Momentum-Space Treatment of Coulomb Interaction in Three-Nucleon Reactions

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Outline

- Screening and renormalization
- Practical realization
- Reliability tests
- Results
- Summary

Previous works: Alt *et al.*

- Quasiparticle approach

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- Quasiparticle approach
- Approximations of hadronic interaction:
  ▷ low partial waves
  ▷ rank-1 separable NN potentials
- Approximations in the treatment of screened Coulomb

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- Unscreened limit: screening-independent results

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$$z_R^{-\frac{1}{2}}\psi_{RL}(r) \approx \psi_{CL}(r)$$

$$z_R = e^{-2i(\sigma_L - \eta_{LR})}$$

Screening and renormalization

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Two-particle screened Coulomb transition matrix

$$T_R = w_R + w_R G_0 T_R$$

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Infinite R limit for on-shell T_R and wave function

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Screening and renormalization

Two-particle screened Coulomb transition matrix

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Infinite R limit for on-shell T_R and wave function

$$T_R z_R^{-1} \xrightarrow[R\to\infty]{} T_C$$

$$(1 + G_0 T_R)|\mathbf{p}\rangle z_R^{-\frac{1}{2}} \xrightarrow[R\to\infty]{} |\psi_C^{(+)}(\mathbf{p})\rangle$$

exists after renormalization with diverging phase factors

$$z_R \xrightarrow[R\to\infty]{} \exp(-2i(\sigma_L - \eta_{LR}))$$

$$\xrightarrow[R\to\infty]{} \exp(-2i\kappa[\ln(2pR) - C/n])$$

Strategy for more complicated systems

- Isolate diverging screened Coulomb contributions in form of on-shell transition matrix and wave function

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- Apply renormalization to obtain unsreened Coulomb limit

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pp transition matrix

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short-range part: finite R

Proton-deuteron scattering

AGS equations

$$U_{\beta\alpha}^{(R)} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\sigma} \bar{\delta}_{\beta\sigma} T_{\sigma}^{(R)} G_0 U_{\sigma\alpha}^{(R)}$$

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long-range part

$$T_{\alpha R}^{pd} = W_{\alpha R}^{pd} + W_{\alpha R}^{pd} G_{\alpha}^{(R)} T_{\alpha R}^{pd}$$

Proton-deuteron scattering

Split into long-range part

$$T^{pd}_{\alpha R} = W^{pd}_{\alpha R} + W^{pd}_{\alpha R} G^{(R)}_{\alpha} T^{pd}_{\alpha R}$$

and Coulomb-distorted short-range part

$$U^{(R)}_{\beta\alpha} = \delta_{\beta\alpha} T^{pd}_{\alpha R} + [1 + T^{pd}_{\beta R} G^{(R)}_{\beta}] \tilde{U}^{(R)}_{\beta\alpha} [1 + G^{(R)}_{\alpha} T^{pd}_{\alpha R}]$$

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Renormalized amplitudes:

$$U_{\beta\alpha} = \delta_{\beta\alpha} T^{pd}_{\alpha C} + \lim_{R\to\infty} Z_R^{-\frac{1}{2}} [U^{(R)}_{\beta\alpha} - \delta_{\beta\alpha} T^{pd}_{\alpha R}] Z_R^{-\frac{1}{2}}$$

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short-range part: finite R

Electromagnetic reactions

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e.m. current matrix elements with screened Coulomb:

$$\langle \psi_{pd}^{(-)(R)} | j^{\mu} | B^{(R)} \rangle$$

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short range: finite R

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$$U^{(R)} = PG_0^{-1} + PT^{(R)}G_0U^{(R)}$$

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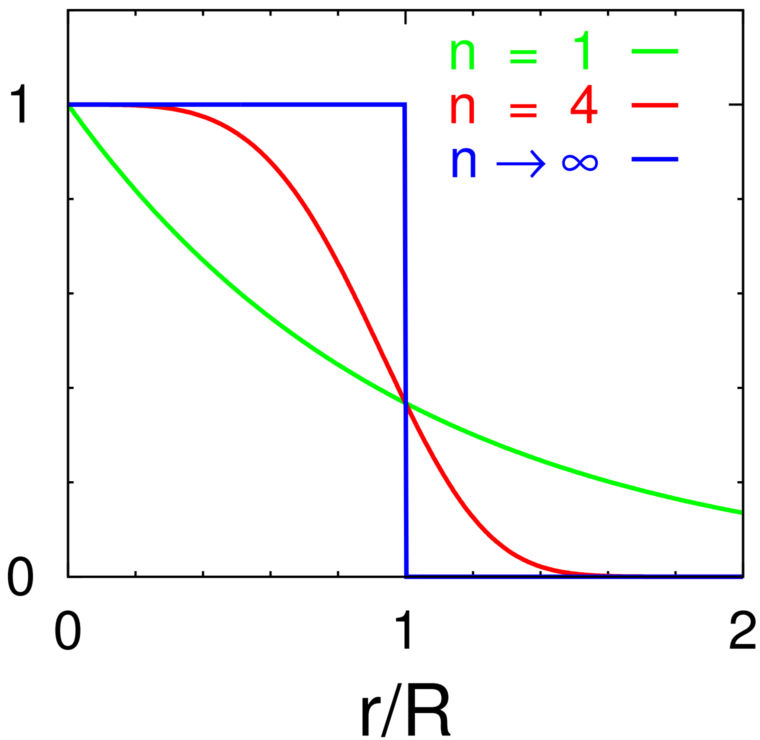
Problem: slow partial-wave convergence

Solution: special choice of screening

perturbation theory for high partial waves

Screened Coulomb potential

$$\frac{w_R(r)}{w(r)} = e^{-\left(\frac{r}{R}\right)^{\mathrm{n}}}$$



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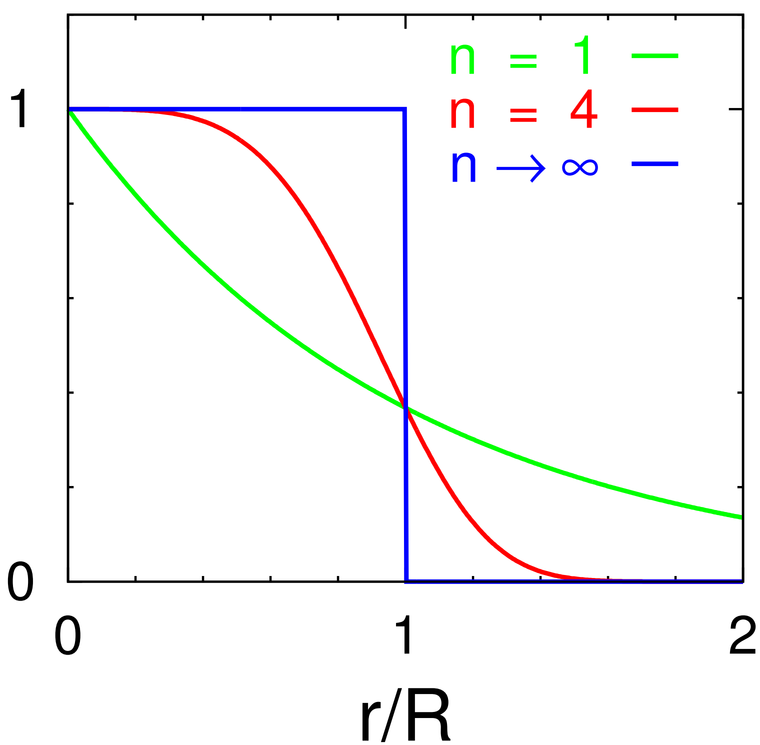

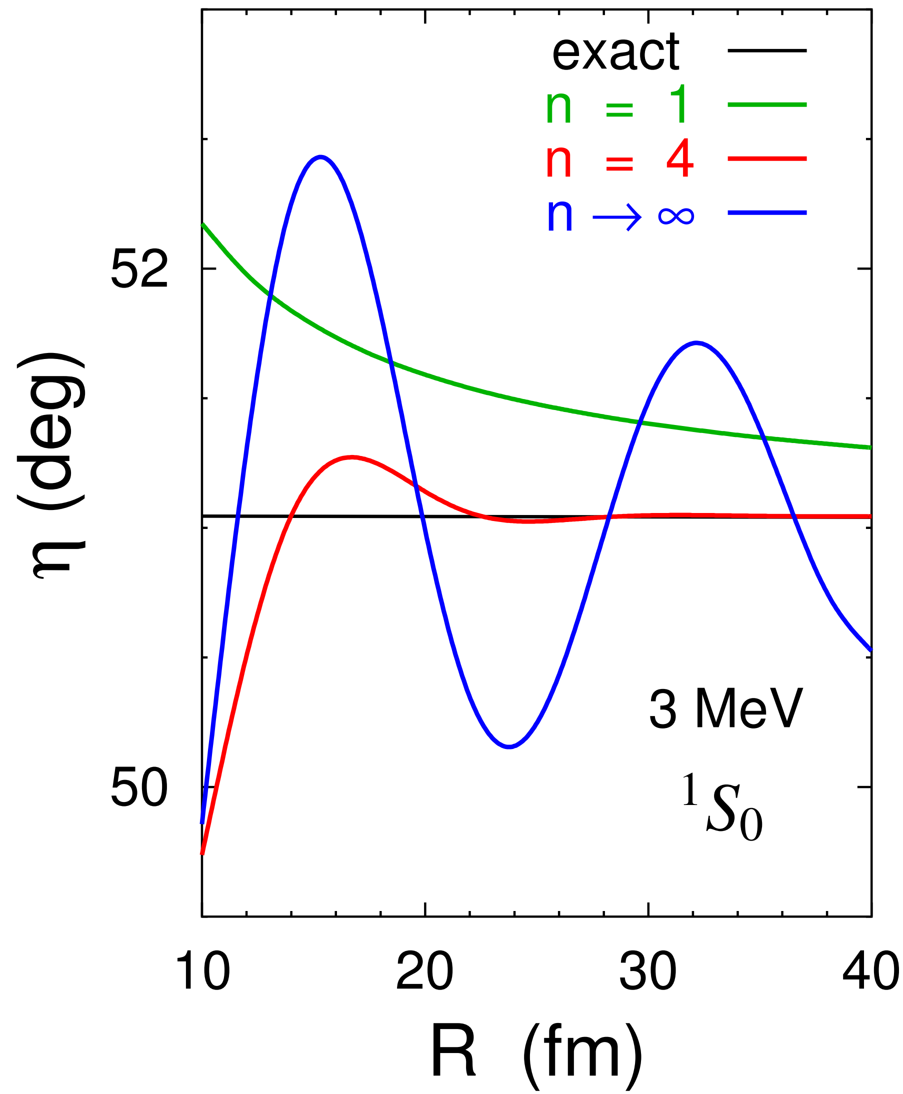

optimal choice: $3 \leq \mathrm{n} \leq 6$

Perturbation theory for high partial waves

$$T \longrightarrow T + \Delta T$$

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1st order in ΔT, all orders in T:

$$\begin{aligned} U &= PG_0^{-1} + PTG_0U \\ \Delta U &= P\Delta TG_0U + PTG_0\Delta U \end{aligned}$$

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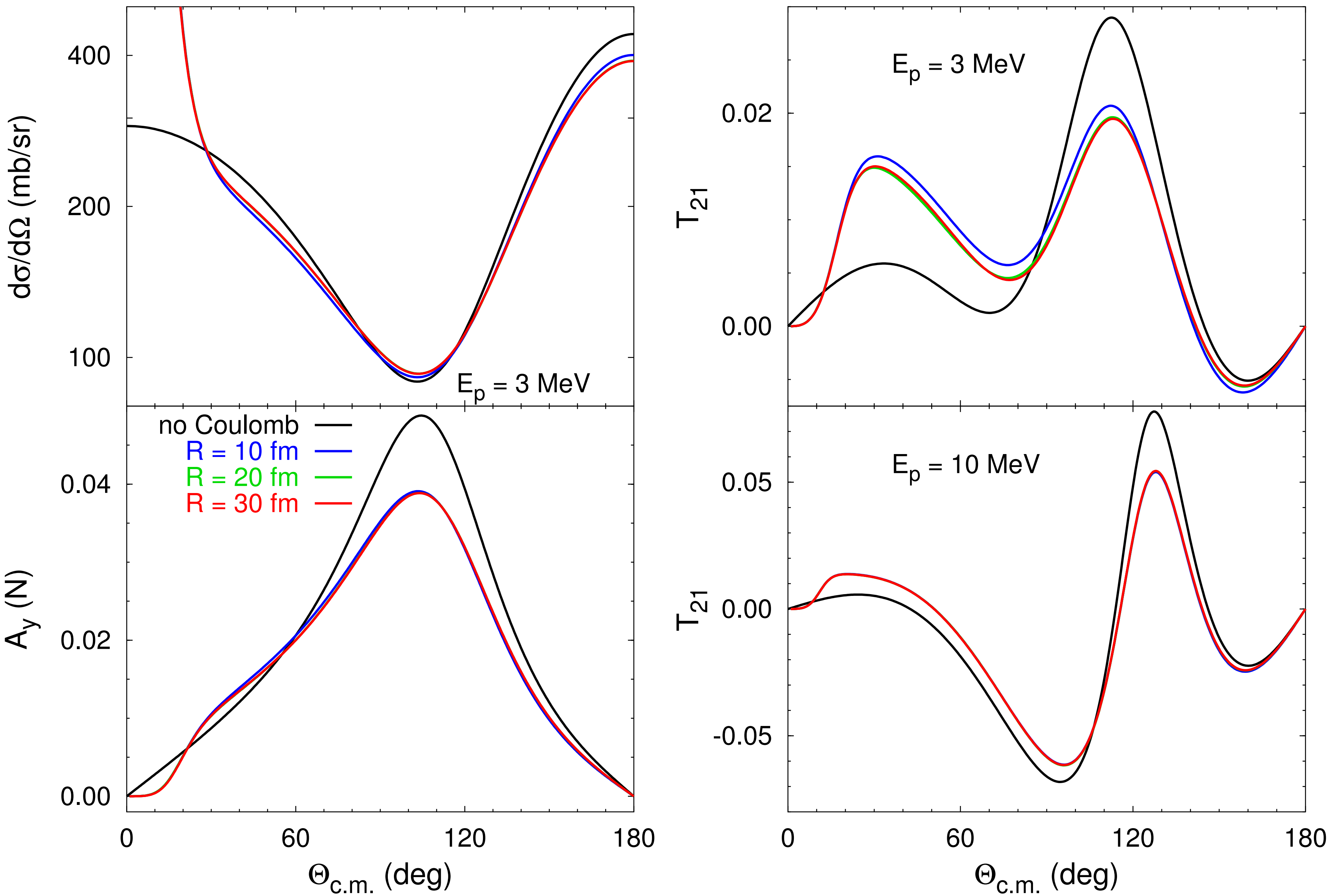
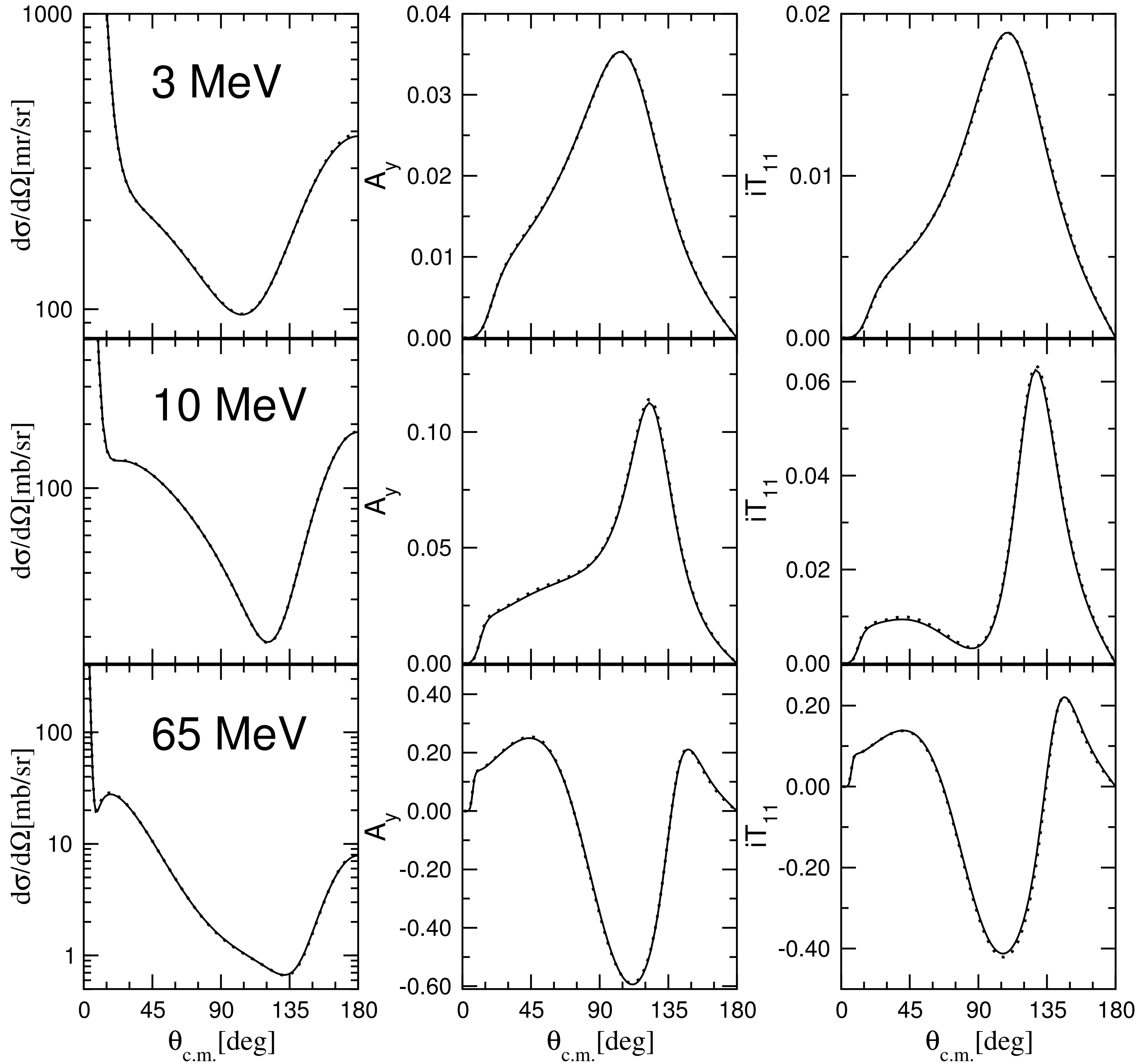
Practical calculations: $L_T \approx L_{\Delta T}/2$

Screening and renormalization:
Convergence test

Convergence with R: pd elastic scattering



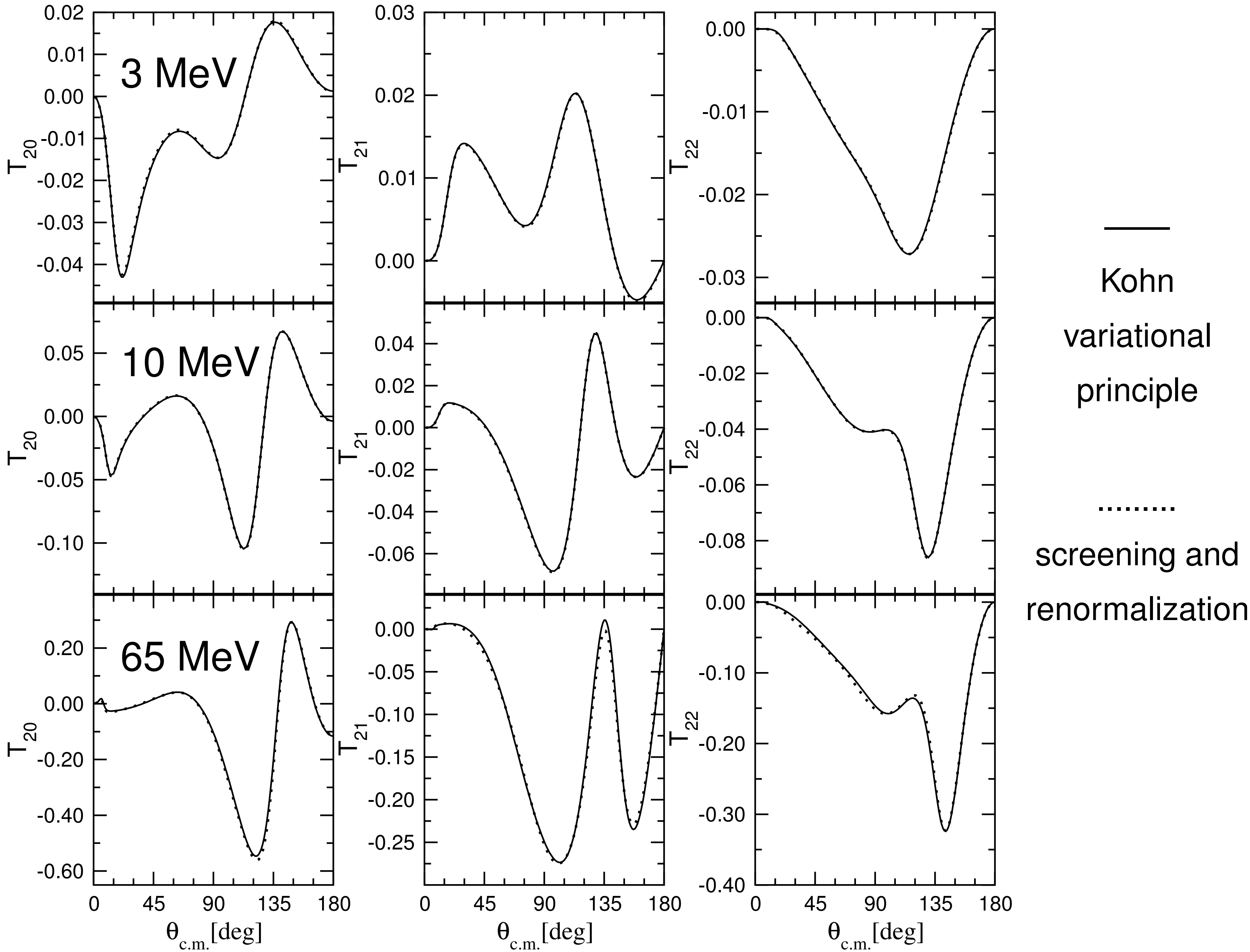
pd elastic scattering:
Comparison with configuration-space results



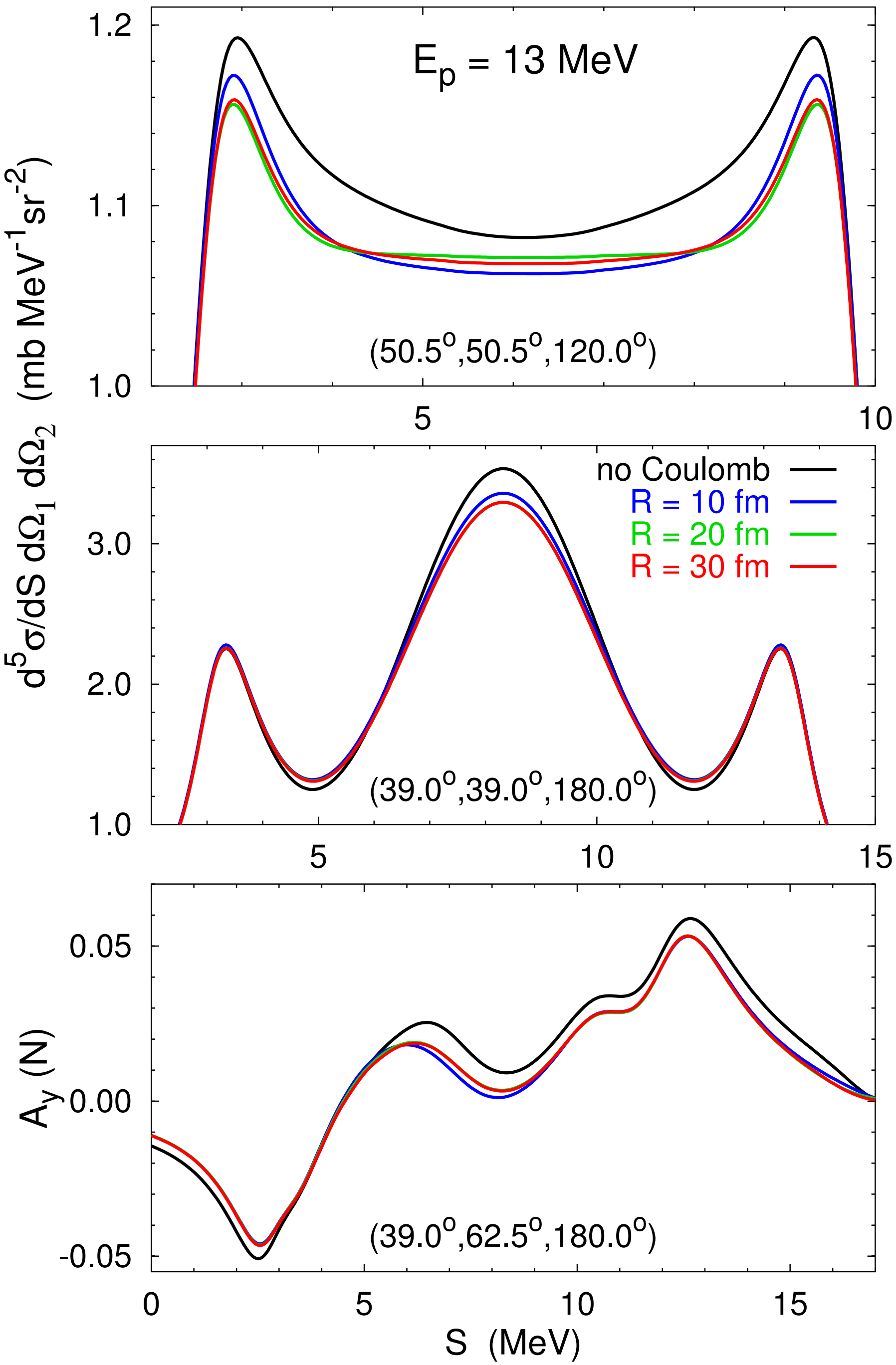
—— Kohn variational principle

……… screening and renormalization

pd elastic scattering: Comparison with configuration-space results



Convergence with *R*: *pd* breakup



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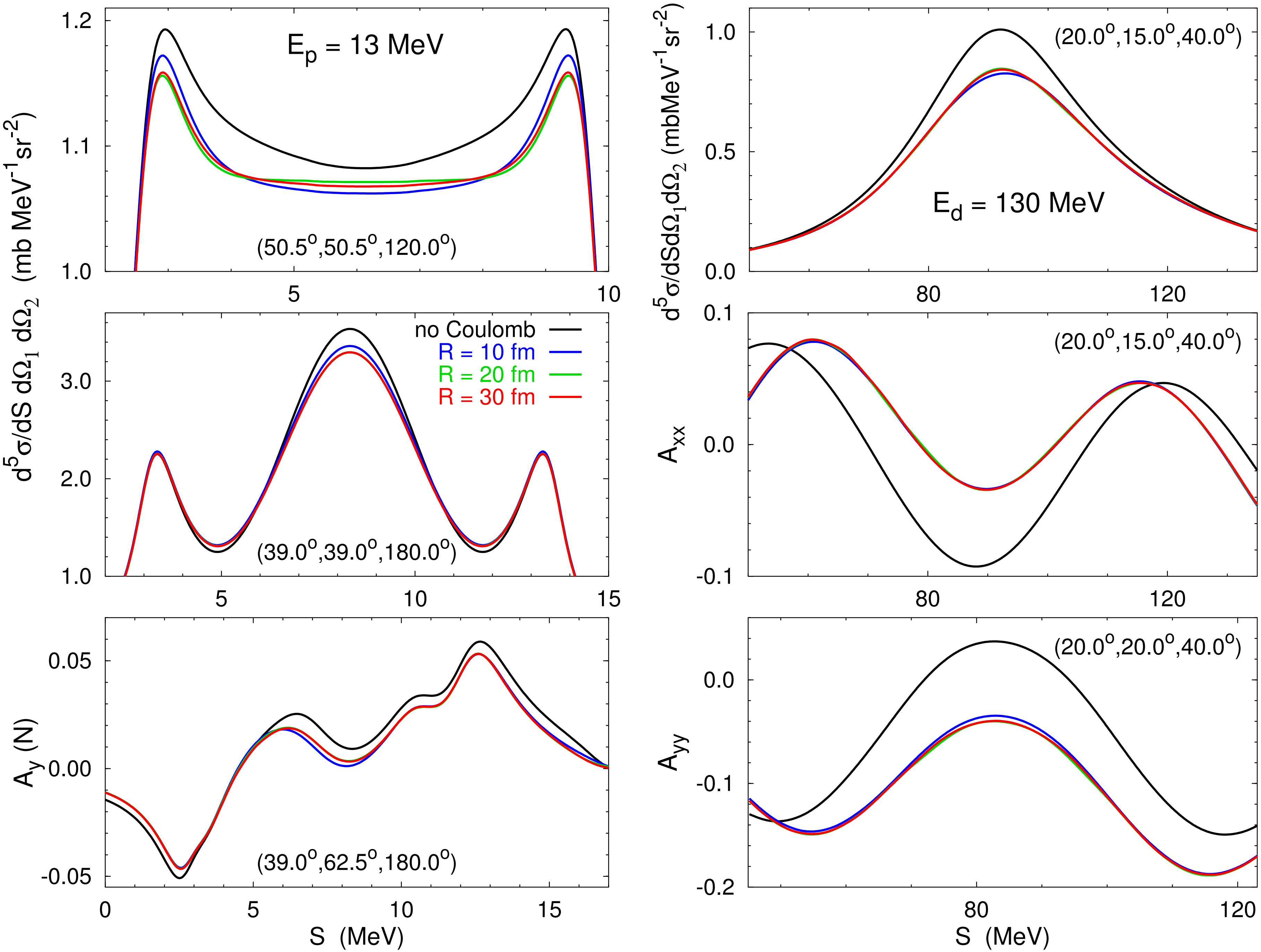
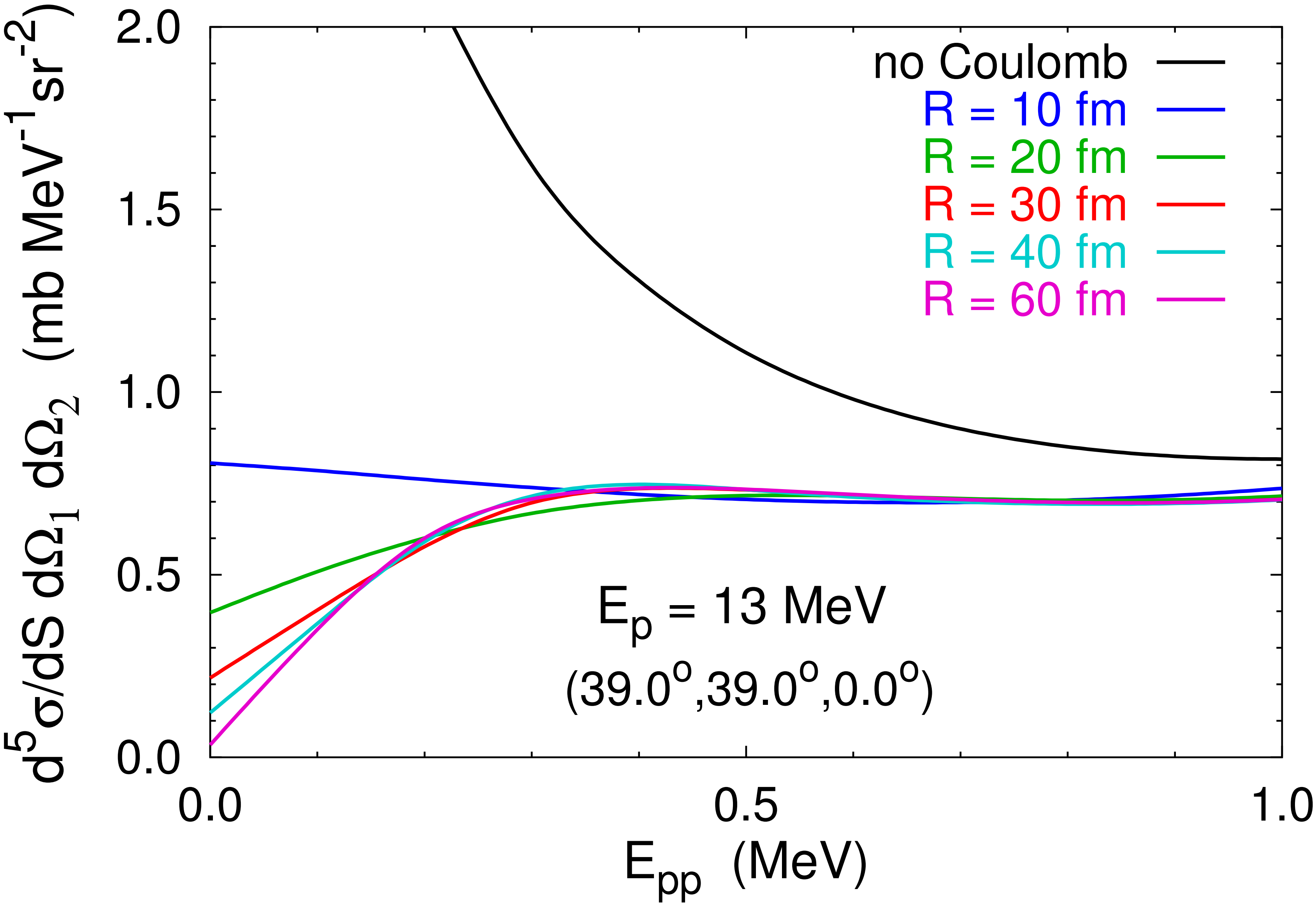
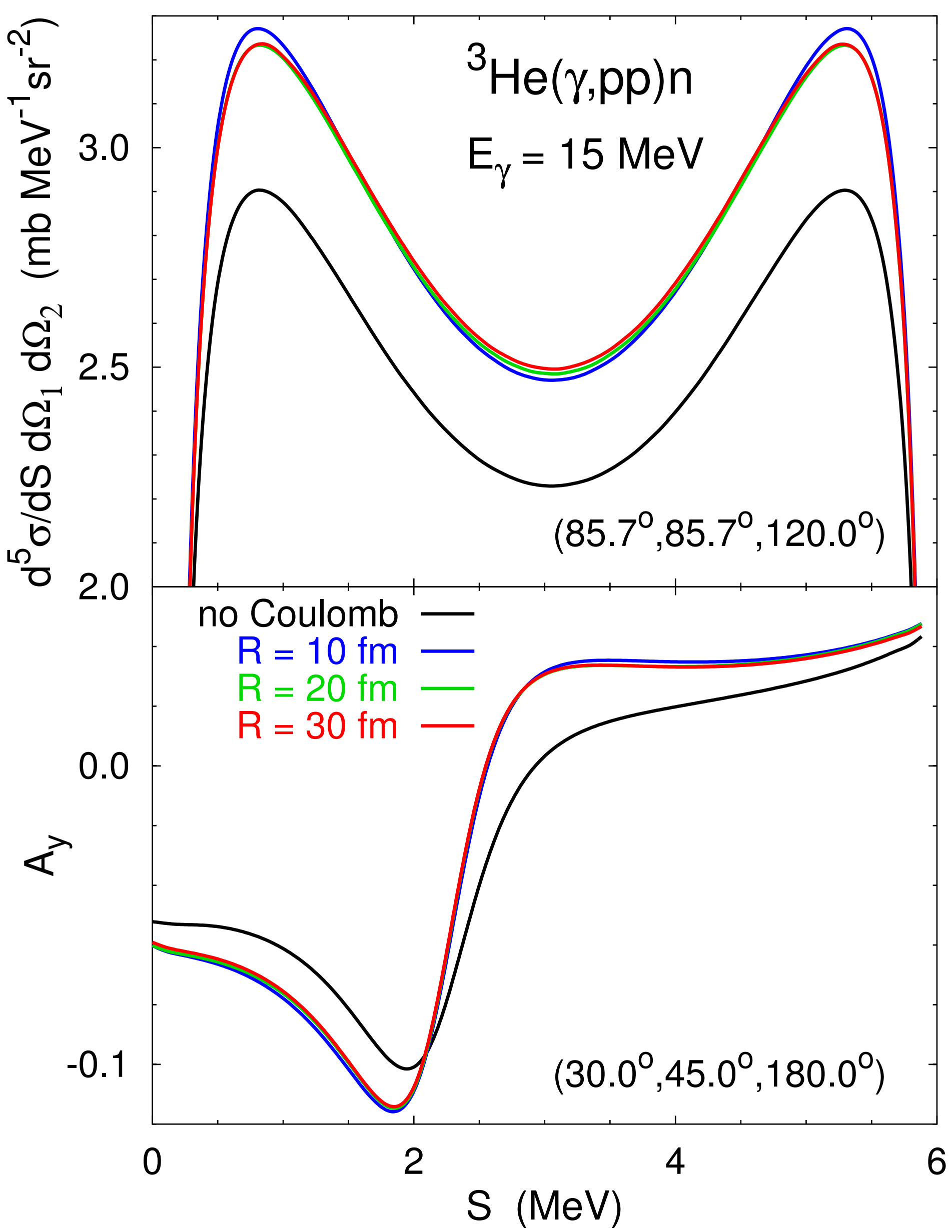


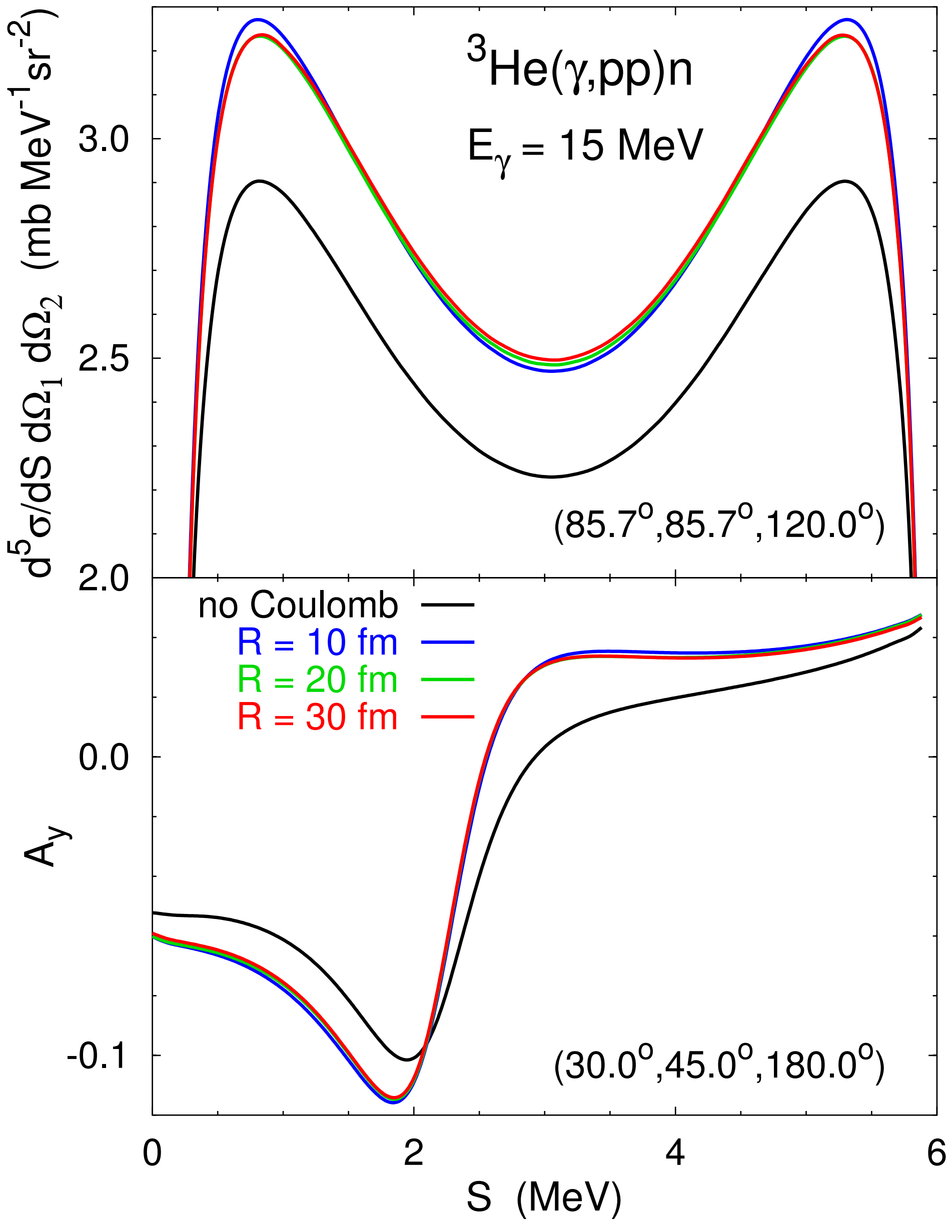
Convergence with *R*: *pd* breakup in *pp*-FSI kinematics

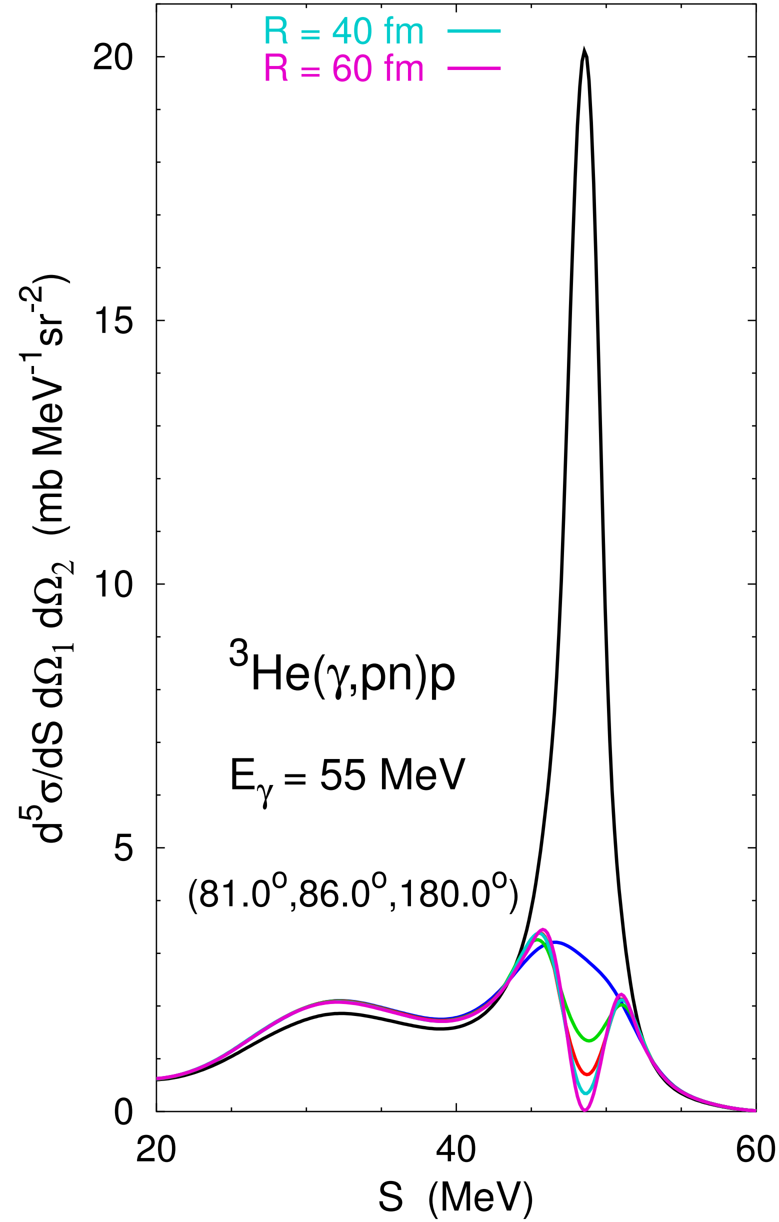


Convergence with *R*: 3*N* photodisintegration of ^{3}He



Convergence with R: $3N$ photodisintegration of ^{3}He





Results

CD Bonn + Δ

CD Bonn + Δ + Coulomb

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} Coulomb effect

} Δ effect

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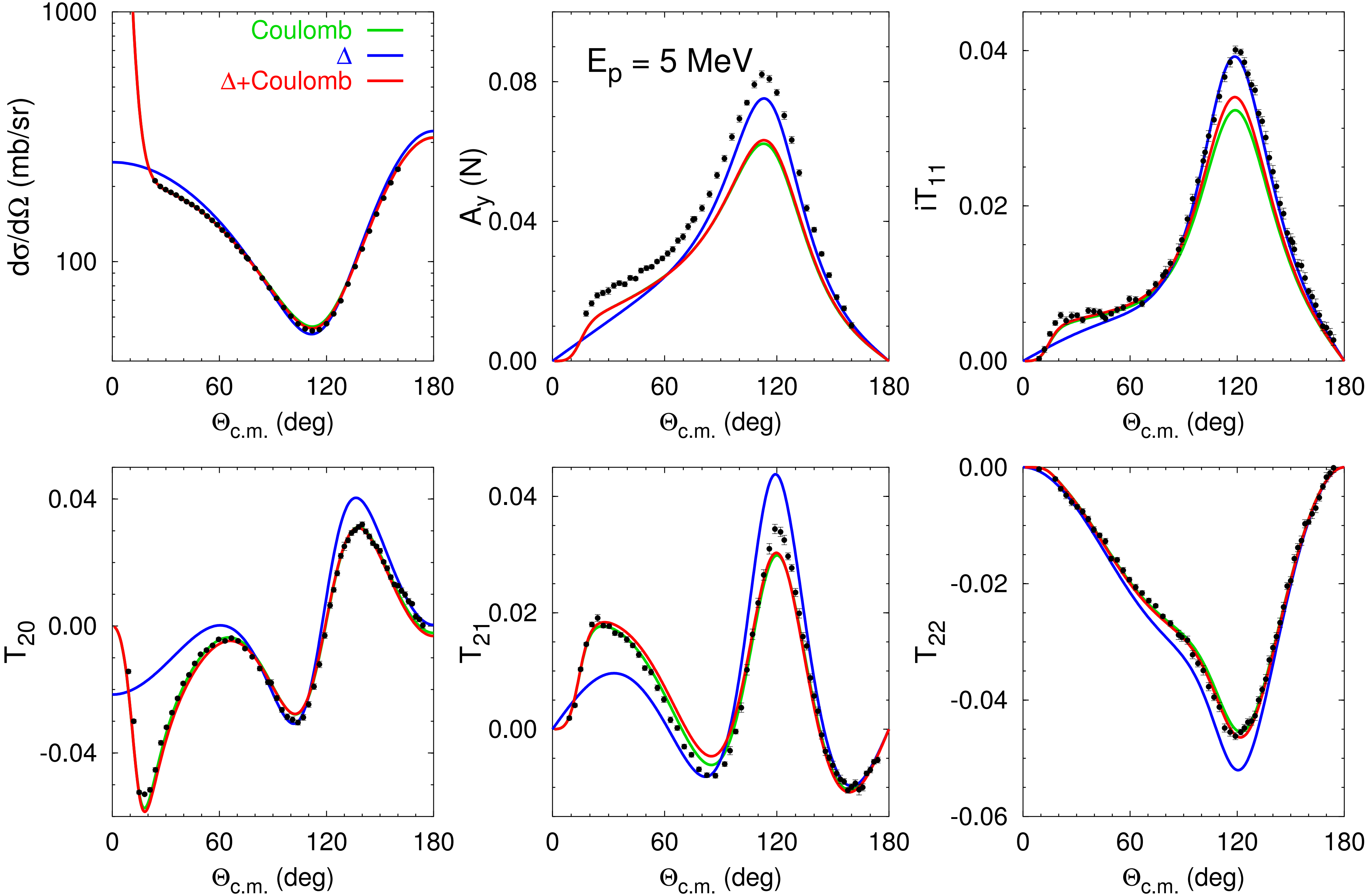
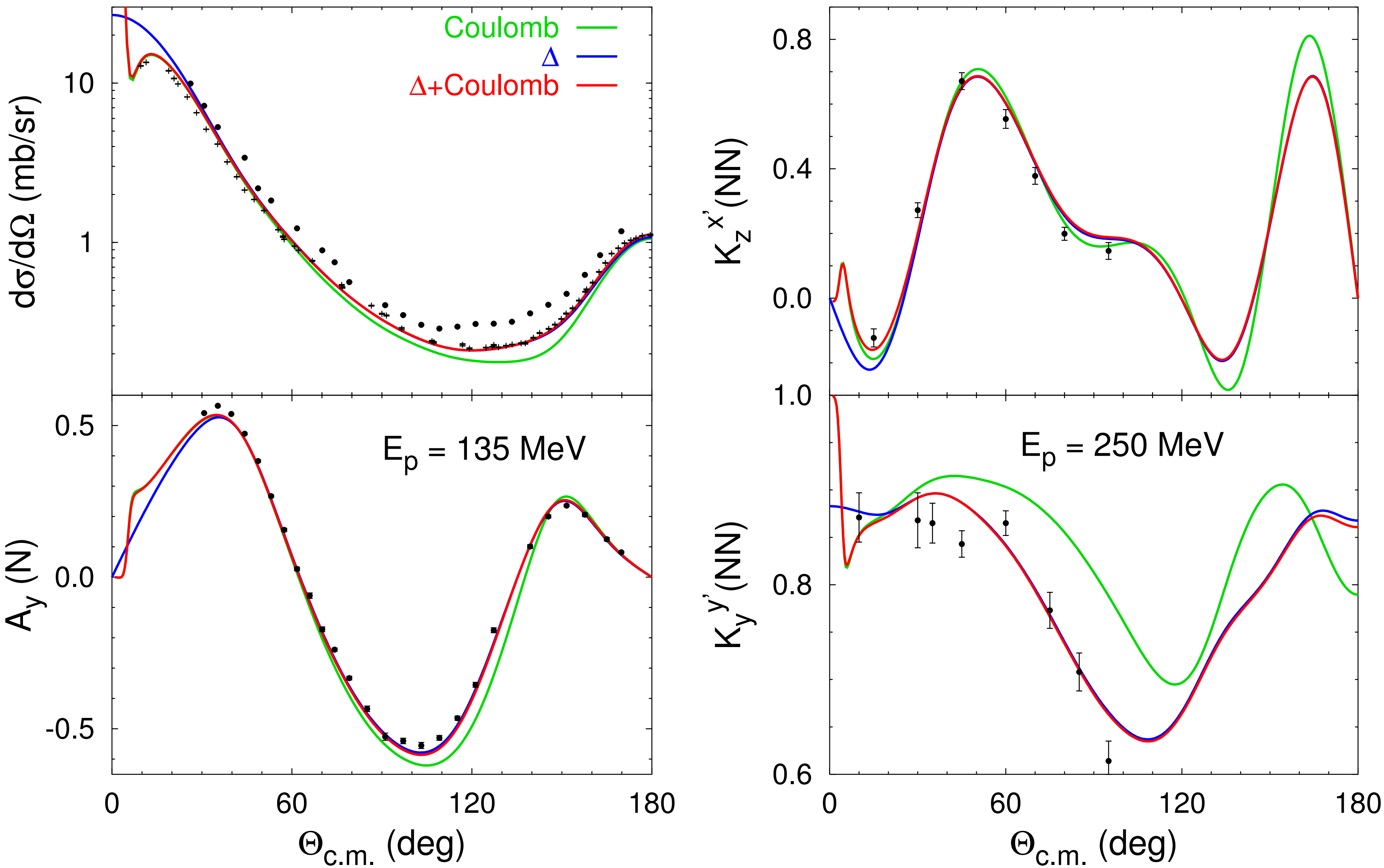
Δ isobar:

- effective 3NF
  - ▷ Fujita-Miyazawa, Illinois, ...
  - ▷ π, ρ, ω, σ exchanges
- effective 2N and 3N currents

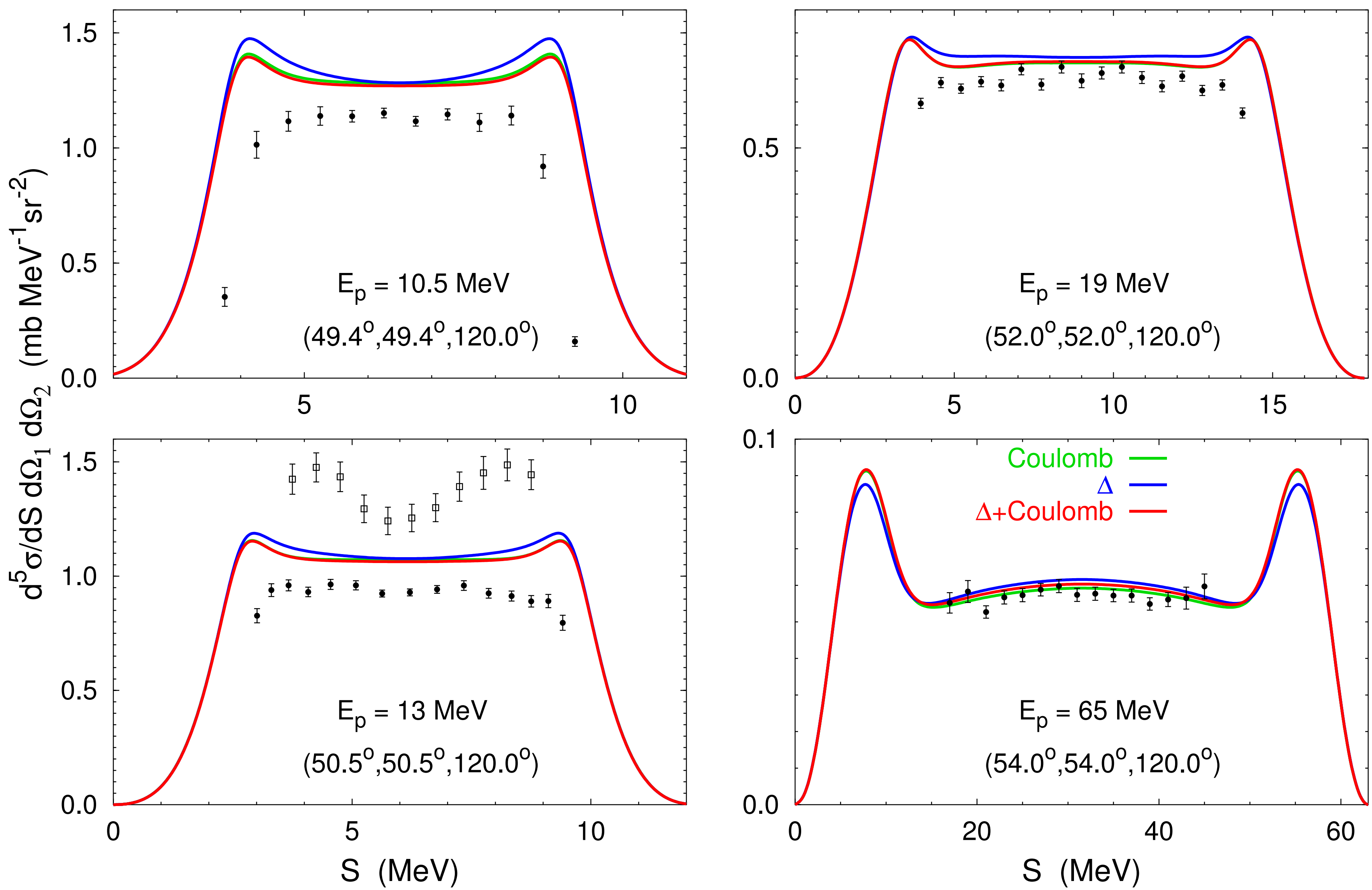
pd elastic scattering at low energies



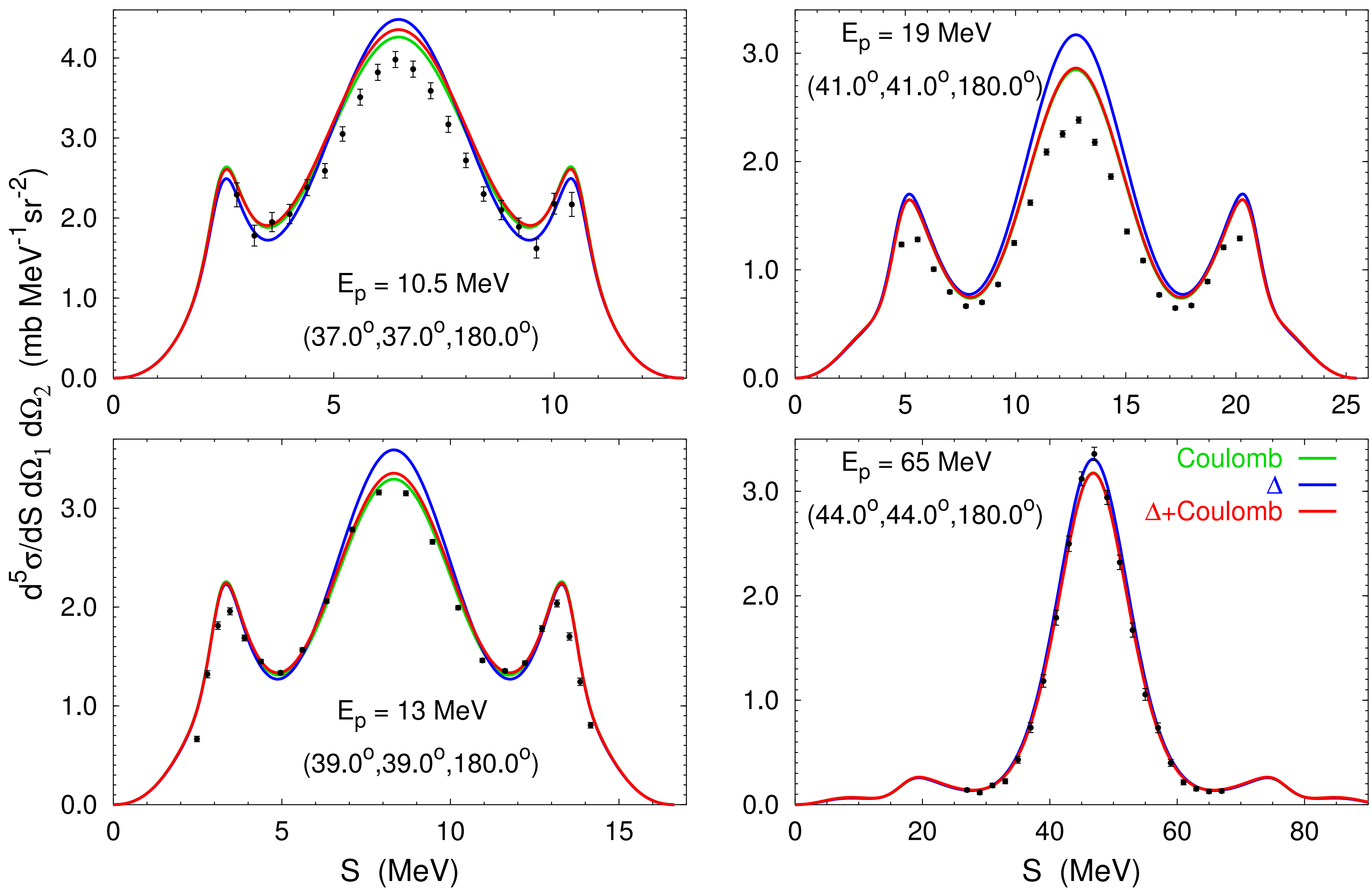
pd elastic scattering at higher energies



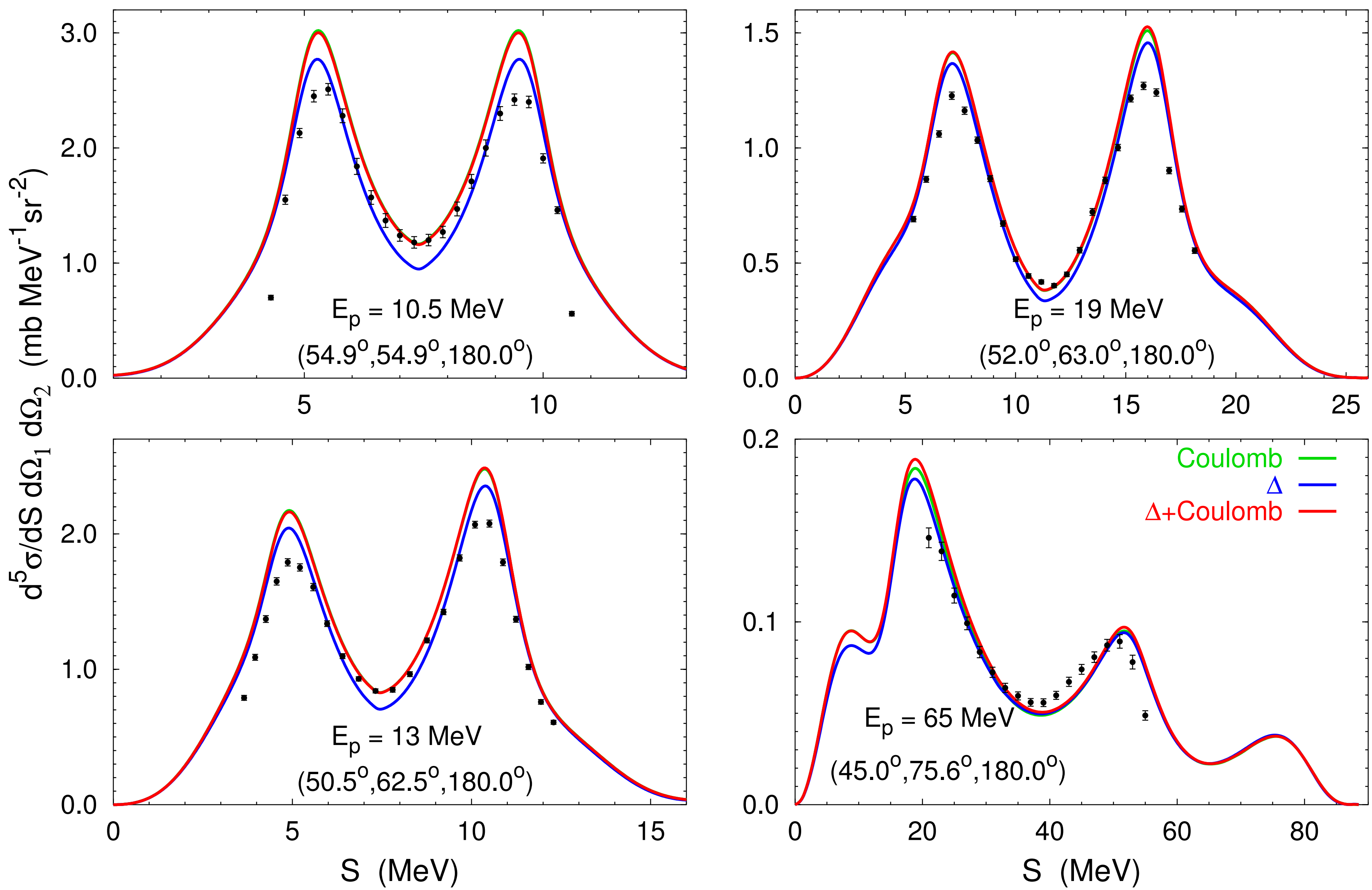
pd breakup: space-star configurations



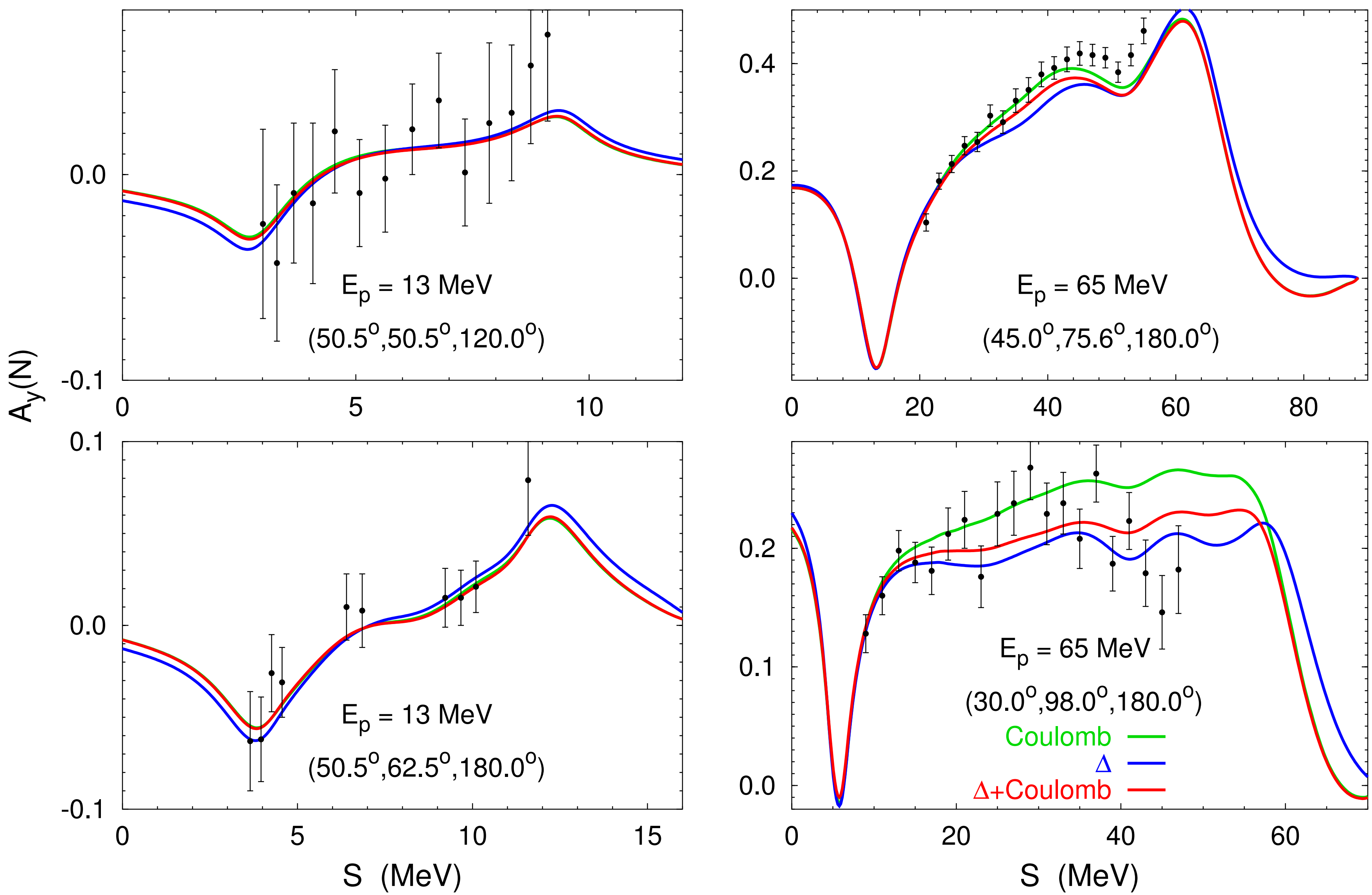
pd breakup: QFS configurations



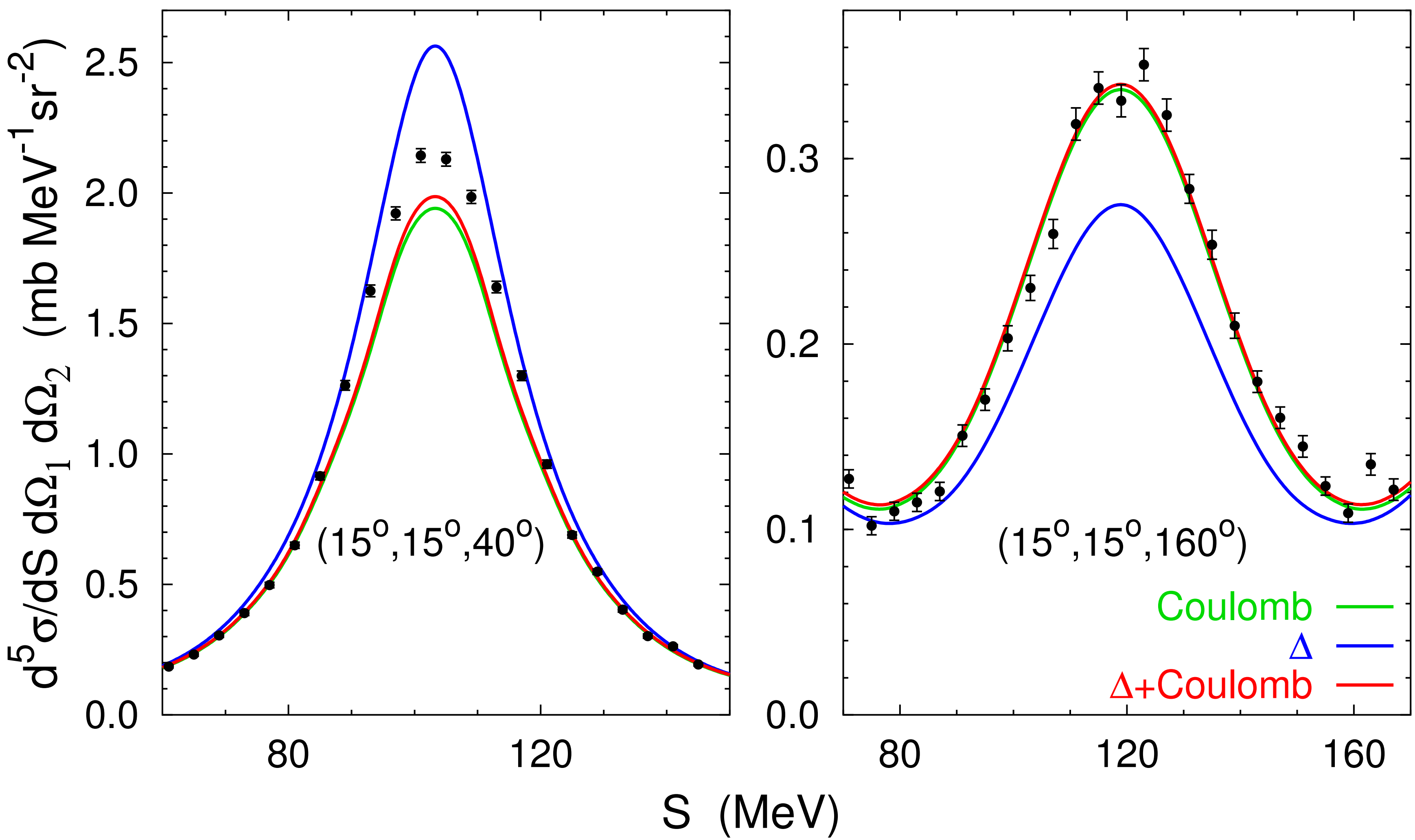
pd breakup: collinear configurations



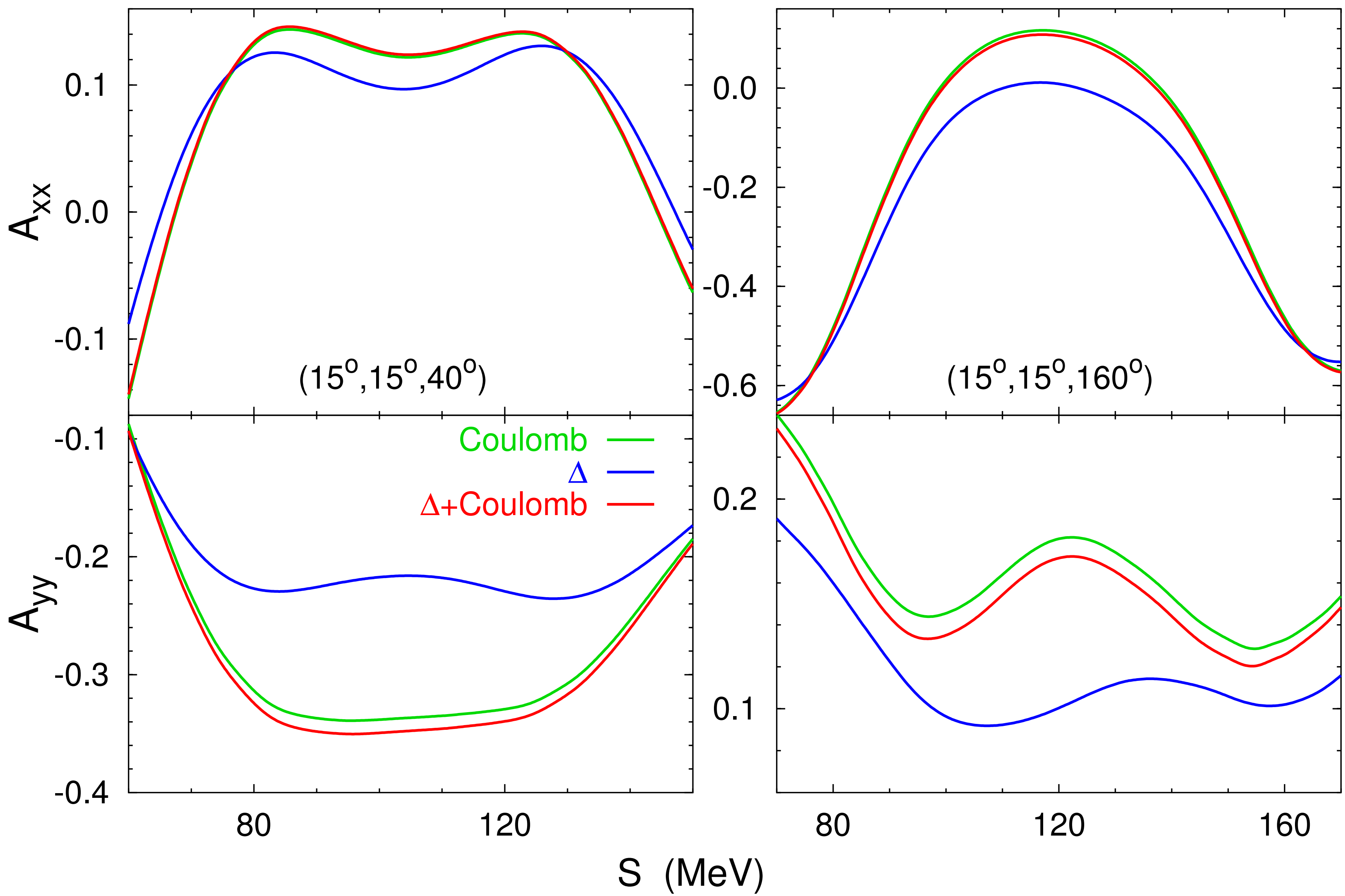
pd breakup: nucleon analyzing power



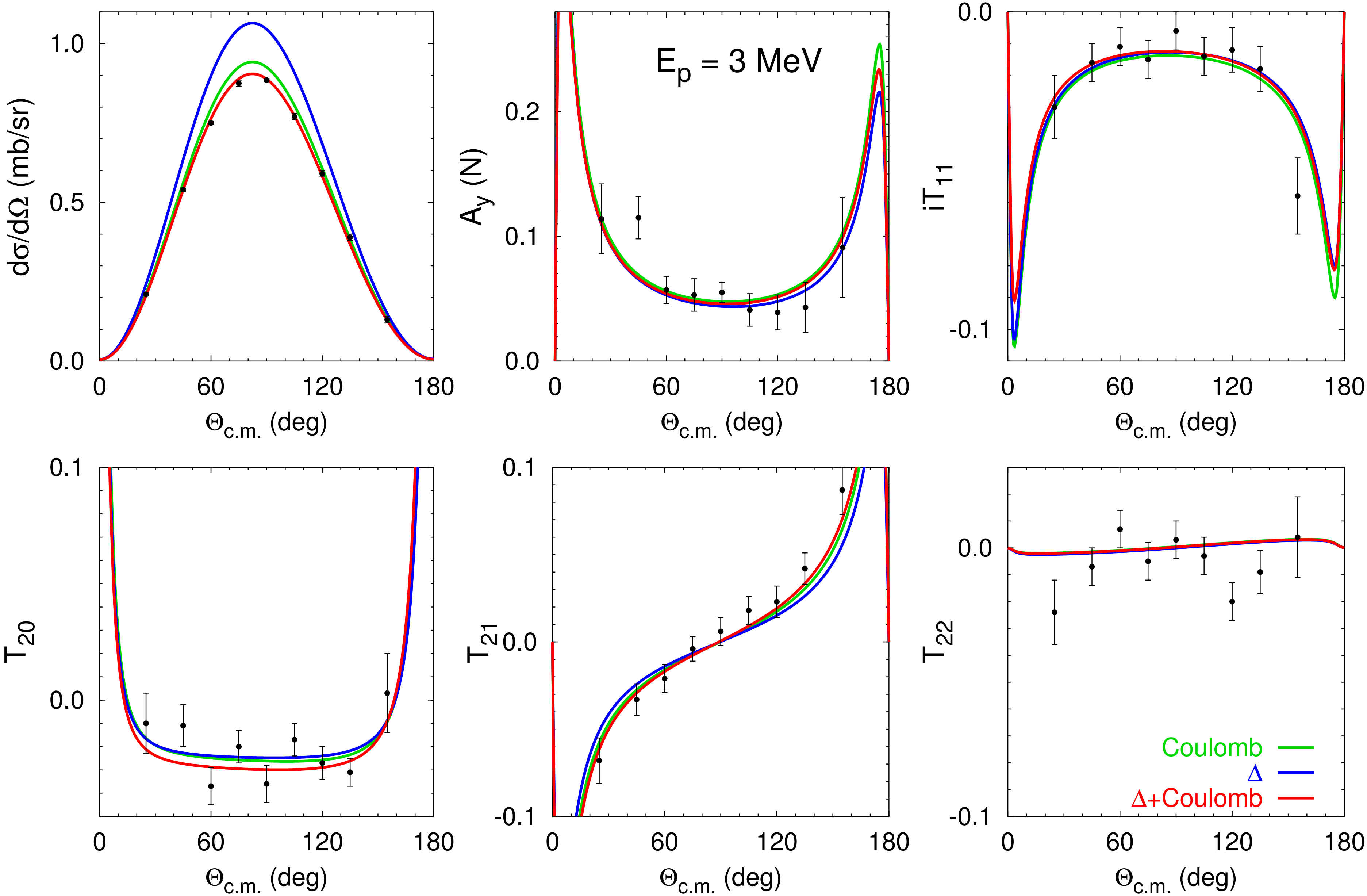
dp breakup at $E_d = 130$ MeV



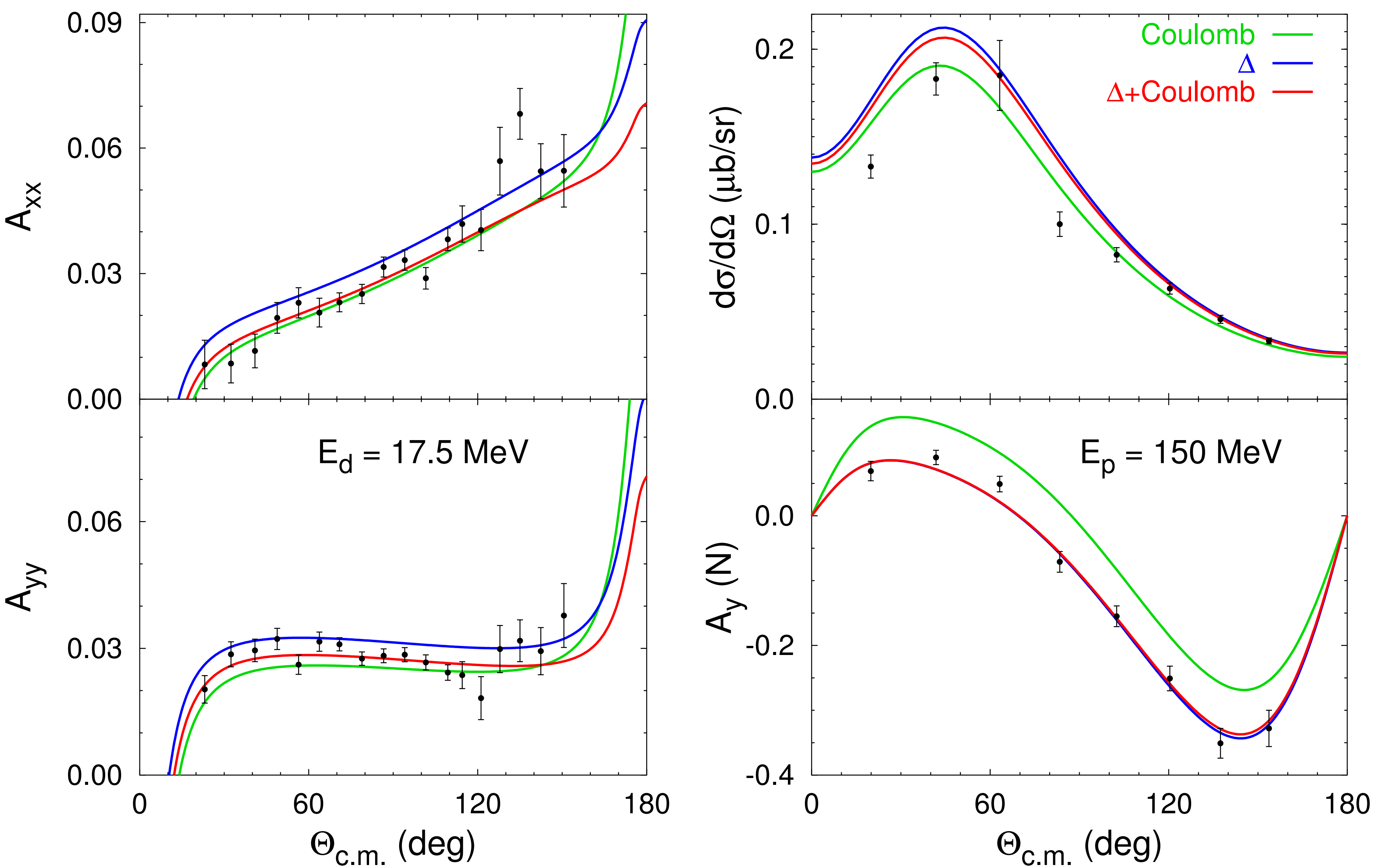
dp breakup: deuteron analyzing powers



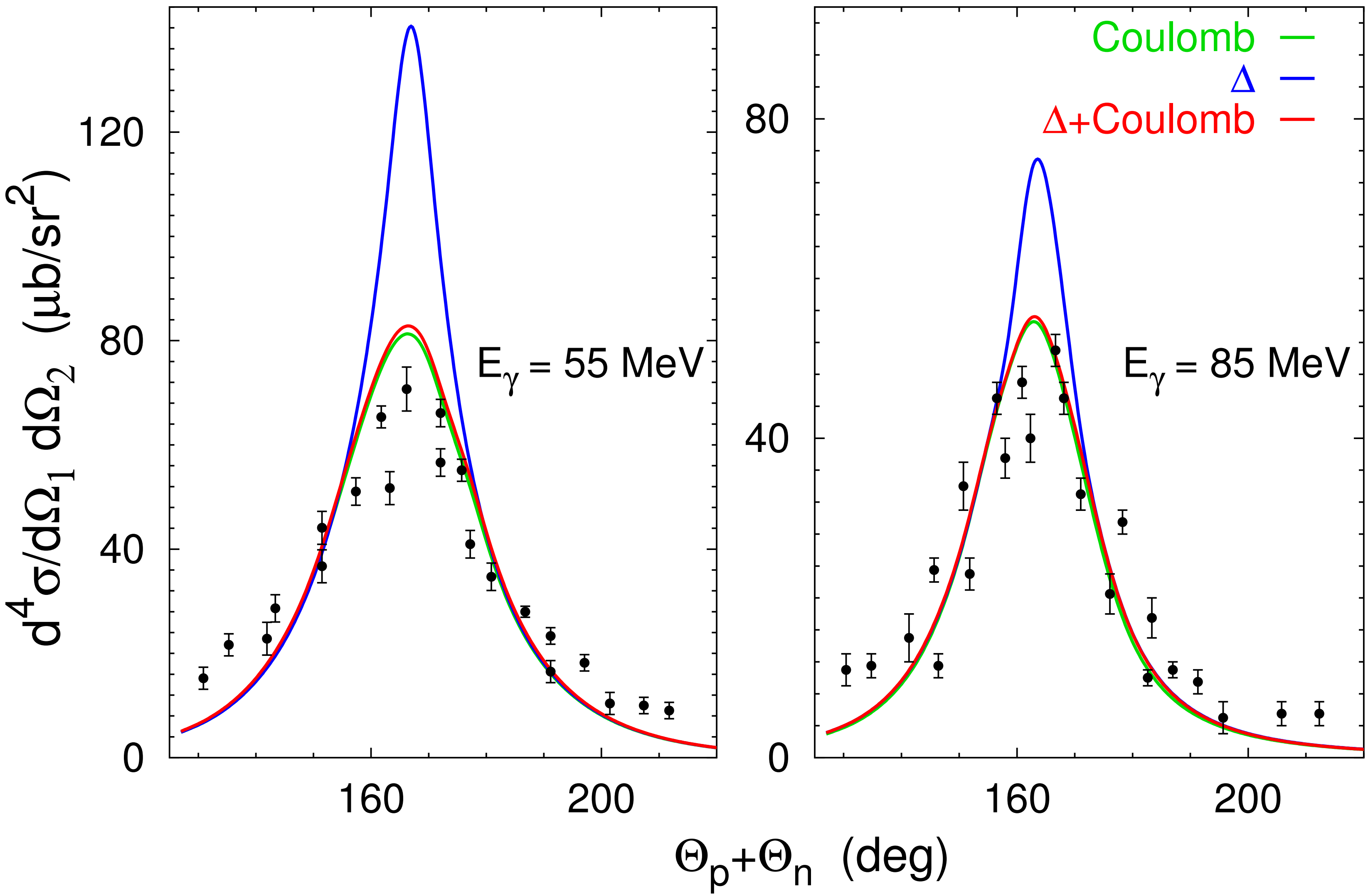
pd radiative capture at low energies



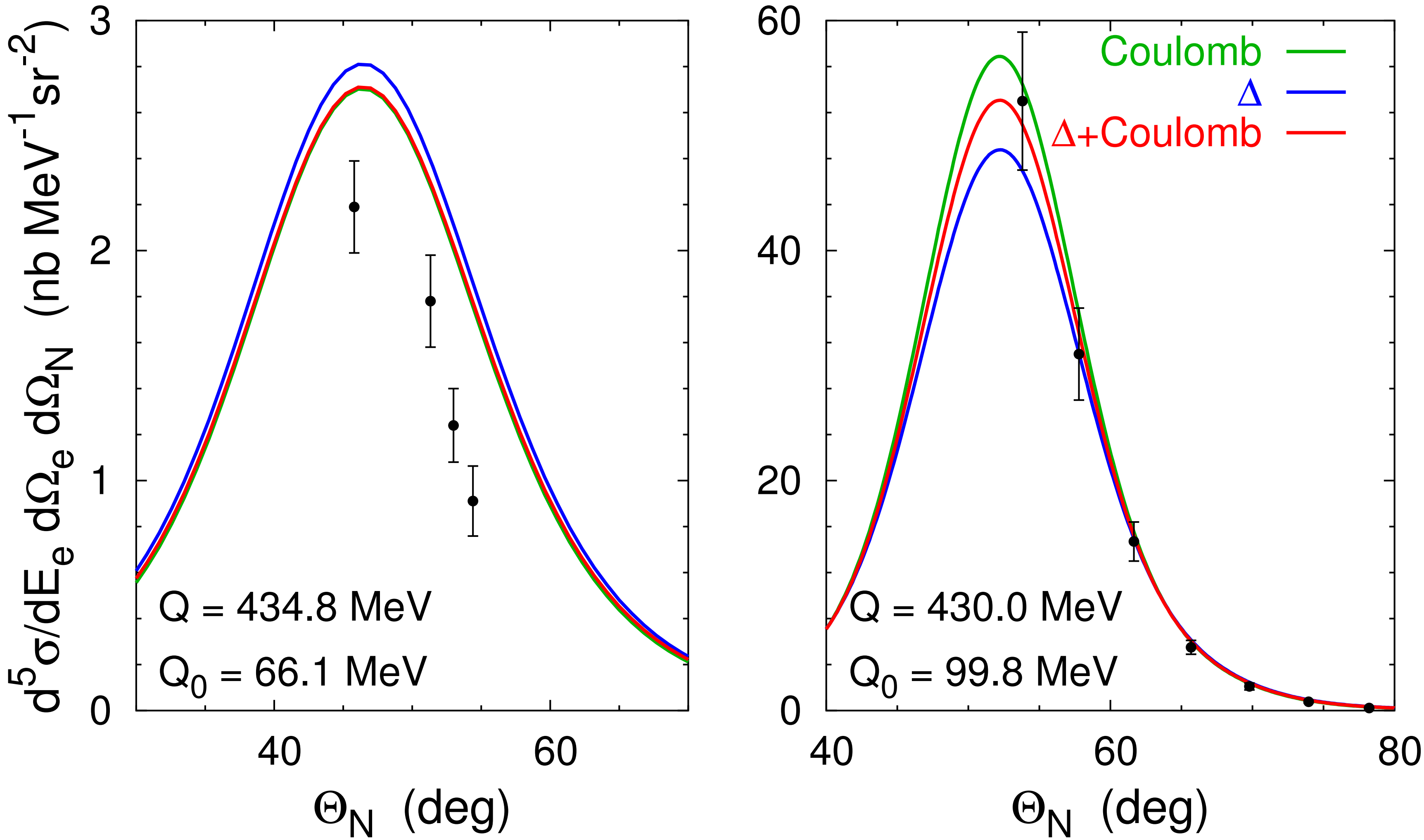
pd radiative capture



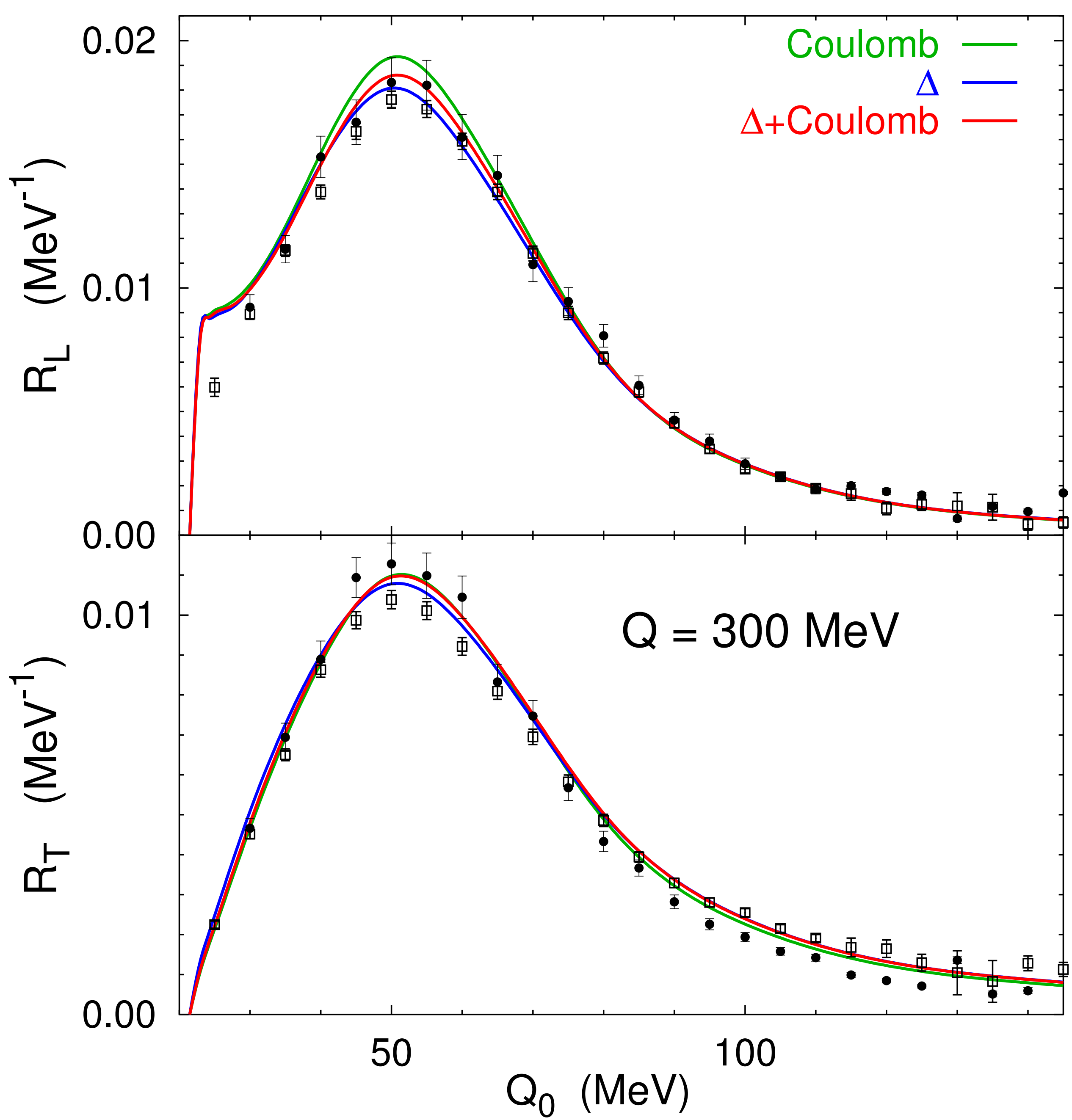
Three-nucleon photodisintegration $^3He(\gamma, pn)p$



Two-body electrodisintegration of ^{3}He



^{3}He(e,e'): inclusive response functions



Comparison with previous works

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new type of screened Coulomb	Yukawa screening

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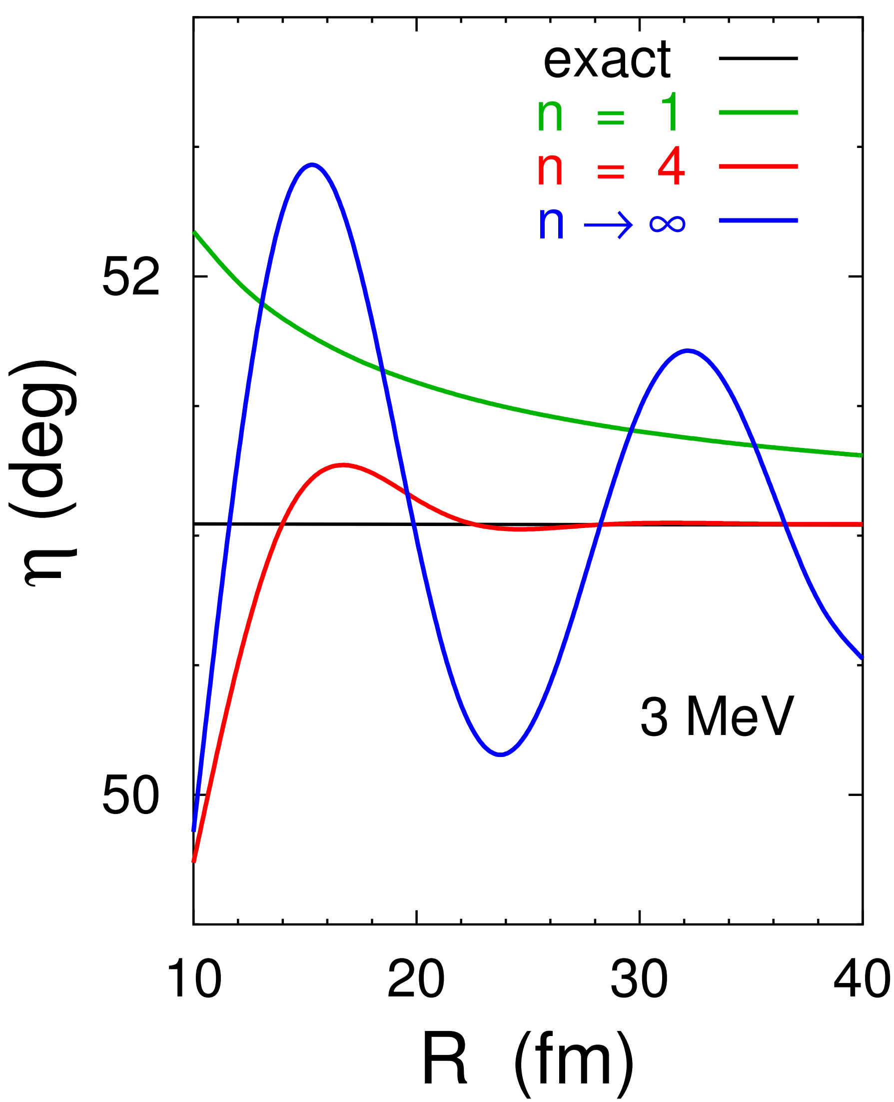
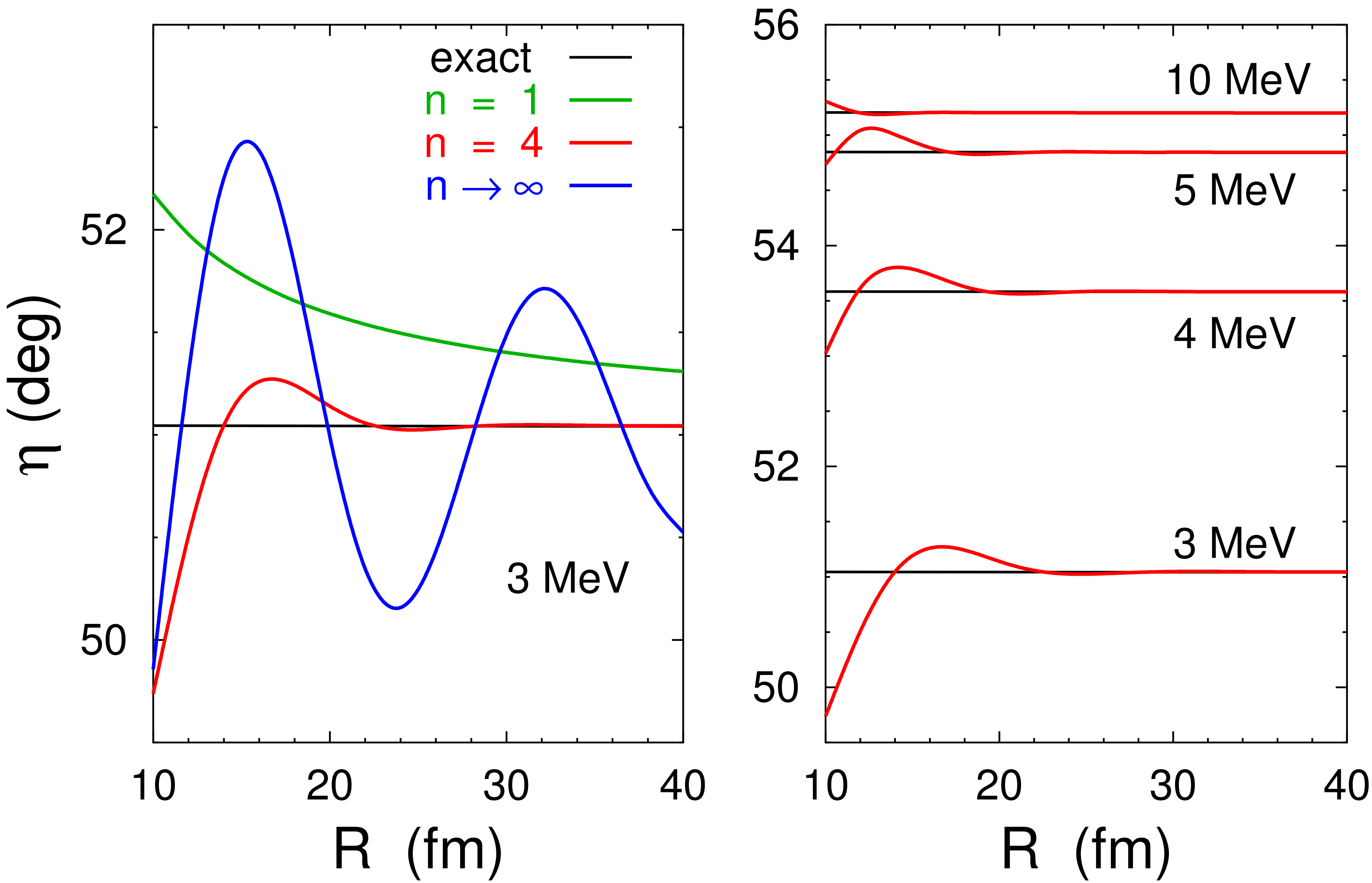
  complicated dependence on energy and momentum transfer

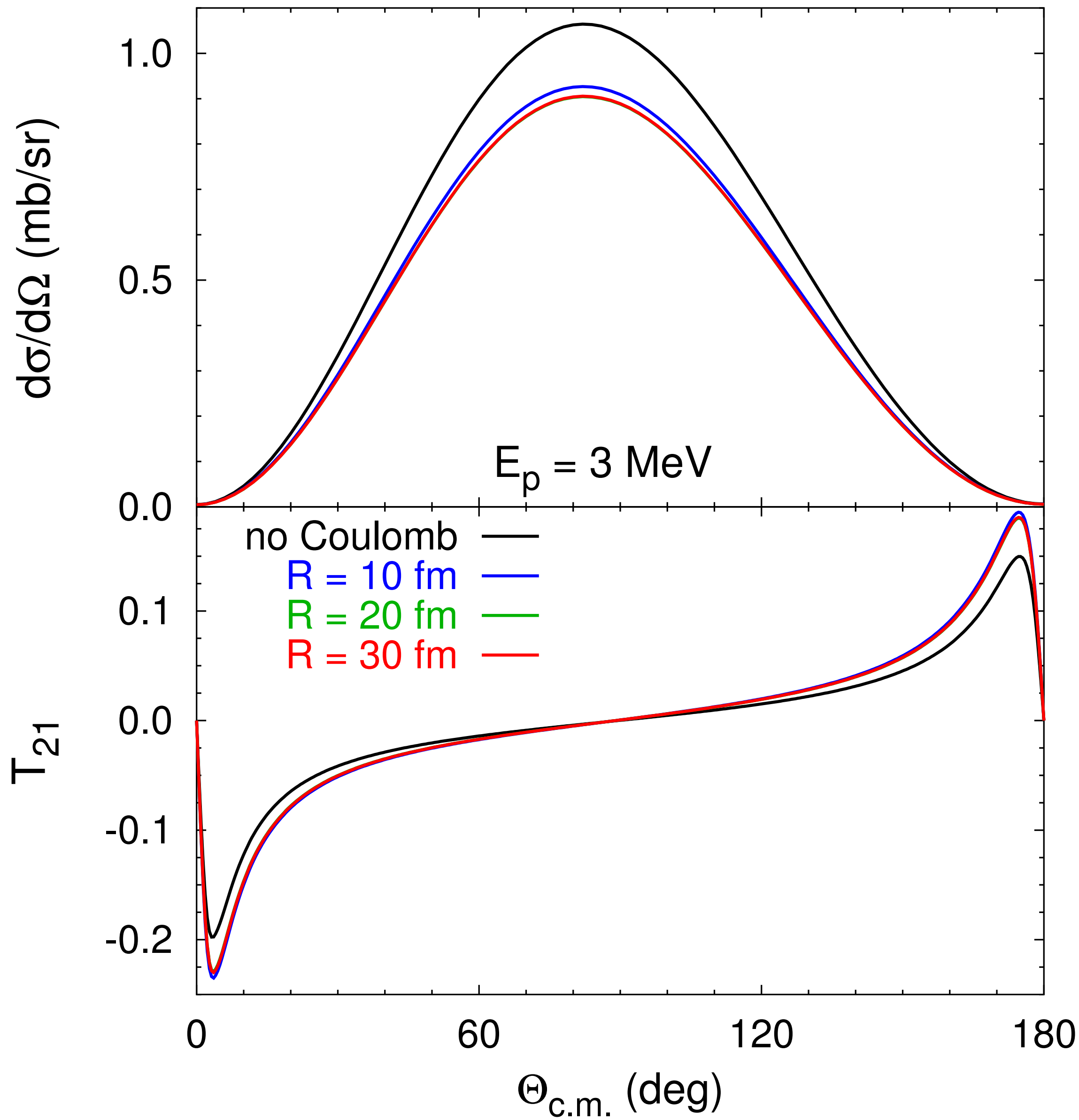
pp scattering: $^1S_0$ phase shift



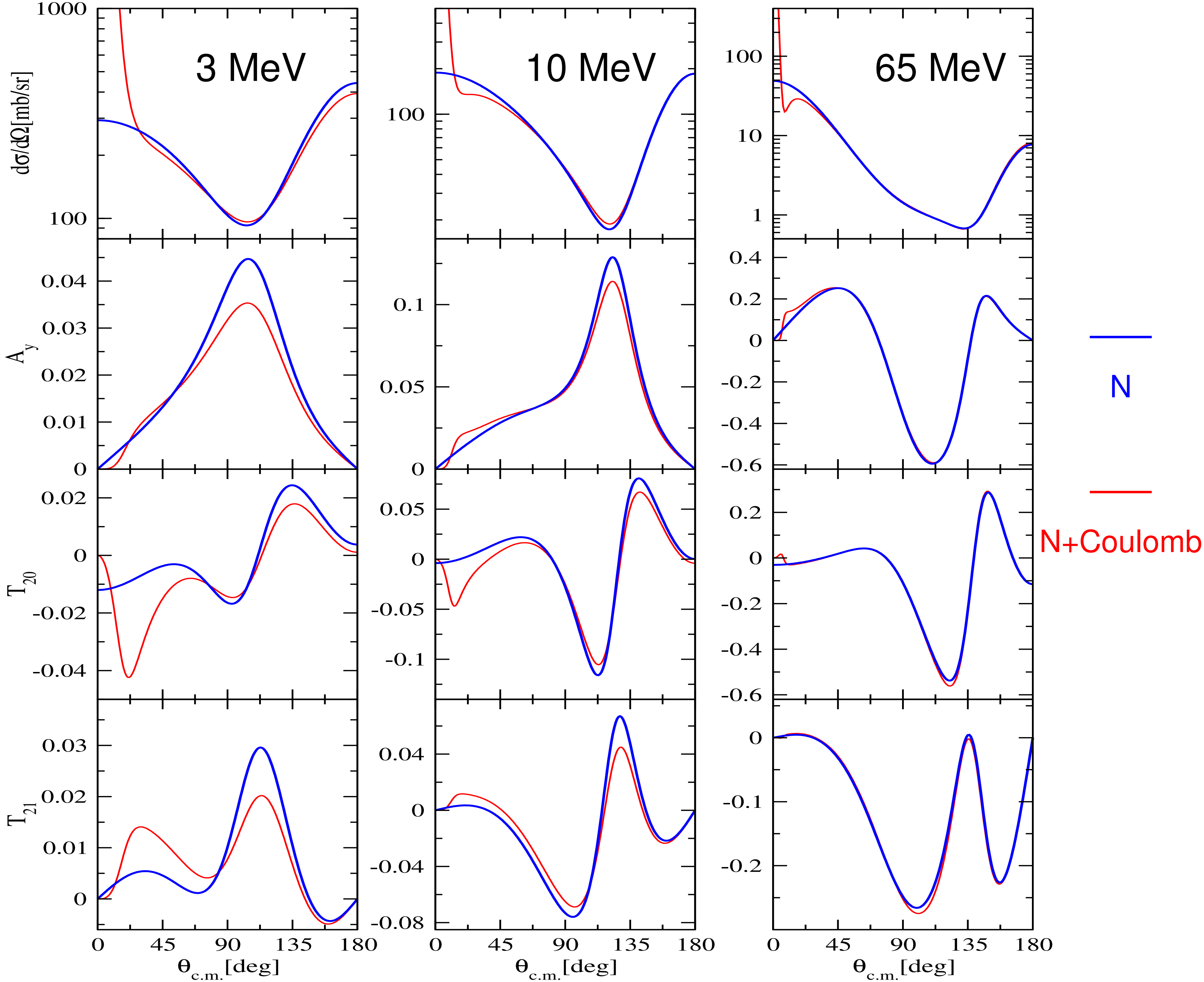
pp scattering: $^1S_0$ phase shift



Convergence with *R*: *pd* radiative capture



pd elastic scattering: energy dependence of Coulomb effect



A_y-puzzle and Doleschall potential