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Quark-Nucleon Di-quark-exchange Potentials

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Abstract

The quark-nucleon (QN) potential due to di-quark exchange is derived for a medium of mixed nuclear-quark matter. The diquark-exchange is of the axial-vector type. The potential is repulsive for S-waves, P-waves, and other partial waves. Parameterizing the strength of this interaction as a function of the quark density such as to increase with the deconfining rate it can become rather significant in neutron stars. With sufficient strength the QN-potential creates a "domain-wall" between nuclear matter (nucleons and nuclei) and the quark/nucleon-core in a neutron star. The latter occurs in the deconfinement region.

I. INTRODUCTION

At present the mixed nuclear-quark matter is a much attended topic, see [1]. In this note we derive the quark-nucleon (QN) potential due to di-quark D) exchange.

In considering meson coupling to quarks as well as to nucleons leads in a natural way to coupled channel treatment in a mixture of nuclear and quark matter. The tri-quark presentation of the nucleon [2], *i.e.* $N \sim \eta_N = (\tilde{q}C\gamma^\mu q)\gamma_5\gamma_\mu q$ suggests the reactions $N \leftrightarrow 3Q$, which takes place in high density matter. For chemical reactions $A \leftrightarrow B$ there is no confinement barrier in contrast to $N \leftrightarrow 3Q$, which complicates the thermodynamic treatment. Therefore, in connecting low and high density systems taking into account of confinement-deconfinement (in a phenomenological way) is essential for a realistic description.

It turns out that for the treatment of the Lagrangian with the tri-quark field $\eta_N(x)$ in *e.g.* the functional form of the partition function for matter is difficult to handle. This problem is circumvented by avoiding third powers in the quark fields by the introduction of an auxiliary colored di-quark field $\chi^a_\mu(x)$ [12], which upon quantization leads to di-quarks D. Apart from this technical reason it may be that di-quark configurations play a real physical rôle.

In [3] nucleon and quark imixed matter is discussed in the context of the MF-approach in matter in the framework of the grand-partition functional. Here, we restrict ourselves to the nucleons and the quarks, working out the QN-potential in this paper. The resulting repulsion may be seen as not *ad hoc*, but as a natural consequence of the deconfimenent mechanism. *Exchange of these di-quarks leads in all (S-, P-, etc) waves to a repulsive interaction between the quark and a nucleon.*

We study the confinement-deconfinement in mixed nuclear and quark matter based on the transition $N \leftrightarrow Q + D$ in a phenomenological way. The model we use is a contracting sphere of mixed neutron and quark matter. As an application of the diquark-exchange QN-potential we considered the interaction of an infalling nucleon scattering with a quark-sphere. It is found that the nucleons are reflected from the surface of the sphere. The same will happen to infalling nuclei. This means that, with a sufficiently strength, controlled by the parameters λ_3, Λ, a "domain-wall' is created by the QN-potential. The same will happen with sphere containing a mixture of quark and nucleons which occurs in the deconfining region.

The contents of this paper is as follows. In section II the reprentation of the triquark states for nucelons is given as well as the Lagrangian for the description of the mixed matter of quarks and nucleons with the nucleon tri-quark transition. Furthermore the di-quark are introduced. In section III the nucleon-quark potentials are derived from a contact interaction based on di-quark (D) exchange between nucleons and quarks. In section IV an analysis of the QN-potential parameters and figures are given in order to estimate the possible significance of the results. An illustration is given of nucleon and quark densities in a homologous collapse. Section V contains a summary and conclusions. In Appendix A, conform Ref. [3], the Lagrangian for mixed nuclear-quark matter is described in the MF-approximation. The grand-partition function is given, again in the MF-approximation, and the interaction betwee quarks and nucleons via the di-quark field is derived. In Appendix B the Feynman propagator is derived following the standard field-theoretican method for the auxiliary di-quark field. It is found to be zero. In Appendix C the special features of exchange-potentials aer given.

II. MIXED MATTER: QUARKS AND NUCLEONS

In mixed quark-nucleon matter there are, depending on the densities, transitions between nucleons and tri-quarks. For the tri-quark system we choose the operator, see [2]

$$\eta_N(x) = \big(\tilde{q}^a(x)C\gamma^\mu q^b(x)\big)\gamma_5\gamma_\mu q^c(x)f^{abc} \tag{2.1}$$

where C is the charge conjugation operator in Dirac space, which has in the PD-representation the properties $C^{-1}\gamma^\mu C = -\gamma^{\mu T}$, $C = -C^{-1} = -C^\dagger = -C^T$. For the proton and neutron this is

$$\eta_p(x) = (\tilde{u}^a(x)C\gamma^\mu u^b(x))\gamma_5\gamma_\mu d^c(x)\varepsilon^{abc}, \quad (2.2a)$$
$$\eta_n(x) = (\tilde{d}^a(x)C\gamma^\mu d^b(x))\gamma_5\gamma_\mu u^c(x)\varepsilon^{abc}, \quad (2.2b)$$

where a,b,c denote the $SU_c(3)$ color indices of the quark fields.
In a mean-field (MF) theory with a scalar and vector fields σ, ω the Lagrangian density consists of four parts

$$\mathcal{L} = \mathcal{L}_Q + \mathcal{L}_N + \mathcal{L}_M + \mathcal{L}_{int}, \quad (2.3a)$$
$$\mathcal{L}_Q = \bar{q}(x)\Big[i\gamma_\mu\left(\partial^\mu + \frac{i}{3}g_\omega\omega^\mu\right) - \left(m_Q - \frac{1}{3}g_\sigma\sigma + \frac{1}{3}g_P\chi_P\right)\Big]\, q(x) \quad (2.3b)$$
$$\mathcal{L}_N = \bar{\psi}(x)\Big[i\gamma_\mu\left(\partial^\mu + ig_\omega\omega^\mu\right) - \left(m_N - g_\sigma\sigma + g_P\chi_P\right)\Big]\,\psi(x) \quad (2.3c)$$
$$\mathcal{L}_M = +\frac{1}{2}\left(\partial^\mu\sigma\partial_\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - U(\sigma) + \frac{1}{2}\mathcal{M}^2\chi_P^2, \quad (2.3d)$$
$$\mathcal{L}_{int} = -\lambda_3\{\bar{\psi}(x)\,\eta_N + \bar{\eta}_N\,\psi(x)\}.. \quad (2.3e)$$

with a scalar self-interaction, see *e.g.* [10]. With this interaction $N \leftrightarrow 3Q$ the baryon number density $\rho_B = B = \rho_N + \rho_Q/3$ is conserved. The nucleon-triquark coupling λ_3 has the dimension $[\lambda_3] = [\mathrm{MeV}]^{-2}$, and via the baryon density ρ_B dependence contains the confinement-deconfinement transition information. A direct way to treat this sytem in MFT would be to introduce the auxiliary tri-quark field η_N via the Lagrangian density

$$\mathcal{L}_\eta \sim \bar{\eta}\eta - \left[\bar{\eta}(q^a(x)C\gamma^\mu q^b(x))\gamma_5\gamma_\mu q^c(x)\varepsilon^{abc} + h.c.\right]$$

which via the E.L:. equations gives for the composite field $\eta = (\tilde{q}^a(x)C\gamma^\mu q^b(x))\gamma_5\gamma_\mu q^c(x)\varepsilon^{abc}$.
However, the occurrence of a triple-quark field makes a handling of the partition fuctions Z_G very complicated. In the tri-quark nucleon presentation (2.1) the contraction of the indices, indiacted by (...), suggest to introduce instead the diquark field.

A. Mixed matter: Mean-field with Di-quarks

For the interaction Lagrangian in (2.3) with the tri-quark field $\eta_N(x)$ the functional form of the partition function is difficult to handle. In order to avoid third powers in the quark fields we write $\eta_N(x)$ in terms of the (bosonic) di-quark field $\chi_\mu^a(x)$ as

$$\eta_N(x) = (\hbar c)^2\gamma_5\gamma^\mu q^a(x)\cdot\chi_\mu^a(x),\ \chi_\mu^a(x) \equiv \varepsilon^{abc}\tilde{q}^b(x)C\gamma_\mu q^c(x)/(\hbar c)^2. \quad (2.4)$$

Introduction this auxiliary di-quark field χ_μ^a via the Lagrangian density [12]

$$\mathcal{L}_\chi = \bar{\lambda}_3\Big\{\chi_\mu^{a\dagger}(x)\chi^{\mu a}(x) - \left[\chi_\mu^{a\dagger}(x)(\tilde{q}^b(x)C\gamma^\mu q^c(x))f^{abc} + h.c.\right]\Big\} \quad (2.5)$$

gives via the E.L. equation $\chi_\mu^a(x) \sim (\tilde{q}^b(x)C\gamma_\mu q^c(x))f^{abc}$.
The total Lagrangian is the Lagrangian in (2.3 with the addition of $\mathcal{L}_\chi$ and for $N \leftrightarrow 3Q$ interaction $\mathcal{L}_{int}$ the substitution $N \leftrightarrow 3Q$ transitions in the form

$$\mathcal{L}_{int} \rightarrow -\bar{\lambda}_3\{(\bar{\psi}(x)\gamma_5\gamma^\mu q^a)\,\chi_\mu^a + h.c.\}, \quad (2.6)$$

where $\bar{\lambda}_3 = (\hbar c)^2\lambda_3$. From now on we use the notation $\lambda_3 \equiv \bar{\lambda}_3$.

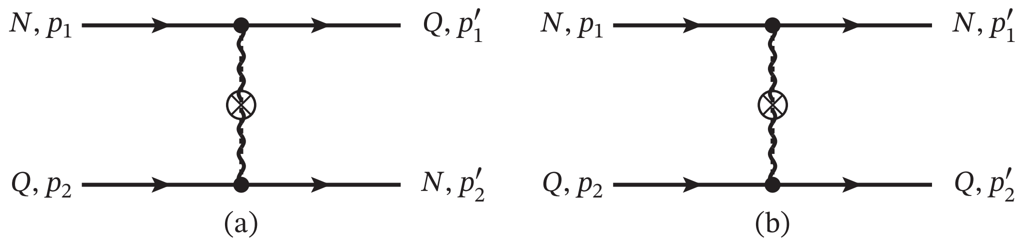

FIG. 1: Diquark-exchange (a) $NQ \to QN$ and (b) $NQ \to NQ$ transitions.

B. Di-quark Isospin

As the proton and neutron presentation (2.2) show the diquark field has isospin one. So, $\chi^a_\mu(x)$ is an isovector vector-field. Therefore, we introduce the fields

$$\mathbf{D}^a_\mu(x) \equiv \boldsymbol{\chi}^a_\mu(x) = \varepsilon^{abc}\tilde{Q}^b(x)C\gamma_\mu\boldsymbol{\tau}Q^c(x)/(\hbar c)^2, \tag{2.7}$$

where $Q = (u, d)$ is the isospin-spinor SU(2) doublet. The di-quark NQ-vertex is given by the interaction Lagrangian (2.6)

$$\mathcal{L}^{(1)}_{int} = -\lambda_3\{(\bar{\psi}(x)\gamma_5\gamma^\mu\boldsymbol{\tau}q^a)\cdot\mathbf{D}^a_\mu(x) + h.c.\}, \tag{2.8}$$

III. DI-QUARK EXCHANGE NUCLEON-QUARK INTERACTION

Since $\mathcal{L}_\chi$ does not have a "kinetic energy" term the $\chi^a_\mu(x)$ does not propagate, leading to "contact term" interactions only. *Note: in Appendix B it is shown explicitly that indeed the Feynman propagator is zero, i.e. no propagation, which not unusual for an auxiliary field, see Ref. [12].* In second-order the (effective) interaction Lagrangian for the di-quark exchange is

$$\mathcal{L}^{(2)}_{QN,QN} = -\lambda_3^2(\bar{\psi}\gamma_5\gamma_\mu\boldsymbol{\tau}Q)\cdot(\bar{Q}\gamma_5\gamma^\mu\boldsymbol{\tau}\psi)/\mathcal{M}^2, \tag{3.1}$$

which represents the lowest order contact $NQ \to QN$ interaction, where $\mathcal{M} = \hbar c$.

In Fig. 1 panel (a) represents the exchange potential V_e and panel (b) according to $\mathcal{L}^{(2)}$ represents the direct potential V_d.

From first-order the interaction, $V \sim -L_{int}$, Eqn. (3.1) gives for panel (a)

$$V(p_1', s_1', p_2', s_2'; p_1, s_1, p_2, s_2) = -2\lambda_3^2\left[\bar{u}_Q(p_1', s_1')\gamma_5\gamma^\mu\boldsymbol{\tau}u_N(p_1, s_1)\right]\cdot\left[\bar{u}_N(p_2', s_2')\gamma_5\gamma_\mu\boldsymbol{\tau}u_Q(p_2, s_2)\right] \tag{3.2}$$

where a color factor 2 is included. Using Pauli-spinor matrix elements

$$\bar{u}(\mathbf{p}'\gamma_5\gamma_0u(\mathbf{p}) = -\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\frac{\boldsymbol{\sigma}\cdot\mathbf{p}'}{\mathcal{E}'} + \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{\mathcal{E}}\right], \tag{3.3}$$

$$\bar{u}(\mathbf{p}')\gamma_5\boldsymbol{\gamma}u(\mathbf{p}) = -\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\boldsymbol{\sigma} + \frac{(\boldsymbol{\sigma}\cdot\mathbf{p}')\,\boldsymbol{\sigma}\,(\boldsymbol{\sigma}\cdot\mathbf{p})}{\mathcal{E}'\mathcal{E}}\right] \approx -\boldsymbol{\sigma}, \tag{3.4}$$

where M',M are the quark or the nucleon mass, and $\mathcal{E} = E_p + M$. Note that the leading term from the vertex factors [....][....] in (3.2) is $-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$. In momentum space we write $\widetilde{V}_{QN} \equiv \widetilde{V}^{(a)}_{QN} + \widetilde{V}^{(b)}_{QN}$ and obtain

$$
\begin{aligned}
\widetilde{V}^{(a)}_{QN} &= -2\lambda_3^2\Bigg[\left(1-\frac{2\mathbf{k}^2}{3M_QM_N}+\frac{3(\mathbf{q}^2+\mathbf{k}^2/4)}{2M_QM_N}\right)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\frac{1}{4M_QM_N}\left((\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})-\frac{1}{3}\mathbf{k}^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right)\\
&\qquad +\frac{i}{4M_QM_N}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{q}\times\mathbf{k}\Bigg]\cdot\widetilde{g}(\mathbf{k}^2), \qquad (3.5a)\\
\widetilde{V}^{(b)}_{QN} &= +2\lambda_3^2\Bigg[\frac{(M_N-M_Q)^2}{4M_N^2M_Q^2}\left\{(\mathbf{q}^2+\mathbf{k}^2/4)-\mathbf{k}^2/2\right\}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\\
&\quad -\frac{i}{4}\left(\frac{M_N^2-M_Q^2}{8M_N^2M_Q^2}\right)(1+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{n}\Bigg]\cdot\widetilde{g}(\mathbf{k}^2), \qquad (3.5b)
\end{aligned}
$$

where $\widetilde{g}(\mathbf{k}^2)=\exp\left(-\mathbf{k}^2/\Lambda^2\right)/\mathcal{M}^2$. Here, we added the gaussian cut-off and a scale parameter $\mathcal{M}$.
Note: $V^{(a)}_{QN}$ is similar to axial-vector exchange in NN and YN. $V^{(b)}_{QN}$ is the "extra term" proportional to the M_N-M_Q mass difference, which is not small in the QN-potential.

In configuration space, taking into account the exchange character of the potential we have a factor $P_f P_\sigma$. Since the physical strates satisfy $P_f P_\sigma P_x = -1$, this leads to a factor $-P_x$ and a sign-change in the antisymmetric spin-orbit. We obtain for the central, spin-spin, tensor, and spin-orbit QD-potentials, see *e.g.* Ref. [14],

$$\begin{aligned} V^{(a)}_{QN}(r) &= -2\lambda_3^2\,\frac{\Lambda}{8\pi}\left[\left(\phi^0_C(r) - \frac{\Lambda^2}{6M_N M_Q}\phi^1_C(r)\right)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) - \frac{3}{4M_Q M_N}\left(\boldsymbol{\nabla}^2\phi^0_C(r) + \phi^0_C(r)\boldsymbol{\nabla}^2\right)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\right. \\ &\quad \left. -\frac{\Lambda^2}{16M_N M_Q}\,\phi^0_T(r)\,S_{12} + \frac{\Lambda^2}{8M_N M_Q}\phi^0_{SO}(r)\,\mathbf{L}\cdot\mathbf{S}\right](\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\,P_x, \qquad (3.6a) \\ V^{(b)}_{QN}(r) &= -2\lambda_3^2\,\frac{\Lambda}{8\pi}\left[\frac{(M_N-M_Q)^2}{4M_N M_Q}\left\{+\frac{\Lambda^2}{8M_N M_Q}\phi^1_C(r) + \frac{1}{2M_N M_Q}\left(\boldsymbol{\nabla}^2\phi^0_C(r) + \phi^0_C(r)\boldsymbol{\nabla}^2\right)\right\}\cdot\right. \\ &\quad \left.\times(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) - \frac{\Lambda^2}{4M_N M_Q}\frac{(M_N^2-M_Q^2)}{4M_N M_Q}\phi^0_{SO}(r)\cdot\frac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{L}\right](\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\,P_x, \qquad (3.6b) \end{aligned}$$

where

$$\begin{aligned} \phi^0_C(r) &= \frac{1}{\sqrt{\pi}}\frac{\Lambda^2}{\mathcal{M}^2}\ \exp\left[-\frac{1}{4}\Lambda^2 r^2\right], && (3.7a) \\ \phi^1_C(r) &= \frac{2}{\sqrt{\pi}}\frac{\Lambda^2}{\mathcal{M}^2}\ \left(3-\Lambda^2 r^2/2\right)\exp\left[-\frac{1}{4}\Lambda^2 r^2\right], && (3.7b) \\ \phi^0_T(r) &= \frac{1}{6\sqrt{\pi}}\frac{\Lambda^2}{\mathcal{M}^2}\ (\Lambda r)^2\exp\left[-\frac{1}{4}\Lambda^2 r^2\right], && (3.7c) \\ \phi^0_{SO}(r) &= \frac{2}{\sqrt{\pi}}\frac{\Lambda^2}{\mathcal{M}^2}\ \exp\left[-\frac{1}{4}\Lambda^2 r^2\right]. && (3.7d) \end{aligned}$$

We introduced a gaussian cut-off with the parameter Λ. This parameter is a free parameter and can be used to tune the di-quark exchange potential which is also the case with λ_3. The non-local potential is

$$V^{(n.l.)}(r) = -\left[\boldsymbol{\nabla}^2\frac{\phi(r)}{2M_{red}} + \frac{\phi(r)}{2M_{red}}\boldsymbol{\nabla}^2\right]P_x,\ \text{with}\ \phi(\mathrm{r}) = -\frac{\lambda_3^2}{4\pi}\frac{3\Lambda}{4\mathrm{M_Q M_N}}\phi^0_{\mathrm{C}}(\mathrm{r})\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\,(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \qquad (3.8)$$

For the statistical average S-wave potential we obtain from Eq. (3.6)

$$\bar{V}(CQM) = \frac{1}{4}V(^1S_0) + \frac{3}{4}V(^3S_1) = +\frac{3\lambda_3^2}{4\pi}\,\Lambda\left(\phi^0_C(r) - \frac{\Lambda^2}{6M_N M_Q}\left\{1 - \frac{3(M_N-M_Q)^2}{16M_N M_Q}\right\}\phi^1_C(r)\right) \qquad (3.9)$$

which result comes from $(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) = -3$ for both $^1S_0$ and $^3S_1$.

The confinement-deconfinement transition can be parametrized as $\lambda_3 \to \gamma_D\lambda_3$ with *e.g.*

$$\gamma_D(\rho_N,\rho_D) = \left[\exp\{+\gamma_3\,(\rho_N/\rho_D - 1)\} - 1\right]\,\theta(\rho_N-\rho_D), \qquad (3.10)$$

where ρ_D is the deconfinement threshold. In [1] a similar form is used for the density dependence of the constituent quark mass.

Notes: *1. The S-wave quark-nucleon repulsion (3.9) is repulsive and becomes strong for high densities. 2. The* $^1P_1$*-wave has* $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 P_x = -9$ *giving strong repulsion. For* $^3P_J(J = 0,1,2)$

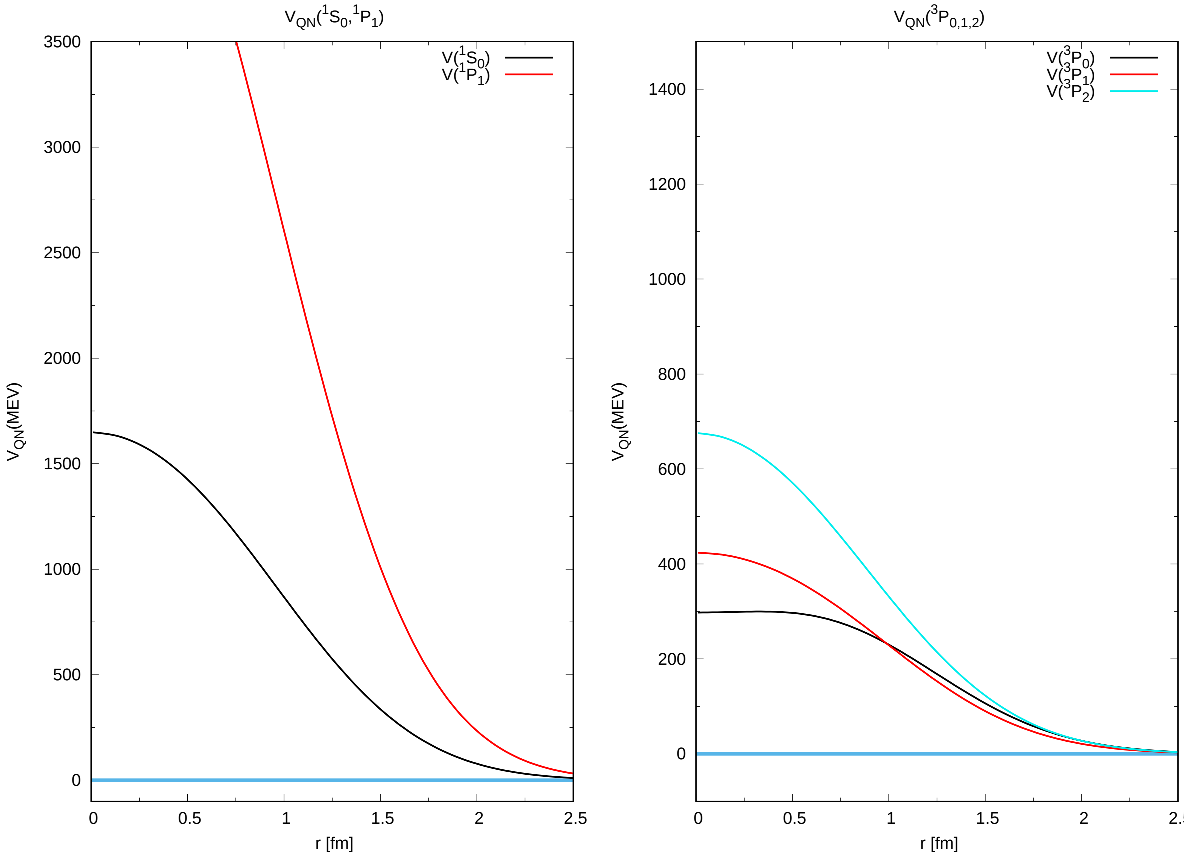

FIG. 2: Quark-nucleon di-quark-exchange S- and P-wave potentials. Parameters: $\lambda_3/\sqrt{4\pi} = 1.0, \mathcal{M} = \hbar c, \Lambda =$ 400 MeV.

the spin-isospin and the exchange operator give a factor -1, giving again a (weaker) repulsion. 3. The di-quark exchange potential gives a repulsive wall for the nucleons between the nucleon- and quark-phase.

In Fig. 2 the S- and P-wave di=quark exchange quark-nucleon potentials are shown. The parameters are: $\lambda_3 = 5.4, m_\chi = 2m_Q$. (Note that $V(^3S_1) = V(^1S_0)$, so there is in fact only one S-wave potential.) In Fig. 3 the contributions from the $V_\sigma^0 \sim \phi_\sigma^0(r)$, $V_\sigma^1 \sim \phi_\sigma^1(r)$ and $V_\sigma^2 \sim \phi_\sigma^1(r)$ terms are shown. We note that the volume integral of $V_\sigma^{1,2} = 0$, so that its effect is diminished.

IV. ANALYSIS AND RESULTS

Analysis S-wave potential $V_{QN} \equiv V_{QN}^{(a)}$:

1. The S-wave potential is explicitly

$$V_{QN}(L=0)(r) = 3\lambda_3^2 \frac{\Lambda}{4\pi\sqrt{\pi}} \frac{\Lambda^2}{\mathcal{M}^2}\left[1 - \frac{3\Lambda^2}{4M_N M_Q}\left(1 - \frac{1}{6}\Lambda^2 r^2\right)\right] \exp\left[-\frac{1}{4}\Lambda^2 r^2\right]. \tag{4.1}$$

2. At r=0

$$V_{QN}(r=0) = \frac{3\lambda_3^2}{4\pi\sqrt{\pi}} \frac{\Lambda^3}{\mathcal{M}^2} \tag{4.2}$$

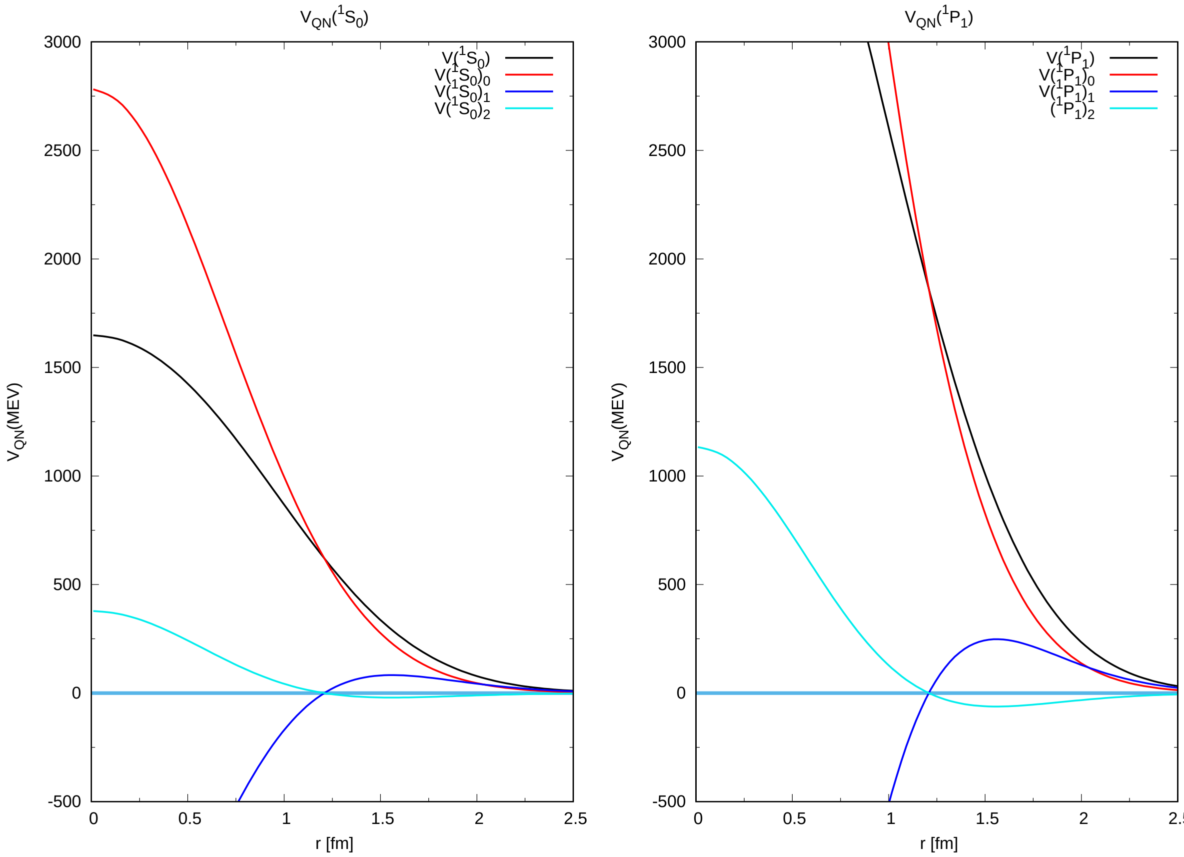

FIG. 3: Quark-nucleon di-quark-exchange $V(^1S_0)$ and $V(^1P_1)$ contributions. Parameters: $\lambda_3/\sqrt{4\pi} = 1.0, \mathcal{M} = \hbar c, \Lambda = 400$ MeV.

3. Zero of the potential

$$V_{QN} \,:\, r_0 = \sqrt{6}\sqrt{1 - \frac{4M_Q M_N}{3\Lambda^2}}\,\Lambda^{-1}. \tag{4.3}$$

For $\Lambda \leq 2M_N/3 \approx 627$ MeV there is no zero, giving gauss-like potentials.

4. **Strength comparison:** The volume integrals V_{int} for S-waves of the D-quark exchange and ω vector-exchange are

$$V_{int}(D) = -\frac{2\lambda_3^2}{\mathcal{M}^2}\langle \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\rangle = \frac{6\lambda_3^2}{\mathcal{M}^2}\,,\; V_{int}(\omega) = \frac{g_\omega^2}{m_\omega^2}. \tag{4.4}$$

For equal strength we have $\lambda_3/g_\omega = (\mathcal{M}/m_\omega)/\sqrt{6} \approx 0.10$ for $\mathcal{M} = \hbar c = 197.32$ MeV, $m_\omega = 783$ MeV and $g_\omega/\sqrt{4\pi} = 3.1149$ in ESC16-model.

5. **Supernova Collapse:** The deconfining interaction Lagrangians $\mathcal{L}_{int}$ in (2.3e) and (2.6) not only are importasnt for neutron stars, but may also play a role in supernova phenomena. In Fig. 5 an illustration of the neutron and quark densities is shown for a homologous (exponential) supernova collapse. As a function of time $n_B(t) = n_B(t_0)\exp\left[3H(t-t_0)\right]$, where t_0 is the contraction

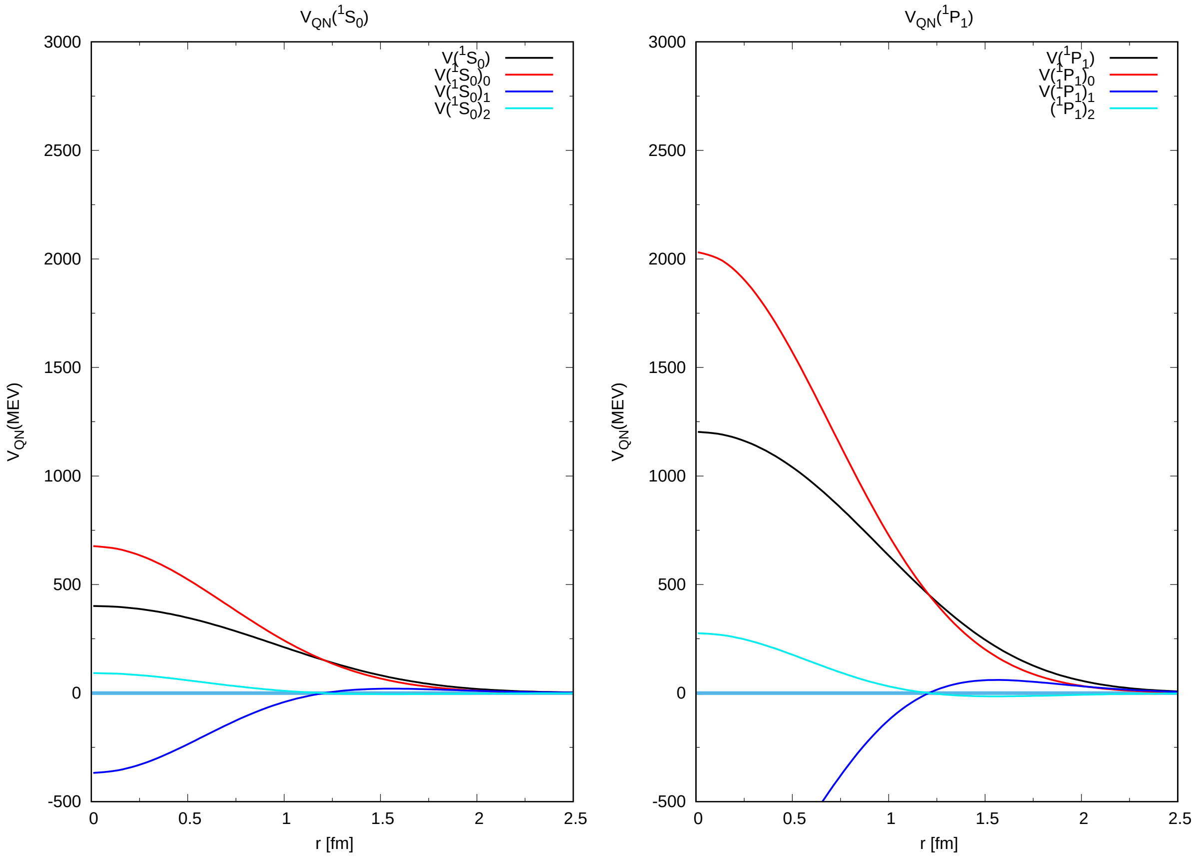

FIG. 4: Quark-nucleon di-quark-exchange $V(^1S_0)$ and $V(^1P_1)$ contributions. Parameters: $\lambda_3/\sqrt{4\pi} = 1.0, \mathcal{M} = 400, \Lambda = 400$ MeV, or equivalent $\lambda_3/\sqrt{4\pi} = 2.35, \mathcal{M} = M_p, \Lambda = 400$ MeV.

starting time. The dependence of the deconfinement coupling λ_3 on the density is like in (3.10). Let t_D the time deconfinement sets in. Then, for $t \leq t_D$ one has $n_N(t) = n_B(t), n_Q(t) = 0$. For $t \geq t_D$ the densities satisfy $dn_N/dt = 3Hn_N - \Gamma n_N, dn_Q/dt = 3Hn_Q + 3\Gamma n_N$, where Γ is the deconfinement transition width. Solving these equations [17] leads to

$$\rho_B \geq \rho_D : \rho_N = \rho_B \left(\frac{\rho_B}{\rho_D}\right)^{-\Gamma/3H}, \; \rho_Q = 3\rho_B \left[1 - \left(\frac{\rho_B}{\rho_D}\right)^{-\Gamma/3H}\right].$$

In Fig. 5 the decay width $\Gamma(N \to 3Q) = 14$ MeV corresponds to the mass difference $\Delta m = M_N - 3M_Q = 4.63$ MeV, and $\lambda_3/\sqrt{4\pi} = 1.0$ is treated as density independent.
The time-evolution is shown in Table I, which is according to the Wilson's scenario [18, 19]. Here R=R(t) denotes the radius of the collapsing inner part of the star. At $t_0 = 273$ ms the radius R= 17.7 km, the mass of the (homologous) collapsing core $M_{h.c.} \approx M_{Ch} \approx 0.79 M_\odot$, and nuclear density $\rho_c = 1.29 \times 10^{14}$ g/cm^3.

Since the velocities in the inner part are smaller than the velocity of the sound v_s there is almost instant pressure communication, which results in uniform densities inside the inner part for each time. The inner region is totally dominated by quark-matter indicating a separation of the quarkyonic- and the nucleonic-phase, which is likely also to occur in neutron stars.

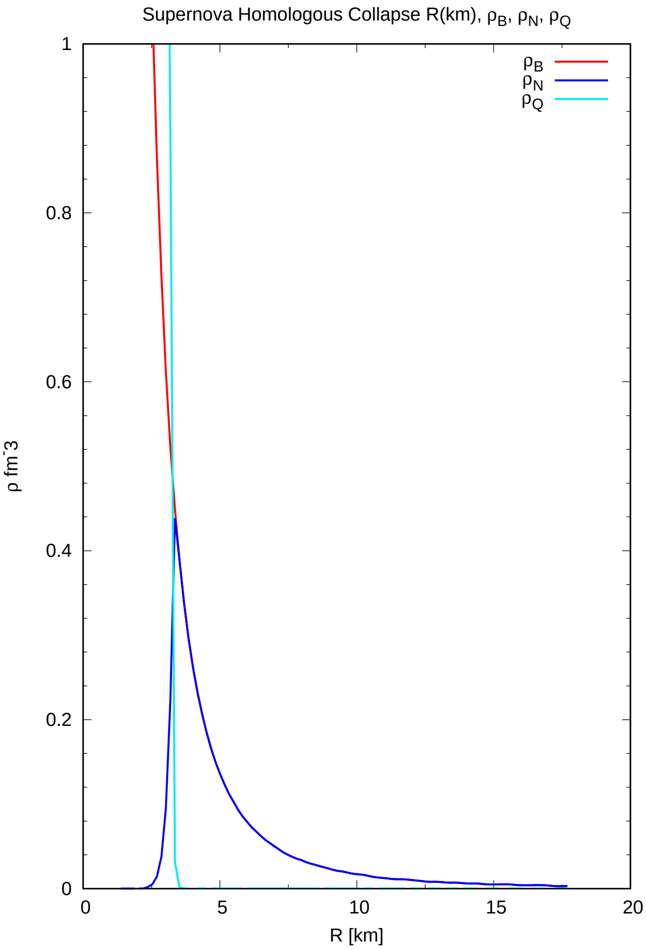

FIG. 5: *Homologous collapse supernova a la Wilson [18, 19],* $H = 2.579s^{-1}, \Gamma = 14MeV$, $\rho_0 = 0.153 fm^{-3}, \rho_D = 3\rho_0$.

V. SUMMARY AND CONCLUSION

Summarizing: *1. The S-wave quark-nucleon potential (3.9) is repulsive and becomes strong for high densities. 2. The* $^1P_1$*-wave has* $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 P_x = -9$ *giving strong repulsion. For* $^3P_J(J = 0, 1, 2)$ *the spin-isospin and the exchange operator give a factor -1, giving again a (weaker) repulsion. 3. The di-quark exchange potential gives a repulsive wall for the nucleons between the nucleon- and quark-phase.*

The introduction of the di-quark field is natural in view of the p and n presentation in [2]. Technically it has the advantage that this facilitates the study of nuclear/quark matter using the grand-partition function Z_G in the MFT.

Potentially, the QN-potential can be important for *e.g.* G-matrix calculations of neutron matter, see *e.g* [1]. The coupling λ_3 can be investigated in such calculations. The density dependent treatment of the $N \leftrightarrow 3Q$ coupling is important for a realistic description of the confinement-deconfinement process.

t(s)	R(km)	ρ_B/ρ_0	ρ_N/ρ_0	ρ_Q/ρ_0
0.273	17.700	0.306e-02	0.306e-02	0.000e+00
0.313	15.965	0.417e-02	0.417e-02	0.000e+00
0.363	14.033	0.614e-02	0.614e-02	0.000e+00
0.413	12.336	0.904e-02	0.904e-02	0.000e+00
0.463	10.843	0.133e-02	0.133e-02	0.000e+00
0.513	9.531	0.196e-01	0.196e-01	0.000e+00
0.563	8,378	0.289e-01	0.289e-01	0.000e+00
0.613	7.364	0.425e-01	0.425e-01	0.000e+00
0.663	6.473	0.626e-01	0.626e-01	0.000e+00
0.713	5.690	0.921e-01	0.921e-01	0.000e+00
0.763	5.002	0.136e-01	0.136e-01	0.000e+00
0.813	4.397	0.200e+00	0.200e+00	0.000e+00
0.863	3.865	0.294e+00	0.294e+00	0.000e+00
0.923	3.311	0.468e+00	0.452e+00	0.466e-01
0.943	3.144	0.546e+00	0.399e+00	0.441e+00
0.963	2.986	0.637e+00	0.352e+00	0.856e+00
1.013	2.625	0.938e+00	0.257e+00	0.204e+01
1.063	2.307	0.138e+01	0.188e+00	0.358e+01
1.113	2.028	0.203e+01	0.138e+00	0.569e+01
1.163	1.783	0.300e+01	0.101e+00	0.868e+01
1.183	1.639	0.350e+01	0.887e-01	0.102e+02
1.213	1.567	0.441e+01	0.735e-01	0.130e+02
1.233	1.488	0.515e+01	0.649e-01	0.152e+02
1.263	1.377	0.649e+01	0.538e-01	0.193e+02

TABLE I: Homologous Exponential Collapse

Appendix A: Mixed matter: Mean-field with Di-quarks

For the interaction Lagrangian in (2.3) with the tri-quark field $\eta_N(x)$ the functional form of the partition function is difficult to handle. In order to avoid third powers in the quark fields we write $\eta_N(x)$ in terms of the (bosonic) di-quark field $\chi^a_\mu(x)$ as

$$\eta_N(x) = (\hbar c)^2\gamma_5\gamma^\mu q^a(x)\cdot\chi^a_\mu(x),\ \chi^a_\mu(x) \equiv \varepsilon^{abc}\tilde{q}^b(x)C\gamma_\mu q^c(x)/(\hbar c)^2. \tag{A1}$$

Introduction this auxiliary di-quark field χ^a_μ via the Lagrangian density

$$\mathcal{L}_\chi = \bar{\lambda}_3^2\Big\{P\chi^{a\dagger}_\mu(x)\chi^{\mu a}(x) - \big[\chi^{a\dagger}_\mu(x)\big(\tilde{q}^b(x)C\gamma^\mu q^c(x)\big)f^{abc} + h.c.\big]\Big\} \tag{A2}$$

gives via the E.L. equation $\chi^a_\mu(x) = \big(\tilde{q}^b(x)C\gamma_\mu q^c(x)\big)f^{abc}$. *Sign in $\mathcal{L}_\chi$ is chosen such that Z_G is defined!* The total Lagrangian is the Lagrangian in (2.3 with the addition of $\mathcal{L}_\chi$ and for $N \leftrightarrow 3Q$ interaction $\mathcal{L}_{int}$ the substitution $N \leftrightarrow 3Q$ transitions in the form

$$\mathcal{L}_{int} \ \rightarrow\ -\lambda_3\{(\bar{\psi}(x)\gamma_5\gamma^\mu q^a)\,\chi^a_\mu + h.c.\}, \tag{A3}$$

The total Lagrangian is the Lagrangian in (2.3 with the addition of $\mathcal{L}_\chi$ and the new interaction Lagrangian

In the MF-approximation the momentum space equations for the nucleon and quark fields become

$$\left[\gamma_\mu(k^\mu - g_\omega\omega^\mu) - (m_N - g_\sigma\sigma + g_P\chi_P)\right]\ \psi(k) = \lambda_3\langle\chi^a_0\rangle\gamma_0\gamma_5 q^a(k) \rightarrow 0, \tag{A4a}$$

$$\left[\gamma_\mu(k^\mu - \frac{1}{3}g_\omega\omega^\mu) - (m_Q - \frac{1}{3}g_\sigma\sigma + \frac{1}{3}g_P\chi_P)\right]\ q^a(k) = \lambda_3\langle\chi^a_0\rangle\,\gamma_0\gamma_5\psi(k) \rightarrow 0. \tag{A4b}$$

The partition function becomes

$$Z_G \ =\ \int[d\bar{\psi}][d\psi][d\bar{q}][dq][d\sigma][d\omega_\mu]\int\mathcal{D}\chi^{a\mu}\mathcal{D}\chi^{a\dagger}_\mu\exp\Big[\int_0^\beta d\tau\int d^3x\,\cdot$$
$$\times\big(\mathcal{L}_N + \mathcal{L}_Q + \mathcal{L}_M + \mathcal{L}_{int} + \mathcal{L}_\chi + \mu_N\psi^\dagger\psi + \mu_Q q^\dagger q + \mu_D\chi^{a\dagger}_\mu\chi^{a\mu}\big)\Big],$$

where we introduced the chemical potential for the di-quarks D. Since in the ground state $\langle\chi^a_\mu\rangle = 0$ we put $\mu_D = 0$ in the following. For simplicity in this section we leave out the scalar self-interactions $U(\sigma)$, the pomeron and instanton fields χ_P, χ_I, *i.e.*

$$\mathcal{L}_Q \ \rightarrow\ \bar{q}(x)\Big[i\gamma_\mu\left(\partial^\mu + \frac{i}{3}g_\omega\omega^\mu\right) - \left(m_Q - \frac{1}{3}g_\sigma\sigma\right)\Big]\,q(x) \tag{A5a}$$

$$\mathcal{L}_N \ \rightarrow\ \bar{\psi}(x)\Big[i\gamma_\mu\left(\partial^\mu + ig_\omega\omega^\mu\right) - \left(m_N - g_\sigma\sigma\right)\Big]\,\psi(x) \tag{A5b}$$

$$\mathcal{L}_M \ \rightarrow\ +\frac{1}{2}\left(\partial^\mu\sigma\partial_\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \tag{A5c}$$

Making the MF-approximation for the scalar and vector fields the grand partition function becomes

$$\mathcal{Z}_G \ =\ \int\mathcal{D}\sigma\mathcal{D}\omega_\mu\int\mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}q\mathcal{D}\bar{q}^a\int\mathcal{D}\chi^{a\mu}\mathcal{D}\chi^{a\dagger}_\mu\exp\Big[i\int_0^\beta d\tau\int d^3x\,\mathcal{L}\Big] \tag{A6a}$$

$$\Rightarrow\ \exp\Big[\beta V(-\frac{1}{2}m_\sigma^2\bar{\sigma}^2 + \frac{1}{2}m_\omega^2\bar{\omega}_0^2)\Big]\int\mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}q\mathcal{D}\bar{q}^a\int\mathcal{D}\chi^{a\mu}\mathcal{D}\chi^{a\dagger}_\mu\exp\Big[i\int_0^\beta d\tau\int d^3x\,\mathcal{L}\Big] \tag{A6b}$$

where for the meson fields the MF approximation $\sigma(x) = \bar{\sigma}$ and $\omega_\mu(x) = \bar{\omega}_0$ is used. Furthermore, $\bar{\mathcal{L}} = \mathcal{L}^*_{NQ} + \mathcal{L}_\chi + \mathcal{L}_{int}$ with

$$\bar{\mathcal{L}}_{NQ} = \mathcal{L}^*_{NQ} + \mu^*\psi^\dagger(x)\psi(x) + \mu^*_Q q^{a\dagger}(x)q^a(x), \tag{A7a}$$

$$\mathcal{L}^*_{NQ} = \bar{\psi}(x)\left[i\gamma_\mu\partial^\mu - m^*_N\right]\psi(x) + \bar{q}(x)\left[i\gamma_\mu\partial^\mu - m^*_Q\right]q(x), \tag{A7b}$$

where

$$m^*_N = m_N - g_\sigma\bar{\sigma} \;\; , \;\; m^*_Q = m_Q - \frac{1}{3}g_\sigma\bar{\sigma}, \tag{A8a}$$

$$\mu^*_N = \mu_N - g_\omega\bar{\omega}_0 \;\; , \;\; \mu^*_Q = \mu_Q - \frac{1}{3}g_\omega\bar{\omega}_0. \tag{A8b}$$

The terms in $\mathcal{L}_\chi + \mathcal{L}_{int}$ are schematically, apart from an overall factor $\bar{\lambda}_3^2$,

$$\mathcal{L}_\chi + \mathcal{L}_{int} \sim \quad \chi^\dagger_\mu\chi^\mu - \chi^\dagger_\mu B^\mu - B^\dagger_\mu\chi^\mu = (\chi_\mu - B_\mu)^\dagger(\chi^\mu - B^\mu) - B^\dagger_\mu B^\mu, \tag{A9}$$

with $B_\mu = \left[\bar{\psi}(x)\gamma_5\gamma_\mu q(x)\right]/(\hbar c)^2$. The integration over the di-quark fields gives in the exponential of Z_G the term

$$-\bar{\lambda}_3^2\, B^\dagger_\mu B^\mu = -\bar{\lambda}_3^2(\bar{\psi}\gamma_5\gamma_\mu q)(\bar{q}\gamma_5\gamma^\mu\psi) = -\bar{\lambda}_3^2\Big[-(\bar{\psi}\psi)(\bar{q}q)$$
$$-\frac{1}{2}(\bar{\psi}\gamma^\mu\psi)(\bar{q}\gamma_\mu q) - \frac{1}{2}(\bar{\psi}\gamma_5\gamma^\mu\psi)(\bar{q}\gamma_5\gamma_\mu q) + (\bar{\psi}\gamma_5\psi)(\bar{q}\gamma_5 q)\Big], \tag{A10}$$

where $\bar{\lambda}_3 \equiv \lambda_3/(\hbar c)$, and in the form of the r.h.s. the Fierz-identities are used.
In $\mathcal{Z}_G$ the Lagrangian now becomes $\bar{\mathcal{L}} = \bar{\mathcal{L}}_N + \bar{\mathcal{L}}_Q + \bar{\mathcal{L}}'_{int}$ with

$$\bar{\mathcal{L}}'_{ints} = -\bar{\lambda}_3^2\, B^\dagger_\mu B^\mu = -\bar{\lambda}_3^2\,\left(\bar{\psi}\gamma_5\gamma_\mu q\right)\left(\bar{q}\gamma_5\gamma^\mu\psi\right). \tag{A11}$$

In the ground state translation-, rotation- and parity-invariance for homegeneous nucleon and quark matter, the MF-approximation leads to

$$\langle(\bar{\psi}\gamma_5\gamma_\mu q)(\bar{q}\gamma_5\gamma^\mu\psi)\rangle = -\langle\bar{\psi}\psi\rangle\langle\bar{q}q\rangle - \frac{1}{2}\langle\psi^\dagger\psi\rangle\langle q^\dagger q\rangle$$
$$= -\rho_s(N)\rho_s(Q) - \frac{1}{2}\rho_N\rho_Q \equiv -\rho_{NQ}, \tag{A12}$$

and the partition function becomes

$$\mathcal{Z}_G = \exp\left[\beta V\left(-\frac{1}{2}m_\sigma^2\bar{\sigma}^2 + \frac{1}{2}m_\omega^2\bar{\omega}_0^2 - \bar{\lambda}_3^2\rho_{NQ}\right)\right]\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}q\mathcal{D}\bar{q}^a \exp\Big[\int_0^\beta d\tau\int d^3x\,\bar{\mathcal{L}}_{N,Q}\Big]$$
$$\equiv \exp\left[\beta V\left(-\frac{1}{2}m_\sigma^2\bar{\sigma}^2 + \frac{1}{2}m_\omega^2\bar{\omega}_0^2 + \bar{\lambda}_3^2\rho_{NQ}\right)\right] Z_G(NQ), \tag{A13}$$

with $Z_G(NQ) = Z_{G,N}\, Z_{G,Q}$.

The contribution of the $N \leftrightarrow 3Q$ interaction to the pressure density is

$$p_{int} = \bar{\lambda}_3^2\rho_{NQ} = +\bar{\lambda}_3^2\left[\rho_s(N)\rho_s(Q) + \frac{1}{2}\rho_N\rho_Q\right], \tag{A14a}$$

$$\bar{\lambda}_3^2 \to \gamma_D\bar{\lambda}_3^2,\; \gamma_D = \left[1 - \exp\{-\gamma_1(\rho_N/\rho_D - 1)\}\right]\,\theta(\rho_N - \rho_D). \tag{A14b}$$

Here $\gamma_D(\rho_N)$ describes the transition between the confined and the deconfined phase. The contribution to the energy density is $\epsilon_{int} \to -k_B T\,(\partial/\partial V)\ln Z_{int} = -\bar{\lambda}_3^2\rho_{NQ}$. *So, the λ_3-coupling lowers the energy and*

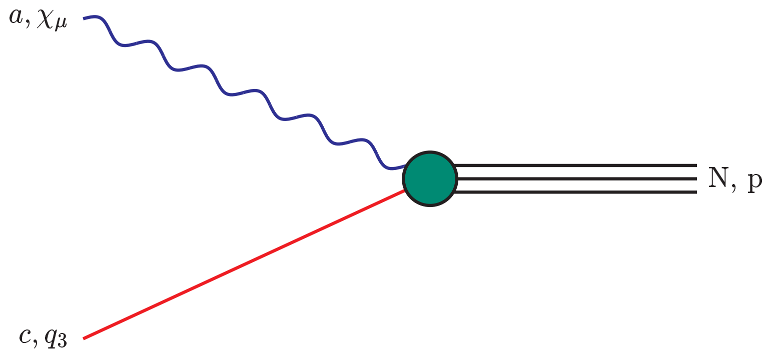

FIG. 6: Diquark-Quark-Nucleon Vertex

increases the pressure.

With $m_{N,Q} \equiv m^*_{N,Q}$ and $\mu_{N,Q} \equiv \mu^*_{N,Q}$ the pressure, energy, entropy, and particle densities are

$$p_{FG}(N+Q) = \frac{T}{V}\ln Z_{FG}(N+Q) = T\sum_{i=N,Q}\int^{\infty}\frac{d^3p}{(2\pi)^3}\ln\left(1+e^{-\beta(\omega_i-\mu_i)}\right), \tag{A15a}$$

$$\epsilon_{FG}(N+Q) = \sum_{i=N,Q}\int^{\infty}\frac{d^3p}{(2\pi)^3}\frac{\omega_i}{e^{\beta(\omega_i-\mu_i)}+1}, \tag{A15b}$$

$$s_{FG}(N+Q) = (1/T)\sum_{i=N,Q}\int^{\infty}\frac{d^3p}{(2\pi)^3}\frac{(\omega_i-\mu_i)}{e^{\beta(\omega_i-\mu_i)}+1} + \ln Z_{FG}(N) + \ln Z_{FG}(Q), \tag{A15c}$$

$$n_{FG}(N+Q) = \sum_{i=N,Q}\int^{\infty}\frac{d^3p}{(2\pi)^3}\frac{1}{e^{\beta(\omega_i-\mu_i)}+1}. \tag{A15d}$$

The energy densities are

$$\epsilon_N = -\frac{1}{V}\frac{\partial \ln Z_N}{\partial\beta} + \mu^*_N\frac{N_N}{V},\ \epsilon_Q = -\frac{1}{V}\frac{\partial \ln Z_Q}{\partial\beta} + \mu^*_Q\frac{N_Q}{V}. \tag{A16}$$

The MF-contribution from the mesons and $N \to 3Q$ to the entropy density is $s_{mesons} + s_{int} = 0$.

Chemical equilibrium requires $\mu_N = 3\mu_Q$ or $E_{F,N} = 3E_{F,Q}$, which implies for the CQM that $k_{F,N} = 3k_{F,Q}$. Using $\rho_N = \gamma_N[k^3_{F,N}/(6\pi^2)]$ and $\rho_Q = \gamma_Q[k^3_{F,Q}/(6\pi^2)]$ gives the relation $\rho_Q/\rho_N = (\gamma_Q/\gamma_N)(k_{F,Q}/k_{F,N})^3$. Considering neutron matter and symmetric u,d quark matter one has $\gamma_N = 2, \gamma_Q = 6$, so that $\rho_Q/\rho_N = 2/9$.

Varying the $\bar{\sigma}$ and $\bar{\omega}_0$, the equilibrium configuration is attained when P is an extremum. So

$$\bar{\sigma} = -\left(\frac{g_\sigma}{m_\sigma^2}\right)\frac{\partial P_{FG}}{\partial m_N^*} = -\left(\frac{g_\sigma}{3m_\sigma^2}\right)\frac{\partial P_{FG}}{\partial m_Q^*}, \tag{A17a}$$

$$\bar{\omega}_0 = \left(\frac{g_\omega}{m_\omega^2}\right)\frac{\partial P_{FG}}{\partial \mu_N^*} = \left(\frac{g_\omega}{3m_\omega^2}\right)\frac{\partial P_{FG}}{\partial \mu_Q^*}, \tag{A17b}$$

using the CQM where $m_M^* = 3m_Q^*$ and $\mu_N^* = 3\mu_Q^*$. Also

$$\bar{\omega}_0 = \frac{g_\omega}{m_\omega^2}\, n,\ \bar{\sigma} = \frac{g_\sigma}{m_\sigma^2}\, n_s, \tag{A18}$$

where

$$n_s = \gamma_n \int \frac{d^3p}{(2\pi)^3}\frac{m_N^*}{E_N^*}\left(\left[e^{\beta(E_N^*-\mu_N^*)}+1\right]^{-1} + \left[e^{\beta(E_N^*+\mu_N^*)}+1\right]^{-1}\right), \tag{A19}$$

with $E_N^* = \sqrt{p^2 + m_N^{*2}}$. The equation (A18) is the well known self-consistent equation to be solved for m_N^*, as can be seen from the alternative form $m_N^* = m_N - (g_\sigma^2/m_\sigma^2)\, n_s$.

Appendix B: Diquark Feynman Propagator

The diquark field Feynman propagator is

$$i(\Delta_F)_{\mu\nu}(x'-x) = (0|\chi_\mu(x')\chi_\nu^\dagger(x)|0)\,\theta(t'-t) + (0|\chi_\mu^\dagger(x)\chi_\nu^\dagger(x')|0)\,\theta(t-t') \tag{B1}$$

where the diquark fields are

$$\chi_\mu(x) = q^b(x)C\gamma_\mu q^c(x)\, f^{abc}\ ,\ \chi_\mu^\dagger(x) = \bar{q}^a(x)xC\gamma_\mu C\,\bar{q}^b(x)\, f^{abc} \tag{B2}$$

where the Dirac indices are contracted, and C is the charge conjugation Dirac matrix which satisfies $C^{-1}\gamma_\mu C = -\gamma_\mu^T$ [4]. Suppressing the color indices, the plane wave expansion of the quark field, see *e.g.* [4], reads

$$q(x) = \sum_s \int \frac{d^3p}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_p}}\left[b(p,s)\,u(p,s)\,e^{-ip.x} + d^\dagger(p,s)\,v(p,s)\,e^{ip.x}\right]. \tag{B3}$$

The annihilation and creation operators $b(p,s)$ and $d^\dagger(p,s)$ satisfy the anti-commutation relations $\{b(p,s), b^\dagger(p',s')\} = \delta_{s,s'}\,\delta^{(3)}(\mathbf{p}-\mathbf{p'})$ etc.

1. Convolution product di-quark field: First we consider the convolution form of the di-quark field

$$\chi_\mu(x) = \int d^4y\, q(y)\, C\, \gamma_\mu\, q(x-y) \tag{B4}$$

Using the notation $d^3\tilde{p} \equiv [d^3p/(2\pi)^{3/2}]\sqrt{m/E_p}$, the vacuum expectations become

$$(0|\chi_\mu(x')\,\chi_\nu^\dagger(x)|0) = \int d^4y \int d^4z \sum_{s,r,s',r'} \int d^3\tilde{p} \int d^3\tilde{p}' \int d^3\tilde{q} \int d^3\tilde{q}'$$
$$(0|\left[b_{p,s}\,u_{p,s}e^{-ip\cdot y} + d^\dagger_{p,s}v_{p,s}e^{ip\cdot y}\right] C\gamma_\mu \left[b_{p',s'}u_{p',s'}e^{-ip'\cdot(x'-y)} + d^\dagger_{p',s'}v_{p',s'}e^{ip'\cdot(x'-y)}\right]\cdot$$
$$\times\left[b^\dagger\bar{u}_{q,r}e^{-iq\cdot(x-z)} + d_{q,r}\bar{v}_{q,r}e^{iq\cdot(x-z)}\right]\gamma_\nu C\left[b^\dagger_{q',r'}\bar{u}_{q',r'}e^{-iq'\cdot z} + d_{q',r'}\bar{v}_{q',r'}e^{iq'\cdot z}\right]|0). \tag{B5}$$

The non-vanishing vacuum expectation values are

$$(0|b_{p,s}b_{p',s'}b^\dagger_{q,r}b^\dagger_{q',s'}|0) = \delta_{s',r}\delta^3(p'-q)\,\delta_{s,r'}\delta^3(p-q') - \delta_{s',r'}\delta^3(p'-q')\delta_{s,r}\delta^3(p-q), \tag{B6}$$
$$(0|d^\dagger_{p,s}d^\dagger_{p',s'}d_{q,r}d_{q',s'}|0) = \delta_{s',r}\delta^3(p'-q)\,\delta_{s,r'}\delta^3(p-q')i - \delta_{s',r'}\delta^3(p'-q')\delta_{s,r}\delta^3(p-q). \tag{B7}$$

The d^4y and d^4z integrations leads to the expression

$$(0|\chi_\mu(x')\,\chi_\nu^\dagger(x)|0)_1 = (2\pi)^8 \sum_{s,r,s',r'} \int d^3\tilde{p} \int d^3\tilde{p}' \int d^3\tilde{q} \int d^3\tilde{q}'$$
$$\times\Big[\left[u_{p,s}C\,\gamma_\mu u_{p',s'}\right]\left[\bar{u}_{q,r}\,\gamma_\nu\,C\,\bar{u}_{q',r'}\right]\delta^4(p-p')\delta^4(q-q')\,e^{-ip\cdot x'}e^{iq\cdot x}\,.$$
$$\left[\delta_{s',r}\delta^3(p'-q)\,\delta_{s,r'}\delta^3(p-q') - \delta_{s',r'}\delta^3(p'-q')\delta_{s,r}\delta^3(p-q)\right]\Big] =$$
$$(2\pi)^8 \sum_{s,r,s',r'} \int \frac{d^3p}{(2\pi)^3}\frac{m}{E_p} \int \frac{d^3q}{(2\pi)^3}\frac{m}{E_q}\, e^{-ip\cdot(x'-x)}\,\delta^3(p-q)\cdot$$
$$\times\Big\{\left[u_{p,s}C\,\gamma_\mu u_{p,s'}\right]\left[\bar{u}_{q,r}\gamma_\nu C\,\bar{u}_{q,r'}\right]\delta_{s'r}\delta_{sr'} - \left[u_{p,s}C\,\gamma_\mu u_{p,s'}\right]\left[\bar{u}_{q,r}\gamma_\nu C\,\bar{u}_{q,r'}\right]\delta_{s'r'}\delta_{sr}\Big\} =$$
$$(2\pi)^8 \sum_{s,r} \int \frac{d^3p}{(2pi)^3}\frac{m}{E_p} \int \frac{d^3q}{(2\pi)^3}\frac{m}{E_q}\, e^{-ip\cdot(x'-x)}\,\delta^3(p-q)\cdot$$
$$\times\Big\{\left[u_{p,s}C\,\gamma_\mu u_{p,r}\right]\left[\bar{u}_{q,r}\gamma_\nu C\,\bar{u}_{q,s}\right] - \left[u_{p,s}C\,\gamma_\mu u_{p,r}\right]\left[\bar{u}_{q,s}\gamma_\nu C\,\bar{u}_{q,r}\right]\Big\} =$$
$$(2\pi)^8 \sum_{s,r} \int \frac{d^3p}{(2\pi)^6}\left(\frac{m}{E_p}\right)^2 e^{-ip\cdot(x'-x)}\,.$$
$$\times\Big\{\left[u_{p,s}C\,\gamma_\mu u_{p,r}\right]\left[\bar{u}_{p,r}\gamma_\nu C\,\bar{u}_{p,s}\right] - \left[u_{p,s}C\,\gamma_\mu u_{p,r}\right]\left[\bar{u}_{p,s}\gamma_\nu C\,\bar{u}_{p,r}\right]\Big\}. \tag{B8}$$

Now,

$$u^T(p,s) = -\bar{v}(p,s)\,C\;,\;\bar{u}^T(p,s) = C^{-1}v(p,s) \tag{B9}$$

which gives

$$\left[u^T_{p,s}C\gamma_\mu u_{p,r}\right] = +\left[\bar{v}(p,s)\gamma_\mu u(p,r)\right]\;,\;\left[\bar{u}_{p,r}\gamma_\nu C\,\bar{u}_{p,s}\right] = +\left[\bar{u}_{p,r}\gamma_\nu v_{p,s}\right]$$

leading to

$$(0|\chi_\mu(x')\,\chi_\nu^\dagger(x)|0)_1 = (2\pi)^8 \sum_{s,r,s',r'} (2\pi)^8 \sum_{s,r} \int \frac{d^3p}{(2\pi)^6}\left(\frac{m}{E_p}\right)^2 e^{-ip\cdot(x'-x)}\,.$$
$$\times\Big\{\left[\bar{v}_{p,s}\gamma_\mu u_{p,r}\right]\left[\bar{u}_{p,r}\gamma_\nu v_{p,s}\right] - \left[\bar{v}_{p,s}\gamma_\mu u_{p,r}\right]\left[\bar{u}_{p,s}\gamma_\nu v_{p,r}\right]\Big\} = 0. \tag{B10}$$

This follows from derivation of the Gordon-decomposition for $[\bar{u}_{p,s}\gamma_\mu v_{p,r}] = 0$ and $[\bar{v}_{p,s}\gamma_\mu u_{p,r}] = 0$!

2. Normal product di-quark field: With the di-quark field $\chi_\mu(x)$ as defined in (B2) we have instead of

$$(0|\chi_\mu(x')\,\chi_\nu^\dagger(x)|0) = \sum_{s,r,s',r'} \int d^3\tilde{p} \int d^3\tilde{p}' \int d^3\tilde{q} \int d^3\tilde{q}'$$

$$(0|\left[b_{p,s}\,u_{p,s}e^{-ip\cdot x'} + d^\dagger_{p,s}v_{p,s}e^{ip\cdot x'}\right] C\gamma_\mu \left[b_{p',s'}u_{p',s'}e^{-ip'\cdot x'} + d^\dagger_{p',s'}v_{p',s'}e^{ip'\cdot x'}\right]\cdot$$

$$\times\left[b^\dagger\bar{u}_{q,r}e^{-iq\cdot x} + d_{q,r}\bar{v}_{q,r}e^{iq\cdot x}\right]\gamma_\nu C\left[b^\dagger_{q',r'}\bar{u}_{q',r'}e^{-iq'\cdot x} + d_{q',r'}\bar{v}_{q',r'}e^{iq'\cdot x}\right]|0). \tag{B11}$$

Restriction to the non-vanishing vacuum expectationa in (B7) we have

$$(0|\chi_\mu(x')\,\chi_\nu^\dagger(x)|0) = \sum_{s,r,s',r'} \int d^3\tilde{p} \int d^3\tilde{p}' \int d^3\tilde{q} \int d^3\tilde{q}'$$

$$\Big[(0|b_{p,s}b_{p',s'}b^\dagger_{q,r}b^\dagger_{q',s'}|0)\,[u_{p,s}C\gamma_\mu u_{p',s'}][\bar{u}_{q,r}\gamma_\nu C\bar{u}_{q',r'}]e^{-i(p+p')\cdot x'}e^{-i(q+q')\cdot x}$$

$$+(0|d^\dagger_{p,s}d^\dagger_{p',s'}d_{q,r}d_{q',s'}|0)\,[v_{p,s}C\gamma_\mu v_{p',s'}][\bar{v}_{q,r}\gamma_\nu C\bar{v}_{q',r'}]e^{+i(p+p')\cdot x'}e^{+i(q+q')\cdot x}\Big] \tag{B12}$$

Next we use

$$\int \frac{d^3p}{(2\pi)^3E_p} = \int \frac{d^4p}{(2\pi)^4}\delta(p^2-m_Q^2)\theta(p_0) \equiv \int d^4\tilde{p},$$

and arrive at the expression

$$(0|\chi_\mu(x')\,\chi_\nu^\dagger(x)|0) = \sum_{s,r,s',r'} \int d^4\tilde{p} \int d^4\tilde{p}' \int d^4\tilde{q} \int d^4\tilde{q}'$$

$$\Big[(0|b_{p,s}b_{p',s'}b^\dagger_{q,r}b^\dagger_{q',s'}|0)\,[u_{p,s}C\gamma_\mu u_{p',s'}][\bar{u}_{q,r}\gamma_\nu C\bar{u}_{q',r'}]e^{-i(p+p')\cdot x'}e^{-i(q+q')\cdot x}$$

$$+(0|d^\dagger_{p,s}d^\dagger_{p,s'}d_{q,r}d_{q,s'}|0)\,[v_{p,s}C\gamma_\mu v_{p,s'}][\bar{v}_{q,r}\gamma_\nu C\bar{v}_{q,r'}]e^{+i(p+p')\cdot x'}e^{+i(q+q')\cdot x}\Big] \tag{B13}$$

Then, the first term in (B11) gives the contribution

$$(0|\chi_\mu(x')\,\chi_\nu^\dagger(x)|0)_1 = \sum_{s,r,s',r'} \int d^4\tilde{p} \int d^4\tilde{p}' \int d^4\tilde{q} \int d^4\tilde{q}'\, e^{-i(p+p')\cdot x'}e^{+i(q+q')\cdot x}\,.$$

$$\times\Big[\left[u_{p,s}C\,\gamma_\mu u_{p',s'}\right]\left[\bar{u}_{q,r}\,\gamma_\nu\,C\,\bar{u}_{q',r'}\right]\left[\delta_{s',r}\delta^4(p'-q)\,\delta_{s,r'}\delta^4(p-q') - \delta_{s',r'}\delta^4(p'-q')\delta_{s,r}\delta^4(p-q)\right] =$$

$$\sum_{s,r,s',r'} \int d^4\tilde{p} \int d^4\tilde{q}\, e^{-i(p+q)\cdot(x'-x)}\Big\{\delta_{s,r'}\delta_{s',r}\left[\bar{v}_{p,s}\gamma_\mu u_{q,s'}\right]\left[\bar{u}_{q,r}\,\gamma_\nu\,v_{p,r'}\right]$$

$$-\delta_{s',r'}\delta_{s,r}\left[\bar{v}_{p,s}\gamma_\mu u_{q,s'}\right]\left[\bar{u}_{p,r}\,\gamma_\nu\,v_{q,r'}\right]\Big\} \tag{B14}$$

Similarly, for the second term in (B11) we obtain

$$(0|\chi_\mu(x')\,\chi_\nu^\dagger(x)|0)_2 = \sum_{s,r,s',r'} \int d^4\tilde{p} \int d^4\tilde{q}\, e^{-i(p+q)\cdot(x'-x)}\Big\{\delta_{s,r'}\delta_{s',r}\left[\bar{u}_{p,s}\gamma_\mu u_{q,s'}\right]\left[\bar{v}_{q,r}\,\gamma_\nu\,u_{p,r'}\right]$$

$$-\delta_{s',r'}\delta_{s,r}\left[\bar{u}_{p,s}\gamma_\mu v_{q,s'}\right]\left[\bar{v}_{p,r}\,\gamma_\nu\,u_{q,r'}\right]\Big\} \tag{B15}$$

From the Gordon decomposition derivation one gets $\left[\bar{u}_{p,r}\,\gamma_\mu\,v_{q,r'}\right]=0$!
Remark: This why a photon does not couple to pairs, unless there is an additional interaction with another system like for example a nucleus with charge Ze.

3. Conclusion: $(\Delta_F)_{\mu\nu}(x'-x)=0$ **: The di-quark does not propagate. Only a contact interaction between a nucleon and a quark is possible!**

Appendix C: Exchange Potentials

In this section we follow our multi-channel description formalism in the treatment of the exchange potentials [6, 8]. In thias appendix we give a detailed treatment for ΛN matrix elements, which can transcribed directly to the QN matrix elements. K-exchange for $\Lambda N \to N\Lambda$ corresponds to di-quark exchange for $QN \to NQ$.

In the case of the anti-symmetric spin-orbit the exchange potential requires some attention, because their special features. The potentials in configuration space are described in Pauli-spinor space as follows

$$V = V_C + V_\sigma \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + V_T\, S_{12} + V_{SLS}\,\mathbf{L}\cdot\mathbf{S}_+ + V_{ALS}\,\mathbf{L}\cdot\mathbf{S}_- + V_Q\, Q_{12}\,. \tag{C1}$$

Here, the definition of the matrix elements of the spin operators are defined as follows

$$\left(\chi^\dagger_{m'}(\Lambda)\chi^\dagger_{n'}(N)|\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2|\chi^\dagger_m(\Lambda)\chi^\dagger_n(N)\right) \equiv \left(\chi^\dagger_{m'}(\Lambda)|\boldsymbol{\sigma}_1|\chi^\dagger_m(\Lambda)\right)\cdot\left(\chi^\dagger_{n'}(N)|\boldsymbol{\sigma}_1|\chi^\dagger_n(N)\right)\,, \tag{C2}$$

and similarly for the SU(2) and SU(3) operator matrix elements. In Fig. 7 the labels (m,n,m',n') refer to the spin, and the labels $(\alpha,\beta,\alpha',\beta')$ refer to unitary spin, like SU(2) or SU(3). The momenta on line 1 are $\mathbf{p}$ and $\mathbf{p}'$ for respectively the initial and the final state. Likewise, the momenta on line 2 are $-\mathbf{p}$ and $-\mathbf{p}'$ for respectively the initial and the final state.

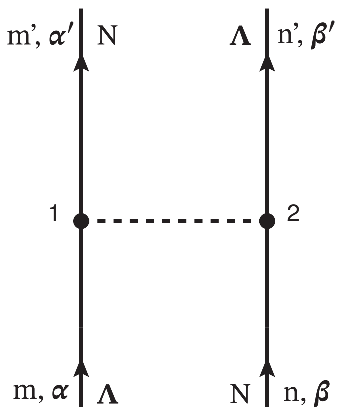

FIG. 7: Particle- and spin-exchange for ΛN

In graph Fig. 7 we encounter the matrix elements

$$(\boldsymbol{\sigma}_1)_{m',m} = \left(\chi^\dagger_{m'}(N)|\boldsymbol{\sigma}_1|\chi^\dagger_m(\Lambda)\right)\ ,\ \ (\boldsymbol{\sigma}_2)_{n',n} = \left(\chi^\dagger_{n'}(\Lambda)|\boldsymbol{\sigma}_2|\chi^\dagger_n(N)\right) \tag{C3}$$

1. Spin-Exchange Potentials

In order to project the exchange potentials on the forms in (C1) we have to rewrite these matrix elements in terms of those occurring in (C2). This can be done using the spin-exchange operator P_σ:

$$P_\sigma = \frac{1}{2}\left(1 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\right) \ . \tag{C4}$$

Properties of this operator are

$$\begin{aligned}
&P_\sigma\dagger = P_\sigma \ , \ P_\sigma^2 = 1 \ , \\
&P_\sigma\, \chi_{1,m}\chi_{2,n} = \chi_{1,n}\chi_{2,m} \ , \\
&P_\sigma\, \sigma_{1,k}\, P_\sigma = \sigma_{2,k} \ , \\
&P_\sigma\, \sigma_{2,k}\, P_\sigma = \sigma_{1,k} \ .
\end{aligned}$$

Similar properties hold for the flavor-exchange operator P_f, but then for the SU(2) isospin operators τ_k, or the SU(3) octet operators λ_k.
In the following we make only explicit the spin labels, but similar operations apply to the SU(2) or SU(3) labels.

Using this spin-exchange operator, we find that

$$\begin{aligned}
&\left(\chi^\dagger_{1,m'}(N)\chi^\dagger_{2,n'}(\Lambda)|\boldsymbol{\sigma}_1 \otimes 1_2 - 1_1 \otimes \boldsymbol{\sigma}_2|\chi^\dagger_{1,m}(\Lambda)\chi^\dagger_{2,n}(N)\right) = \\
&\left(\chi^\dagger_{2,n'}(N)\chi^\dagger_{1,m'}(\Lambda)|P^\dagger_\sigma\left(\boldsymbol{\sigma}_1 \otimes 1_2 - 1_1 \otimes \boldsymbol{\sigma}_2\right)P_\sigma\, P_\sigma|\chi^\dagger_{1,m}(\Lambda)\chi^\dagger_{2,n}(N)\right) = \\
&-\left(\chi^\dagger_{1,m'}(\Lambda)\chi^\dagger_{1,n'}(N)|\left(\boldsymbol{\sigma}_1 \otimes 1_2 - 1_1 \otimes \boldsymbol{\sigma}_2\right)\, P_\sigma|\chi^\dagger_{1,m}(\Lambda)\chi^\dagger_{2,n}(N)\right) \ .
\end{aligned} \tag{C5}$$

above, we added the subscripts 1 and 2 to indicate explicitly the baryon line that is involved.

2. Spin- and Strangeness-Exchange Potentials

In addition to the spin-exchange, we also have the flavor-exchange operator P_f active here. So, in total we have to apply $-P_\sigma\, P_f = P_x$, i.e. the space-exchange operator. This latter relation follows from the anti-symimetry of the two-baryon states, which implies that only states with $P_f P_\sigma P_x = -1$ are physical. All this implies

1. For the ALS-potential derived in K-exchange etc. one has in (C1), considering both spin- and flavor-exchange, the operator

$$\text{ALS} \Rightarrow \frac{1}{2}\left(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2\right) \cdot \mathbf{L}\, P_x \tag{C6}$$

2. For the SLS-potential derived in K-exchange etc. one has in (C1), considering both spin- and flavor-exchange, the operator $P_f P_\sigma$, but since

$$\begin{aligned}
\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\, \sigma_{1,k} &= \sigma_{2,k} + i\epsilon_{klm}\, \sigma_{1,l}\sigma_{2,m} \ , \\
\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\, \sigma_{2,k} &= \sigma_{1,k} + i\epsilon_{klm}\, \sigma_{2,l}\sigma_{1,m} \ ,
\end{aligned}$$

one derives easily that

$$P_\sigma\left(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2\right) \cdot \mathbf{L} \ = \ \left(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2\right) \cdot \mathbf{L} \,, \tag{C7}$$

and therefore, similarly to (C5) we have, with the inclusion of the flavor labels,

$$\Big(\chi^{\dagger}_{1,m'\alpha'}(N)\chi^{\dagger}_{2,n'\beta'}(\Lambda)|\boldsymbol{\sigma}_1\otimes 1_2 + 1_1\otimes\boldsymbol{\sigma}_2|\chi^{\dagger}_{1,m\alpha}(\Lambda)\chi^{\dagger}_{2,n\beta}(N)\Big) =$$
$$\Big(\chi^{\dagger}_{2,n'\beta'}(N)\chi^{\dagger}_{1,m'\alpha'}(\Lambda)|P^{\dagger}_f P^{\dagger}_\sigma\Big(\boldsymbol{\sigma}_1\otimes 1_2 + 1_1\otimes\boldsymbol{\sigma}_2\Big)|\chi^{\dagger}_{1,m\alpha}(\Lambda)\chi^{\dagger}_{2,n\beta}(N)\Big) =$$
$$\Big(\chi^{\dagger}_{1,m'\alpha'}(\Lambda)\chi^{\dagger}_{1,n'\beta'}(N)|\,(\boldsymbol{\sigma}_1\otimes 1_2 + 1_1\otimes\boldsymbol{\sigma}_2)\;P_f\,|\chi^{\dagger}_{1,m\alpha}(\Lambda)\chi^{\dagger}_{2,n\beta}(N)\Big)\ . \tag{C8}$$

So, for the SLS-potential derived in K-exchange etc. one has in (C1), considering both spin- and flavor-exchange, the operator

$$\text{SLS} \Rightarrow \frac{1}{2}\,(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{L}\,P_f \tag{C9}$$

This treatment for the SLS-potential also applies to the central-, spin-spin-, tensor-, and quadratic-spin-orbit potentials as well, of course.

We conclude this section by noticing that we have found, using our multi-channel set-up the same prescriptions for the treatment of the flavor-exchange potentials as in [6]. For the treatment of the ALS-potential for $S=\pm1$-exchange, our prescription here is more clear. For example in the case of the coupled $^1P_1 -^3 P_1$ system our prescription is unambiguous, and given by the P_x-operator, which is the same for both partial-waves coupled in this case.

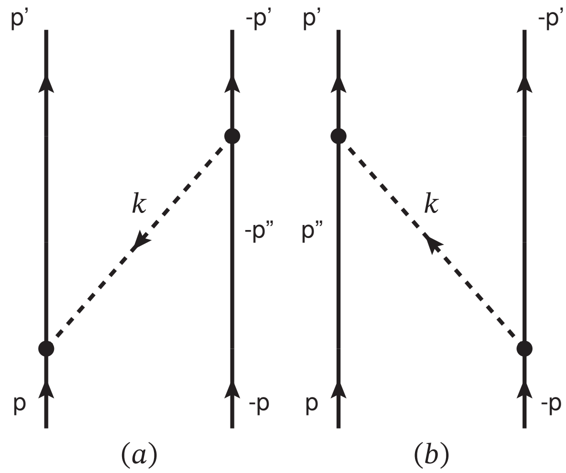

FIG. 8: K- and K*-exchange time-ordered graphs (a) and (b).

Appendix D: Gaussian-Fourier

$$\int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} e^{-\mathbf{k}^2/\Lambda^2} = \frac{m^3}{\pi\sqrt{\pi}}\, m^3 \exp(-m^2r^2), \tag{D1a}$$

$$\int \frac{d^3k}{(2\pi)^3} k_i k_j\, e^{i\mathbf{k}\cdot\mathbf{r}} e^{-\mathbf{k}^2/\Lambda^2} = \frac{2m^5}{\pi\sqrt{\pi}} \left(\delta_{ij} - \frac{1}{2}(mr)^2\, \hat{r}_i \hat{r}_j\right) \exp(-m^2r^2), \tag{D1b}$$

$$\int \frac{d^3k}{(2\pi)^3} \mathbf{k}^2\, e^{i\mathbf{k}\cdot\mathbf{r}} e^{-\mathbf{k}^2/\Lambda^2} = \frac{2m^5}{\pi\sqrt{\pi}} \left(3 - 2m^2r^2\right) \exp(-m^2r^2), \tag{D1c}$$

$$\int \frac{d^3k}{(2\pi)^3} \boldsymbol{\sigma}_1 \cdot \mathbf{k}\, \boldsymbol{\sigma}_2 \cdot \mathbf{k}\, e^{i\mathbf{k}\cdot\mathbf{r}} e^{-\mathbf{k}^2/\Lambda^2} = \frac{2m^5}{\pi\sqrt{\pi}} \left(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - 2m^2r^2 \boldsymbol{\sigma}_1 \cdot \hat{\mathbf{r}} \boldsymbol{\sigma}_2 \cdot \hat{\mathbf{r}}\right) \exp(-m^2r^2)$$

$$= \frac{2m^5}{\pi\sqrt{\pi}} \left[\left(1 - \frac{2}{3}m^2r^2\right) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \frac{2}{3}(m^2r^2)\, S_{12}\right] \exp(-m^2r^2), \tag{D1d}$$

$$\int \frac{d^3k}{(2\pi)^3} \left[\boldsymbol{\sigma}_1 \cdot \mathbf{k}\, \boldsymbol{\sigma}_2 \cdot \mathbf{k} - \frac{1}{3}\mathbf{k}^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\right] e^{i\mathbf{k}\cdot\mathbf{r}} e^{-\mathbf{k}^2/\Lambda^2} = -\frac{2m^5}{\pi\sqrt{\pi}} \frac{2}{3}(mr)^2 \exp(-m^2r^2)\, S_{12}, \tag{D1e}$$

$$\int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} i\mathbf{S} \cdot (\mathbf{q} \times \mathbf{k}) e^{-\mathbf{k}^2/\Lambda^2} = \frac{2m^5}{\pi\sqrt{\pi}} \exp(-m^2r^2)\, \mathbf{L} \cdot \mathbf{S}, \tag{D1f}$$

where $m = \Lambda/2$.

$$\int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} e^{-\mathbf{k}^2/\Lambda^2} = \frac{\Lambda^3}{8\pi\sqrt{\pi}} \exp(-\frac{1}{4}\Lambda^2r^2), \tag{D2a}$$

$$\int \frac{d^3k}{(2\pi)^3} k_i k_j\, e^{i\mathbf{k}\cdot\mathbf{r}} e^{-\mathbf{k}^2/\Lambda^2} = \frac{\Lambda^5}{16\pi\sqrt{\pi}} \left(\delta_{ij} - \frac{1}{8}(\Lambda r)^2\, \hat{r}_i \hat{r}_j\right) \exp(-\frac{1}{4}\Lambda^2r^2), \tag{D2b}$$

$$\int \frac{d^3k}{(2\pi)^3} \mathbf{k}^2\, e^{i\mathbf{k}\cdot\mathbf{r}} e^{-\mathbf{k}^2/\Lambda^2} = \frac{\Lambda^5}{16\pi\sqrt{\pi}} \left(3 - \Lambda^2r^2/2\right) \exp(-\frac{1}{4}\Lambda^2r^2), \tag{D2c}$$

$$\int \frac{d^3k}{(2\pi)^3} \left[\boldsymbol{\sigma}_1 \cdot \mathbf{k}\, \boldsymbol{\sigma}_2 \cdot \mathbf{k} - \frac{1}{3}\mathbf{k}^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\right] e^{i\mathbf{k}\cdot\mathbf{r}} e^{-\mathbf{k}^2/\Lambda^2} = -\frac{\Lambda^5}{16\pi\sqrt{\pi}} \frac{1}{6}(\Lambda r)^2 \exp(-\frac{1}{4}\Lambda^2r^2)\, S_{12}, \tag{D2d}$$

$$\int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} \left[i\mathbf{S} \cdot (\mathbf{q} \times \mathbf{k})\right] e^{-\mathbf{k}^2/\Lambda^2} = \frac{\Lambda^5}{16\pi\sqrt{\pi}} \exp(-\frac{1}{4}\Lambda^2r^2)\, \mathbf{L} \cdot \mathbf{S}, \tag{D2e}$$

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