdot\chi'^\dagger_i\cdot\left[\boldsymbol{\sigma}_i+\frac{\boldsymbol{\sigma}_i\cdot\mathbf{k'}_i\ \boldsymbol{\sigma}_i\ \boldsymbol{\sigma}_i\cdot\mathbf{k}_i}{(E'_i+m_i)(E_i+m_i)}\right]\ \chi_i \\ &\approx \chi'^\dagger_i\left[\boldsymbol{\sigma}_i+\frac{1}{4m_i^2}\left\{\mathbf{k}'_i(\boldsymbol{\sigma}_i\cdot\mathbf{k}_i)+\mathbf{k}_i(\boldsymbol{\sigma}_i\cdot\mathbf{k}'_i)-(\mathbf{k}'_i\cdot\mathbf{k}_i)\boldsymbol{\sigma}_i-i(\mathbf{k}'_i\times\mathbf{k}_i)\right\}\right]\ \chi_i \\ &= \chi'^\dagger_i\left[\boldsymbol{\sigma}_i+\frac{1}{16m_i^2}\left\{2\mathbf{Q}_i(\boldsymbol{\sigma}_i\cdot\mathbf{Q}_i)-2\mathbf{k}(\boldsymbol{\sigma}_i\cdot\mathbf{k})-(\mathbf{Q}_i^2-\mathbf{k}^2)\boldsymbol{\sigma}_i+2i(\mathbf{Q}_i\times\mathbf{k})\right\}\right]\ \chi_i \\ &\Rightarrow \chi'^\dagger_i\left[\boldsymbol{\sigma}_i+\frac{1}{2m_i^2}\frac{1}{3R_N^2}\ \boldsymbol{\sigma}_i-\frac{1}{4m_i^2}\frac{1}{R_N^2}\boldsymbol{\sigma}_i\right. \\ &\left.+\frac{1}{16m_i^2}\left\{\frac{8}{9}\mathbf{q}(\boldsymbol{\sigma}_i\cdot\mathbf{q})-2\mathbf{k}(\boldsymbol{\sigma}_i\cdot\mathbf{k})-(\frac{4}{9}\mathbf{q}^2-\mathbf{k}^2)\boldsymbol{\sigma}_i+\frac{4i}{3}(\mathbf{q}\times\mathbf{k})\right\}\right]\ \chi_i \end{aligned} \tag{7.4}$$

Summing over the quarks gives

$$\begin{aligned}\boldsymbol{\Gamma}_{5,CQM} &= \sum_{i=1,3}[\bar{u}_i(\mathbf{k'}_i)\boldsymbol{\gamma}_i\gamma_5 u_i(\mathbf{k}_i)] = \chi'^\dagger_N\left[\left(1-\frac{1}{12(m_iR_N)^2}\right)\boldsymbol{\sigma}\right. \\ &\left.+\frac{1}{16m_i^2}\left\{\frac{8}{9}\mathbf{q}(\boldsymbol{\sigma}\cdot\mathbf{q})-2\mathbf{k}(\boldsymbol{\sigma}\cdot\mathbf{k})-(\frac{4}{9}\mathbf{q}^2-\mathbf{k}^2)\ \boldsymbol{\sigma}+4i(\mathbf{q}\times\mathbf{k})\right\}\right]\ \chi_N. \end{aligned} \tag{7.5}$$

The direct coupling to the nucleons gives

$$\begin{aligned}\boldsymbol{\Gamma}_{5,NN} &= [\bar{u}_N(\mathbf{p}')\boldsymbol{\gamma}\gamma_5 u_N(\mathbf{p})] = \sqrt{\frac{E'+M'}{2M'}\ \frac{E+M}{2M}}\cdot\chi'^\dagger_N\ \left[\boldsymbol{\sigma}+\frac{(\boldsymbol{\sigma}\cdot\mathbf{p}')\ \boldsymbol{\sigma}\ (\boldsymbol{\sigma}\cdot\mathbf{p})}{(E'+M')(E+M)}\right]\ \chi_N \\ &\approx \sqrt{\frac{E'+M'}{2M'}\ \frac{E+M}{2M}}\cdot\chi'^\dagger_N\left[\boldsymbol{\sigma}+\frac{1}{4M'M}\left\{\mathbf{p}'(\boldsymbol{\sigma}\cdot\mathbf{p})+\mathbf{p}(\boldsymbol{\sigma}\cdot\mathbf{p}')-(\mathbf{p}'\cdot\mathbf{p})\boldsymbol{\sigma}-i(\mathbf{p}'\times\mathbf{p})\right\}\right]\ \chi_N \\ &\Rightarrow \left[\boldsymbol{\sigma}+\frac{1}{4M'M}\left\{2\mathbf{q}(\boldsymbol{\sigma}\cdot\mathbf{q})-\frac{1}{2}\mathbf{k}(\boldsymbol{\sigma}\cdot\mathbf{k})-\left(\mathbf{q}^2-\mathbf{k}^2/4\right)\ \boldsymbol{\sigma}+i(\mathbf{q}\times\mathbf{k})\right\}\right]. \end{aligned} \tag{7.6}$$

Similarly to the scalar- and vector-meson, the last expression is to be compared to $\boldsymbol{\Gamma}_{5,CQM}$ in (7.5).

For "constituent" quarks with $m_i = M/3$ the result (7.5) reads

$$\begin{aligned}
\mathbf{\Gamma}_{5,CQM} &\Rightarrow \chi_N'^\dagger \left[\left(1-\frac{3}{4(MR_N)^2}\right)\boldsymbol{\sigma}\right.\\
&\qquad \left. +\frac{1}{4M'M}\left\{2\mathbf{q}(\boldsymbol{\sigma}\cdot\mathbf{q}) - \frac{9}{2}\mathbf{k}(\boldsymbol{\sigma}\cdot\mathbf{k}) - (\mathbf{q}^2 - \frac{9}{4}\mathbf{k}^2)\ \boldsymbol{\sigma} + 9i(\mathbf{q}\times\mathbf{k})\right\}\right]\ \chi_N\\
&= \chi_N'^\dagger \left[\left(1-\frac{3}{4(MR_N)^2}+\frac{\mathbf{k}^2}{2M'M}\right)\boldsymbol{\sigma} - \frac{\mathbf{k}(\boldsymbol{\sigma}\cdot\mathbf{k})}{M'M}\right.\\
&\qquad \left. +\frac{1}{4M'M}\left\{2\mathbf{q}(\boldsymbol{\sigma}\cdot\mathbf{q}) - \frac{1}{2}\mathbf{k}(\boldsymbol{\sigma}\cdot\mathbf{k}) - (\mathbf{q}^2 - \mathbf{k}^2/4)\ \boldsymbol{\sigma} + 9i(\mathbf{q}\times\mathbf{k})\right\}\right]\ \chi_N
\end{aligned} \tag{7.7}$$

3. $\mathbf{\Gamma}_5$-vertex(continued A): Next, we impose for the quarks the conservation of the axial current. The current is

$$J^a_\mu = g_a\bar{\psi}\gamma_\mu\gamma_5\psi + \frac{if_a}{\mathcal{M}}\partial_\mu(\bar{\psi}\gamma_5\psi), \tag{7.8}$$

and $\partial\cdot J^A = 0$ imposes the relation

$$f_a = \left(\frac{m^2_{A_1}}{2m_Q\mathcal{M}}\right)^{-1} g_a. \tag{7.9}$$

Taking $m_{A_1} = \sqrt{2}m_\rho \approx 2\sqrt{2}m_Q$ the axial current becomes

$$J^a_\mu = g_a\left[\bar{\psi}\gamma_\mu\gamma_5\psi + \frac{i}{4m_Q}\partial_\mu(\bar{\psi}\gamma_5\psi)\right]. \tag{7.10}$$

The f_a-contributions to the axial-vertex are

$$\mu = 0\ :\ \sim (E'-E) \sim (M'Mm_Q)^{-1} \approx 0, \tag{7.11a}$$

$$\mu = i\ :\ -\frac{1}{4m_Q}\ \mathbf{k}\ [\bar{u}(\mathbf{k}'_i)\gamma_5 u(\mathbf{k}_i)] \Rightarrow +\sqrt{\frac{E'_i+m'_i}{2m'_i}\ \frac{E_i+m_i}{2m_i}}\cdot\chi_i'^\dagger\left[\frac{1}{8m_i^2}\ \mathbf{k}\ (\boldsymbol{\sigma}_i\cdot\mathbf{k})\right]\chi_i. \tag{7.11b}$$

Taking this f_a-contributions into account we obtain for "constituent" quarks:

$$\mathbf{\Gamma}_{5,NN} \Rightarrow \chi_N'^\dagger\left[\ \boldsymbol{\sigma} + \frac{1}{4M'M}\left\{2\mathbf{q}(\boldsymbol{\sigma}\cdot\mathbf{q}) - \left(\mathbf{q}^2 - \mathbf{k}^2/4\right)\ \boldsymbol{\sigma} + i(\mathbf{q}\times\mathbf{k})\right\}\right]\ \chi_N, \tag{7.12a}$$

$$\begin{aligned}
\mathbf{\Gamma}_{5,CQM} &\Rightarrow \chi_N'^\dagger\left[\left(1-\frac{3}{4(MR_N)^2}+\frac{\mathbf{k}^2}{2M'M}\right)\boldsymbol{\sigma}\right.\\
&\qquad\left.+\frac{1}{4M'M}\left\{2\mathbf{q}(\boldsymbol{\sigma}\cdot\mathbf{q}) - (\mathbf{q}^2-\mathbf{k}^2/4)\ \boldsymbol{\sigma} + 9i(\mathbf{q}\times\mathbf{k})\right\}\right]\ \chi_N
\end{aligned} \tag{7.12b}$$

Here, we omitted the factor $\sqrt{(E'+M')(E+M)/4M'M}$ for the same reasom as for the scalar- and vector-meson. **Remark $\mathbf{\Gamma}_{5,CQM}$**: (i) for $R_N \approx 1fm$ the term $3/4(MR_N)^2 \approx 3/100 \ll 1$ and may be neglected, (ii) the $\mathbf{k}^2/(2M'M)$ term can be removed by taking into account the zero in the vertices (see above), and (iii) the $\mathbf{k}(\boldsymbol{\sigma}\cdot\mathbf{k})/M'M$-term has been removed by adding an f_a-coupling at the quark-level in a way compatible with axial-current conservation.
The change in the zero is as follows: we write the zero in the form

$$\left(1-\mathbf{k}^2/U^2\right)\left(1+\mathbf{k}^2/2M_N^2\right) \approx 1-\mathbf{k}^2/\bar{U}^2,\ \bar{U} = U/\sqrt{1-U^2/2M_N^2}.$$

So, there (only) remains the problem with the spin-orbit terms! For the solution see the next paragraphs.

4. $\mathbf{\Gamma}_5$-vertex(continued B): We note that $\mathbf{\Gamma} = \sum_{i=1}^3 \bar{u}_i\boldsymbol{\gamma}_i\gamma_5 u_i\ \langle\bar{u}_N\mathbf{\Sigma}_N u_N\rangle$ for non-relativistic quarks, i.e. it measures the contribution of the quarks to the nucleon spin. In the parton model it appeared that a large portion of the nucleon spin has to come from gluonic and quark orbital angular momentum contributions [26]. In the ESC-model we ascribe the meson-couplings to the quark-antiquark pair creation process. To account for a modification for the axial-vector mesons we consider the following additional phenomenological interaction at the quark level [28]

$$\begin{aligned} \Delta\mathcal{H} &= \frac{ig'_a}{\mathcal{M}^2}\, \epsilon^{\mu\nu\alpha\beta}\, (\partial_\alpha\bar{\psi})\gamma_\nu(\partial_\beta\psi)\; A_\mu \\ &\Rightarrow\; \Delta\Gamma_5^\mu = \frac{ig'_a}{\mathcal{M}^2}\; [\bar{u}(p')\gamma_\nu u(p)]\; \epsilon^{\mu\nu\alpha\beta} p'_\alpha p_\beta. \end{aligned} \tag{7.13}$$

Now, we assume that $\mathcal{M} \sim M_N$. Then, if $\nu = n = 1,2,3$ the vertex is $\propto 1/M^3 \approx 0$. So, the only important contribution is given for $\nu = 0$. In this case, summing over the (valence) quarks,

$$\begin{aligned} \Delta\mathbf{\Gamma}^\mu_{5,CQM} &= \sum_{i=1}^{3} \Delta\mathbf{\Gamma}^\mu_{5,i} = -\frac{ig'_a}{\mathcal{M}^2} \sum_{i=1}^{3} [\bar{u}(k'_i)\gamma_{i,0} u(k_i)]\; (\mathbf{Q}_i \times \mathbf{k}) + O(1/M^3) \\ &\Rightarrow\; -\frac{2ig'_a}{M'M}\sqrt{\frac{E'+M'}{2M'}\,\frac{E+M}{2M}} \cdot \left[\chi'^\dagger_N \chi_N\right]\; (\mathbf{q}\times\mathbf{k}). \end{aligned} \tag{7.14}$$

By choosing $g'_a = g_a$, where g_a is the axial coupling constant at the quark level, the axial-vertex becomes $\mathbf{\Gamma}_{5,CQM} \sim \mathbf{\Gamma}_{5,NN}$.

5. **Orbital Angular Momentum interpretation**: In the parton model it appeared that a large portion of the nucleon spin comes from orbital quark motion and gluonic contributions [26]. The orbital angular momentum of the quarks is present for the non-forward matrix element, i.e. $\mathbf{p} \neq \mathbf{p}'$. Therefore we consider the following form of the additional interaction at the quark level [31]

$$\Delta\mathcal{H}' \;=\; g''_a\; \epsilon^{\mu\nu\alpha\beta}\; \left[\bar{\psi}(x)\mathcal{L}_{\nu\alpha\beta}\psi(x)\right]\; A_\mu, \tag{7.15}$$

where [44]

$$\mathcal{L}_{\nu\alpha\beta} = i\gamma_\nu \left(x_\alpha \frac{\partial}{\partial x^\beta} - x_\beta \frac{\partial}{\partial x^\alpha}\right) \tag{7.16}$$

is the orbital part of $\mathcal{M}_{\nu\alpha\beta}$, the angular momentum density operator. The vertex for the NNA$_1$-coupling is given by

$$\begin{aligned} \langle p', s'|\Delta H'|p, s; k, \rho\rangle &= \int d^4x \langle p', s'|\Delta\mathcal{H}'|p, s; k, \rho\rangle \sim \varepsilon_\mu(k,\rho)\; \epsilon^{\mu\nu\alpha\beta}\, \cdot \\ &\times \int d^4x\; e^{-ik\cdot x}\; \langle p', s'|i\bar{\psi}(x)\gamma_\nu\, (x_\alpha\nabla_\beta - x_\beta\nabla_\alpha)\, \psi(x)|p, s\rangle \end{aligned} \tag{7.17}$$

As pointed out in the previous paragraph the dominant contribution comes from $\nu = 0$. For this we have to evaluate the integral

$$J_{ab} = i\int d^4x\; e^{-ik\cdot x}\; \langle p', s'|i\psi^\dagger(x)\, (x_a\nabla_b - x_b\nabla_a)\, \psi(x)|p, s\rangle \tag{7.18}$$

Since we have only quarks, focussing on quark i=1, the quark field operator is

$$\psi_i(x) \Rightarrow \sum_s \int \frac{d^3k_i}{(2\pi)^{3/2}} \sqrt{\frac{m_Q}{\mathcal{E}(k_i)}}\; b(k_i, s_i)\; u(k_i, s_i)\; e^{-ik_i\cdot x}\; e^{-\alpha(\mathbf{k}_i^2 - \mathbf{k}_i\cdot\mathbf{p}/2)} \tag{7.19}$$

where $\alpha = 2R_N^2/3$. Using this in (7.18) we get

$$\begin{aligned}
J_{ab} &= \left[u^\dagger(k_i', s')\ u(k_i, s)\right] \int d^4x\ e^{i(k_i'-k_i-k)\cdot x}\ (x_a k_{i,b} - x_b k_{i,a})\, e^{-\alpha(\mathbf{k}_i'^2+\mathbf{k}_i^2)} e^{\alpha(\mathbf{k}_i'\cdot\mathbf{p}'+\mathbf{k}_i\cdot\mathbf{p})/2} \\
&= \left[u^\dagger(k_i', s')\ u(k_i, s)\right] \int d^4x\ e^{i(k_i'-k_i-k)\cdot x}\ (x_a k_{i,b} - x_b k_{i,a})\, e^{-\alpha(\mathbf{Q}^2-2\mathbf{q}\cdot\mathbf{Q})/2} \\
&= (2\pi)\delta(E'-E-k^0)\left[u^\dagger(k_i', s')\ u(k_i, s)\right] \int d^3x\ e^{-i(\mathbf{p}'-\mathbf{p}-\mathbf{k})\cdot\mathbf{x}}\ (x_a k_{i,b} - x_b k_{i,a})\, e^{-\alpha(\mathbf{Q}^2-2\mathbf{q}\cdot\mathbf{Q})/2} \\
&= -(2\pi i)\delta(E'-E-k^0)\left[u^\dagger(k_i', s')\ u(k_i, s)\right] \int d^3x\ \left[(\nabla_{p,a} k_{i,b} - \nabla_{p,b} k_{i,a})\, e^{-i(\mathbf{p}'-\mathbf{p}-\mathbf{k})\cdot\mathbf{x}}\right]\ e^{-\alpha(\mathbf{Q}^2-2\mathbf{q}\cdot\mathbf{Q})/2} \\
&= +(2\pi)^4 i\delta(E'-E-k^0)\delta^{(3)}(\mathbf{p}'-\mathbf{p}-\mathbf{k})\ \cdot(\alpha/2)\ \left[u^\dagger(k_i', s')\ u(k_i, s)\right]\cdot \\
&\quad \times (Q_a k_{i,b} - Q_b k_{i,a})\ e^{-\alpha(\mathbf{Q}^2-2\mathbf{q}\cdot\mathbf{Q})/2} \\
&\Rightarrow +(2\pi)^4 i\delta(E'-E-k^0)\delta^{(3)}(\mathbf{p}'-\mathbf{p}-\mathbf{k})\ \cdot(\alpha/3)\ \left[u^\dagger(k_i', s')\ u(k_i, s)\right]\cdot \\
&\quad \times (q_a k_{i,b} - q_b k_{i,a})\ e^{-\alpha(\mathbf{q}^2-2\mathbf{q}\cdot\mathbf{Q})/2}
\end{aligned} \tag{7.20}$$

Substition in (7.17) gives

$$\begin{aligned}
\langle p', s'|\Delta H'|p, s; k, \rho\rangle &\approx +(2\pi)^4 i\delta(E'-E-k^0)\delta^{(3)}(\mathbf{p}'-\mathbf{p}-\mathbf{k})\ g_a''(\alpha/3)\ \varepsilon_m(k,\rho)\cdot \\
&\quad \times \left[u^\dagger(k_i', s')\ u(k_i, s)\right]\ \epsilon_{mab}\,(q_a k_{i,b} - q_b k_{i,a})\ e^{-\alpha(\mathbf{q}^2-2\mathbf{q}\cdot\mathbf{Q})/2} \\
&\Rightarrow +(2\pi)^4 i\delta^{(4)}(p'-p-k)\ g_a''(2\alpha/3)\ \varepsilon_m(k,\rho)\cdot \\
&\quad \times \left[u^\dagger(k_i', s')\ u(k_i, s)\right]\ \boldsymbol{\varepsilon}(k,\rho)\cdot\mathbf{q}\times\mathbf{k}\ e^{-\alpha(\mathbf{q}^2-2\mathbf{q}\cdot\mathbf{Q})/2}
\end{aligned} \tag{7.21}$$

This leads to

$$\Delta\mathbf{\Gamma}'^m_{5,CQM} \propto ig_a''(4R_N^2/3)\sqrt{\frac{E'+M'}{2M'}\ \frac{E+M}{2M}}\cdot\left[\chi_N'^\dagger\chi_N\right]\ (\mathbf{q}\times\mathbf{k}). \tag{7.22}$$

which is equivalent to the result (7.14) for

$$g_a'' = -\frac{3g_a'}{2(MR_N)^2} = -\frac{3g_a}{8(MR_N)^2}. \tag{7.23}$$

Therefore, we can give the extra quark-coupling for the axial-vector vertex the interpretation as representing the orbital angular momentum of the three quarks in a nucleon (baryon) in the non-forward matrix element. In this sense it is related to the "spin-crisis" [26].

The "spin-cisis" in the quark-parton model revealed the importance of the orbital angular momentum and the gluonic content of the nucleon. At low energy the similar "crisis" shows up quite naturally in the axial-vector coupling. Taking the orbital angular momentum of the quarks into account nicely connects the "constituent" quark model with the axial-vector vertex at the nucleon level. Interesting would be to analyze this phenomenon in the IMF.

VIII. CONCLUSIONS AND DISCUSSION

We have shown that for all meson-nucleon-nucleon couplings the Pauli-expansion structure of the vertices can be reproduced by the "constituent" quark model. For the scalar, the vector, and axial-vector mesons it required extra couplings at the quark level in order to achieve this compatibility: (a) In the central part for scalar and vector mesons an extra interaction is necessary on the quark level to produce the correct $\mathbf{k}^2/M'M$ terms at the nucleon level; (b) Using $\delta^3(\mathbf{K}-\mathbf{k})$ at the meson vertex leads to "spurious" $1/R_N^2$-terms in the central parts for (i) the scalar- and vector-meson vertex, and (ii) the axial-vector vertex. As demonstrated in subsection IV C such terms can eliminated by the introduction of a gaussian like distribution in $\mathbf{K}$. Therefore, these terms are omitted. This leads, at least for terms up to $1/M'M$, to the conclusion:

The Idea/conjecture made in the Introduction, asserting that based on the Lorentz structure the ratio's of spin-spin, tensor-, and spin-orbit-vertices and the central potentials as given by the nucleon-level potentials are independent of the internal structure, can be realized completely in the CQM.

For the axial-vector coupling we have to introduce next to the usual $\gamma_\mu\gamma_5$-coupling a new coupling related to the orbital angular-momentum contents related to the transverse motion of the quarks in the nucleon. This is in line with the quark-parton model, where the so-called "spin-crisis" can be solved by invoking such and/or a *gluonic* contribution to the spin of the nucleon.

[In passing we note that an important non-zero *gluonic* contribution would be in line with the soft-core NN-models (OBE and ESC) contain the pomeron-exchange potential which also has a *gluonic* interpretation [32, 33]. The same is true for the multi-pomeron repulsion in nuclear matter [33, 34].]

The "constituent" quark model (CQM) is understood in a fundamental way by spontaneous dynamical chiral-symmetry breaking. The instanton solutions in QCD lead to a complex vacuum structure, which can be described by the instanton-liquid model. The pseudoscalar Nambu-Goldstone bosons are ordinary $Q\bar{Q}$-states with a small mass due to the strong instanton induced attraction. For other $Q\bar{Q}$-states there is not such a strong attraction giving vector- and scalar-meson masses of about $2m_Q \approx 750$ MeV. Strong-coupling QCD comes close to an understanding of the phenomenology of the CQM [35]. Another approach to derive the CQM is that of the Light-Front QCD of Wilson and collaborators [36].

In connection with the latter approach we note that putting $\mathbf{p}$ and $\mathbf{p}'$ in the xy-plane and going to the infinite-momentum-frame (I.M.F.) along the z-axis, translates directly our results for the meson-vertices to the quark-parton model. There, our impulse approximation makes perfect sense and our results may be considered as realistic. Thus, one would expect that the CQM-vertices correspond neatly to those at the nucleon-level. However, also here one expects to find an orbital contribution to the spin due to the transverse motion of the quarks, in view of the "spin-crisis".

Finally, the results in these notes can readily be extended to baryons.

APPENDIX A: OVERLAP INTEGRAL I

We consider the nucleon-nucleon graph for meson-exchange between the consituent quarks of the two nucleons. In the following we will use, instead of indices a,b,c, the indices i=1,2,3.

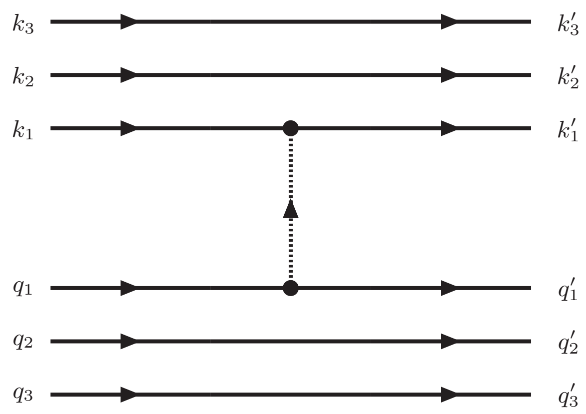

FIG. 4: External and internal momenta for meson-exchange

In Fig. 4 we have given the momenta for the initial and final nucleons, and the assigned momenta of the quarks. From momentum conservation we have

$$p_1 = k_1 + k_2 + k_3 \quad , \quad p_2 = q_1 + q_2 + q_3 \ ,$$
$$p'_1 = k'_1 + k'_2 + k'_3 \quad , \quad p'_2 = q'_1 + q'_2 + q'_3 \ , \tag{A1}$$

For meson-exchange with $p_1 - p'_1 = p'_2 - p_2 \equiv k$, we have for the matrix-element of the potential

$$\begin{aligned}
\langle p'_1 p'_2|V|p_1 p_2\rangle \ = \ & \int \prod_{i=1,3} d^3k_i \ \delta\left(\mathbf{p}_1 - \sum_i \mathbf{k}_i\right) \cdot \int \prod_{j=1,3} d^3k'_j \ \delta\left(\mathbf{p'}_1 - \sum_j \mathbf{k'}_j\right) \cdot \\
\times & \int \prod_{i=1,3} d^3q_i \ \delta\left(\mathbf{p}_2 - \sum_i \mathbf{q}_i\right) \cdot \int \prod_{j=1,3} d^3q'_j \ \delta\left(\mathbf{p'}_2 - \sum_j \mathbf{q'}_j\right) \cdot \\
\times & \ \tilde{\psi}^*_{p'_1}\left(\mathbf{k'}_1, \mathbf{k'}_2, \mathbf{k'}_3\right) \ \tilde{\psi}^*_{p'_2}\left(\mathbf{q'}_1, \mathbf{q'}_2, \mathbf{q'}_3\right) \cdot \tilde{\psi}_{p_1}\left(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3\right) \ \tilde{\psi}_{p_2}\left(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3\right) \ \cdot \\
\times & \ \delta^3\left(\mathbf{k'}_2 - \mathbf{k}_2\right) \ \delta^3\left(\mathbf{k'}_3 - \mathbf{k}_3\right) \ \delta^3\left(\mathbf{q'}_2 - \mathbf{q}_2\right) \ \delta^3\left(\mathbf{q'}_3 - \mathbf{q}_3\right) \ \cdot \\
\times & \ \frac{\gamma\left(k; k'_1, k_1\right) \ \gamma\left(k; q'_1, q_1\right)}{\mathbf{k}^2 + m_M^2} \cdot \delta^3\left(\mathbf{k} - \mathbf{k}'_1 + \mathbf{k}_1\right) \ \delta^3\left(\mathbf{k} + \mathbf{q}'_1 - \mathbf{q}_1\right) \ .
\end{aligned} \tag{A2}$$

In (A2) the γ's denote the vertex functions. Using the gaussian wave function of equation (3.1), we find for the exponent, denoted by f_{NN}, taking into account that the momenta of the 'spectator quarks 2 and 3 do not change, the expression

$$\begin{aligned}
f_{NN} \ = \ \exp & \left[-\frac{R_N^2}{6}\left\{(k_1 - k_2)^2 + (k_1 - k_3)^2 + (k_2 - k_3)^2\right.\right. \\
& + (q_1 - q_2)^2 + (q_1 - q_3)^2 + (q_2 - q_3)^2 \\
& + (k'_1 - k_2)^2 + (k'_1 - k_3)^2 + (k_2 - k_3)^2 \\
& \left.\left. + (q'_1 - q_2)^2 + (q'_1 - q_3)^2 + (q_2 - q_3)^2\right\}\right] \\
= \ \exp & \left[-\frac{R_N^2}{6}\left\{2\left(k_1^2 + k_1'^2\right) - 2\left(k_2 + k_3\right)\cdot\left(k_1 + k'_1\right)\right.\right. \\
& + 2\left(k_2^2 + k_3^2\right) + 2\left(k_2 - k_3\right)^2 \\
& + 2\left(q_1^2 + q_1'^2\right) - 2\left(q_2 + q_3\right)\cdot\left(q_1 + q'_1\right) \\
& \left.\left. + 2\left(q_2^2 + q_3^2\right) + 2\left(q_2 - q_3\right)^2\right\}\right] \ .
\end{aligned} \tag{A3}$$

In (A3) $k_1 \equiv \mathbf{k}_1$ etc. Introducing the 3-momenta

$$\begin{aligned}
P_{23} = k_2 + k_3 \quad &, \quad R_{23} = q_2 + q_3 \ , \\
K_{23} = k_2 - k_3 \quad &, \quad Q_{23} = q_2 - q_3 \ ,
\end{aligned} \tag{A4}$$

for the 'spectator quarks' and the 3-momenta

$$\begin{aligned}
\mathbf{k} = \mathbf{k}'_1 - \mathbf{k}_1 \quad &, \quad \mathbf{k}_1 = \frac{1}{2}\left(\mathbf{Q} - \mathbf{k}\right) \\
\mathbf{Q} = \mathbf{k}_1 + \mathbf{k}'_1 \quad &, \quad \mathbf{k}'_1 = \frac{1}{2}\left(\mathbf{Q} + \mathbf{k}\right) \\
\mathbf{k} = \mathbf{q}_1 - \mathbf{q}'_1 \quad &, \quad \mathbf{q}_1 = \frac{1}{2}\left(\mathbf{S} + \mathbf{k}\right) \\
\mathbf{S} = \mathbf{q}_1 + \mathbf{q}'_1 \quad &, \quad \mathbf{q}'_1 = \frac{1}{2}\left(\mathbf{S} - \mathbf{k}\right) .
\end{aligned} \tag{A5}$$

For the 'active quarks', we can rewrite f_N with the result

$$\begin{aligned} f_{NN} &= \exp\left[-\frac{R_N^2}{6}\left\{\left(\mathbf{Q}^2+\mathbf{k}^2\right)-2\mathbf{P}_{23}\cdot\mathbf{Q}\right.\right. \\ &\qquad +\left(\mathbf{P}_{23}^2+\mathbf{K}_{23}^2\right)+2\mathbf{K}_{23}^2 \\ &\qquad +\left(\mathbf{S}^2+\mathbf{k}^2\right)-2\mathbf{R}_{23}\cdot\mathbf{S} \\ &\qquad \left.\left.+\left(\mathbf{R}_{23}^2+\mathbf{Q}_{23}^2\right)+2\mathbf{Q}_{23}^2\right\}\right] . \end{aligned} \tag{A6}$$

In terms of the new variables defined in (A4) and (A5) the integration over the quark-momenta becomes

$$\begin{aligned} &\left(\frac{1}{8}\right)^4 \int d^3Q\ d^3S\ d^3P_{23}\ d^3K_{23}\ d^3R_{23}\ d^3Q_{23}\cdot \\ &\times\delta^{(3)}\left(p_1+\frac{1}{2}(k-Q)-P_{23}\right)\ \delta^{(3)}\left(p_2-\frac{1}{2}(k+S)-R_{23}\right)\cdot \\ &\times\delta^{(3)}\left(p_1'-\frac{1}{2}(k+Q)-P_{23}\right)\ \delta\left(p_2'+\frac{1}{2}(k-S)-R_{23}\right) \end{aligned} \tag{A7}$$

From these δ-function constraints one immediately gets

$$\begin{aligned} &\delta^{(3)}\left(p_1'-p_1-k\right)\ \delta^{(3)}\left(p_2'-p_2+k\right)= \\ &\delta^{(3)}\left(p_1'-p_1-k\right)\ \delta^{(3)}\left(p_1'+p_2'-p_1-p_2\right) \end{aligned} \tag{A8}$$

i.e. overall 3-momentum conservation and the fixing of $\mathbf{k}$ in terms of the external momenta.
Next we go over to the CM-variables. We have

$$\begin{aligned} \mathbf{p}_1 &= -\mathbf{p}_2=\mathbf{p}\ \ ,\ \ \mathbf{k}=\mathbf{p}'-\mathbf{p}\ \ ,\ \ \mathbf{p}=\mathbf{q}-\frac{1}{2}\mathbf{k}\ \ , \\ \mathbf{p}_1' &= -\mathbf{p}_2'=\mathbf{p}'\ \ ,\ \ \mathbf{q}=\frac{1}{2}(\mathbf{p}+\mathbf{p}')\ \ ,\ \ \mathbf{p}'=\mathbf{q}+\frac{1}{2}\mathbf{k}\ . \end{aligned} \tag{A9}$$

Then using (A4), we find for the expression between the curly brackets in (A5) the following expression

$$\begin{aligned} \left\{\ldots\right\} &= \left\{2\left(\mathbf{q}^2+\mathbf{k}^2\right)+\frac{9}{4}\left(\mathbf{Q}^2+\mathbf{S}^2\right)-3\mathbf{q}\cdot(\mathbf{Q}-\mathbf{S})\right. \\ &\qquad \left.+3\left(\mathbf{K}_{23}^2+\mathbf{Q}_{23}^2\right)\right\} \end{aligned} \tag{A10}$$

Now since the potential matrix elements will not depend on K_{23} and Q_{23}, apart from the appearance of these momenta in the exponential, we can integrate these variables out, with the result:

$$\int d^3K_{23}\ d^3Q_{23}\ \exp\left[-\frac{R_N^2}{2}\left(\mathbf{K}_{23}^2+\mathbf{Q}_{23}^2\right)\right]\Rightarrow\left(\frac{2\pi}{R_N^2}\right)^3\ . \tag{A11}$$

Collecting all results of the section, we find

$$\begin{aligned} \langle p_1'p_2'|V|p_1p_2\rangle &= \left(\frac{1}{8}\right)^4\left(\frac{2\pi}{R_N^2}\right)^3\ \mathcal{N}^4\int d^3Q\ d^3S\cdot \\ &\times\exp\left[-\frac{R_N^2}{6}\left\{\frac{9}{4}\left(\mathbf{Q}^2+\mathbf{S}^2\right)+2\left(\mathbf{q}^2+\mathbf{k}^2\right)-3\mathbf{q}\cdot(\mathbf{Q}-\mathbf{S})\right\}\right]\cdot \\ &\times V_{QQ}(\mathbf{Q},\mathbf{S};\mathbf{q},\mathbf{k})\ \delta^{(3)}\left(\mathbf{k}_1'+\mathbf{q}_1'-\mathbf{k}_1-\mathbf{q}_1\right), \end{aligned} \tag{A12}$$

where $V_{QQ}(\mathbf{Q},\mathbf{S};\mathbf{q},\mathbf{k})$ denotes the QQ-potential which contains the QQM-vertices and the meson propagator.

APPENDIX B: OVERLAP INTEGRAL II

We consider the nucleon-nucleon graph for meson-exchange between the consituent quarks of the two nucleons. In the following we will use, instead of indices a,b,c, the indices i=1,2,3. As shown in Fig. 5 we assume some momentum transfer beteen quark 1 and the pair quark 2 and quark 3.

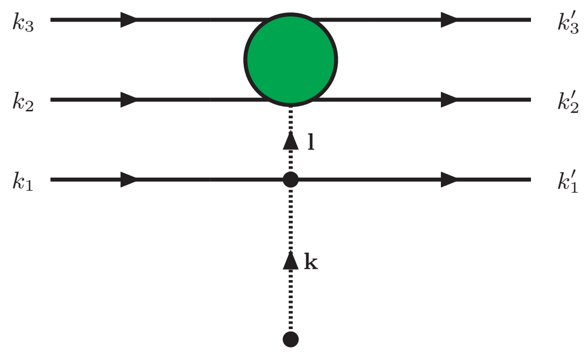

FIG. 5: External and internal momenta for meson-exchange

In Fig. 5, as in Fig. 4, we have given the momenta for the initial and final nucleons, and the assigned momenta of the quarks. From momentum conservation we have

$$p = k_1 + k_2 + k_3 \quad , \quad p' = k'_1 + k'_2 + k'_3. \tag{B1}$$

For meson-exchange with $p' - p \equiv k$, we have for the QQM-vertex

$$\begin{aligned}\langle p'|\Gamma|p\rangle &= \int \prod_{i=1,3} d^3k_i \, \delta\left(\mathbf{p} - \sum_i \mathbf{k}_i\right) \cdot \int \prod_{j=1,3} d^3k'_j \, \delta\left(\mathbf{p}' - \sum_j \mathbf{k}'_j\right) \cdot \\ &\times \; \tilde{\psi}^*_{p'}\left(\mathbf{k}'_1, \mathbf{k}'_2, \mathbf{k}'_3\right) \cdot \tilde{\psi}_p\left(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3\right) \cdot \gamma(\mathbf{k}, \mathbf{l}; \mathbf{k}'_1, \mathbf{k}_1) \cdot \\ &\times \; \delta^3\left(\mathbf{k}'_3 + \mathbf{k}'_2 - \mathbf{k}_3 - \mathbf{k}_2 - \mathbf{l}\right) \, \delta^3\left(\mathbf{k} - \mathbf{l} - \mathbf{k}'_1 + \mathbf{k}_1\right)\end{aligned} \tag{B2}$$

Similar to Appendix A we introduce the combinations

$$\mathbf{Q} = \mathbf{k}'_1 + \mathbf{k}_1 \quad , \quad \mathbf{K} = \mathbf{k}'_1 - \mathbf{k}_1, \tag{B3a}$$

$$\mathbf{P}_{23} = \mathbf{k}_2 + \mathbf{k}_3 \quad , \quad \mathbf{P}'_{23} = \mathbf{k}'_2 + \mathbf{k}'_3, \tag{B3b}$$

$$\mathbf{K}'_{23} = \mathbf{k}'_2 - \mathbf{k}'_3 \quad , \quad \mathbf{K}_{23} = \mathbf{k}_2 - \mathbf{k}_3. \tag{B3c}$$

Furthermore, we use the customary definitions $\mathbf{q} = (\mathbf{p}' + \mathbf{p})/2, \mathbf{k} = \mathbf{p}' - \mathbf{p}$, and note that $\mathbf{K} = \mathbf{k}'_1 - \mathbf{k}_1 = \mathbf{k} - \mathbf{l}$, and $\mathbf{P}'_{23} = \mathbf{P}_{23} + \mathbf{l}$. The Gaussian exponentials of the wave functions contain, see (3.1),

$$\begin{aligned} h_N &= (k_1 - k_2)^2 + (k_1 - k_3)^2 + (k_2 - k_3)^2, \\ h'_N &= (k'_1 - k'_2)^2 + (k'_1 - k'_3)^2 + (k'_2 - k'_3)^2, \end{aligned}$$

Using the definitions above and

$$\begin{aligned} \mathbf{P}'_{23} &= \mathbf{P}_{23} + \mathbf{l} = \mathbf{P}_{23} + (\mathbf{k} - \mathbf{K}) = \mathbf{p} + \mathbf{k} - \frac{1}{2}(\mathbf{Q} + \mathbf{K}), \\ \mathbf{P}_{23} &= \mathbf{p} - \mathbf{k}_1 = \mathbf{p} - \frac{1}{2}(\mathbf{Q} - \mathbf{K}), \end{aligned}$$

one finds

$$
\begin{aligned}
h_N &= \frac{1}{4}\left[(\mathbf{Q}-\mathbf{K}-\mathbf{P}_{23}-\mathbf{K}_{23})^2+(\mathbf{Q}-\mathbf{K}-\mathbf{P}_{23}+\mathbf{K}_{23})^2+4\mathbf{K}_{23}^2\right] \\
&= \frac{1}{2}\left[(\mathbf{Q}-\mathbf{K})^2+\mathbf{P}_{23}^2+3\mathbf{K}_{23}^2-2(\mathbf{Q}-\mathbf{K})\cdot\mathbf{P}_{23}\right] \\
&= \frac{1}{2}\left[\frac{9}{4}(\mathbf{Q}-\mathbf{K})^2-3(\mathbf{Q}-\mathbf{K})\cdot(\mathbf{q}-\mathbf{k}/2)+(\mathbf{q}^2+\mathbf{k}^2/4)+3\mathbf{K}_{23}^2\right],
\end{aligned}
$$

and

$$
\begin{aligned}
h'_N &= \frac{1}{4}\left[(\mathbf{Q}+\mathbf{K}-\mathbf{P'}_{23}-\mathbf{K'}_{23})^2+(\mathbf{Q}+\mathbf{K}-\mathbf{P'}_{23}+\mathbf{K'}_{23})^2+4\mathbf{K'}^2_{23}\right] \\
&= \frac{1}{2}\left[(\mathbf{Q}+\mathbf{K})^2+\mathbf{P'}^2_{23}+3\mathbf{K'}_{23}-2(\mathbf{Q}+\mathbf{K})\cdot\mathbf{P'}_{23}\right] \\
&= \frac{1}{2}\left[\frac{9}{4}(\mathbf{Q}+\mathbf{K})^2-3(\mathbf{Q}+\mathbf{K})\cdot(\mathbf{q}+\mathbf{k}/2)+(\mathbf{q}^2+\mathbf{k}^2/4)+3\mathbf{K'}^2_{23}\right].
\end{aligned}
$$

Summing gives

$$
h'_N+h_N = \frac{9}{4}(\mathbf{Q}^2+\mathbf{K}^2)-3\mathbf{Q}\cdot\mathbf{q}-\frac{3}{2}\mathbf{K}\cdot\mathbf{k}+\left(\mathbf{q^2}+\frac{\mathbf{1}}{\mathbf{4}}\mathbf{k^2}\right)+\frac{3}{2}(\mathbf{K'}^2_{23}+\mathbf{K}^2_{23}),
$$

and, with performing the $d^3K'_{23}$ and d^3K_{23} integrations,

$$
f_N=\exp\left[-\frac{R_N^2}{6}\left\{h'_N+h_N\right\}\right] \;\Rightarrow\; \left(\frac{4\pi}{R_N^2}\right)^3 \exp\left[-\frac{R_N^2}{6}\left\{\frac{9}{4}(\mathbf{Q}^2+\mathbf{K}^2)-3\mathbf{Q}\cdot\mathbf{q}-\frac{3}{2}\mathbf{K}\cdot\mathbf{k}+\left(\mathbf{q}^2+\frac{1}{4}\mathbf{k}^2\right)\right\}\right]. \quad \text{(B4)}
$$

Note that for $\mathbf{K}=\mathbf{k}$, after the K'_{23}, K_{23} integrations:

$$
h'_N+h_N \;\Rightarrow\; \frac{9}{4}\mathbf{Q}^2-3\mathbf{q}\cdot\mathbf{Q}+(\mathbf{q}^2+\mathbf{k}^2),
$$

which corresponds to the expression in Eqn. (3.12). Furthermore, the $\mathbf{k},\mathbf{K}$ dependence differs from (4.19) in the integrand by a factor

$$
\gamma(\mathbf{k},\mathbf{l}) = \exp\left[-\frac{R_N^2}{6}\left\{-3\mathbf{K}\cdot\mathbf{k}+3\mathbf{k}^2\right\}\right]=\exp\left[-\frac{R_N^2}{2}(\mathbf{k}\cdot\mathbf{l})\right]=\exp\left[-\frac{R_N^2}{8}\left\{(\mathbf{k}+\mathbf{l})^2-(\mathbf{k}-\mathbf{l})^2\right\}\right],
$$

which has consequences in particular for the spin-orbit coupling, giving 1/3 instead of 1. Including this factor in the vertex $\langle p'|\Gamma|p\rangle$ in (B2) makes it identical to (4.20), and leads to the expression for Γ_{CQM} given Eqn. (4.21) !

Remark: Consider the general Gauss integral:

$$\begin{aligned} J &= \int\int d^3Q\ d^3K\ \exp\Big[-\Big\{\alpha\mathbf{Q}^2 + \beta\mathbf{K}^2 + \gamma\mathbf{Q}\cdot\mathbf{K} + a\mathbf{Q}\cdot\mathbf{V} + b\mathbf{K}\cdot\mathbf{W}\Big\}\Big] \\ &= \left(\frac{\pi}{\alpha}\right)^{3/2} \int d^3K\ \exp\left[-\left(\beta - \frac{\gamma^2}{4\alpha}\right)\mathbf{K}^2 - \left(b\mathbf{W} - \frac{a\gamma}{2\alpha}\mathbf{V}\right)\cdot\mathbf{K}\right]\cdot\exp\left[+\frac{a^2}{4\alpha}\ \mathbf{V}^2\right] \\ &= \left(\frac{\pi}{\alpha}\right)^{3/2}\left(\frac{4\pi\alpha}{4\alpha\beta - \gamma^2}\right)^{3/2}\exp\left[\alpha\left(b\mathbf{W} - \frac{a\gamma}{2\alpha}\mathbf{V}\right)^2 \Big/ (4\alpha\beta - \gamma^2)\right]\cdot\exp\left[+\frac{a^2}{4\alpha}\ \mathbf{V}^2\right] \end{aligned}$$

The factor in front $(4\alpha\beta - \gamma^2)^{-3/2}$ determines the possible ”spurious” terms. One has

$$\begin{aligned} \mathbf{Q}^2 &: -\frac{d}{d\alpha} \to 6\beta\left(4\alpha\beta - \gamma^2\right)^{-5/2}, \\ \mathbf{K}^2 &: -\frac{d}{d\beta} \to 6\alpha\left(4\alpha\beta - \gamma^2\right)^{-5/2}. \end{aligned}$$

This implies that for a potential term $\propto \left(\mathbf{Q}^2 - \mathbf{K}^2\right)$ *the ”spurious” terms cancel when* $\alpha = \beta$*!*
The example worked out in this Appendix satifies this condition.

APPENDIX C: QUARK SUMMATION

The nucleons are part of the irrep **56** of SU(6). These states have the following structure [37]

$$|N\rangle \sim \frac{1}{\sqrt{2}}\left(\phi_{M,S}\ \chi_{M,S} + \phi_{M,A}\ \chi_{M,A}\right) \equiv (\mathbf{8},\mathbf{2}). \tag{C1}$$

Here $\phi_{M,S}$ and $\phi_{M,A}$ denote the three-quark isospin states with mixed symmetric and ant-symmetric character [37]. likewise for the spin states $\chi_{M,S}$ and $\chi_{M,A}$.
Since the total wave function is symmetric for the spin matrix elements one has

$$\sum_{i=1}^{3}\langle...|\boldsymbol{\sigma}_i|...\rangle \to 3\langle...|\boldsymbol{\sigma}_3|...\rangle. \tag{C2}$$

To find the proper factor we evaluate the proton matrix element:

$$\langle P,+|\sum_{i=1}^{3}\sigma_{i,z}|P,+\rangle \ = \ 3\langle P,+|\sigma_{3,z}|P,+\rangle = \frac{3}{2}\left\{\langle\chi^P_{M,S}|\sigma_{3,z}|\chi^P_{M,S}\rangle + \langle\chi^P_{M,A}|\sigma_{3,z}|\chi^P_{M,A}\rangle\right\}, \tag{C3}$$

where we used the orthonormality of the mixed states

$$\langle\chi_{M,S}|\chi_{M,S}\rangle = 1, \quad \langle\chi_{M,A}|\chi_{M,A}\rangle = 1, \quad \langle\chi_{M,S}|\chi_{M,A}\rangle = 0. \tag{C4}$$

Explicit evaluation:

$$\begin{aligned} \langle\chi^P_{M,S}(+)|\sigma_{3,z}|\chi^P_{M,S}(+)\rangle &= \frac{1}{6}\langle(+-+)\oplus(-++)\ominus 2(++-)|\sigma_{3,z}|(+-+)\oplus(-++)\ominus 2(++-)\rangle \\ &= \frac{1}{6}[1+1-4] = -\frac{1}{3}, \qquad\qquad \text{(C5a)} \\ \langle\chi^P_{M,A}(+)|\sigma_{3,z}|\chi^P_{M,A}(+)\rangle &= \frac{1}{2}\langle(+-+)\oplus(-++)|\sigma_{3,z}|(+-+)\oplus(-++)\rangle \\ &= \frac{1}{2}[1+1] = +1. \qquad\qquad \text{(C5b)} \end{aligned}$$

These results imply the relation

$$\langle P,+|\sum_{i=1}^{3}\boldsymbol{\sigma}_i|P,+\rangle = \langle P,+|\boldsymbol{\sigma}_N|P,+\rangle. \tag{C6}$$

It is now trivial to see that

$$\langle P,+|\sum_{i=1}^{3}\mathbf{1}_{op,i}|P,+\rangle = 3\langle P,+|\mathbf{1}_{op,N}|P,+\rangle. \tag{C7}$$

APPENDIX D: MOMENTUM-SPACE MESON-QUARK-QUARK VERTICES

1. Pauli-reduction Dirac-spinor Γ-matrix elements

The transition from Dirac spinors to Pauli spinors is given here, without approximations. We use the notations $\mathcal{E} = E + M$ and $\mathcal{E}' = E' + M'$, where $E = E(p, M)$ and $E' = E(p', M')$. Also, we omit, on the right-hand side in the expressions below, the final and initial Pauli spinors $\chi'^{\dagger}$ and χ respectively, which are self-evident.

$$\begin{aligned}
\bar{u}(\mathbf{p}')u(\mathbf{p}) &= +\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\left(1-\frac{\mathbf{p}'\cdot\mathbf{p}}{\mathcal{E}'\mathcal{E}}\right) - i\frac{\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}}{\mathcal{E}'\mathcal{E}}\right], &\text{(D1a)}\\
\bar{u}(\mathbf{p}')\gamma_5 u(\mathbf{p}) &= -\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\frac{\boldsymbol{\sigma}\cdot\mathbf{p}'}{\mathcal{E}'} - \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{\mathcal{E}}\right], &\text{(D1b)}\\
\bar{u}(\mathbf{p}')\gamma^0 u(\mathbf{p}) &= +\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\left(1+\frac{\mathbf{p}'\cdot\mathbf{p}}{\mathcal{E}'\mathcal{E}}\right) + i\frac{\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}}{\mathcal{E}'\mathcal{E}}\right], &\text{(D1c)}\\
\bar{u}(\mathbf{p}')\boldsymbol{\gamma}\ u(\mathbf{p}) &= +\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\left(\frac{\mathbf{p}'}{\mathcal{E}'} + \frac{\mathbf{p}}{\mathcal{E}}\right) + i\left(\frac{\boldsymbol{\sigma}\times\mathbf{p}'}{\mathcal{E}'} - \frac{\boldsymbol{\sigma}\times\mathbf{p}}{\mathcal{E}}\right)\right], &\text{(D1d)}\\
\bar{u}(\mathbf{p}')\gamma_5\gamma^0 u(\mathbf{p}) &= -\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\frac{\boldsymbol{\sigma}\cdot(\mathbf{p}'}{\mathcal{E}'} + \frac{\boldsymbol{\sigma}\cdot(\mathbf{p}}{\mathcal{E}}\right], &\text{(D1e)}\\
\bar{u}(\mathbf{p}')\gamma_5\boldsymbol{\gamma}\ u(\mathbf{p}) &= -\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\boldsymbol{\sigma} + \frac{(\boldsymbol{\sigma}\cdot\mathbf{p}')\ \boldsymbol{\sigma}\ (\boldsymbol{\sigma}\cdot\mathbf{p})}{\mathcal{E}'\mathcal{E}}\right]\\
&= -\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\left(1-\frac{\mathbf{p}'\cdot\mathbf{p}}{\mathcal{E}'\mathcal{E}}\right)\boldsymbol{\sigma} - i\frac{\mathbf{p}'\times\mathbf{p}}{\mathcal{E}'\mathcal{E}}\right.\\
&\qquad \left.+\frac{1}{\mathcal{E}'\mathcal{E}}\left(\boldsymbol{\sigma}\cdot\mathbf{p}\ \mathbf{p}' + \boldsymbol{\sigma}\cdot\mathbf{p}'\ \mathbf{p}\right)\right] \approx -\boldsymbol{\sigma},, &\text{(D1f)}
\end{aligned}$$

where we defined $\mathbf{k} = \mathbf{p}' - \mathbf{p}$, $\mathbf{q} = (\mathbf{p}' + \mathbf{p})/2$, and $\kappa_V = f_V/g_V$.

Using the the Gordon decomposition

$$i\ \bar{u}(p')\ \sigma^{\mu\nu}(p'-p)_\nu u(p) = \bar{u}(p')\left\{(M'+M)\gamma^\mu - (p'+p)^\mu\right\}\ u(p) \tag{D2}$$

one obtains for the complete vector-vertex

$$\bar{u}(p')\Gamma^\mu_V u(p) \equiv \bar{u}(p')\left[\gamma^\mu + \frac{i}{2\mathcal{M}}\kappa_V \sigma^{\mu\nu}(p'-p)_\nu\right]u(p)$$

$$= \bar{u}(p')\left[\left(1+\frac{M'+M}{2\mathcal{M}}\kappa_V\right)\gamma^\mu - \frac{\kappa_V}{2\mathcal{M}}(p'+p)_\mu\right]u(p) \Longrightarrow$$

$$\mu = 0 \ : \ +\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\left(1+\frac{M'+M}{2\mathcal{M}}\kappa_V\right)\left(1+\frac{\boldsymbol{\sigma}\cdot\mathbf{p}'\ \boldsymbol{\sigma}\cdot\mathbf{p}}{\mathcal{E}'\mathcal{E}}\right) - \frac{\kappa_V}{2\mathcal{M}}(E'+E)\left(1-\frac{\boldsymbol{\sigma}\cdot\mathbf{p}'\ \boldsymbol{\sigma}\cdot\mathbf{p}}{\mathcal{E}'\mathcal{E}}\right)\right], \quad \text{(D3a)}$$

$$\mu = i \ : \ +\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\left(1+\frac{M'+M}{2\mathcal{M}}\kappa_V\right)\left\{\left(\frac{\mathbf{p}'}{\mathcal{E}'}+\frac{\mathbf{p}}{\mathcal{E}}\right)+i\left(\frac{\boldsymbol{\sigma}\times\mathbf{p}'}{\mathcal{E}'}-\frac{\boldsymbol{\sigma}\times\mathbf{p}}{\mathcal{E}}\right)\right\} - \frac{\kappa_V}{2\mathcal{M}}(\mathbf{p}'+\mathbf{p})\left(1-\frac{\boldsymbol{\sigma}\cdot\mathbf{p}'\ \boldsymbol{\sigma}\cdot\mathbf{p}}{\mathcal{E}'\mathcal{E}}\right)\right]. \quad \text{(D3b)}$$

---

2. 1/M-expansion Γ-matrix elements

The exact transition from Dirac spinors to Pauli spinors is given in Appendix D 1. From the expressions in D 1, keeping only terms up to order $1/M$, and setting the scaling mass $\mathcal{M} = M$, we find that the vertex operators in Pauli-spinor space for the NNm vertices are given by

$$\bar{u}(\mathbf{p}')u(\mathbf{p}) = \left[\left(1-\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}\right) - \frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right], \quad \text{(D4a)}$$

$$\bar{u}(\mathbf{p}')\gamma_5 u(\mathbf{p}) = -\frac{1}{2M}[\boldsymbol{\sigma}\cdot(\mathbf{p}'-\mathbf{p})] = -\frac{1}{2M}[\boldsymbol{\sigma}\cdot\mathbf{k}], \quad \text{(D4b)}$$

$$\bar{u}(\mathbf{p}')\gamma^0 u(\mathbf{p}) = \left[\left(1+\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}\right) + \frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right], \quad \text{(D4c)}$$

$$\bar{u}(\mathbf{p}')\boldsymbol{\gamma}\ u(\mathbf{p}) = \frac{1}{2M}\left[(\mathbf{p}'+\mathbf{p}) + i\boldsymbol{\sigma}\times(\mathbf{p}'-\mathbf{p})\right], \quad \text{(D4d)}$$

$$\bar{u}(\mathbf{p}')\gamma_5\gamma^0 u(\mathbf{p}) = -\frac{1}{2M}[\boldsymbol{\sigma}\cdot(\mathbf{p}'+\mathbf{p})] = -\frac{1}{M}[\boldsymbol{\sigma}\cdot\mathbf{q}], \quad \text{(D4e)}$$

$$\bar{u}(\mathbf{p}')\gamma_5\boldsymbol{\gamma}\ u(\mathbf{p}) = -\left[\boldsymbol{\sigma} + \frac{1}{4M^2}(\boldsymbol{\sigma}\cdot\mathbf{p}')\ \boldsymbol{\sigma}\ (\boldsymbol{\sigma}\cdot\mathbf{p})\right] = -\left[\left(1-\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}\right)\boldsymbol{\sigma} - \frac{i}{4M^2}\mathbf{p}'\times\mathbf{p} + \frac{1}{4M^2}\left(\boldsymbol{\sigma}\cdot\mathbf{p}\ \mathbf{p}' + \boldsymbol{\sigma}\cdot\mathbf{p}'\ \mathbf{p}\right)\right] \approx -\boldsymbol{\sigma},, \quad \text{(D4f)}$$

where we defined $\mathbf{k} = \mathbf{p}' - \mathbf{p}$, $\mathbf{q} = (\mathbf{p}' + \mathbf{p})/2$, and $\kappa_V = f_V/g_V$. In passing we note that the inclusion of the $1/M^2$-terms is necessary in order to get spin-orbit potentials, like in the case of the OBE-potentials.

For the magnetic-coupling we use the Gordon decomposition

$$i\ \bar{u}(p')\ \sigma^{\mu\nu}(p'-p)_\nu u(p) = \bar{u}(p')\left\{2M\gamma^\mu - (p'+p)^\mu\right\}\ u(p) \quad \text{(D5)}$$

We get

$$i\ \bar{u}(p')\ \sigma^{\mu\nu}(p'-p)_\nu u(p) \Longrightarrow$$

$$\mu = 0 \ : \ -M\left[\left(1-\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}\right) + \frac{(p'^2+p^2)}{2M^2} - \frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right], \quad \text{(D6a)}$$

$$\mu = i \ : \ -\left[\frac{1}{2}(\mathbf{p}'+\mathbf{p}) - \frac{i}{2}\boldsymbol{\sigma}\times(\mathbf{p}'-\mathbf{p})\right]. \quad \text{(D6b)}$$

For the vector-vertex with direct and derivative coupling one has

$$
\begin{aligned}
\bar{u}(p')\Gamma^\mu_V u(p) &\equiv \bar{u}(p')\left[\gamma^\mu + \frac{i}{2M}\kappa_V \sigma^{\mu\nu}(p'-p)_\nu\right] u(p) \\
&= \bar{u}(p')\left[(1+\kappa_V)\gamma^\mu - \frac{\kappa_V}{2M}(p'+p)_\mu\right] u(p) \Longrightarrow \\
\mu = 0 \ \ :& \ \ \left[(1+\kappa_V)\left(1+\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}+\frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right)\right. \\
& \left. -\kappa_V \frac{E_{p'}+E_p}{2M}\left(1-\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}-\frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right)\right] \approx \\
& \left[1+(1+2\kappa_V)\left\{\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}+\frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right\} - \kappa_V \frac{\mathbf{p}'^2+\mathbf{p}^2}{4M^2}\right], \qquad \text{(D7a)} \\
\mu = i \ \ :& \ \ \frac{1}{M}\left[\frac{1}{2}(\mathbf{p}'+\mathbf{p}) + \frac{i}{2}(1+\kappa_V)\boldsymbol{\sigma}\times(\mathbf{p}'-\mathbf{p})\right]. \qquad \text{(D7b)}
\end{aligned}
$$

In terms of the magnetic and electric couplings, $g_V = G_M + G_E$ and $f_V = -G_e$, we have $g_V\kappa_V = -G_E, g_V(1+\kappa_V) = G_M, g_V(1+2\kappa_V) = G_M - G_E$. This gives

$$
\begin{aligned}
g_V\bar{u}(p')\Gamma^\mu_V u(p) &\equiv g_V\bar{u}(p')\left[\gamma^\mu + \frac{i}{2M}\kappa_V \sigma^{\mu\nu}(p'-p)_\nu\right] u(p) \\
\mu = 0 \ \ :& \ \ \left[(G_M+G_E)+(G_M-G_E)\left\{\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}+\frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right\} + G_E\frac{\mathbf{p}'^2+\mathbf{p}^2}{4M^2}\right], \qquad \text{(D8a)} \\
\mu = i \ \ :& \ \ \frac{1}{M}\left[\frac{1}{2}(\mathbf{p}'+\mathbf{p})(G_M+G_E) + \frac{i}{2}G_M\boldsymbol{\sigma}\times(\mathbf{p}'-\mathbf{p})\right]. \qquad \text{(D8b)}
\end{aligned}
$$

3. Meson-vertices in Pauli-spinor space

The transition from Dirac spinors to Pauli spinors is reviewed in Appendix C of [38]. Following this reference and keeping only terms up to order $(1/M)^2$, we find that the vertex operators in Pauli-spinor space for the QQm vertices are given by

$$
\begin{aligned}
\bar{u}(\mathbf{p}')\Gamma^{(1)}_P u(\mathbf{p}) &= -i\frac{f_P}{m_\pi}\left[\boldsymbol{\sigma}_1\cdot\mathbf{k} \pm \frac{\omega}{2M}\boldsymbol{\sigma}_1\cdot(\mathbf{p}'+\mathbf{p})\right], \qquad \text{(D9a)} \\
\bar{u}(\mathbf{p}')\Gamma^{(1)}_V u(\mathbf{p}) &= g_V\left[\left\{\left(1+\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}\right) - \frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right\}\ \phi^0_V\right. \\
& \left. -\frac{1}{2M}\left\{(\mathbf{p}'+\mathbf{p}) + i(1+\kappa_V)\boldsymbol{\sigma}_1\times\mathbf{k}\right\}\cdot\boldsymbol{\phi}_V\right], \qquad \text{(D9b)} \\
\bar{u}(\mathbf{p}')\Gamma^{(1)}_A u(\mathbf{p}) &= g_A\left[-\frac{1}{2M}\left\{\boldsymbol{\sigma}\cdot(\mathbf{p}'+\mathbf{p})\right\}\ \phi^0_A\right. \\
& \left. +\left\{\boldsymbol{\sigma} + \frac{1}{4M^2}(\boldsymbol{\sigma}\cdot\mathbf{p}')\ \boldsymbol{\sigma}\ (\boldsymbol{\sigma}\cdot\mathbf{p})\right\}\cdot\boldsymbol{\phi}_A\right], \qquad \text{(D9c)} \\
\bar{u}(\mathbf{p}')\Gamma^{(1)}_S u(\mathbf{p}) &= g_S\left[\left(1-\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}\right) - \frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right], \qquad \text{(D9d)}
\end{aligned}
$$

where we defined $\mathbf{k} = \mathbf{p}' - \mathbf{p}$ and $\kappa_V = f_V/g_V$. In the pseudovector vertex, the upper (lower) sign stands for creation (absorption) of the pion at the vertex. In passing we note that the inclusion of the $1/M^2$-terms is necessary in order to get spin-orbit potentials, like in the case of the OBE-potentials.

For the complete vector-meson coupling to the quarks we have, writing $\Gamma_V = \Gamma_V^{(m)} + \Gamma_V^{(e)}$,

$$\bar{u}(\mathbf{p}')\Gamma_V^{(m)}u(\mathbf{p}) = G_{m,v}\left[\left\{\left(1+\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}\right)+\frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right\}\ \phi_V^0 + \frac{1}{2M}\left\{(\mathbf{p}'+\mathbf{p})+i\boldsymbol{\sigma}_1\times\mathbf{k}\right\}\cdot\boldsymbol{\phi}_V\right], \tag{D10a}$$

$$\begin{aligned}\bar{u}(\mathbf{p}')\Gamma_V^{(e)}u(\mathbf{p}) &= G_{m,e}\left[\frac{\mathcal{E}'+\mathcal{E}}{\mathcal{M}}\left\{\left(1-\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}\right)-\frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right\}\ \phi_V^0 + \frac{(\mathbf{p}'+\mathbf{p})}{\mathcal{M}}\left\{\left(1-\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}\right)-\frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right\}\right\}\cdot\boldsymbol{\phi}_V\right] \\ &\approx G_{m,e}\left[2\frac{M}{\mathcal{M}}\left\{\left(1+\frac{\mathbf{p}'^2-\mathbf{p}'\cdot\mathbf{p}+\mathbf{p}^2}{4M^2}\right)-\frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right\}\ \phi_V^0+\frac{(\mathbf{p}'+\mathbf{p})}{\mathcal{M}}\right\}\cdot\boldsymbol{\phi}_V\right]\end{aligned} \tag{D10b}$$

The extra QQ axial-coupling has the vertex

$$\bar{u}(\mathbf{p}')\Gamma_A^{(o)}u(\mathbf{p}) = \frac{g_a'}{\mathcal{M}^2}\left[\frac{1}{M}\left\{(\mathbf{p}'\cdot\mathbf{p}-\mathbf{p}^2)\boldsymbol{\sigma}\cdot\mathbf{p}'+(\mathbf{p}'\cdot\mathbf{p}-\mathbf{p}'^2)\boldsymbol{\sigma}\cdot\mathbf{p}\right\}\ \phi_A^0 - 2i\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\phi}_A\right]. \tag{D11}$$

APPENDIX E: PAULI-SPINOR INVARIANTS FOR NUCLEON-NUCLEON POTENTIALS

Because of rotational invariance and parity conservation, the $\mathcal{V}$-matrix, which is a 4×4-matrix in Pauli-spinor space, can be expanded into the following set of in general 8 spinor invariants, see for example Ref. [10]. Introducing [39]

$$\mathbf{q}=\frac{1}{2}(\mathbf{p}'+\mathbf{p})\ ,\ \mathbf{k}=\mathbf{p}'-\mathbf{p}\ ,\ \mathbf{n}=\mathbf{p}\times\mathbf{p}', \tag{E1}$$

with, of course, $\mathbf{n}=\mathbf{q}\times\mathbf{k}$, we choose for the operators P_j in spin-space

$$\begin{aligned}&P_1=1,\quad P_2=\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2,\\ &P_3=(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})-\frac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\mathbf{k}^2,\\ &P_4=\frac{i}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{n},\quad P_5=(\boldsymbol{\sigma}_1\cdot\mathbf{n})(\boldsymbol{\sigma}_2\cdot\mathbf{n}),\\ &P_6=\frac{i}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{n},\\ &P_7=(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{k})+(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{q}),\\ &P_8=(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{k})-(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{q}).\end{aligned} \tag{E2}$$

Here we follow Ref. [40], where in contrast to Ref. [32], we have chosen P_3 to be a purely 'tensor-force' operator. The expansion in Pauli spinor-invariants reads

$$\mathcal{V}(\mathbf{p}',\mathbf{p})=\sum_{j=1}^{8}\ \widetilde{V}_j(\mathbf{p}'^2,\mathbf{p}^2,\mathbf{p}'\cdot\mathbf{p})\ P_j(\mathbf{p}',\mathbf{p})\ . \tag{E3}$$

APPENDIX F: EXTENDED-SOFT-CORE QQ-POTENTIALS IN MOMENTUM SPACE

The potential of the ESC-model contains the contributions from (i) One-boson-exchanges, (ii) Uncorrelated Two-Pseudo-scalar exchange, and (iii) Meson-Pair-exchange. In this section we review the potentials and indicate the changes with respect to earlier papers on the OBE- and ESC-models. The spin-1 meson-exchange is an important ingredient for the baryon-baryon force. In the ESC08-model we treat the vector-mesons and the axial-vector mesons according to the Proca- [41] and the B-field [42, 43] formalism respectively. For details, we refer to Appendix F.

1. One-Boson-Exchange Interactions in Momentum Space

The OBE-potentials are the same as given in [32, 40], with the exception of (i) the zero in the scalar form factor, and (ii) the axial-vector-meson potentials. Here, we review the OBE-potentials briefly, and give those potentials which are not included in the above references. The local interaction Hamilton densities for the different couplings are [44]

a) Pseudoscalar-meson exchange ($J^{PC} = 0^{-+}$)

$$\mathcal{H}_{PV} = \frac{f_{PV}}{m_{\pi^+}} \bar{\psi}\gamma_\mu\gamma_5\psi\partial^\mu\phi_P. \tag{F1}$$

This is the pseudovector coupling, and the relation with the pseudoscalar coupling is $g_P = 2M_B/m_{\pi^+}$, where M_B is the nucleon or hyperon mass.

b) Vector-meson exchange ($J^{PC} = 1^{--}$)

$$\mathcal{H}_V = g_V \bar{\psi}\gamma_\mu\psi\phi_V^\mu + \frac{f_V}{4\mathcal{M}} \bar{\psi}\sigma_{\mu\nu}\psi(\partial^\mu\phi_V^\nu - \partial^\nu\phi_V^\mu), \tag{F2}$$

where $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$, and the scaling mass $\mathcal{M}$, will be taken to be the proton mass.

c) Axial-vector-meson exchange ($J^{PC} = 1^{++}$, 1^{st} kind):

$$\mathcal{H}_A = g_A[\bar{\psi}\gamma_\mu\gamma_5\psi]\phi_A^\mu + \frac{if_A}{\mathcal{M}}[\bar{\psi}\gamma_5\psi]\ \partial_\mu\phi_A^\mu. \tag{F3}$$

In ESC04 the g_A-coupling was included, but not the derivative f_A-coupling [45]. Also, in ESC04 we used a local-tensor approximation (LTA) for the $(\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q})$ operator. Here, we improve on that considerably by avoiding such rather crude approximation. The details of our new treatment are given in Appendix E.

d) Axial-vector-meson exchange ($J^{PC} = 1^{+-}$, 2^{nd} kind):

$$\mathcal{H}_B = \frac{if_B}{m_B}[\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi]\ \partial_\nu\phi_B^\mu\ . \tag{F4}$$

In ESC04 this coupling was not included. Like for the axial-vector mesons of the 1^{st}-kind we include an SU(3)-nonet with members $b_1(1235), h_1(1170), h_1(1380)$. In the quark-model they are $Q\bar{Q}(^1P_1)$-states.

e) Scalar-meson exchange ($J^{PC} = 0^{++}$):

$$\mathcal{H}_S = g_S[\bar{\psi}\psi]\phi_S + \frac{f_S}{\mathcal{M}}[\bar{\psi}\gamma_\mu\psi]\ \partial^\mu\phi_S, \tag{F5}$$

which is the most general interaction. In ESC04 the possibility of the derivative f_S-coupling was not considered. By partial integration it is clear that the derivative vertex is proportional to the baryon mass difference and therefore there can only be expected sizable effects for κ-exchange. However, it is easily seen that for example for the $\Lambda N \leftrightarrow \Sigma N$ it leads to a coupled-channel problem with a (non-real) hermitean potential. This can be handled in principle, but complicates the solution and moreover this coupling is not needed. Therefore, we take $f_S = 0$.

f) Pomeron-exchange ($J^{PC} = 0^{++}$): The vertices for this 'diffractive'-exchange have the same Lorentz structure as those for scalar-meson-exchange.

g) Odderon-exchange ($J^{PC} = 1^{--}$):

$$\mathcal{H}_O = g_O[\bar{\psi}\gamma_\mu\psi]\phi_O^\mu + \frac{f_O}{4\mathcal{M}}[\bar{\psi}\sigma_{\mu\nu}\psi](\partial^\mu\phi_O^\nu - \partial^\nu\phi_O^\mu). \tag{F6}$$

Since the gluons are flavorless, Odderon-exchange is treated as an SU(3)-singlet. Furthermore, since the Odderon represents a Regge-trajectory with an intercept equal to that of the Pomeron, and is supposed not to contribute for small $\mathbf{k}^2$, we include a factor $\mathbf{k}^2/\mathcal{M}^2$ in the coupling.

Including form factors $f(\mathbf{x}'-\mathbf{x})$, the interaction hamiltonian densities are modified to

$$H_X(\mathbf{x}) = \int d^3x'\, f(\mathbf{x}'-\mathbf{x})\mathcal{H}_X(\mathbf{x}'), \tag{F7}$$

for $X = P,\ V,\ A$, and S (P = pseudo-scalar, V = vector, A = axial-vector, and S = scalar). The potentials in momentum space are the same as for point interactions, except that the coupling constants are multiplied by the Fourier transform of the form factors.

In the derivation of the V_i we employ the same approximations as in [32, 40], i.e.

1. We expand in $1/M$: $E(p) = \left[\mathbf{k}^2/4 + \mathbf{q}^2 + M^2\right]^{\frac{1}{2}}$ $\approx M + \mathbf{k}^2/8M + \mathbf{q}^2/2M$ and keep only terms up to first order in $\mathbf{k}^2/M$ and $\mathbf{q}^2/M$. This except for the form factors where the full $\mathbf{k}^2$-dependence is kept throughout the calculations. Notice that the gaussian form factors suppress the high $\mathbf{k}^2$-contributions strongly.

2. In the meson propagators $(-(p_1-p_3)^2 + m^2) \approx (\mathbf{k}^2 + m^2)$.

3. When two different baryons are involved at a BBM-vertex their average mass is used in the potentials and the non-zero component of the momentum transfer is accounted for by using an effective mass in the meson propagator (for details see [40]).

Due to the approximations we get only a linear dependence on $\mathbf{q}^2$ for V_1. In the following, separating the local and the non-local parts, we write

$$V_i(\mathbf{k}^2,\mathbf{q}^2) = V_{ia}(\mathbf{k}^2) + V_{ib}(\mathbf{k}^2)(\mathbf{q}^2 + \frac{1}{4}\mathbf{k}^2), \tag{F8}$$

where in principle $i = 1, 8$.

The OBE-potentials are now obtained in the standard way (see e.g. [32, 40]) by evaluating the BB-interaction in Born-approximation. We write the potentials V_i of Eqs. (F8) in the form

$$V_i(\mathbf{k}^2,\mathbf{q}^2) = \sum_X \Omega_i^{(X)}(\mathbf{k}^2)\cdot\Delta^{(X)}(\mathbf{k}^2, m^2, \Lambda^2). \tag{F9}$$

Furthermore for $X = P, V$

$$\Delta^{(X)}(\mathbf{k}^2, m^2, \Lambda^2) = e^{-\mathbf{k}^2/\Lambda^2} / \left(\mathbf{k}^2 + m^2\right), \tag{F10}$$

and for $X = S, A$ a zero in the form factor

$$\Delta^{(S)}(\mathbf{k}^2, m^2, \Lambda^2) = \left(1 - \mathbf{k}^2/U^2\right)\ e^{-\mathbf{k}^2/\Lambda^2} / \left(\mathbf{k}^2 + m^2\right), \tag{F11}$$

and for $X = D, O$

$$\Delta^{(D)}(\mathbf{k}^2, m^2, \Lambda^2) = \frac{1}{\mathcal{M}^2} e^{-\mathbf{k}^2/(4m^2_{P,O})}. \tag{F12}$$

In the latter expression $\mathcal{M}$ is a universal scaling mass, which is again taken to be the proton mass. The mass parameter m_P controls the $\mathbf{k}^2$-dependence of the Pomeron-, f-, f'-, A_2-, and $K^{\star\star}$-potentials. Similarly, m_O controls the $\mathbf{k}^2$-dependence of the Odderon.

In the following we give the OBE-potentials in momentum-space for the hyperon-nucleon systems. From these those for NN and YY can be deduced easily. We assign the particles 1 and 3 to be hyperons, and particles 2 and 4 to be nucleons. Mass differences among the hyperons and among the nucleons will be neglected.

2. Non-strange Meson-exchange

For the non-strange mesons the mass differences at the vertices are neglected, we take at the YYM- and the NNM-vertex the average hyperon and the average nucleon mass respectively. This implies that we do not include contributions to the Pauli-invariants P_7 and P_8. For vector-, and diffractive OBE-exchange we refer the reader to Ref. [40], where the contributions to the different $\Omega_i^{(X)}$'s for baryon-baryon scattering are given in detail.

(a) Pseudoscalar-meson exchange:

$$\Omega_{2a}^{(P)} = -g_{13}^{p} g_{24}^{p} \left(\frac{\mathbf{k}^2}{12 M_y M_n}\right) \quad , \quad \Omega_{3a}^{(P)} = -g_{13}^{p} g_{24}^{p} \left(\frac{1}{4 M_y M_n}\right), \tag{F13a}$$

$$\Omega_{2b}^{(P)} = +g_{13}^{p} g_{24}^{p} \left(\frac{\mathbf{k}^2}{24 M_y^2 M_n^2}\right) \quad , \quad \Omega_{3b}^{(P)} = +g_{13}^{p} g_{24}^{p} \left(\frac{1}{8 M_y^2 M_n^2}\right), . \tag{F13b}$$

PV-formulas:

$$\Omega_{2a}^{(P)} = -f_{13}^{pv} f_{24}^{pv} \left(\frac{\mathbf{k}^2}{3 m_{\pi^+}^2}\right) \quad , \quad \Omega_{3a}^{(P)} = -f_{13}^{pv} f_{24}^{pv} \left(\frac{1}{m_{\pi^+}^2}\right), \tag{F13c}$$

$$\Omega_{2b}^{(P)} = +f_{13}^{pv} f_{24}^{pv} \left(\frac{\mathbf{k}^2}{6 m_{\pi^+}^2 M_y M_n}\right) \quad , \quad \Omega_{3b}^{(P)} = +f_{13}^{pv} f_{24}^{pv} \left(\frac{1}{2 m_{\pi^+}^2 M_y^2 M_n^2}\right), . \tag{F13d}$$

(b) Vector-meson exchange:

$$\begin{aligned}
\Omega_{1a}^{(V)} &= \left\{ g_{13}^{v} g_{24}^{v} \left(1 - \frac{\mathbf{k}^2}{2 M_y M_n}\right) - g_{13}^{v} f_{24}^{v} \frac{\mathbf{k}^2}{4 \mathcal{M} M_n} - f_{13}^{v} g_{24}^{v} \frac{\mathbf{k}^2}{4 \mathcal{M} M_y} \right. \\
&\quad \left. + f_{13}^{v} f_{24}^{v} \frac{\mathbf{k}^4}{16 \mathcal{M}^2 M_y M_n} \right\}, \quad \Omega_{1b}^{(V)} = g_{13}^{v} g_{24}^{v} \left(\frac{3}{2 M_y M_n}\right), \\
\Omega_{2a}^{(V)} &= -\frac{2}{3} \mathbf{k}^2\, \Omega_{3a}^{(V)}, \quad \Omega_{2b}^{(V)} = -\frac{2}{3} \mathbf{k}^2\, \Omega_{3b}^{(V)}, \\
\Omega_{3a}^{(V)} &= \left\{ (g_{13}^{v} + f_{13}^{v} \frac{M_y}{\mathcal{M}})(g_{24}^{v} + f_{24}^{v} \frac{M_n}{\mathcal{M}}) - f_{13}^{v} f_{24}^{v} \frac{\mathbf{k}^2}{8 \mathcal{M}^2} \right\} / (4 M_y M_n), \\
\Omega_{3b}^{(V)} &= -(g_{13}^{v} + f_{13}^{v} \frac{M_y}{\mathcal{M}})(g_{24}^{v} + f_{24}^{v} \frac{M_n}{\mathcal{M}}) / (8 M_y^2 M_n^2), \\
\Omega_{4}^{(V)} &= -\left\{ 12 g_{13}^{v} g_{24}^{v} + 8 (g_{13}^{v} f_{24}^{v} + f_{13}^{v} g_{24}^{v}) \frac{\sqrt{M_y M_n}}{\mathcal{M}} - f_{13}^{v} f_{24}^{v} \frac{3 \mathbf{k}^2}{\mathcal{M}^2} \right\} / (8 M_y M_n) \\
\Omega_{5}^{(V)} &= -\left\{ g_{13}^{v} g_{24}^{v} + 4 (g_{13}^{v} f_{24}^{v} + f_{13}^{v} g_{24}^{v}) \frac{\sqrt{M_y M_n}}{\mathcal{M}} + 8 f_{13}^{v} f_{24}^{v} \frac{M_y M_n}{\mathcal{M}^2} \right\} / (16 M_y^2 M_n^2) \\
\Omega_{6}^{(V)} &= -\left\{ (g_{13}^{v} g_{24}^{v} + f_{13}^{v} f_{24}^{v} \frac{\mathbf{k}^2}{4 \mathcal{M}^2}) \frac{(M_n^2 - M_y^2)}{4 M_y^2 M_n^2} - (g_{13}^{v} f_{24}^{v} - f_{13}^{v} g_{24}^{v}) \frac{1}{\sqrt{\mathcal{M}^2 M_y M_n}} \right\}.
\end{aligned} \tag{F14}$$

(c) Scalar-meson exchange:

$$\begin{aligned}
\Omega_{1}^{(S)} &= -g_{13}^{s} g_{24}^{s} \left(1 + \frac{\mathbf{k}^2}{4 M_y M_n} - \frac{\mathbf{q}^2}{2 M_y M_n}\right) \\
\Omega_{1b}^{(S)} &= +g_{13}^{s} g_{24}^{s} \frac{1}{2 M_y M_n} \quad , \quad \Omega_{4}^{(S)} = -g_{13}^{s} g_{24}^{s} \frac{1}{2 M_y M_n} \\
\Omega_{5}^{(S)} &= g_{13}^{s} g_{24}^{s} \frac{1}{16 M_y^2 M_n^2} \quad , \quad \Omega_{6}^{(S)} = -g_{13}^{s} g_{24}^{s} \frac{(M_n^2 - M_y^2)}{4 M_y M_n}.
\end{aligned} \tag{F15}$$

(d) Axial-vector-exchange $J^{PC} = 1^{++}$:

$$\Omega^{(A)}_{2a} = -g^a_{13}g^a_{24}\left[1-\frac{2\mathbf{k}^2}{3M_yM_n}\right] + \left[\left(g^A_{13}f^A_{24}\frac{M_n}{\mathcal{M}} + f^A_{13}g^A_{24}\frac{M_y}{\mathcal{M}}\right) - f^A_{13}f^A_{24}\frac{\mathbf{k}^2}{2\mathcal{M}^2}\right]\ \frac{\mathbf{k}^2}{6M_yM_n}$$

$$\Omega^{(A)}_{2b} = -g^a_{13}g^a_{24}\left(\frac{3}{2M_yM_n}\right)$$

$$\Omega^{(A)}_{3} = -g^a_{13}g^a_{24}\left[\frac{1}{4M_yM_n}\right] + \left[\left(g^A_{13}f^A_{24}\frac{M_n}{\mathcal{M}} + f^A_{13}g^A_{24}\frac{M_y}{\mathcal{M}}\right) - f^A_{13}f^A_{24}\frac{\mathbf{k}^2}{2\mathcal{M}^2}\right]\ \frac{1}{2M_yM_n}$$

$$\Omega^{(A)}_{4} = -g^a_{13}g^a_{24}\left[\frac{1}{2M_yM_n}\right]\ ,\ \Omega^{(A)}_{6} = -g^a_{13}g^a_{24}\left[\frac{(M_n^2-M_y^2)}{4M_y^2M_n^2}\right]$$

$$\Omega^{(A)'}_{5} = -g^a_{13}g^a_{24}\left[\frac{2}{M_yM_n}\right] \tag{F16}$$

Here, we used the B-field description with $\alpha_r = 1$, see Appendix F. The detailed treatment of the potential proportional to P'_5, i.e. with $\Omega^{(A)'}_5$, is given in [46], Appendix B.

(e) Axial-vector mesons with $J^{PC} = 1^{+-}$:

$$\Omega^{(B)}_{2a} = +f^B_{13}f^B_{24}\frac{(M_n+M_y)^2}{m_B^2}\left(1-\frac{\mathbf{k}^2}{4M_yM_n}\right)\left(\frac{\mathbf{k}^2}{12M_yM_n}\right),\ \Omega^{(B)}_{2b} = +f^B_{13}f^B_{24}\frac{(M_n+M_y)^2}{m_B^2}\left(\frac{\mathbf{k}^2}{8M_y^2M_n^2}\right)$$

$$\Omega^{(B)}_{3a} = +f^B_{13}f^B_{24}\frac{(M_n+M_y)^2}{m_B^2}\left(1-\frac{\mathbf{k}^2}{4M_yM_n}\right)\left(\frac{1}{4M_yM_n}\right),\ \Omega^{(B)}_{3b} = +f^B_{13}f^B_{24}\frac{(M_n+M_y)^2}{m_B^2}\left(\frac{3}{8M_y^2M_n^2}\right). \tag{F17}$$

(f) Diffractive-exchange (pomeron, f, f', A_2):
The Ω^D_i are the same as for scalar-meson-exchange Eq.(F15), but with $\pm g^S_{13}g^S_{24}$ replaced by $\mp g^D_{13}g^D_{24}$, and except for the zero in the form factor.

(g) Odderon-exchange: The Ω^O_i are the same as for vector-meson-exchange Eq.(refeq2), but with $g^V_{13} \to g^O_{13}$, $f^V_{13} \to f^O_{13}$ and similarly for the couplings with the 24-subscript.

As in Ref. [40] in the derivation of the expressions for $\Omega^{(X)}_i$, given above, M_y and M_n denote the mean hyperon and nucleon mass, respectively $M_y = (M_1+M_3)/2$ and $M_n = (M_2+M_4)/2$, and m denotes the mass of the exchanged meson. Moreover, the approximation $1/M_N^2 + 1/M_Y^2 \approx 2/M_nM_y$, is used, which is rather good since the mass differences between the baryons are not large.

3. One-Boson-Exchange Interactions in Configuration Space I

In configuration space the BB-interactions are described by potentials of the general form

$$V = \Big\{V_C(r) + V_\sigma(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + V_T(r)S_{12} + V_{SO}(r)\mathbf{L}\cdot\mathbf{S} + V_Q(r)Q_{12} + V_{ASO}(r)\ \frac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{L} - \frac{1}{2M_yM_n}\left(\boldsymbol{\nabla}^2V^{n.l.}(r) + V^{n.l.}(r)\boldsymbol{\nabla}^2\right)\Big\}\cdot\mathcal{P}, \tag{F18a}$$

$$V^{n.l.} = \Big\{\varphi_C(r) + \varphi_\sigma(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + \varphi_T(r)S_{12}\Big\}\cdot\mathcal{P}, \tag{F18b}$$

where for non-strange mesons $\mathcal{P} = 1$, and

$$S_{12} = 3(\boldsymbol{\sigma}_1\cdot\hat{r})(\boldsymbol{\sigma}_2\cdot\hat{r}) - (\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2), \tag{F19a}$$

$$Q_{12} = \frac{1}{2}\left[(\boldsymbol{\sigma}_1\cdot\mathbf{L})(\boldsymbol{\sigma}_2\cdot\mathbf{L}) + (\boldsymbol{\sigma}_2\cdot\mathbf{L})(\boldsymbol{\sigma}_1\cdot\mathbf{L})\right], \tag{F19b}$$

$$\phi(r) = \phi_C(r) + \phi_\sigma(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2, \tag{F19c}$$

For the basic functions for the Fourier transforms with gaussian form factors, we refer to Refs. [32, 40]. For the details of the Fourier transform for the potentials with P_5', which occur in the case of the axial-vector mesons with $J^{PC}=1^{++}$, we refer to Appendix E.
(a) Pseudoscalar-meson-exchange:

$$V_{PS}(r) = \frac{m}{4\pi}\left[g^p_{13}g^p_{24}\frac{m^2}{4M_yM_n}\left(\frac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\ \phi^1_C + S_{12}\phi^0_T\right)\right]\mathcal{P}, \tag{F20a}$$

$$V^{n.l.}_{PS}(r) = \frac{m}{4\pi}\left[g^p_{13}g^p_{24}\frac{m^2}{4M_yM_n}\left(\frac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\ \phi^1_C + S_{12}\phi^0_T\right)\right]\mathcal{P}. \tag{F20b}$$

(b) Vector-meson-exchange:

$$\begin{aligned}
V_V(r) = & \frac{m}{4\pi}\left[\left\{g^v_{13}g^v_{24}\left[\phi^0_C + \frac{m^2}{2M_yM_n}\phi^1_C\right]\right.\right. \\
& + \left[g^v_{13}f^v_{24}\frac{m^2}{4\mathcal{M}M_n} + f^v_{13}g^v_{24}\frac{m^2}{4\mathcal{M}M_y}\right]\phi^1_C + f^v_{13}f^v_{24}\frac{m^4}{16\mathcal{M}^2M_yM_n}\phi^2_C\Big\} \\
& + \frac{m^2}{6M_yM_n}\left\{\left[\left(g^v_{13}+f^v_{13}\frac{M_y}{\mathcal{M}}\right)\cdot\left(g^v_{24}+f^v_{24}\frac{M_n}{\mathcal{M}}\right)\right]\phi^1_C + f^v_{13}f^v_{24}\frac{m^2}{8\mathcal{M}^2}\phi^2_C\right\}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) \\
& - \frac{m^2}{4M_yM_n}\left\{\left[\left(g^v_{13}+f^v_{13}\frac{M_y}{\mathcal{M}}\right)\cdot\left(g^v_{24}+f^v_{24}\frac{M_n}{\mathcal{M}}\right)\right]\phi^0_T + f^v_{13}f^v_{24}\frac{m^2}{8\mathcal{M}^2}\phi^1_T\right\}S_{12} \\
& - \frac{m^2}{M_yM_n}\left\{\left[\frac{3}{2}g^v_{13}g^v_{24} + (g^v_{13}f^v_{24}+f^v_{13}g^v_{24})\frac{\sqrt{M_yM_n}}{\mathcal{M}}\right]\phi^0_{SO} + \frac{3}{8}f^v_{13}f^v_{24}\frac{m^2}{\mathcal{M}^2}\phi^1_{SO}\right\}\mathbf{L}\cdot\mathbf{S} \\
& + \frac{m^4}{16M_y^2M_n^2}\left\{\left[g^v_{13}g^v_{24} + 4\,(g^v_{13}f^v_{24}+f^v_{13}g^v_{24})\frac{\sqrt{M_yM_n}}{\mathcal{M}} + 8f^v_{13}f^v_{24}\frac{M_yM_n}{\mathcal{M}^2}\right]\right\}\cdot \\
& \times\frac{3}{(mr)^2}\phi^0_TQ_{12} - \frac{m^2}{M_yM_n}\left\{\left[\left(g^v_{13}g^v_{24} - f^v_{13}f^v_{24}\frac{m^2}{\mathcal{M}^2}\right)\frac{(M_n^2-M_y^2)}{4M_yM_n}\right.\right. \\
& - (g^v_{13}f^v_{24} - f^v_{13}g^v_{24})\frac{\sqrt{M_yM_n}}{\mathcal{M}}\Big]\phi^0_{SO}\Big\}\cdot\frac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{L}\Big]\mathcal{P},
\end{aligned} \tag{F21a}$$

$$\begin{aligned}
V^{n.l.}_V(r) = & \frac{m}{4\pi}\left[\frac{3}{2}g^v_{13}g^v_{24}\ \phi^0_C\right. \\
& + \frac{m^2}{6M_yM_n}\left\{\left[\left(g^v_{13}+f^v_{13}\frac{M_y}{\mathcal{M}}\right)\cdot\left(g^v_{24}+f^v_{24}\frac{M_n}{\mathcal{M}}\right)\right]\phi^1_C\right\}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) \\
& - \frac{m^2}{4M_yM_n}\left\{\left[\left(g^v_{13}+f^v_{13}\frac{M_y}{\mathcal{M}}\right)\cdot\left(g^v_{24}+f^v_{24}\frac{M_n}{\mathcal{M}}\right)\right]\phi^0_T\right\}S_{12}\Big]\mathcal{P}.
\end{aligned} \tag{F21b}$$

Note: the non-local tensor and "associated" spin-spin terms are not included in ESC08c-model.

(c) Scalar-meson-exchange:

$$\begin{aligned}
V_S(r) = & -\frac{m}{4\pi}\left[g^s_{13}g^s_{24}\left\{\left[\phi^0_C - \frac{m^2}{4M_yM_n}\phi^1_C\right] + \frac{m^2}{2M_yM_n}\phi^0_{SO}\ \mathbf{L}\cdot\mathbf{S} + \frac{m^4}{16M_y^2M_n^2}\cdot\right.\right. \\
& \times\frac{3}{(mr)^2}\phi^0_TQ_{12} + \frac{m^2}{M_yM_n}\left[\frac{(M_n^2-M_y^2)}{4M_yM_n}\right]\phi^0_{SO}\cdot\frac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{L} \\
& + \frac{1}{4M_yM_n}\left(\boldsymbol{\nabla}^2\phi^0_C + \phi^0_C\boldsymbol{\nabla}^2\right)\Big\}\Big]\mathcal{P}.
\end{aligned} \tag{F22}$$

(d) Axial-vector-meson exchange $J^{PC}=1^{++}$:

$$
\begin{aligned}
V_A(r) = -\frac{m}{4\pi}\Bigg[&\left\{g^a_{13}g^a_{24}\left(\phi^0_C+\frac{2m^2}{3M_yM_n}\phi^1_C\right)+\frac{m^2}{6M_yM_n}\left(g^a_{13}f^a_{24}\frac{M_n}{\mathcal{M}}+f^a_{13}g^a_{24}\frac{M_y}{\mathcal{M}}\right)\phi^1_C\right.\\
&\left.+f^a_{13}f^a_{24}\frac{m^4}{12M_yM_n\mathcal{M}^2}\phi^2_C\right\}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)-\frac{3}{4M_yM_n}g^a_{13}g^a_{24}\left(\boldsymbol{\nabla}^2\phi^0_C+\phi^0_C\boldsymbol{\nabla}^2\right)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\\
&-\frac{m^2}{4M_yM_n}\left\{\left[g^a_{13}g^a_{24}-2\left(g^a_{13}f^a_{24}\frac{M_n}{\mathcal{M}}+f^a_{13}g^a_{24}\frac{M_y}{\mathcal{M}}\right)\right]\phi^0_T-f^a_{13}f^a_{24}\frac{m^2}{\mathcal{M}^2}\phi^1_T\right\}S_{12}\\
&+\frac{m^2}{2M_yM_n}g^a_{13}g^a_{24}\left\{\phi^0_{SO}\ \mathbf{L}\cdot\mathbf{S}+\frac{m^2}{M_yM_n}\left[\frac{(M_n^2-M_y^2)}{4M_yM_n}\right]\phi^0_{SO}\cdot\frac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{L}\right\}\Bigg]\mathcal{P}. && \text{(F23)}
\end{aligned}
$$

(e) Axial-vector-meson exchange $J^{PC}=1^{+-}$:

$$
\begin{aligned}
V_B(r) \ &= \ -\frac{m}{4\pi}\frac{(M_n+M_y)^2}{m^2}\left[f^B_{13}f^B_{24}\left\{\frac{m^2}{12M_yM_n}\left(\phi^1_C+\frac{m^2}{4M_yM_n}\phi^2_C\right)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\right.\right.\\
&\qquad\left.\left.-\frac{m^2}{8M_yM_n}\left(\boldsymbol{\nabla}^2\phi^1_C+\phi^1_C\boldsymbol{\nabla}^2\right)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)+\left[\frac{m^2}{4M_yM_n}\right]\phi^0_T\ S_{12}\right\}\right]\mathcal{P}, && \text{(F24a)}\\
V_B^{n.l.}(r) \ &= \ -\frac{m}{4\pi}\frac{(M_n+M_y)^2}{m^2}\left[f^B_{13}f^B_{24}\left\{\frac{3m^2}{4M_yM_n}\left(\frac{1}{3}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\ \phi^1_C+S_{12}\ \phi^0_T\right)\right\}\right]\mathcal{P}. && \text{(F24b)}
\end{aligned}
$$

(f) Diffractive exchange:

$$
\begin{aligned}
V_D(r) = \frac{m_P}{4\pi}\Bigg[&g^D_{13}g^D_{24}\frac{4}{\sqrt{\pi}}\frac{m_P^2}{\mathcal{M}^2}\cdot\Bigg[\Bigg\{1+\frac{m_P^2}{2M_yM_n}(3-2m_P^2r^2)+\frac{m_P^2}{M_yM_n}\mathbf{L}\cdot\mathbf{S}\\
&+\left(\frac{m_P^2}{2M_yM_n}\right)^2Q_{12}+\frac{m_P^2}{M_yM_n}\left[\frac{(M_n^2-M_y^2)}{4M_yM_n}\right]\cdot\frac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{L}\Bigg\}\ e^{-m_P^2r^2}\\
&+\frac{1}{4M_yM_n}\left(\boldsymbol{\nabla}^2e^{-m_P^2r^2}+e^{-m_P^2r^2}\boldsymbol{\nabla}^2\right)\Bigg]\Bigg]\mathcal{P}. && \text{(F25)}
\end{aligned}
$$

(g) Odderon-exchange:

$$
\begin{aligned}
V_{O,C}(r) &= +\frac{g_{13}^O g_{24}^O}{4\pi}\frac{8}{\sqrt{\pi}}\frac{m_O^5}{\mathcal{M}^4}\left[\left(3-2m_O^2r^2\right)\right. \\
&\left. -\frac{m_O^2}{M_yM_n}\left(15-20m_O^2r^2+4m_O^4r^4\right)\right]\exp(-m_O^2r^2)\ , \qquad \text{(F26a)} \\
V_{O,n.l.}(r) &= -\frac{g_{13}^O g_{24}^O}{4\pi}\frac{8}{\sqrt{\pi}}\frac{m_O^5}{\mathcal{M}^4}\frac{3}{4M_yM_n}\left\{\boldsymbol{\nabla}^2\left[(3-2m_O^2r^2)\exp(-m_O^2r^2)\right]+\right. \\
&\left.+\left[(3-2m_O^2r^2)\exp(-m_O^2r^2)\right]\boldsymbol{\nabla}^2\right\}\ , \qquad \text{(F26b)} \\
V_{O,\sigma}(r) &= -\frac{g_{13}^O g_{24}^O}{4\pi}\frac{8}{3\sqrt{\pi}}\frac{m_O^5}{\mathcal{M}^4}\frac{m_O^2}{M_yM_n}\left[15-20m_O^2r^2+4m_O^4r^4\right]\exp(-m_O^2r^2)\cdot \\
&\times\left(1+\kappa_{13}^O\frac{M_y}{\mathcal{M}}\right)\left(1+\kappa_{24}^O\frac{M_n}{\mathcal{M}}\right), \qquad \text{(F26c)} \\
V_{O,T}(r) &= -\frac{g_{13}^O g_{24}^O}{4\pi}\frac{8}{3\sqrt{\pi}}\frac{m_O^5}{\mathcal{M}^4}\frac{m_O^2}{M_yM_n}\cdot m_O^2r^2\left[7-2m_O^2r^2\right]\exp(-m_O^2r^2)\cdot \\
&\times\left(1+\kappa_{13}^O\frac{M_y}{\mathcal{M}}\right)\left(1+\kappa_{24}^O\frac{M_n}{\mathcal{M}}\right), \qquad \text{(F26d)} \\
V_{O,SO}(r) &= -\frac{g_{13}^O g_{24}^O}{4\pi}\frac{8}{\sqrt{\pi}}\frac{m_O^5}{\mathcal{M}^4}\frac{m_O^2}{M_yM_n}\left[5-2m_O^2r^2\right]\exp(-m_O^2r^2)\cdot \\
&\times\left\{3+\left(\kappa_{13}^O+\kappa_{24}^O\right)\frac{\sqrt{M_yM_n}}{\mathcal{M}}\right\}, \qquad \text{(F26e)} \\
V_{O,Q}(r) &= +\frac{g_{13}^O g_{24}^O}{4\pi}\frac{2}{\sqrt{\pi}}\frac{m_O^5}{\mathcal{M}^4}\frac{m_O^4}{M_y^2M_n^2}\left[7-2m_O^2r^2\right]\exp(-m_O^2r^2)\cdot \\
&\times\left\{1+4\left(\kappa_{13}^O+\kappa_{24}^O\right)\frac{\sqrt{M_yM_n}}{\mathcal{M}}+8\kappa_{13}\kappa_{24}\frac{M_yM_n}{\mathcal{M}^2}\right\}, \qquad \text{(F26f)} \\
V_{O,ASO}(r) &= -\frac{g_{13}^O g_{24}^O}{4\pi}\frac{4}{\sqrt{\pi}}\frac{m_O^5}{\mathcal{M}^4}\frac{m_O^2}{M_yM_n}\left[5-2m_O^2r^2\right]\exp(-m_O^2r^2)\cdot \\
&\times\left\{\frac{M_n^2-M_y^2}{M_yM_n}-4\left(\kappa_{24}^O-\kappa_{13}^O\right)\frac{\sqrt{M_yM_n}}{\mathcal{M}}\right\}. \qquad \text{(F26g)}
\end{aligned}
$$

4. Strange Meson-exchange

The rules for hypercharge nonzero exchange have been given in e.g. Ref. [47]. The potentials for non-zero hypercharge exchange $(K, K^*, \kappa, K_A, K_B)$ are obtained from the expressions given in the previous subsections for non-strange mesons by taking care of the following points: (a) For strange meson exchange $\mathcal{P} = -\mathcal{P}_x\mathcal{P}_\sigma$. (b) In the latter case one has to replace both M_n and M_y by $\sqrt{M_yM_n}$, and reverse the sign of the antisymmetric spin orbit.

APPENDIX G: FOLDING AMPLITUDE SCALAR-EXCHANGE II

In this Appendix the lower vertex in Fig. 2 is worked out for the scalar-meson coupling. This in order to check the signs in the vertex function in comparison with the upper vertex.

The Dirac-spinor part of the scalar-meson QQ-vertex is

$$\begin{aligned}[\bar{u}_i(\mathbf{q'}_i)u_i(\mathbf{q}_i)] &= \sqrt{\frac{E'_i+m'_i}{2m'_i}\frac{E_i+m_i}{2m_i}}\cdot\chi'^{\dagger}_i\cdot\left[1-\frac{\boldsymbol{\sigma}_i\cdot\mathbf{q'}_i}{E'_i+m_i}\frac{\boldsymbol{\sigma}_i\cdot\mathbf{q}_i}{E_i+m_i}\right] \\ &\approx \chi'^{\dagger}_i\left[1-\frac{\mathbf{q}'_i\cdot\mathbf{q}_i}{4m_i^2}-\frac{i}{4m_i^2}\boldsymbol{\sigma}_i\cdot\mathbf{q}'_i\times\mathbf{q}_i\right]\chi_i \\ &= \chi'^{\dagger}_i\left[1-\frac{\mathbf{S}_i^2-\mathbf{k}^2}{16m_i^2}-\frac{i}{8m_i^2}\boldsymbol{\sigma}_i\cdot\mathbf{S}_i\times\mathbf{k}\right]\chi_i. \end{aligned} \tag{G1}$$

Here is used that for the CQM $E_i \approx m_i$. The performance of the **Q**-integral in (G1) gives

$$[\bar{u}_i(\mathbf{q'}_i)u_i(\mathbf{q}_i)] \Rightarrow \chi'^{\dagger}_i\left[1-\left(\frac{1}{4m_i^2R_N^2}+\frac{\mathbf{q}^2}{36m_i^2}\right)+\frac{\mathbf{k}^2}{16m_i^2}+\frac{i}{12m_i^2}\boldsymbol{\sigma}_i\cdot\mathbf{q}\times\mathbf{k}\right]\chi_i \tag{G2}$$

Summing over the quarks leads to the vertex

$$\Gamma_{CQM} = \sum_{i=1-3}[\bar{u}_i(\mathbf{q'}_i)u_i(\mathbf{q}_i)] \Rightarrow 3\left[1-\left(\frac{1}{4m_Q^2R_N^2}+\frac{\mathbf{q}^2}{36m_Q^2}\right)+\frac{\mathbf{k}^2}{16m_Q^2}+\frac{i}{36m_Q^2}\sum_i\boldsymbol{\sigma}_i\cdot\mathbf{q}\times\mathbf{k}\right] \tag{G3}$$

The CQM replacement $m_Q \approx \sqrt{M'M}/3$ leads to

$$\Gamma_{CQM} = 3\left[\left(1-\frac{1}{4m_Q^2R_N^2}\right)-\frac{\mathbf{q}^2}{4M'M}+\frac{9\mathbf{k}^2}{16M'M}+\frac{i}{4M'M}\sum_i\boldsymbol{\sigma}_N\cdot\mathbf{q}\times\mathbf{k}\right], \tag{G4}$$

where we used $\sum_i \boldsymbol{\sigma}_i = \boldsymbol{\sigma}_N$. This assumes that the spin of the nucleon is given by the total spin of the quarks [27]. *Notice that the* $1/R_N^2$*-term in (G4) has the same sign to that of (4.8). Hence, these terms would not cancel in the NN-potential in this simple treatment.*

APPENDIX H: FOLDING TENSOR-EXCHANGE VERTEX

For the coupling of the tensor mesons ($J^{PC} = 2^{++}$) to the quarks, similar to that for the nucleons, we take

$$\mathcal{H}_{fNN} = -\left[\frac{i}{2}\bar{\psi}\left(\gamma^\mu\partial^\nu+\gamma^\nu\partial^\mu\right)\psi\ F_1-\bar{\psi}\partial^\mu\partial^\nu\psi\ F_2\right]\cdot f_{\mu\nu}\ , \tag{H1}$$

where $f_{\mu\nu} = f_{\nu\mu}$, i.e. symmetric, and

$$F_1 = \frac{G_{T,1}}{\mathcal{M}}\ ,\ \text{and}\ F_2 = \frac{G_{T,2}}{\mathcal{M}^2}\ . \tag{H2}$$

The Sach form factors are in terms of the $G_{T,i}$ defined as

$$G_M = G_{T,1}\ ,\ G_E = G_{T,1}-\frac{(t-4M^2)}{4M^2}G_{T,2} \approx G_{T,1}+\left(1+\frac{\mathbf{k}^2}{4M^2}\right)G_{T,2}\ . \tag{H3}$$

The latter are defined for general J in e.g. Rijken, Phd. Thesis (Nijmegen, 1975). This is of importance when we apply the constraints imposed by EXD, which relates the tensor-meson couplings to the vector-meson couplings. As a matter of fact, EXD predicts that $\mathcal{M}F_1 = F_{V,1}$ and $\mathcal{M}F_2 = F_{V,2}$ for the pairs (A_2,ρ) and $(f(1270),\omega)$.

Using the Gordon decomposition, the Pauli- couplings G_i^T are related to the Dirac-couplings g_T, f_T by

$$g_T = G_{T,1}+G_{T,2}\ ,\ f_T = -G_{T,2}\ , \tag{H4}$$

and notice that

$$F_1 + MF_2 = g_T/\mathcal{M}\ ,\ MF_2 = -f_T/\mathcal{M}\ , \tag{H5}$$

which is strictly valid only for $\mathcal{M} = M$.
For the QQf-vertices this gives the factors

$$\begin{aligned}
\bar{u}_i(\mathbf{k_i}')\Gamma_T^{\mu\nu}u_i(\mathbf{k}_i) &\sim \frac{1}{4}\bar{u}_i(\mathbf{k}_i')\left[\left((k_i+k_i')^\nu\gamma^\mu + (k_i+k_i')^\mu\gamma^\nu\right)\ F_1 + (k_i+k_i')^\mu(k+k')^\nu\ F_2\right]u_i(\mathbf{k}_i) \\
&\sim \frac{1}{2}\ (k_i+k_i')^\nu\ \bar{u}_i(\mathbf{k}_i')\left[\gamma^\mu F_1 + \frac{1}{2}(k+k')^\mu\ F_2\right]u_i(\mathbf{k}_i) \\
\bar{u}_j(\mathbf{q}_j')\Gamma_T^{\mu\nu}u_j(\mathbf{q}_j) &\sim \frac{1}{4}\bar{u}_j(\mathbf{q}_j')\left[\left((q+q')^\rho\gamma^\sigma + (q_j+q_j')^\sigma\gamma^\rho\right)\ F_1' + (q_j+q_j')^\rho(q+q')^\sigma\ F_2'\right]u_j(\mathbf{q}_j) \\
&\sim \frac{1}{2}\ (q_j+q_j')^\sigma\ \bar{u}_j(\mathbf{q}_j')\left[\gamma^\rho F_1' + \frac{1}{2}(q_j+q_j')^\rho\ F_2'\right]u_j(\mathbf{q}_j) \qquad \text{(H6)}
\end{aligned}$$

Here the symbol $\sim$ indicates that factors coming from the normalization $\sqrt{(E+m_Q)/2m_Q}$ of the Dirac-spinors have been suppressed. (In the present case, as also for vector- and scalar-exchange, they cancel out when we pass to the level of the Lippmann-Schwinger equation.) The second form of the vertices is equivalent to the first form due to the symmetry of the tnesor field $f_{\mu\nu}$. Notice that, apart from the factor $(k_1'+k_1)^\nu$, the Γ_T matrix element in (H6) is identical to that for the vector meson.
The propagator for the spin-2 mesons contains the projection operator

$$\mathcal{P}_{\mu\nu;\rho\sigma}(k) = \frac{1}{2}\left(P_{\mu\rho}P_{\nu\sigma} + P_{\mu\sigma}P_{\nu\rho}\right) - \frac{1}{3}P_{\mu\nu}P_{\rho\sigma}\ , \qquad \text{(H7)}$$

where $P_{\mu\nu}(k) = -\eta_{\mu\nu} + k_\mu k_\nu/m^2$, with $k = k_i' - k_i = p' - p = q - q'$. On-mass-shell and equal quark masses the $k_\mu k_\nu$-terms in the $P_{\mu\nu}(k)$ do not contribute, so

$$\mathcal{P}_{\mu\nu;\rho\sigma}(k) \Rightarrow \frac{1}{2}\left(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}\right) - \frac{1}{3}\eta_{\mu\nu}\eta_{\rho\sigma}\ . \qquad \text{(H8)}$$

Therefore, we find three contributions to the QQ-potential

$$V_{T,ij} = V_{T,ij}^{(1)} + V_{T,ij}^{(2)} + V_{T,ij}^{(3)}\ , \qquad \text{(H9)}$$

where, denoting $p := k_i, p' := k_i'$ and $q := q_j, q' := q_j'$,

$$\begin{aligned}
V_{T,ij}^{(1)} &= -\frac{1}{8}\ (p+p')\cdot(q+q')\ \bar{u}_i(\mathbf{p}')\left[\gamma^\mu\ F_1 + \frac{1}{2}(p+p')^\mu\ F_2\right]u_i(\mathbf{p})\ \cdot \\
&\quad \times\ \bar{u}_j(\mathbf{q}')\left[\gamma_\mu\ F_1' + \frac{1}{2}(q+q')_\mu\ F_2'\right]u_j(\mathbf{q})\ \times\left[\mathbf{k}^2+m^2\right]^{-1}\ , \\
V_{T,ij}^{(2)} &= -\frac{1}{8}\ \bar{u}_i(\mathbf{p}')\left[\gamma\cdot(q+q')\ F_1 + \frac{1}{2}(p+p')\cdot(q+q')\ F_2\right]u_i(\mathbf{p})\ \cdot \\
&\quad \times\ \bar{u}_j(\mathbf{q}')\left[\gamma\cdot(p+p')\ F_1' + \frac{1}{2}(q+q')\cdot(p+p')\ F_2'\right]u_j(\mathbf{q})\ \times\left[\mathbf{k}^2+m^2\right]^{-1}\ , \\
V_{T,ij}^{(3)} &= +\frac{1}{48}\left[4MF_1 + \left(4M^2-t\right)F_2\right]\left[4M'F_1' + \left(4M'^2-t\right)F_2'\right]\cdot \\
&\quad \times\left[\bar{u}_i(\mathbf{p}')u_i(\mathbf{p})\right]\left[\bar{u}_j(\mathbf{q}')u_j(\mathbf{q})\right]\times\left[\mathbf{k}^2+m^2\right]^{-1}\ , \qquad \text{(H10)}
\end{aligned}$$

where for the third contribution we used the Dirac equation $\gamma\cdot p\ u(p) = m_Q u(p)$. This last contribution is very akin to the scalar potential and the result can be written down almost immediately using the results of Phys.Rev. **D** 17 (1978).
Working out the contribution for $\mu = 0$, similar to that for the scalar- and vector-meson one finds, with $F_{1,2}' = F_{1,2}$

and $m_i = m_j = m_Q$,

$$\begin{aligned}
V^{(1)}_{T,ij} &\approx -\frac{1}{2} m_i^2 \left[\left\{1 + \frac{\mathbf{Q}_i^2 - \mathbf{k}^2}{4m_i^2} + \frac{i}{8m_i^2}\boldsymbol{\sigma}_i \cdot \mathbf{Q}_i \times \mathbf{k}\right\} F_1 \right.\\
&\left. + \left\{\left\{1 - \frac{\mathbf{Q}_1^2 - \mathbf{k}^2}{16m_i^2} + \frac{i}{8m_i^2}\boldsymbol{\sigma}_i \cdot \mathbf{Q}_i \times \mathbf{k}\right\} m_i F_2\right] \cdot \right.\\
&\times\left[\left\{1 + \frac{\mathbf{S}_j^2 - \mathbf{k}^2}{4m_j^2} + \frac{i}{8m_j^2}\boldsymbol{\sigma}_j \cdot \mathbf{S}_j \times \mathbf{k}\right\} F_1 \right.\\
&\left. + \left\{\left\{1 - \frac{\mathbf{S}_j^2 - \mathbf{k}^2}{16m_j^2} + \frac{i}{8m_j^2}\boldsymbol{\sigma}_j \cdot \mathbf{S}_j \times \mathbf{k}\right\} m_j F_2\right]\right.\\
&\Rightarrow -\frac{1}{2} m_Q^2 \left[(F_1 + m_Q F_2) + (F_1 - 4m_Q F_2)\frac{\mathbf{Q}_i^2 - \mathbf{k}^2}{16m_Q^2}\right] \cdot \\
&\times\left[(F_1 + m_Q F_2) + (F_1 - 4m_Q F_2)\frac{\mathbf{S}_i^2 - \mathbf{k}^2}{16m_Q^2}\right]\\
&= -\frac{1}{2} g_T^2 \left(\frac{m_Q^2}{\mathcal{M}^2}\right)\left[1 + (1 + 5f_T/g_T)\left\{\frac{\mathbf{Q}_i^2 - \mathbf{k}^2}{16m_Q^2} + \frac{\mathbf{S}_i^2 - \mathbf{k}^2}{16m_Q^2}\right\} + \ \ldots\right],
\end{aligned} \tag{H11}$$

where we neglect the quadratic terms of the product. Similarly, the second and third terms give

$$\begin{aligned}
V^{(2)}_{T,ij} &\approx -\frac{1}{2}\, g_T^2 \left(\frac{m_Q^2}{\mathcal{M}^2}\right)\left[1 - \frac{\mathbf{Q}_i^2 - \mathbf{k}^2}{16m_i^2} + \frac{i}{8m_i^2}\boldsymbol{\sigma}_i \cdot \mathbf{Q}_i \times \mathbf{k}\right]\left[1 - \frac{\mathbf{S}_j^2 - \mathbf{k}^2}{16m_j^2} - \frac{i}{8m_j^2}\boldsymbol{\sigma}_j \cdot \mathbf{S}_j \times \mathbf{k}\right]\\
&\Rightarrow -\frac{1}{2} g_T^2 \left(\frac{m_Q^2}{\mathcal{M}^2}\right)\left[1 - \frac{\mathbf{Q}_i^2 - \mathbf{k}^2}{16m_Q^2} - \frac{\mathbf{S}_i^2 - \mathbf{k}^2}{16m_Q^2} + \ \ldots\right],
\end{aligned} \tag{H12}$$

and

$$V^{(3)}_{T,ij} \approx +\frac{1}{3}\, g_T^2 \left(\frac{m_Q^2}{\mathcal{M}^2}\right)\left[1 - \frac{\mathbf{Q}_i^2 - \mathbf{k}^2}{16m_Q^2} - \frac{\mathbf{S}_i^2 - \mathbf{k}^2}{16m_Q^2} + \ \ldots\right]. \tag{H13}$$

1. Cancellation $(m_Q R_N)^{-2}$ terms in NN-potential

In the case one sticks to the $\delta^3(\mathbf{K} - \mathbf{k}$ the "spurious" contributions to the central potentals can be (almost) completely eliminated by the inclusion of the tensor mesons, which is illustrated below. From the vertices Γ_{CQM} for scalar, vector, and tensor exchange we get, with $\kappa_T = f_T/g_T$,

$$V_{NN,sc} \sim -g_S^2 \left[\left(1 - \frac{1}{4m_Q^2 R_N^2}\right)^2 - \frac{\mathbf{q}^2 + \mathbf{k}^2/4}{2M^2} + \ldots\right] (\mathbf{k}^2 + m_S^2)^{-1}, \tag{H14a}$$

$$V_{NN,vc} \sim +g_V^2 \left[\left(1 + \frac{1}{4m_Q^2 R_N^2}\right)^2 + \frac{\mathbf{q}^2 + \mathbf{k}^2/4}{2M^2} + \ldots\right] (\mathbf{k}^2 + m_V^2)^{-1}, \tag{H14b}$$

$$V_{NN,tn} \sim -\frac{2}{3} g_T^2 \left[\left(1 + \frac{(1 + 3\kappa_T/2)}{4m_Q^2 R_N^2}\right) + (1 + 3\kappa_T/2)\frac{\mathbf{q}^2 + \mathbf{k}^2/4}{4M^2} + \ldots\right] (\mathbf{k}^2 + m_T^2)^{-1}, \tag{H14c}$$

where the couplings g_S, g_V, and g_T are now NN coupling constants. Neglecting the R_N^{-4} terms, the volume integral of the "spurious" $1/R_N^2$ terms is proportional to

$$I_V \sim \left[\frac{g_S^2}{m_S^2} + \frac{g_V^2}{m_V^2} - \frac{1}{3}(1 + 3\kappa_T/2)\frac{g_T^2}{m_T^2}\right] \tag{H15}$$

For $m_S = m_V = m_T/\sqrt{3}$, and $\kappa_T \approx \kappa_V \approx 3.7$, the vanishing of this part of I_V implies $g_T^2 \approx (2/3)(g_S^2 + g_V^2)$. From QPC-mechanism $g_S \approx g_V := \bar{g}$ leading to $g_t \approx (\sqrt{2/3})\bar{g} \approx \bar{q}$. In the approximation of "contact-approximation this shows that the potentials from the "spurious" terms can be made to vanish.

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[27] This seems at first sight not to be true. Below the treatment of the axial-vector meson coupling will reveal that the spin-crisis suggests that most of the spin is not carried by the quarks, but by the gluons. However, in the consituent-quark model there is no gluon-sea but only quarks. Therefore, we suppose that the coupling of the mesons at the quark-level has different vertices than at the nucleon-level, and maintain that the nucleon spin is the result of the total spin of the constituent quarks.
[28] The hermiticity of $\Delta\mathcal{L} \equiv \Delta J^\mu_a \cdot A_\mu$ follows from $[(\partial_\alpha\bar{\psi})\gamma_\nu(\partial_\beta\psi]^\dagger = [(\partial_\beta\bar{\psi})\gamma_\nu(\partial_\alpha\psi]$. The (-)-sign from $i \to -i$ is compensated by the antisymmetry of $\epsilon^{\mu\nu\alpha\beta} = -\epsilon^{\mu\nu\beta\alpha}$.
The charge-conjugation properties follow from $\mathcal{C}[(\partial_\alpha\bar{\psi})\gamma_\nu(\partial_\beta\psi]\mathcal{C}^{-1} = -[(\partial_\beta\bar{\psi})\gamma_\nu(\partial_\alpha\psi]$, and again this (-)-sign is compensated by the antisymmetry of the ϵ-symbol. So the hadronic current ΔJ^μ_a has $J^{PC} = 1^{++}$, *i.e.* the same as the axial-vector meson field.
The extra Lagrangian can be written as

$$\Delta\mathcal{L} \sim -\epsilon^{\mu\nu\alpha\beta}[\bar{\psi}\gamma_\nu\partial_\beta\psi]\ (\partial_\alpha A_\mu) = -\frac{1}{2}[\bar{\psi}\gamma_\nu\partial_\beta\psi]\ \frac{\partial_\alpha X_\mu}{\partial A^\nu}$$
$$= -\frac{1}{4}\epsilon^{\mu\nu\alpha\beta}[\bar{\psi}\left(\gamma_\mu\partial_\nu - \gamma_\nu\partial_\mu\right)\psi]\ (\partial_\alpha A_\beta - \partial_\beta A_\alpha).$$

Here, $X_\mu \equiv 2\epsilon_{\mu\nu\alpha\beta}A^\nu(\partial^\alpha A^\beta)$ the conserved anomalous axial current [29].
[29] **Note**: The axial-vector vertex is also connected to the famous Adler-Bell-Jackiw *axial anomaly* phenomenon [30]. For the axial isospin currents,

$$\partial_\mu J^{5,a}_\mu = -\frac{g^2}{16\pi^2}\epsilon^{\alpha\beta\mu\nu}G^c_{\alpha\beta}G^d_{\mu\nu}\cdot tr[\tau^a t^c t^d],$$

where $G^c_{\mu\nu}$ is a gluon field strength, τ^a is an isospin matrix, t^c is a color matrix. the trace is $tr[\tau^a t^c t^d] = tr[\tau^a]tr[t^c t^d] = 0$. So the isospin currents are unaffected by the axial anomaly. However, for the isospin singlet this is not the case:

$$\partial_\mu J^5_\mu = -\frac{g^2 n_f}{32\pi^2}\epsilon^{\alpha\beta\mu\nu}G^c_{\alpha\beta}G^d_{\mu\nu} \neq 0.$$

This explains why the strong interactions contain no pseudoscalar meson as light as the pion. We conclude that the anomaly is not connected to a possible modification of the strong axial-vector coupling to the quarks.

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The charge-conjugation properties follow from $\mathcal{C}[\bar{\psi}\gamma_\nu\psi]\mathcal{C}^{-1} = -[\bar{\psi}\gamma_\nu\psi]$, and this (-)-sign is compensated by partial integration to bring the differentiations back in the original form.
The parity properties are checked easily: (i) $\mu = 0$: all indices of the angular momentum operator or space-like, and hence $\mathcal{P}A_0\mathcal{P}^{-1} = -A_0$ is compensated by $\gamma_0\gamma_n\gamma_0 = -\gamma_n$. (ii) $\mu = m$: one easily checks that $[\bar{\psi}\mathcal{M}_{\nu\alpha\beta}\psi]$ is partity invariant due to the restrictions on the indices, again because of the ϵ-factor. So, writing $\Delta\mathcal{L}' = J'_{A,\mu}\ A^\mu$ the current has $(-)^{PC} = 1^{++}$.
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