Microscopic Approaches to Light-Nucleus Reactions

K. Nollett, ANL; J. Carlson, LANL
S. Pieper & R. Wiringa, ANL; G. Hale, LANL
V. R. Pandharipande, Ill; R. Schiavilla JLAB/ODU

- Microscopic Approaches
- Summary of Results to Date
- Scattering Tests of Nuclear Interactions : n - α scattering
- Future: Interaction Tests
  Applications

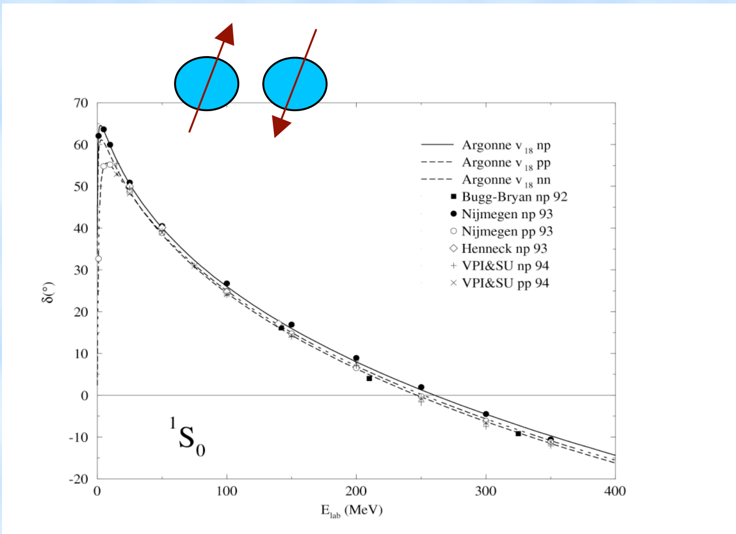
Goals:

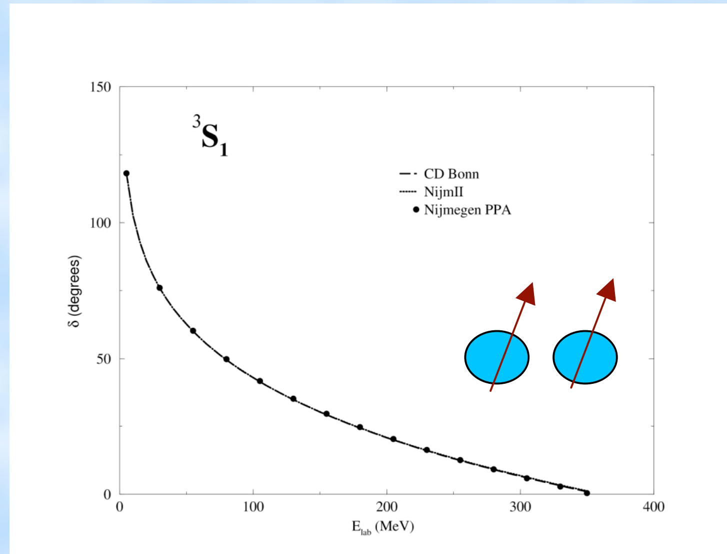
- Understand structure & reactions in few-nucleon systems
- Ties to:

Hadronic Physics (PV) Astrophysics

2- Nucleon System

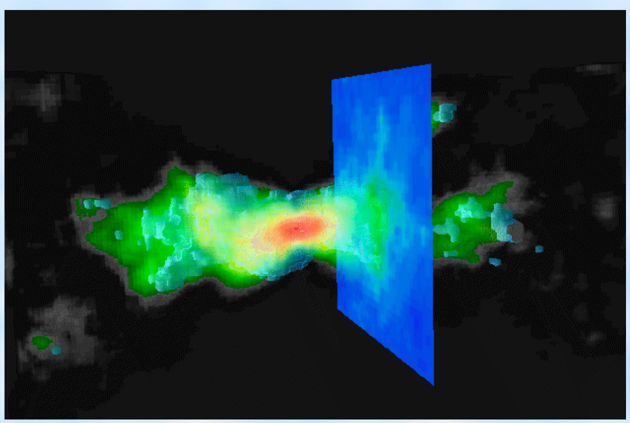
PP & NP scattering yield very precise info on nuclear phase shifts:



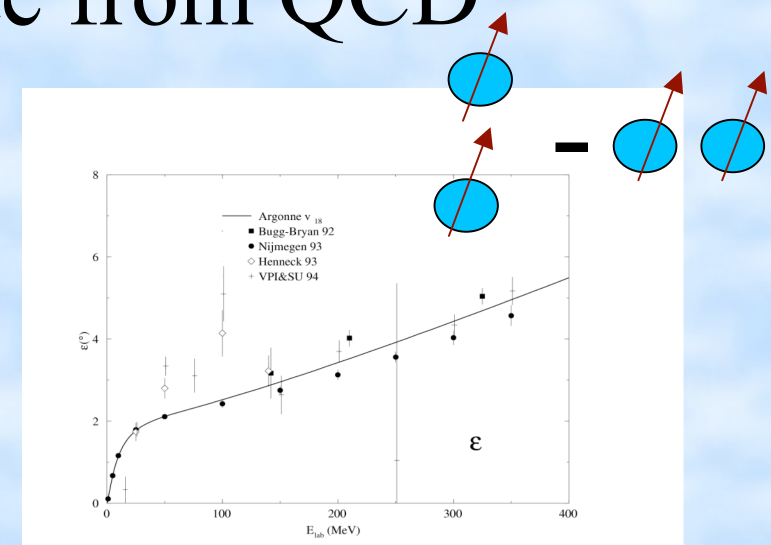


Difficult to calculate from QCD

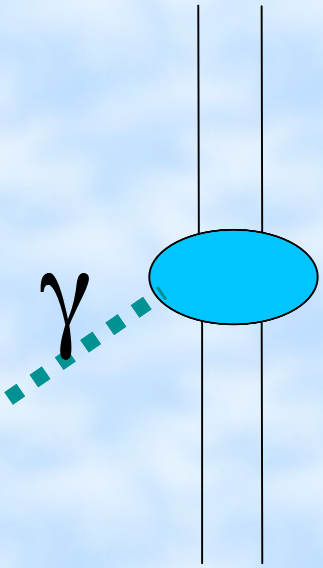
Pion

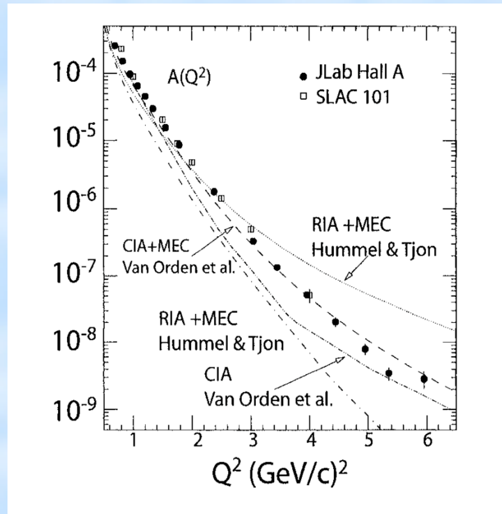


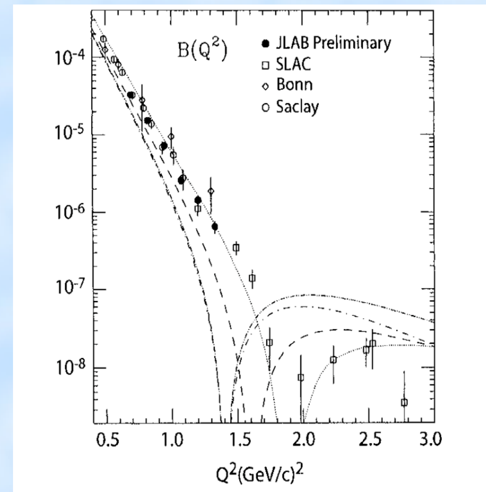
OSU pion propagator

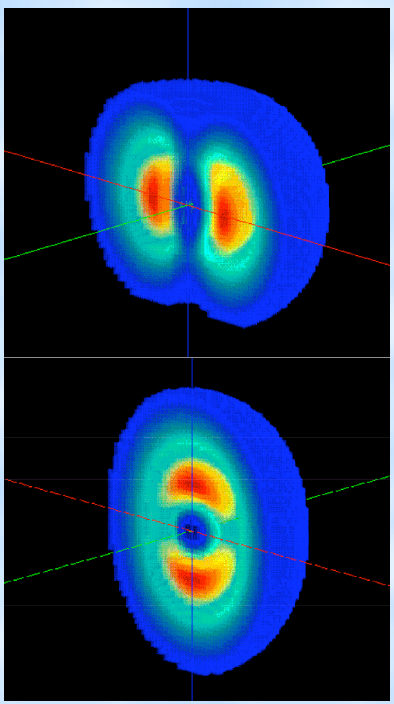


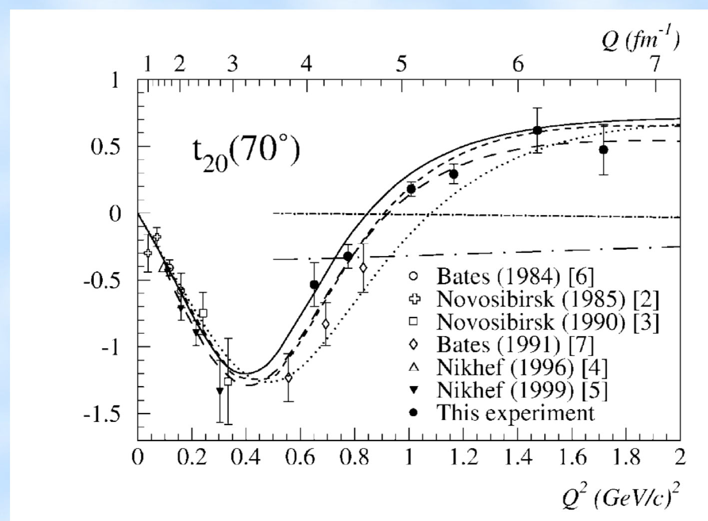
Deuteron Form Factors











How do we solve the Schroedinger Equation ?

Assume non-relativistic nucleons w/ 2- (and 3)-nucleon interactions:

$$|\Psi_T\rangle = \sum_{\sigma,\tau} \chi_\sigma \chi_\tau \phi(\mathbf{R})$$

$$\chi_\sigma = \downarrow_1 \uparrow_2 \ldots \downarrow_A \;\; (2^A \text{ terms})$$ = 256 for A=8

$$\chi_\tau = n_1 n_2 \ldots p_A \;\; (\frac{A!}{N!Z!} \text{ terms})$$ = 70 for ^{8}Be

= 17,920 complex functions in 3A-3 = 21 dimensions

Size of nucleus ~ 10 fm, resolution ~ 0.5 fm $\Rightarrow$ 10^{27} grid points

Variational Monte Carlo (VMC)

$$E = \frac{\langle \Psi_T | H | \Psi_T \rangle}{\langle \Psi_T | \Psi_T \rangle}$$ Minimize Energy

$$\Psi_T = [\mathcal{S} \ (\prod F_{ijk}) \ (\prod F_{ij})] \ \Psi_J$$

$$\Psi_J = \prod f(r_{ij}) | \Phi \rangle$$

Long Distance
(~Shell Model) Structure

Short-Distance Correlations

Spatial Integrals evaluated with
Metropolis Monte Carlo

Results depend upon your knowledge (Intuition).

Green's Function Monte Carlo (GFMC)

$$| \Psi_0 > \; = \; \mathrm{Exp}\,[\, - H \tau \,]\, | \Psi_T >$$

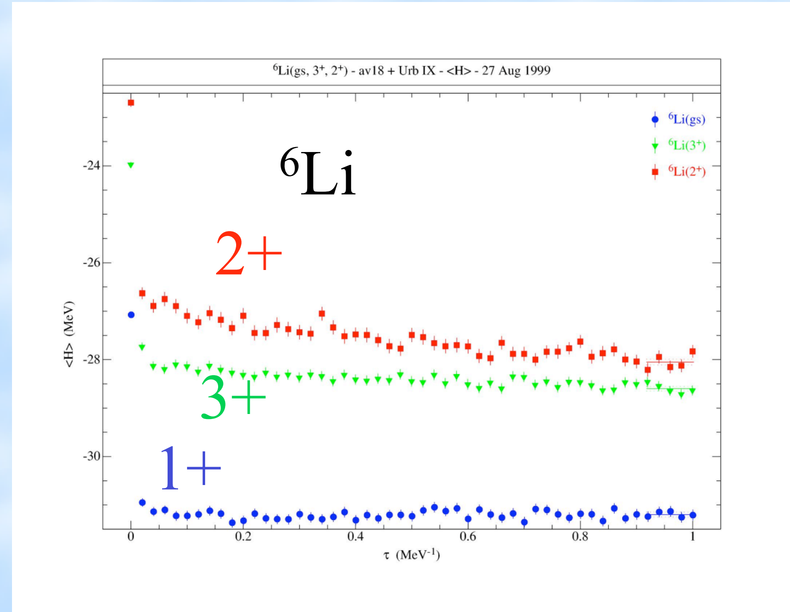
'cooling' algorithm

T= 0 : variational state

T⇒ ∞: ground state

$$\mathrm{Exp}\,[\, - H\, \Delta\tau \,] = \exp\,[\, -V\, \Delta\tau /2 \,]\; \exp\,[\, -T\, \Delta\tau \,]\; \exp\,[\, -V\, \Delta\tau /2 \,]$$

Branching Random Walk



LANL Q computer

Green's function Monte Carlo

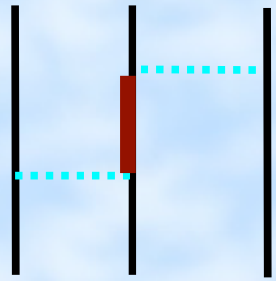
Advantages:

No explicit basis (coordinate space)
Good approximate knowledge used

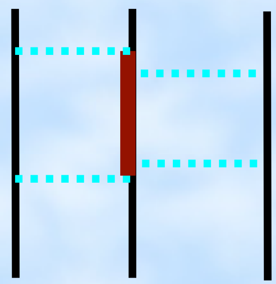
Disadvantages:

Explicit sum over all spin/isospin states
(slow for large A)
Use approximate momentum-dependence
and correct perturbatively
Constrained-path plus unconstrained propagation
used for Fermions

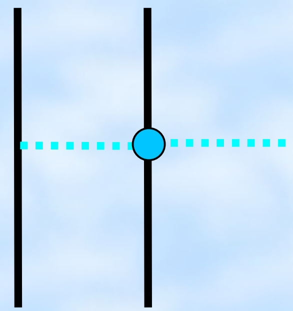
Illinois Models of 3-Nucleon Interaction



3-Nucleon Interaction Required:
Postulated 1st in 1950's
Delta ~ 300 MeV Excited state of Nucleon

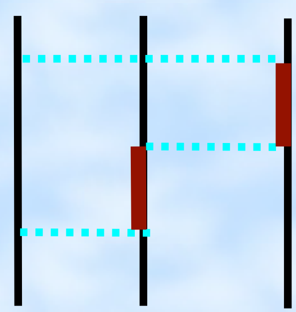


'Short'-range spin-independent TNI
Mimics relativistic effects in NM



S-wave pi-N
Can affect
spin-orbit splitting

Additional Terms in Illinois TNI:
adjusted to L.S splitting and
neutron-rich nuclei (A<9)



Comparatively larger
effects in T=3/2

3-Nucleon Interaction Parameters

TABLE I. Three-body potential parameters used in this paper. Parameters that were not varied in fitting the data are marked with an asterisk.

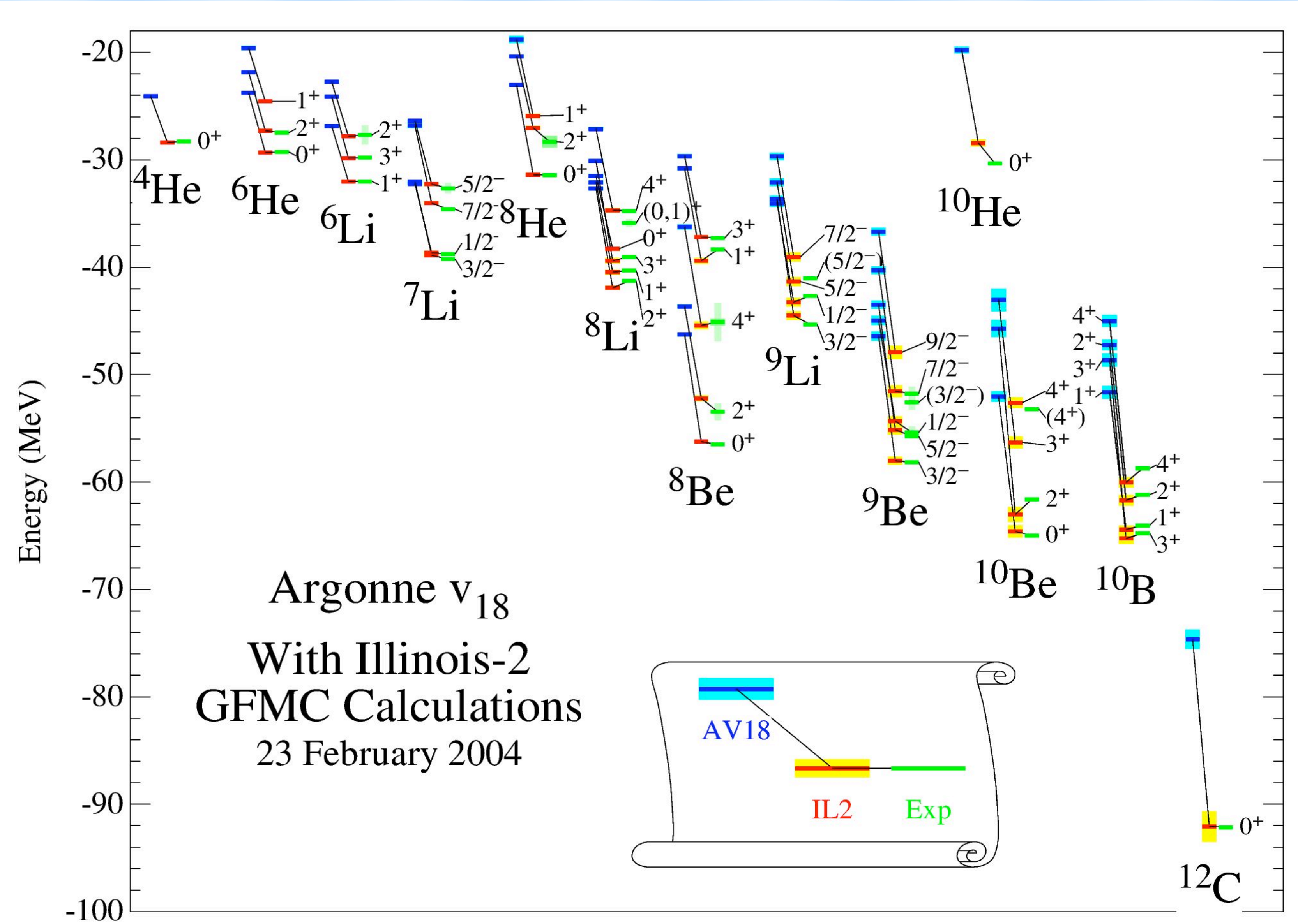
Model	c fm^{-2}	$A_{2\pi}^{PW}$ MeV	$A_{2\pi}^{SW}$ MeV	$A_{3\pi}^{\Delta R}$ MeV	A_R MeV	A_W MeV	A_R^* MeV
UIX	2.1*	−0.0293			0.00480	0*	0.002 91
IL1	2.1*	−0.0385	0.0*	0.0026*	0.00705*	0*	0.004 91
IL2	2.1*	−0.037	−1.0*	0.0026	0.00705	0*	0.004 93
IL3	1.5	−0.07*	−1.0*	0.0065	0.032	0*	0.025 62
IL4	2.1*	−0.028*	−1.0*	0.0021	0.0039	0*	0.001 96
IL5	2.1*	−0.03	−1.0*	0.0021*	0.002*	210	0.0

No unique fit, but:

Additional attraction in isospin-rich nuclei required

Additional spin-orbit splitting required

Spectra



3-nucleon interaction required for accurate spectra
Isospin-dependence information provided by 8He, ...

Comparison w/ NCSM

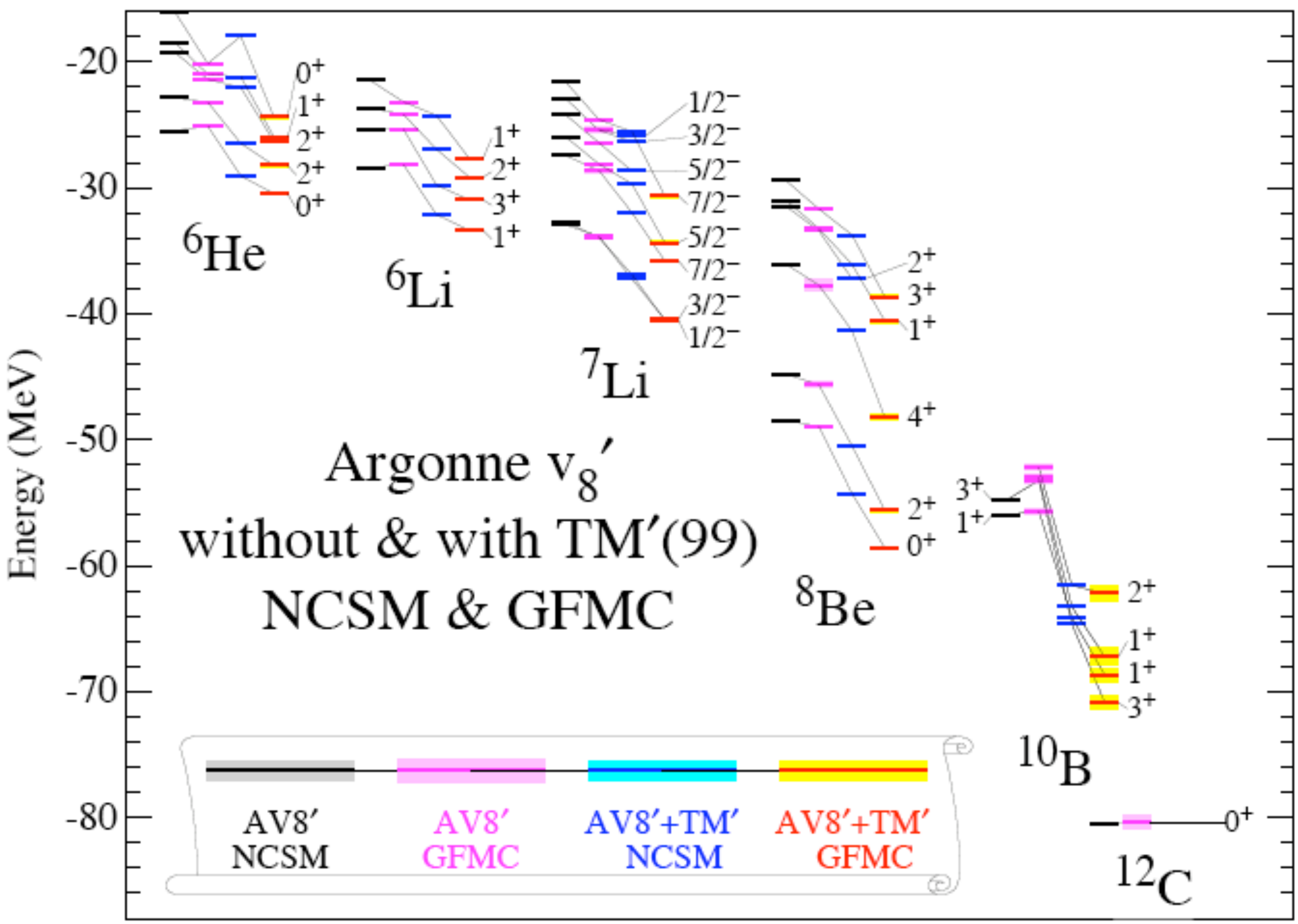
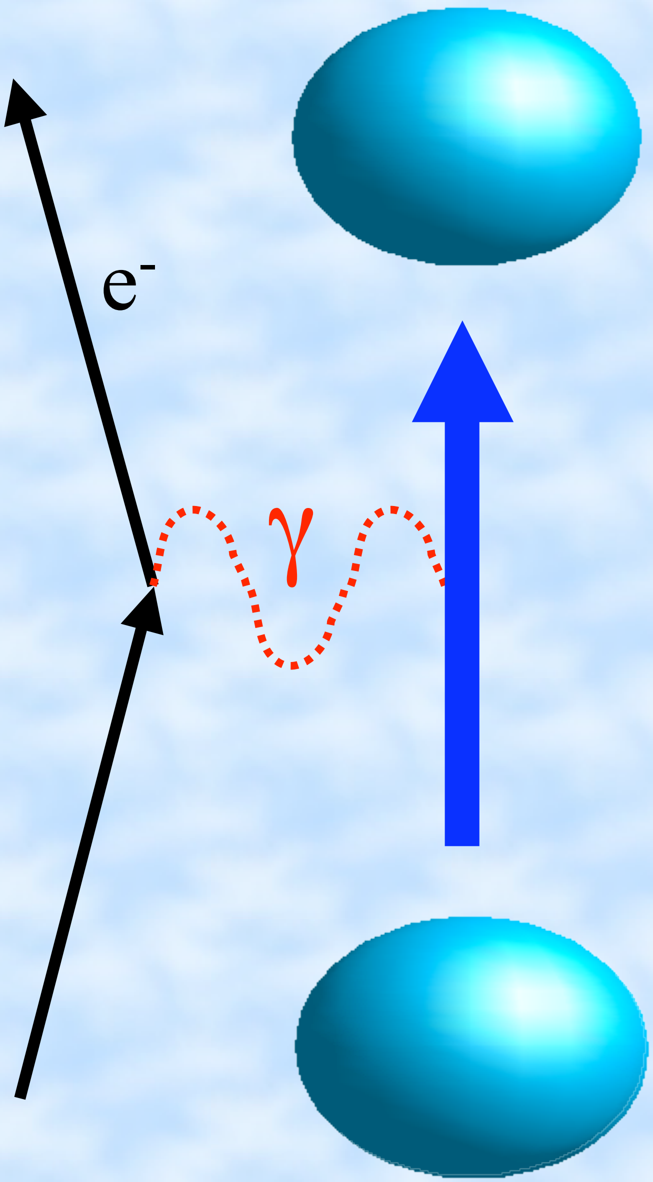
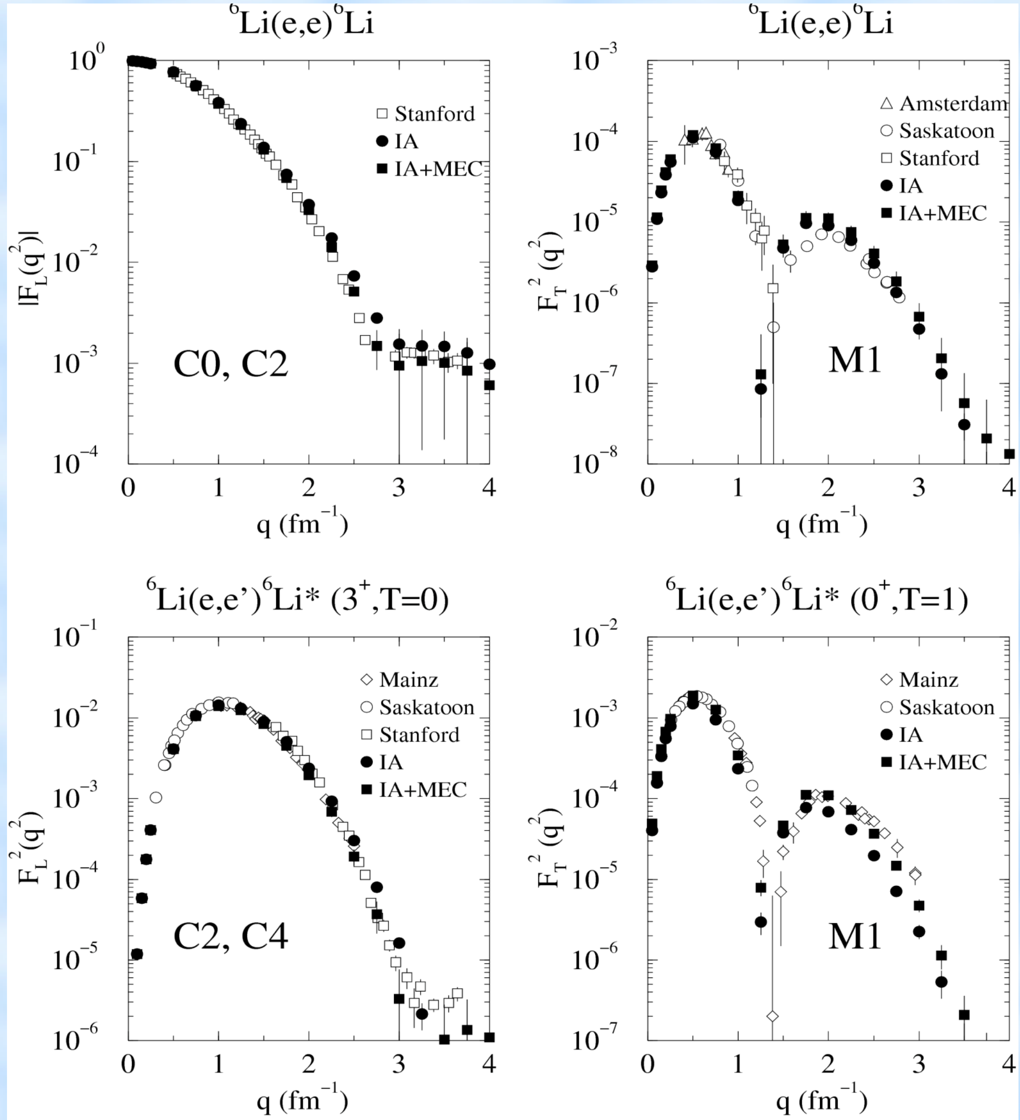


Figure 6. Comparison of NCSM and GFMC energies for the AV8′ and AV8′+TM′ Hamiltonians.

Elastic/Transition Form Factors of ^{6}Li





Ground State

Transition

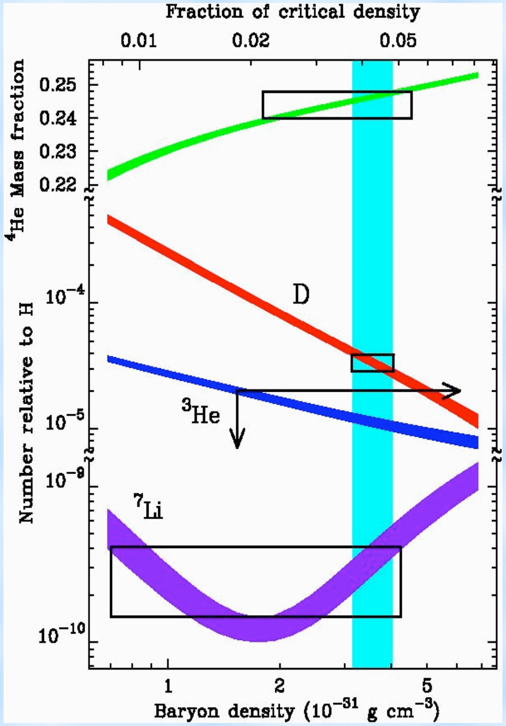
Charge

Mag

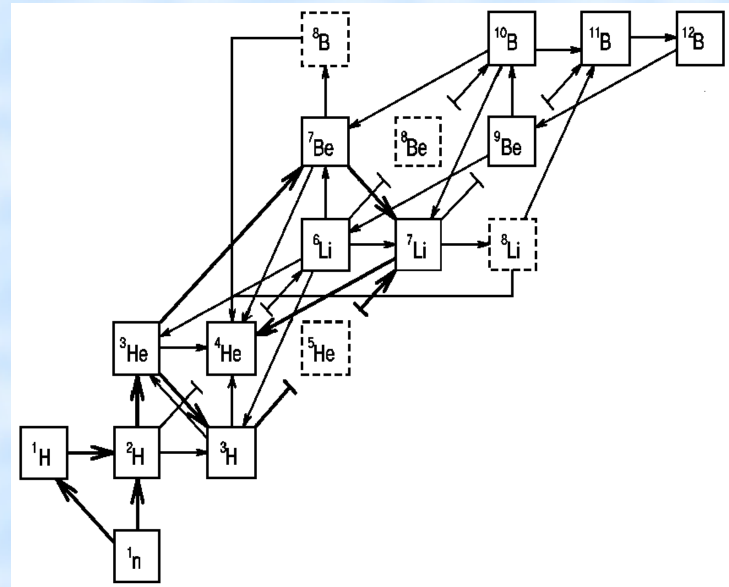
Variational MC
Wiringa, et al

Big-Bang Nucleosynthesis

Abundances

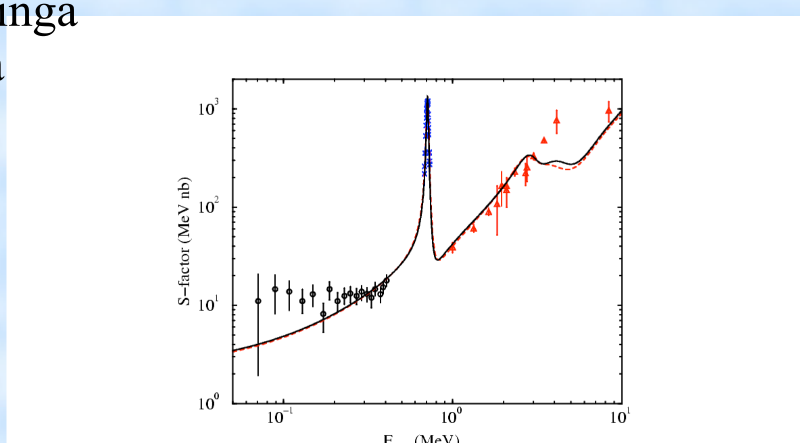


Reaction Network



Astrophysical S-factors

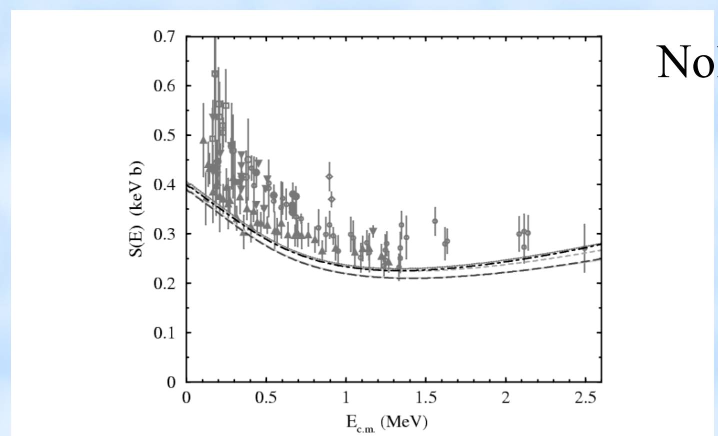
Nollett, Wiringa & Schiavilla



VMC

α (d,γ) ^{6}Li

Nollett

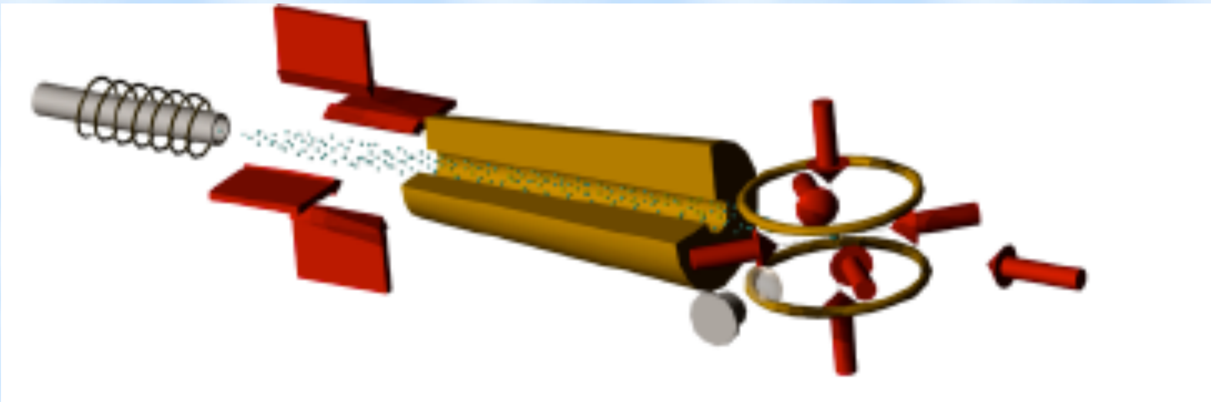


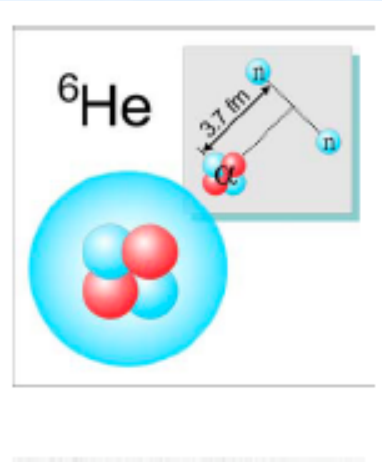
^{3}He (α, γ) ^{7}Be

Neutron-rich Nuclei

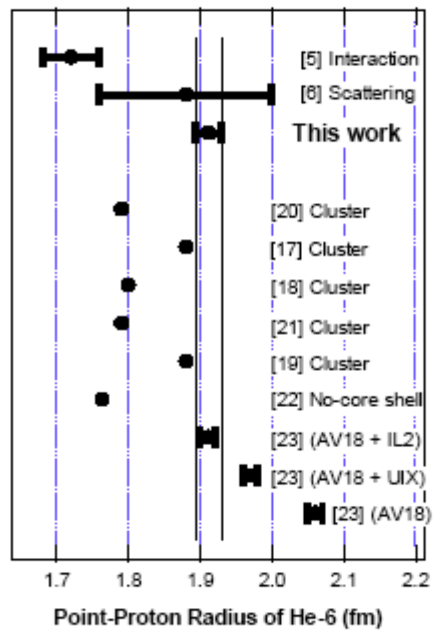
^{6}He charge radius
ANL expt
Jannsens & Lu

Phys.Rev.Lett. 93 (2004) 142501





6He charge radius



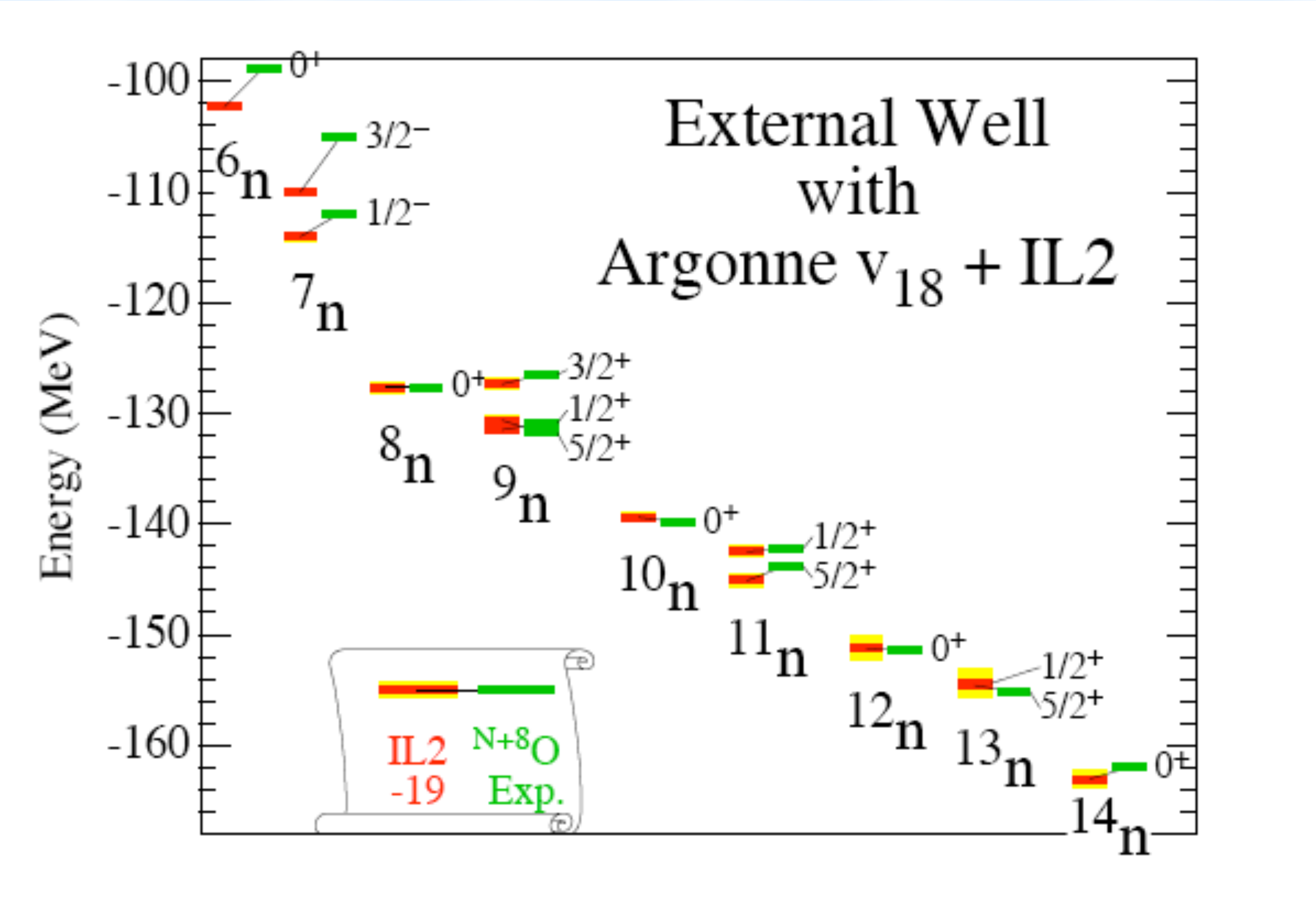
He Isotopes - Charge Radii:

	expt	GFMC
4He	1.676(8)	1.660(10)
6He	2.054(14)	2.08(2)
8He		1.97(3)

Neutron-rich Nuclei

Neutron Drops
Mimic Oxygen isotopes
BCS trial states

S. Pieper,
R. B. Wiringa, and
V. R. Pandharipande



Variational Calculations Of Multiple Excited States

$$|\Psi_J\rangle = \mathcal{A}\left\{\left[\prod_{i<j<k} f^c_{ijk}\right]\left[\prod_{i<j\leq 4} f_{ss}(r_{ij})\right]\sum_{LS[n]}\left(\beta_{LS[n]}\left[\prod_{k\leq 4<l\leq A} f^{LS[n]}_{sp}(r_{kl})\right]\left[\prod_{4<l<m\leq A} f^{LS[n]}_{pp}(r_{lm})\right]|\Phi_A(LS[n]JMTT_3)_{1234:5\cdots A}\rangle\right)\right\}$$

$$|\Phi_A(LS[n]JMTT_3)_{1234:5\cdots A}\rangle = \Big|\Phi_4(0000)_{1234}\left[\prod_{4<l\leq A}\phi^{LS[n]}_p(R_{\alpha l})\right]$$
$$\times\left\{\left[\prod_{4<l\leq A} Y_{1m_l}(\Omega_{\alpha l})\right]_{LM_L[n]}\left[\prod_{4<l\leq A}\chi_l\left(\frac{1}{2}m_s\right)\right]_{SM_S}\right\}_{JM}\left[\prod_{4<l\leq A}\nu_l\left(\frac{1}{2}t_3\right)\right]_{TT_3}\Big\rangle$$

Diagonalize these states in a small (shell-model like) basis:

$$E_{T,ij} = \langle\Psi_T(\beta_i)|H|\Psi_T(\beta_j)\rangle,$$

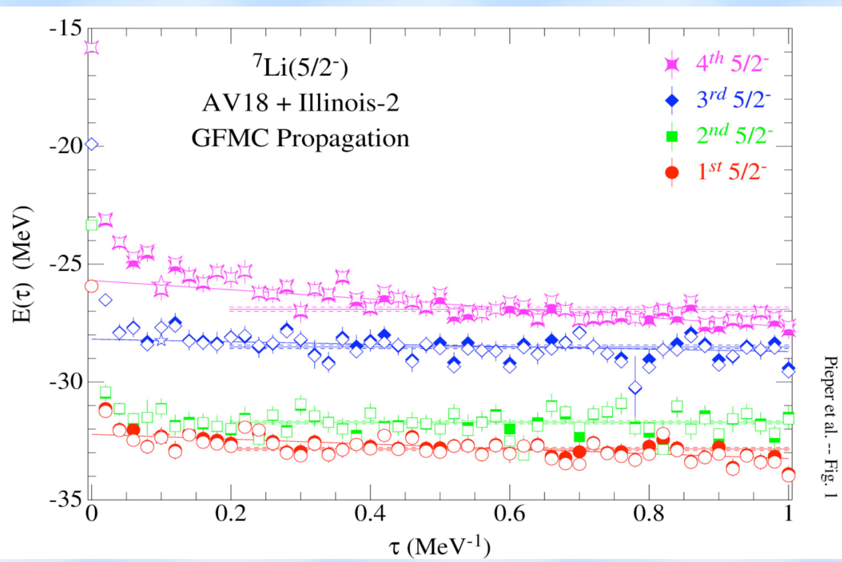
$$N_{T,ij} = \langle\Psi_T(\beta_i)|\Psi_T(\beta_j)\rangle,$$

QMC studies of Light Nuclei

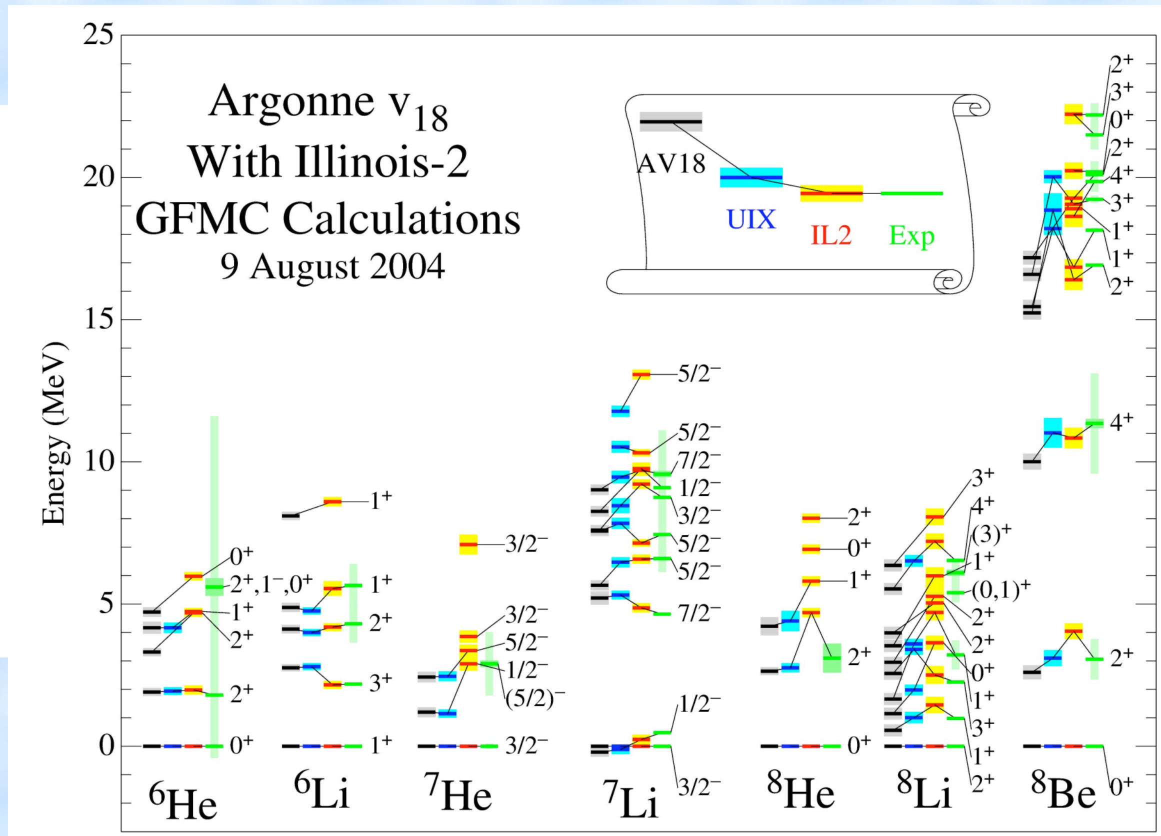
Excited States w/ same quantum numbers

Diagonalize in a small basis:

5/2$^-$ states in ^{7}Li



Spectra in A=6-8



Required for multi-channel, higher energy scattering

Scattering: single channel case
example: n-alpha scattering
Variational Monte Carlo

Enforce a boundary condition:

$$\Psi(\mathbf{R})|_{r>R_0} = \Psi_{c1}\ \Psi_{c2}\ \Psi_{rel}(\mathbf{r}),$$

$$\frac{\nabla_{\mathbf{r}} \cdot \hat{n}\Psi}{\Psi}|_{r=R_0} = \gamma,$$

Variational principle w/ wave functions that satisfy boundary condition:

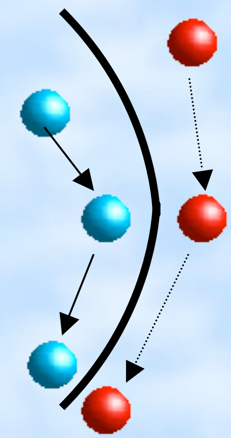
$$|\Psi_T\rangle = \mathcal{A}\left\{\left[1 + \sum_{i<j<k} U_{ijk}\right]\ \mathcal{S}\left[\prod_{i<5} F_{5i} \prod_{i<j<5} F_{ij}\right] \sum_k \beta_k |\Phi_k\rangle\right\}.$$

$$|\Phi\rangle = \mathcal{A}\phi_{s,p}(\hat{r}_{5,\alpha})\mathcal{Y}^{JT}(r_{5,\alpha}, \chi_\sigma(5), \chi_\tau(5))[\uparrow n(1), \uparrow p(2), \downarrow n(3), \downarrow p(4)]$$

GFMC approach to low-energy scattering

Do not have an explicit wavefunction, just paths sampling the wvfn. However, we can use the expression:

$$G(R,R') = G_{c1}\ G_{c2}\ G_{rel},$$

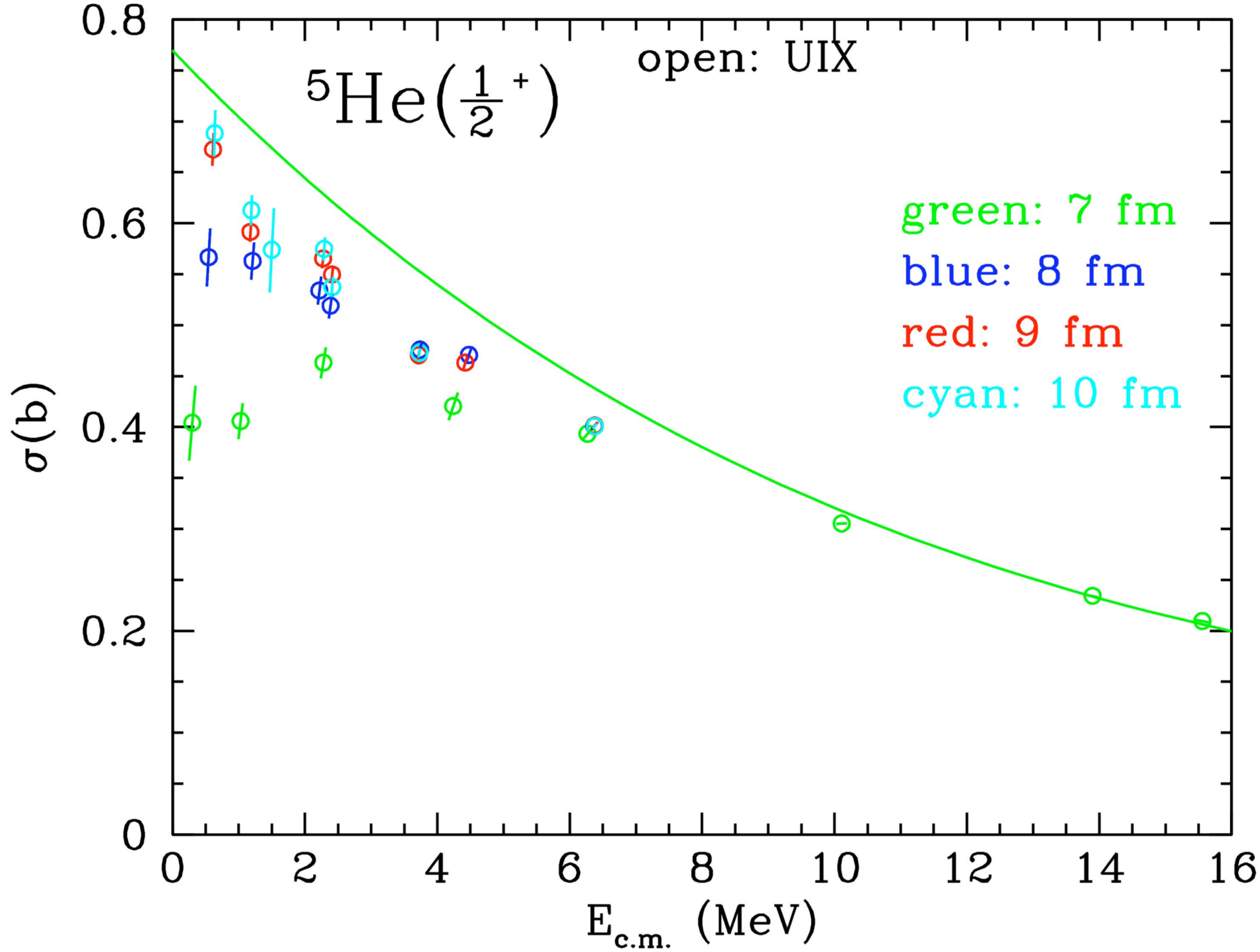


When approaching the surface, at an image at r_e :

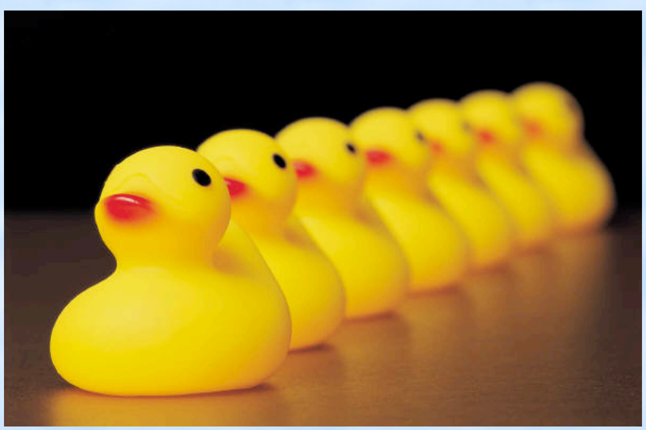
$$|\mathbf{r}||\mathbf{r_e}| = R_0^2,$$

$$\begin{aligned}\Psi_{n+1}(R) \ &= \int dR'_{c1} \int dR'_{c2} \int_{|r'|<R_0} d\mathbf{r}' \ \ G(R,R')\ \Psi_n(R') + \\ &+ \int dR'_{c1} \int dR'_{c2} \int_{|r'_e|>R_0} d\mathbf{r}'_e \ \ G(R,R')\ \Psi_n(R')\end{aligned}$$

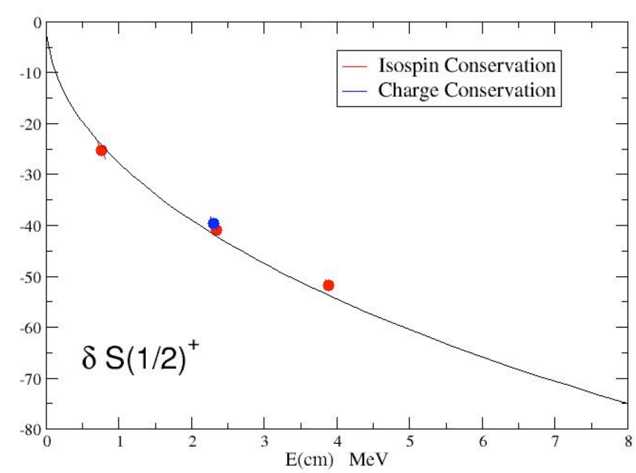
$$\Psi_{n+1}(R) = \int dR' G(R,R')\ [1 + \frac{G(R,R'_e)}{G(R,R')}\ [\frac{r_i}{r}]^3\ [1+\gamma(R'_e - R')\cdot\hat{n}]\ ]\Psi_n(R').$$

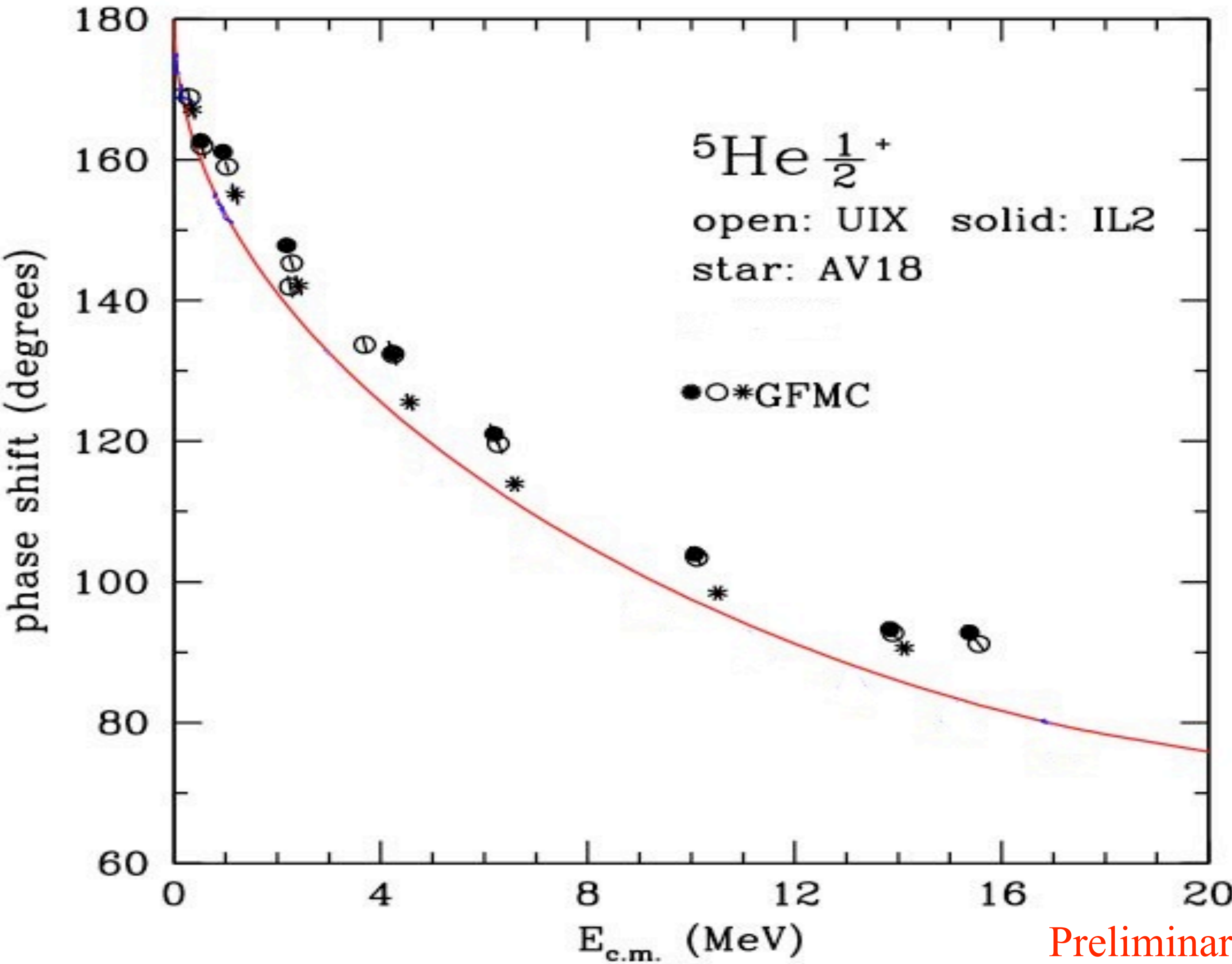


Dependence on matching radius

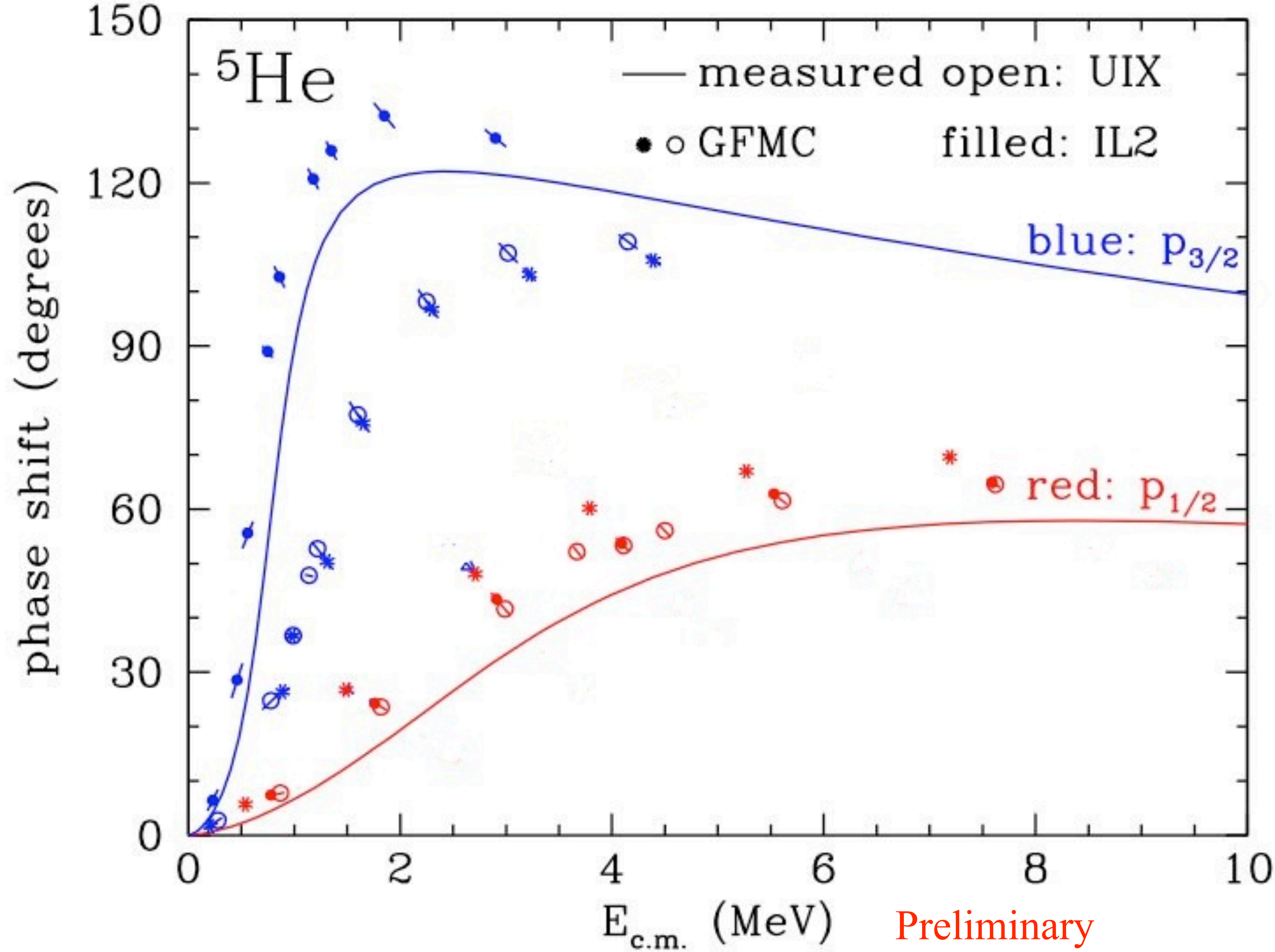


Isospin vs. charge basis





Preliminary

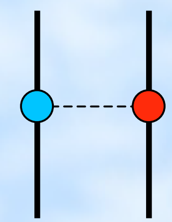


150
120
90
60
30
0
^{5}He
measured open: UIX
GFMC
filled: IL2
blue: $p_{3/2}$
red: $p_{1/2}$
phase shift (degrees)
0
2
4
6
8
10
$E_{c.m.}$ (MeV)
Preliminary

Hadronic PV

Hadronic weak interaction involves PV mixings between s- and p-waves, etc.

DDH model has weak- and strong-couplings;
encodes these mixings

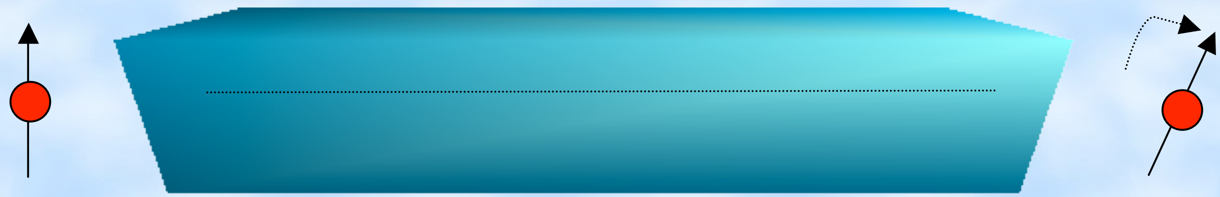


Observables are mixing angles as a function of energy

- p-p longitudinal asymmetry (TRIUMF)
- neutron spin rotation on H and 4He
- np $\Rightarrow$ d γ at LANSCE/SNS
  (measures π NN weak coupling)

Neutron spin rotation

Neutron spin rotation in H, ^{4}He



Phase builds up through coherent forward scattering amplitude.

Hydrogen $d\phi / dz$ (10^{-9}/cm):	Pi-only	DDH	DDH-'best'
AV18	5.21	5.00	7.20
NijmI	5.35	4.85	7.64
CDBonn	5.18	4.55	7.35

Note: sign opposite Born amplitude because of deuteron bound state

Future

- Higher Energy: information on three nucleon interaction
  up to ~20 MeV in alpha-N scattering

- Multiple channel scattering
  In principle clear for 2-body breakup
  Practical considerations: statistics, ...

- Applications:
  BBN reactions
  solar ν reactions
  hadronic PV
  TN burn
  ...