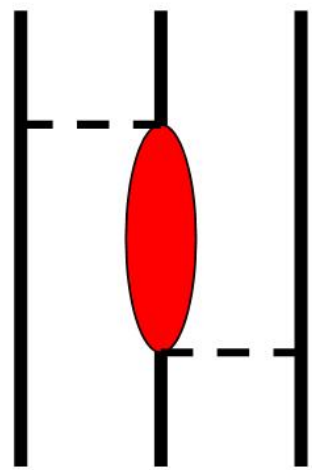
ECT*, Trento, 28 June 2005

Few-nucleon systems and forces in chiral EFT

Evgeny Epelbaum, Jefferson Lab



Outline

Part I. EFT for 2 nucleons: basic concepts & the cutoff issue

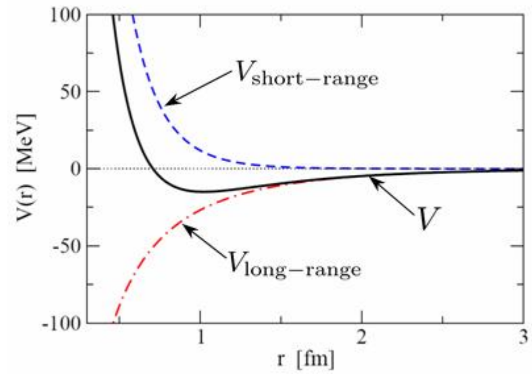
Part II. Few-nucleon forces & systems in chiral EFT: the current status.

Part III. Few-nucleon forces at N^3LO: first results.

Part I. EFT for 2 nucleons: basic concepts

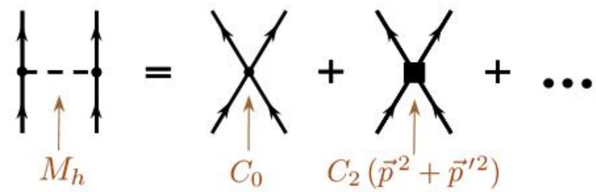
Toy model (à la Lepage)

$$V(\vec{q}) = \underbrace{\frac{\alpha_l}{\vec{q}^{\,2} + M_l^2}}_{M_l = 200\text{ MeV}} + \underbrace{\frac{\alpha_h}{\vec{q}^{\,2} + M_h^2}}_{M_h = 750\text{ MeV}} \quad \longrightarrow \quad V(\vec{r}) = \underbrace{\frac{\alpha_l}{4\pi r} e^{-M_l r}}_{\textit{long-range}} + \underbrace{\frac{\alpha_h}{4\pi r} e^{-M_h r}}_{\textit{short-range}}$$



$$\left.\begin{aligned} \alpha_l &= -1.50 \\ \alpha_h &= 10.81 \end{aligned}\right\} \Longrightarrow \text{S-wave bound state with: } E_B = 2.2229\text{ MeV}$$

At low energy, $q \sim M_l \ll M_h$, the precise structure of $V_{\text{short-range}}$ is irrelevant $\Longrightarrow$ mimic $V_{\text{short-range}}$ by a generic set of point-like interactions:



$$V \rightarrow V_{\text{eff}} = V_{\text{long-range}} + C_0 + C_2\,(\vec{p}^{\,2} + \vec{p}^{\,\prime 2}) + C_4\,\vec{p}^{\,2}\vec{p}^{\,\prime 2} + \ldots$$

T-matrix:

probe high-momentum physics, integral diverges...

- weak interaction, $|\alpha_{l,h}| \ll 1$: $\quad \langle f|T|i\rangle \simeq \langle f|V_{\text{eff}}|i\rangle$
- strong interaction, $|\alpha_{l,h}| \geq 1$: $\quad \langle f|T|i\rangle = \langle f|V_{\text{eff}}|i\rangle + \sum_n \frac{\langle f|V_{\text{eff}}|n\rangle\langle n|V_{\text{eff}}|i\rangle}{E_i - E_n + i\epsilon} + \ldots$

Solution: introduce ultraviolet cutoff Λ : $M_l \ll \Lambda \sim M_h$

Fix $C_{2i}(\Lambda)$ from some low-energy data and make **predictions!**

Use: $k \cot \delta = -\frac{1}{a} + \frac{1}{2} r k^2 + v_2 k^4 + v_3 k^6 + \ldots$

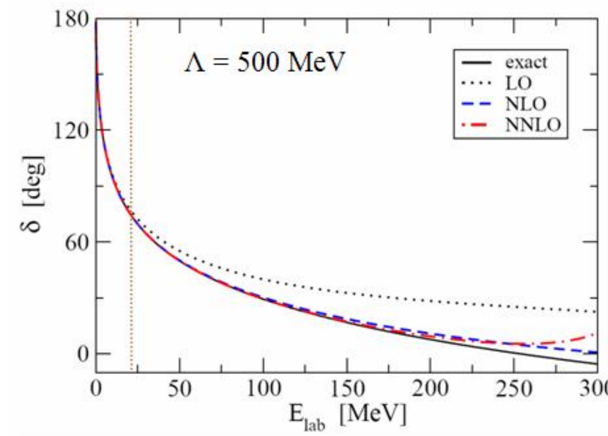
LO: $V_{\rm eff} = V_{\rm long-range} + C_0 \, f_\Lambda(p, p')$, where: $\underbrace{f_\Lambda(p, p')}_{cutoff\ function} = e^{-\frac{\vec p^{\,2} + \vec p^{\,\prime 2}}{\Lambda^2}}$

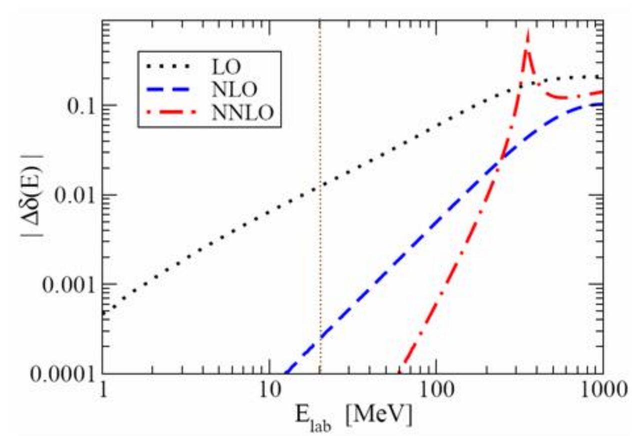
NLO: $V_{\rm eff} = V_{\rm long-range} + \left[C_0 + C_2 (\vec p^{\,2} + \vec p^{\,\prime 2}) \right] f_\Lambda(p, p')$

NNLO: $V_{\rm eff} = V_{\rm long-range} + \left[C_0 + C_2 (\vec p^{\,2} + \vec p^{\,\prime 2}) + C_4 \, \vec p^{\,2} \vec p^{\,\prime 2} \right] f_\Lambda(p, p')$

Convergence for E_B: $E_B = \underbrace{2.1594}_{LO} + \underbrace{0.0638}_{NLO} \underbrace{- 0.0003}_{NNLO} = 2.2229 \text{ MeV}$

Error at order ν: $\Delta\delta(k) \sim (k/\bar\Lambda)^{2\nu}$, $\bar\Lambda \sim 400$ MeV



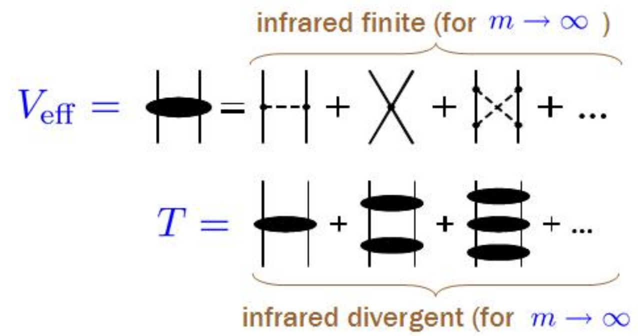


At low energy model independent and systematically improvable approach!

Finite momentum cutoff $\neq$ model dependence

Realistic case: chiral EFT for 2 nucleons à la Weinberg

- Use chiral expansion to derive the NN potential (= set of irreducible diagrams).
- Solve the corresponding dynamical equation to calculate observables.



Problem: chiral potential grows at large momenta (short distances) where it becomes invalid $\longrightarrow$ Shrödinder equation diverges!
How to renormalize the Schrödinger equation?

Detailed answer is given in Lepage's lectures:
"How to renormalize the Schrödinger equation", nucl-th/9706029

Basic idea:

- Use finite cutoff Λ. While the true theory is independent on the value of Λ, the results in EFT are only approximately Λ independent. The residual Λ dependence is removed order-by-order by the corresponding contact terms.
- Which value of Λ should be used? Taking too small Λ will remove the truly long-distance physics and reduce the predictive power of the EFT (errors due to finite Λ typically grow as $(Q/\Lambda)^n$). How about increasing Λ?

> "*Highly nonlinear behavior … results when the cutoff distance a [momentum Λ] is made too small [too large]. …we know that taking $a \to 0$ [$\Lambda \to \infty$] is a bad idea. In general it makes little sense to reduce a below the range r_s [increase Λ beyond r_s^{-1}] of the true potential. When $a < r_s$ [$\Lambda > r_s^{-1}$], high-momentum states are included that are sensitive to structure at distances smaller than r_s. But the structure they see there is almost certainly wrong. Thus taking a smaller than r_s [Λ larger than r_s^{-1}] cannot improve results obtained from effective field theory. In fact, as nonlinearities develop for small a's [large Λ's], results often degrade, or, in more extreme cases, the theory may become unstable or untunable.*"

G. P. Lepage, "How to renormalize the Schrödinger equation", nucl-th/9706029

This viewpoint is adopted in the N^{3}LO analysis by *Epelbaum, Meißner & Glöckle, NPA 747 (2005)*, where Λ was varied in the range 450…600 MeV. Larger Λ's result e. g. in spurious bound states ("highly nonlinear behavior…"). Λ=500 MeV is used in *Entem & Machleidt, PRC 68(2003)*.

skeptic...

Frequently asked questions (FAQ's)

Q: Doesn't a finite cutoff introduce a model dependence?

A: In renormalizable FT's finite Λ would reduce their applicability range to the momentum region $Q < \Lambda$. Effective FT's are anyhow only valid in the low momentum region, $Q < \Lambda_s$. Taking $\Lambda \sim \Lambda_s$ (or larger) does not introduce any ambiguity/model dependence. For explicit examples in CHPT see e.g. *Donoghue, Holstein & Borasoy PRD 59 (1999)*.

Q: How about chiral/gauge invariance?

A: No problem, *see Slavnov, NPB 31 (1971)*, also e. g. *Djukanovic et al., hep-ph/0407170*.

Q: I nevertheless would like to solve the NN problem with the 1π-exchange potential in the limit $\Lambda \to \infty$.

A: Be prepared to have **infinite number** of counter (contact) terms.

Q: Why use $\Lambda \sim 3$ fm^{-1} and not $\Lambda \sim 10\ldots20$ fm^{-1} ? I do not share Lepage's concerns about the "highly nonlinear behavior". I hope, using $\Lambda \sim 10\ldots20$ fm^{-1} will increase the applicability range of the EFT.

arXiv:nucl-th/0506005 v1 2 Jun 2005

Renormalization of One-Pion Exchange and Power Counting

A. Nogga,[1,*] R.G.E. Timmermans,[2,†] and U. van Kolck[3,‡]

Basic findings & conclusions, also presented in the talk by Andreas Nogga

LO potential: $V^{\rm LO} = V_{1\pi} + C_S + C_T\,\vec\sigma_1\vec\sigma_2\,,\qquad V_{1\pi}(\vec r) = \frac{M_\pi^3}{12\pi}\left(\frac{g_A}{2F_\pi}\right)^2 \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,[T(r)\,S_{12} + Y(r)\,\vec\sigma_1\cdot\vec\sigma_2]\,,$

where: $S_{12} = 3(\vec\sigma_1\cdot\hat r)(\vec\sigma_2\cdot\hat r) - \vec\sigma_1\cdot\vec\sigma_2\,,\qquad T(r) = \frac{e^{-M_\pi r}}{M_\pi r}\left[1 + \frac{3}{M_\pi r} + \frac{3}{(M_\pi r)^2}\right],\qquad Y(r) = \frac{e^{-M_\pi r}}{M_\pi r}$

Thus $V_{1\pi}(\vec r)$ is:

- regular in spin-0 channels $^1S_0$, $^1P_1$, $^1D_2$, ...
- singular in spin-1 channels (due to the tensor force)
  - — repulsive in $^3P_1$, $^3F_3$, ... [**stable** for large Λ]
  - — attractive in $^3P_0$, $^3D_2$, $^3P_2$ - $^3D_2$, ... [**unstable** for large Λ]

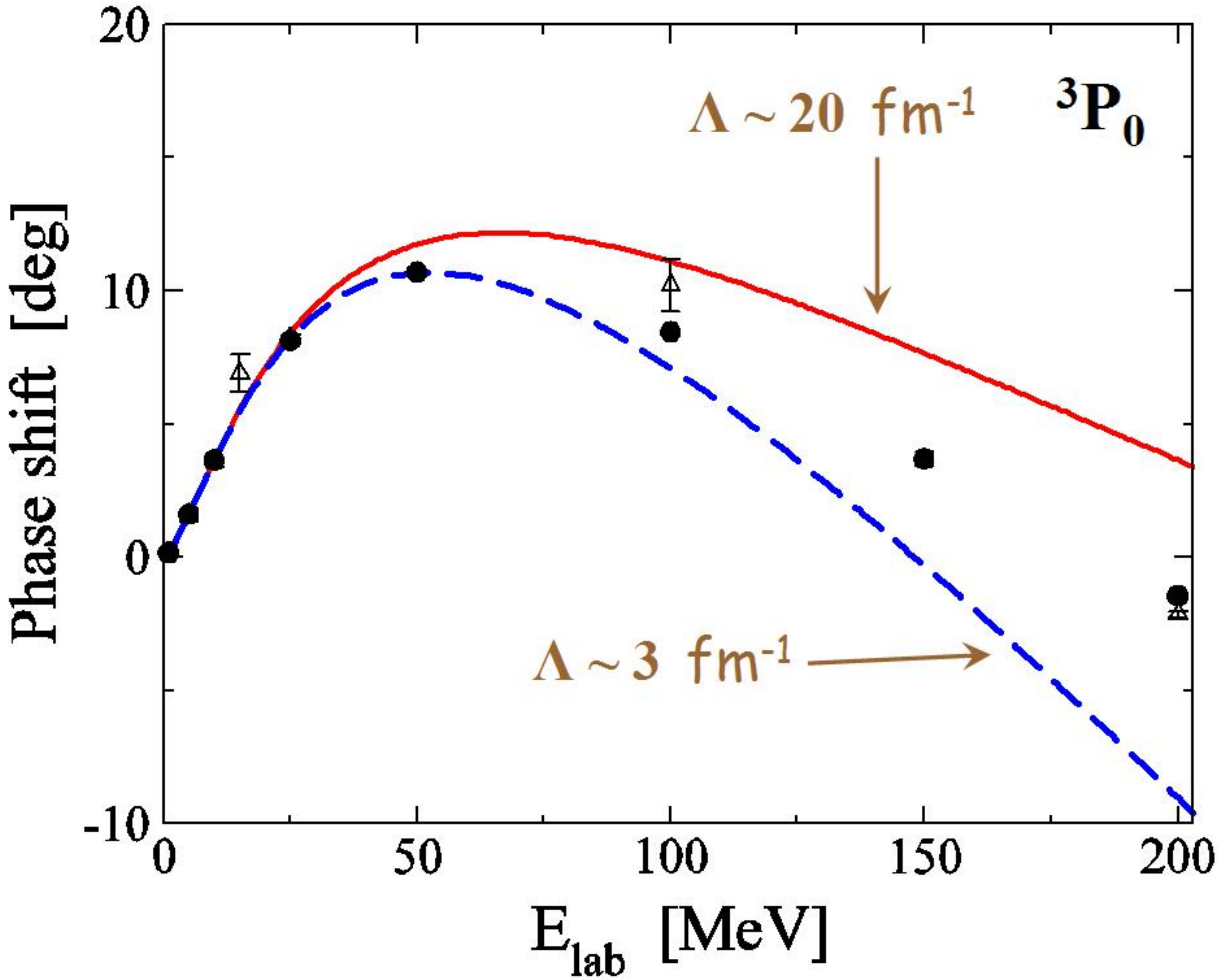
Suggestion by *Nogga et al.*:

- Use large Λ (up to 20 fm^{-1}). Stabilize attractive channels promoting higher-order contact terms to LO.

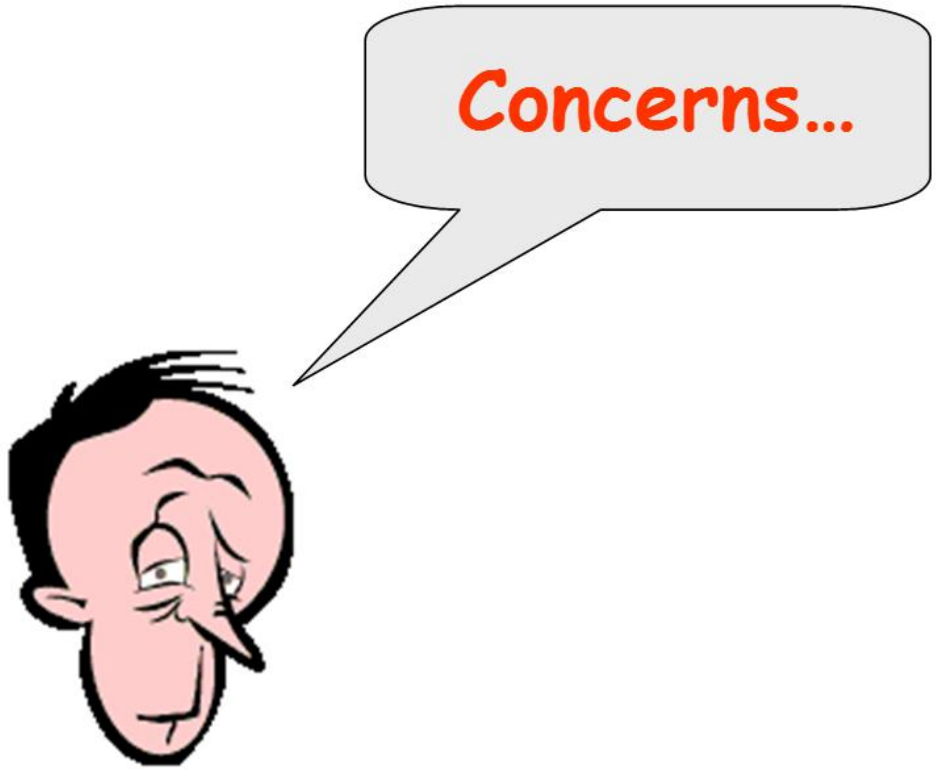
Notice: NN phase shifts with large Λ at NNLO were investigated by *E.E. & Meißner* and presented on the INT Workshop on "*Theories of Nuclear Forces and Few-Nucleon Systems*", 2001.

Example: $^3P_0$ partial wave

(1π-exchange potential + the leading contact term)



(The coefficient of the contact term is fixed from the "scattering length".)

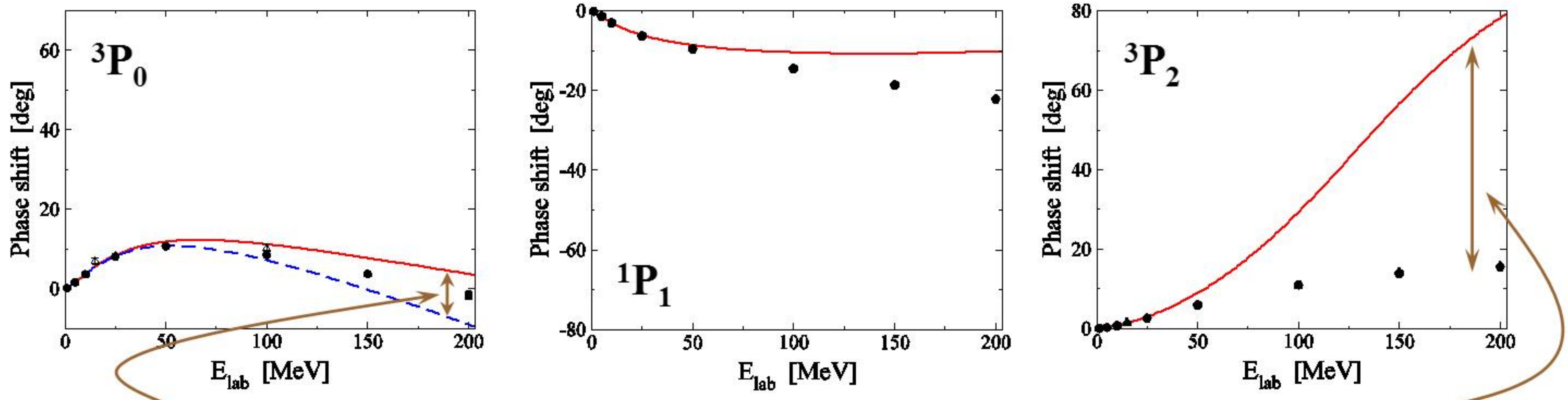

Concerns…

Concern Nr. 1

Even for $\Lambda \sim 20$ fm^{-1} phase shifts are still cutoff dependent. Is this cutoff dependence small or large? How small is small & how large is large?

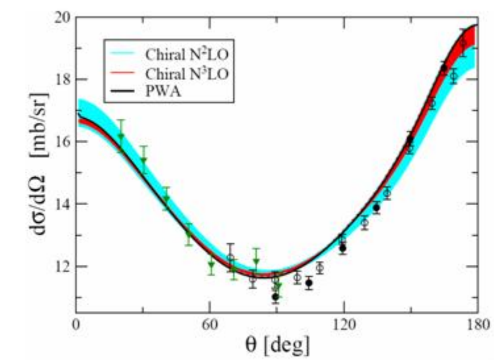
Λ dependence is **small** if it is within the theoretical uncertainty at this order.

What is the theoretical uncertainty at LO?



Why care about if ?

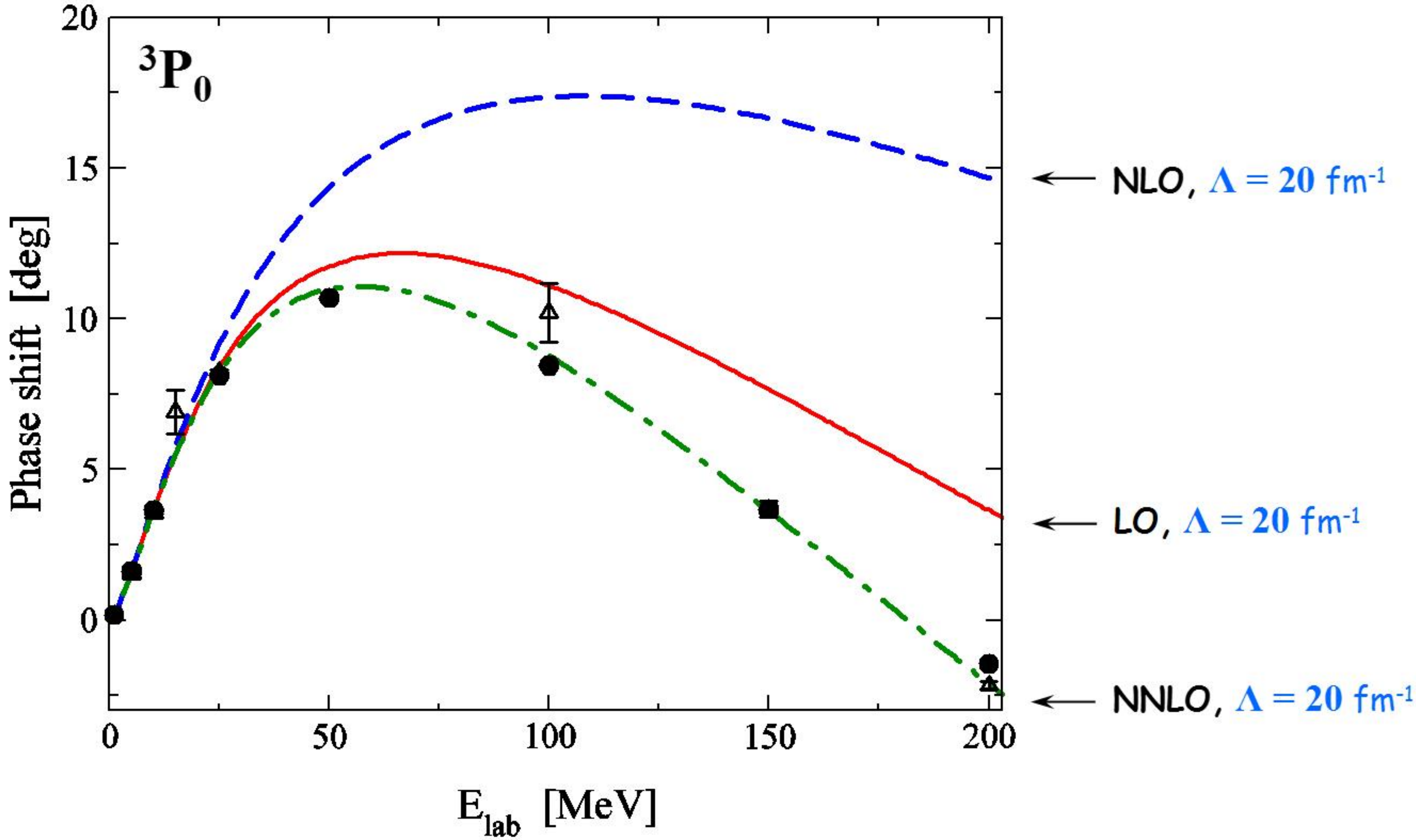
Reducing cutoff dependence in the $^3P_0$ partial wave will **not** improve the description of observables like $d\sigma/d\Omega$



Is not optimizing calculations (i. e. compute only things that are really needed) one of the major advantages of EFT?

Concern Nr. 2

$^3P_0$ phase shift seems to be in a reasonable agreement with the PWA. **Converged result?**



In all cases the potential is singular and attractive. The contact term is fixed to the "scattering length".

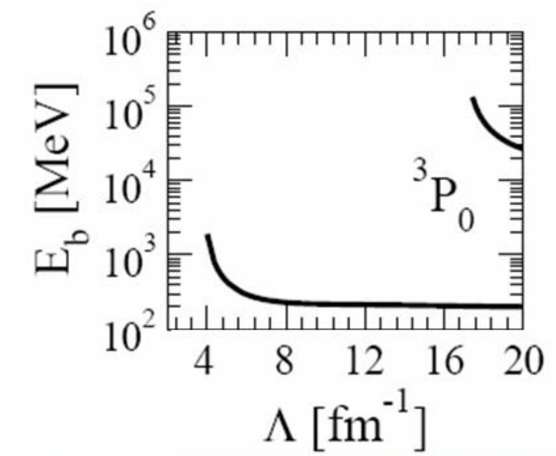
Concern Nr. 3

In the case of 1π-exchange potential one can make Λ arbitrarily large (provided that the attractive channels are stabilized with counter terms). **Shall one then expect significant increase of the applicability range of the EFT?**

No. Increasing the values of Λ in attractive channels leads to spurious bound states. While these bound states are invisible in NN scattering (except in the cases when they appear close to threshold), they affect other processes (>2N systems, γNN, πNN, …). In general, BE's of these spurious states set the scale at which EFT breaks down (i. e. maximal radius of convergence).

BE of the lowest state in the $^3P_0$ channel at LO is $E \sim 200$ MeV. This corresponds to $Q \sim 1.5$ fm^{-1}. This is better for the $^3D_2$ channel: $E \sim 400$ MeV or $Q \sim 2.5$ fm^{-1}.

$\Rightarrow$ $\boxed{\text{cutoff } \Lambda > 2.5 \text{ fm}^{-1} \text{ should be sufficient}}$



(from Nogga et al, nucl-th/0506005)

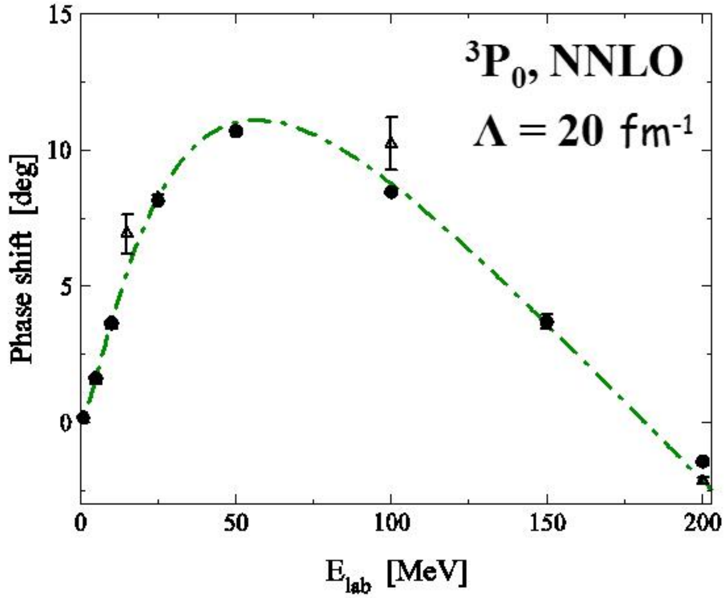
Concern Nr. 4

One can presumably describe NN phase shifts in the attractive channels based on $V_{1\pi} + V_{2\pi}$ and additional counter term(s). **Does one still have predictive power for other observables/systems, i.e. will not one always (?) need to promote higher-order counter terms?**

Example

Suppose one wants to look at the quark/pion mass dependence of the $^3P_0$ phase shift. At NNLO one has 3 deeply bound states for $\Lambda = 20\ \mathrm{fm}^{-1}$, i.e. the potential is **very strong!** It therefore becomes unstable against small perturbations/changes of the parameters.

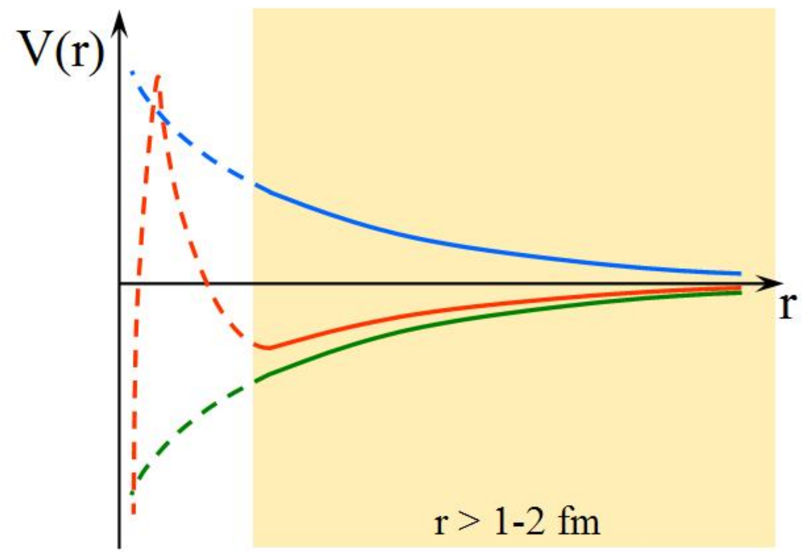
$F_\pi = 92.4$ MeV	$F_\pi^* = 1.1\, F_\pi$
$E_1 = 456$ MeV	$E_1 = 35$ MeV
$E_2 = 22.3$ GeV	$E_2 = 13.0$ GeV
$E_3 = 605$ GeV	$E_3 = 391$ GeV



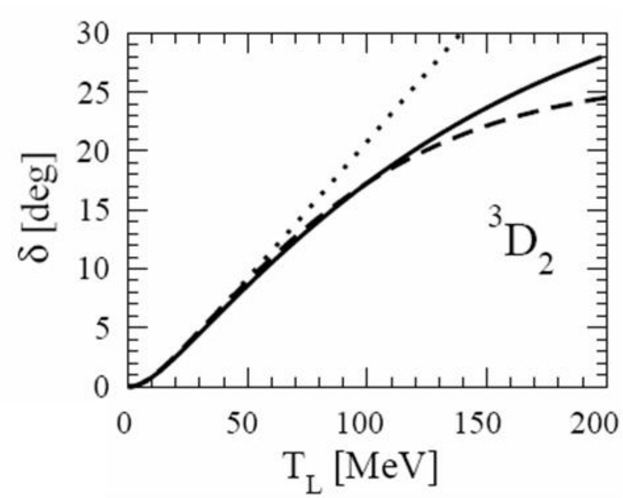
It seems unlikely that one will be able to perform chiral extrapolation without promoting higher-order M_π-dependent counter terms...

Even more concerns…

Does it actually work?



The 1π-exchange potential behaves like $1/r^3$ at short distances. At higher orders the behavior becomes more and more singular. **Be prepared to surprises…**



Example: $^3D_2$ partial wave

LO result, see *Nogga et al., nucl-th/0506005*

$$V^{\rm LO} = V_{1\pi} + C\, p^2\, p'^{\,2}$$

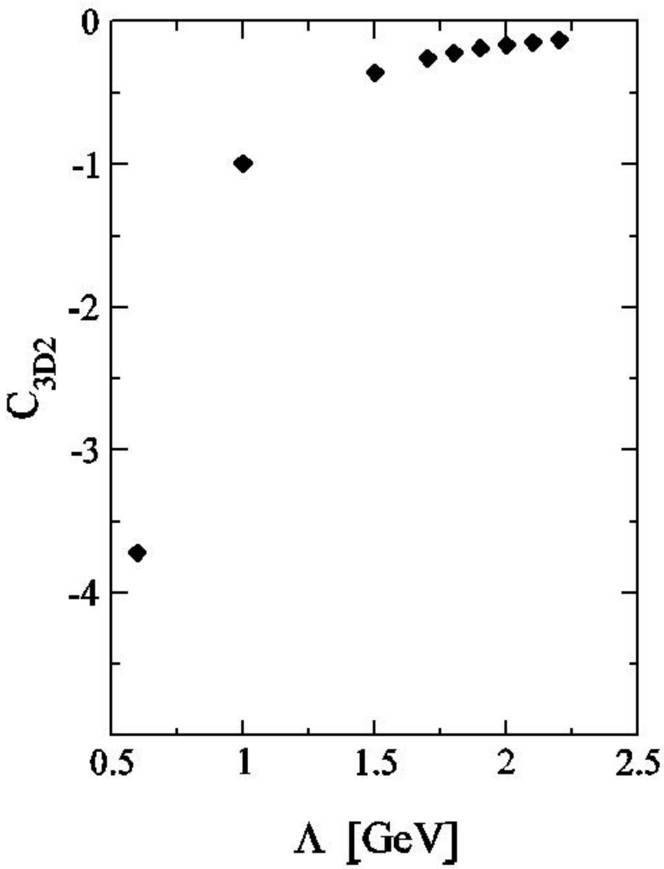
Singular & attractive

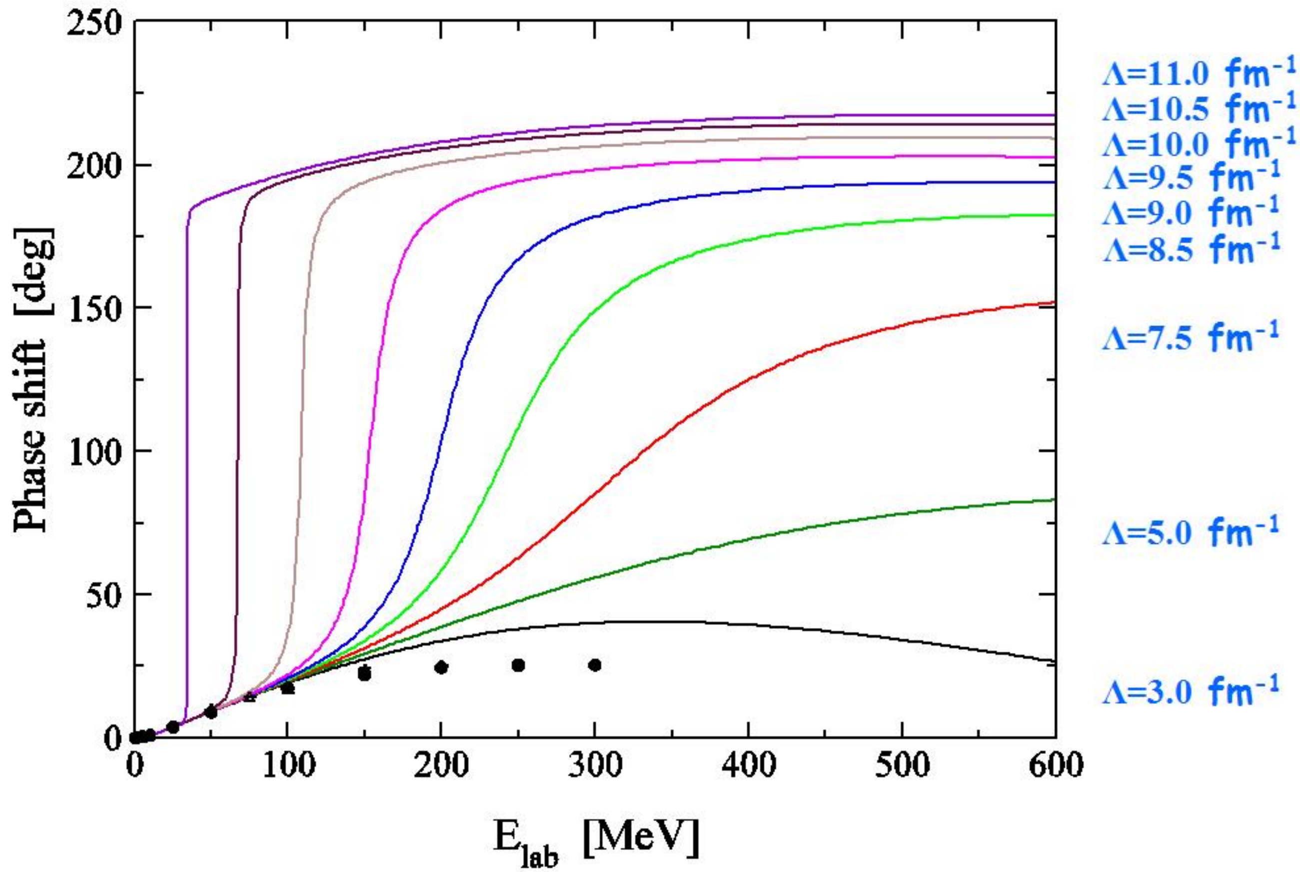
Needed to stabilize the result for δ

Generalization to NLO

$$V^{\rm NLO} = V_{1\pi} + V_{2\pi} + C\, p^2\, p'^{\,2}$$

1/r³, *attractive* **1/r⁵**, *repulsive*



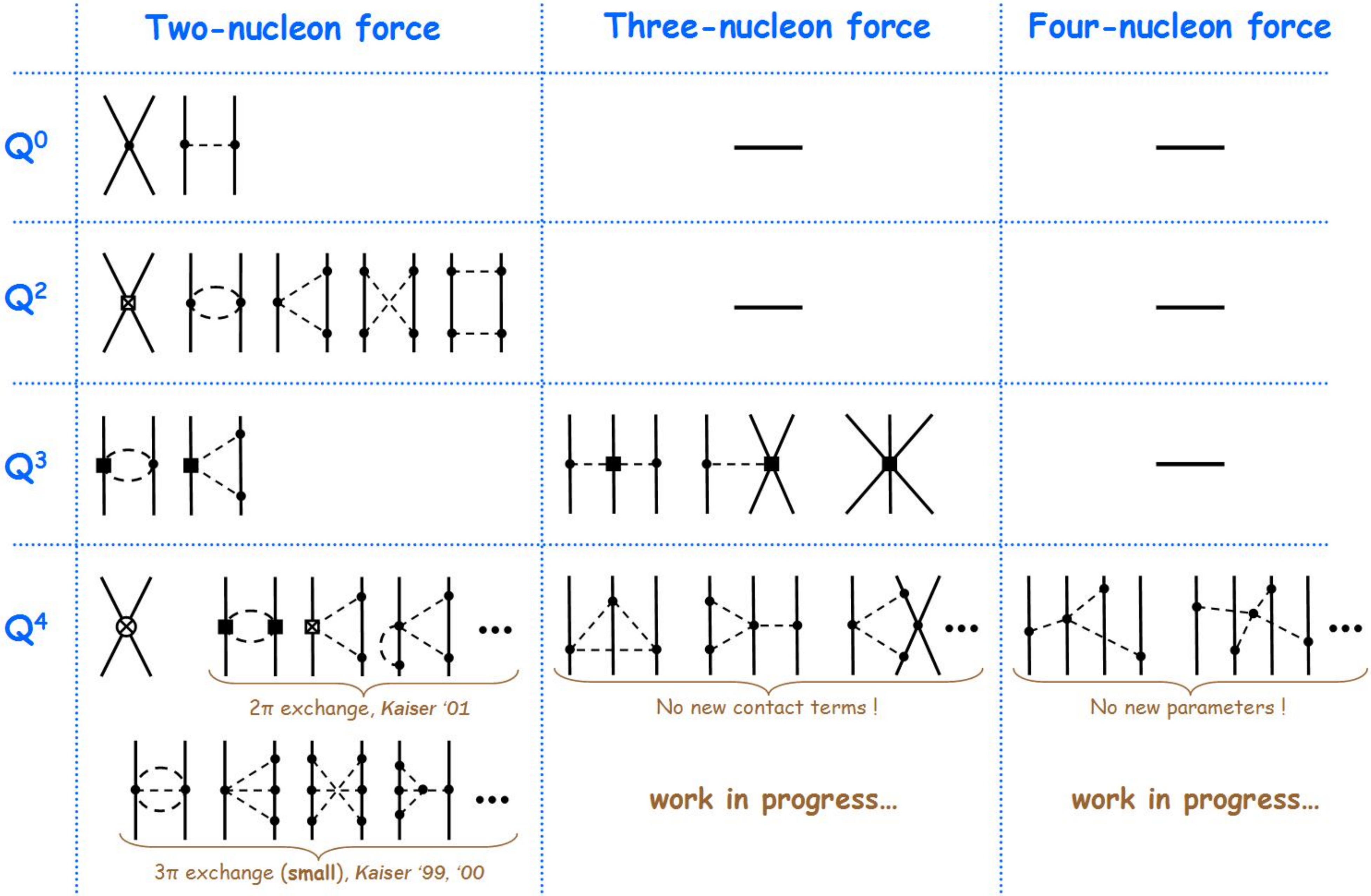


Remember Lepage's: *"…as nonlinearities develop for small **a**'s [large **Λ**'s], results often degrade, or, in more extreme cases, the theory may become unstable or untunable."*

Resonances also occur in many other channels at NLO! Things are better at NNLO (due to the strong attractive 2π-exchange potential). **Resonances expected at N³LO and higher orders!**

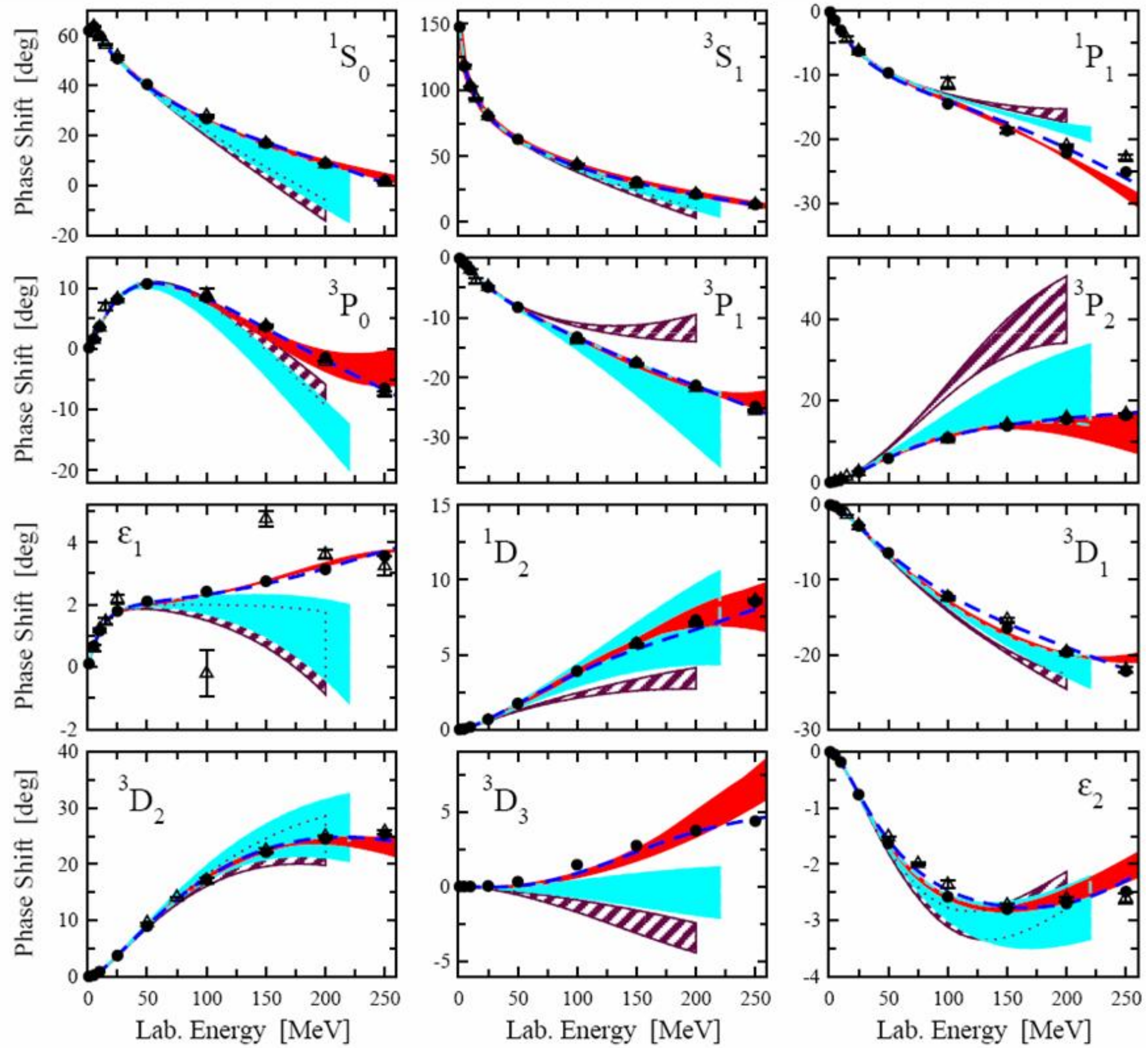
Part II. Few-nucleon forces & systems in chiral EFT: the current status

Few-nucleon forces: isospin-invariant contributions



2 nucleons

Neutron-proton phase shifts up to N^{3}LO



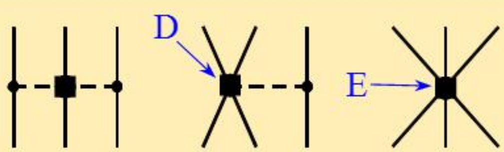
Results from: *Entem & Machleidt, PRC 68 (2003); E.E., Meißner & Glöckle, NPA 747 (2005)*

3 nucleons

In collaboration with:
A.Nogga, W.Glöckle, H.Kamada, Ulf-G.Meißner and H.Witala

No 3NF ⟹ parameter-free
(Epelbaum et al. '01)

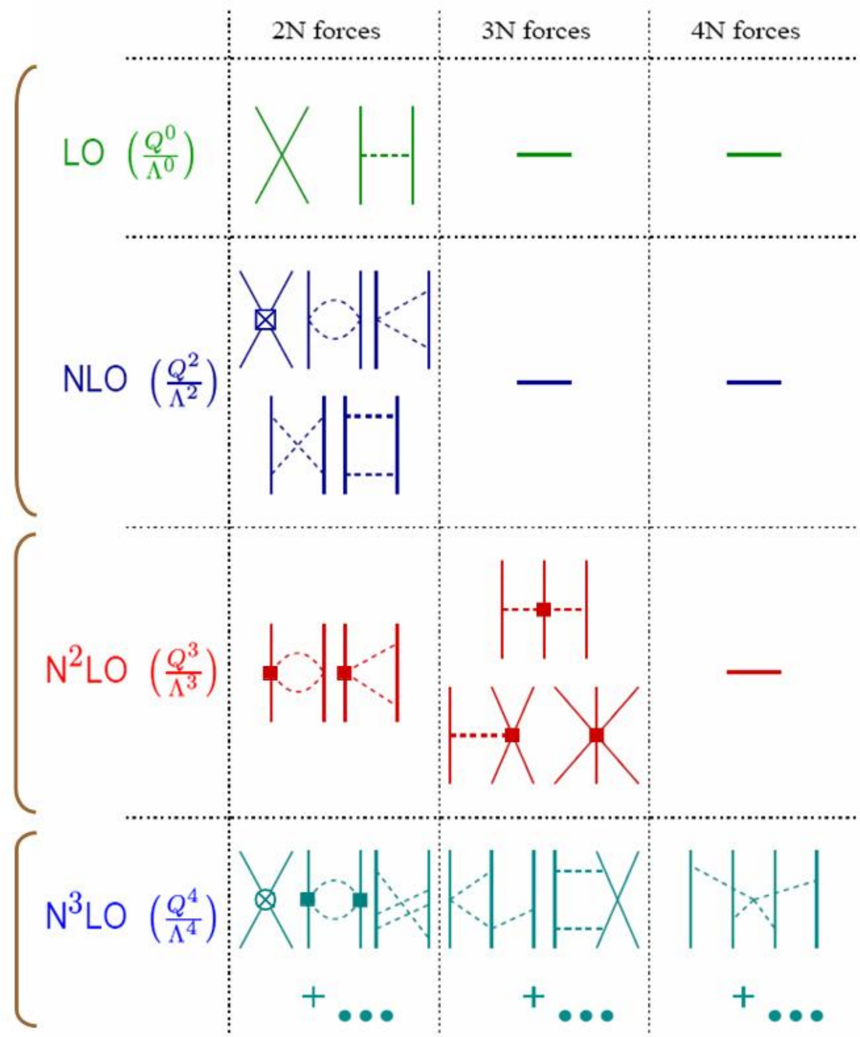
3NF:



D, E fixed from ^{3}H BE and a_{Nd}.
(Epelbaum et al. '02)

in progress...

Hierarchy of nuclear forces

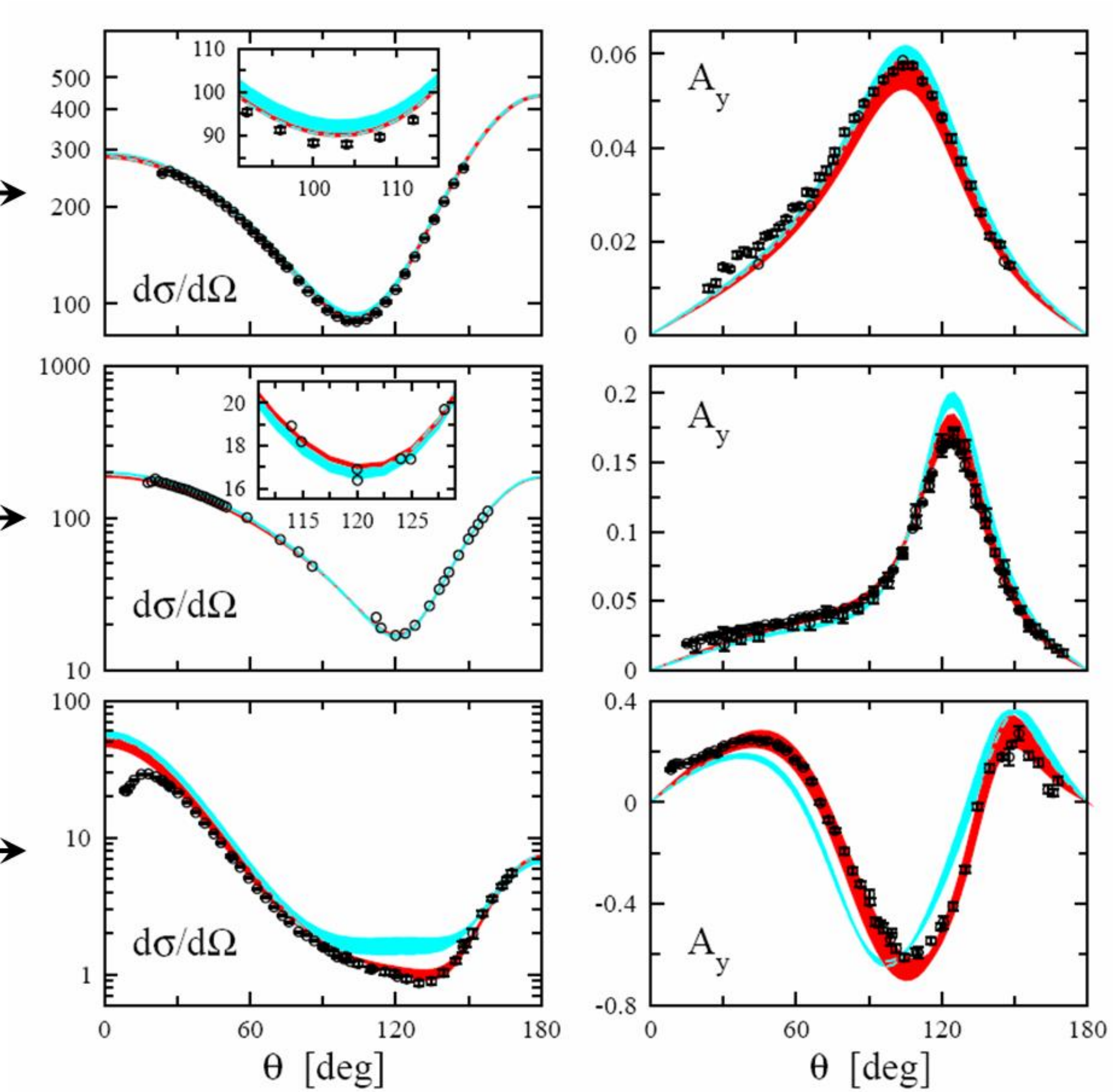


Elastic nucleon-deuteron scattering observables

E=3 MeV →

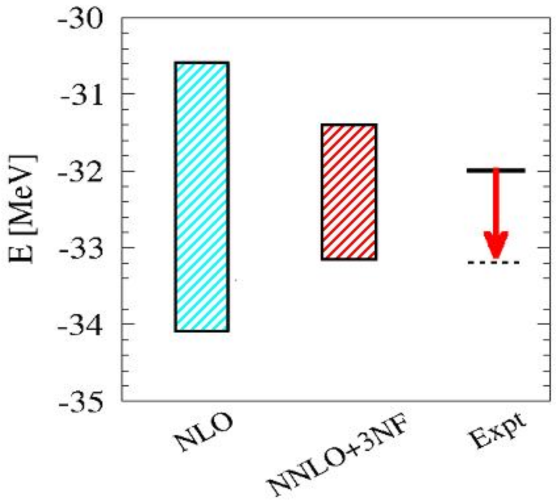
E=10 MeV →

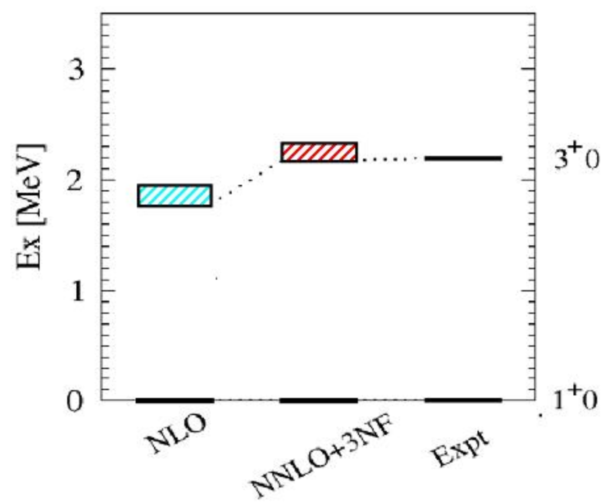
E=65 MeV →



Even more nucleons

Predictions for ^6Li ground and excited states





These results will be updated based on the last generation of chiral forces (spectral-function regularization, *Epelbaum et al., '03, '04*, larger cut-off variation, isospin-breaking corrections, ...)

Further results for even heavier nuclei discussed based on the Idaho N^3LO chiral potential discussed by *Petr Navrátil.*

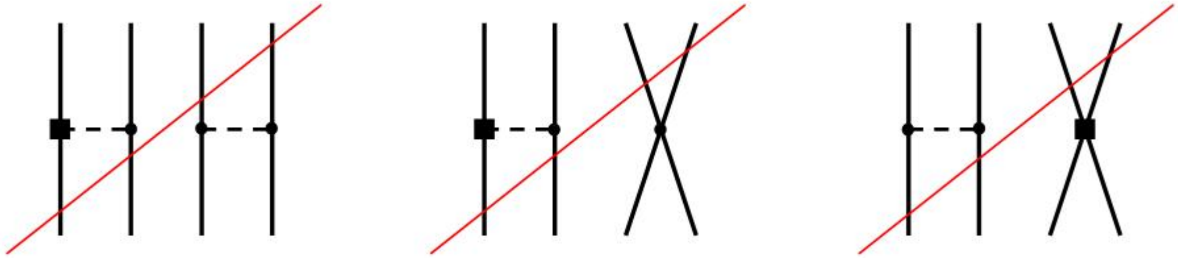
Part III. Few-nucleon forces at N^3LO: first results

Using the **method of unitary transformation**

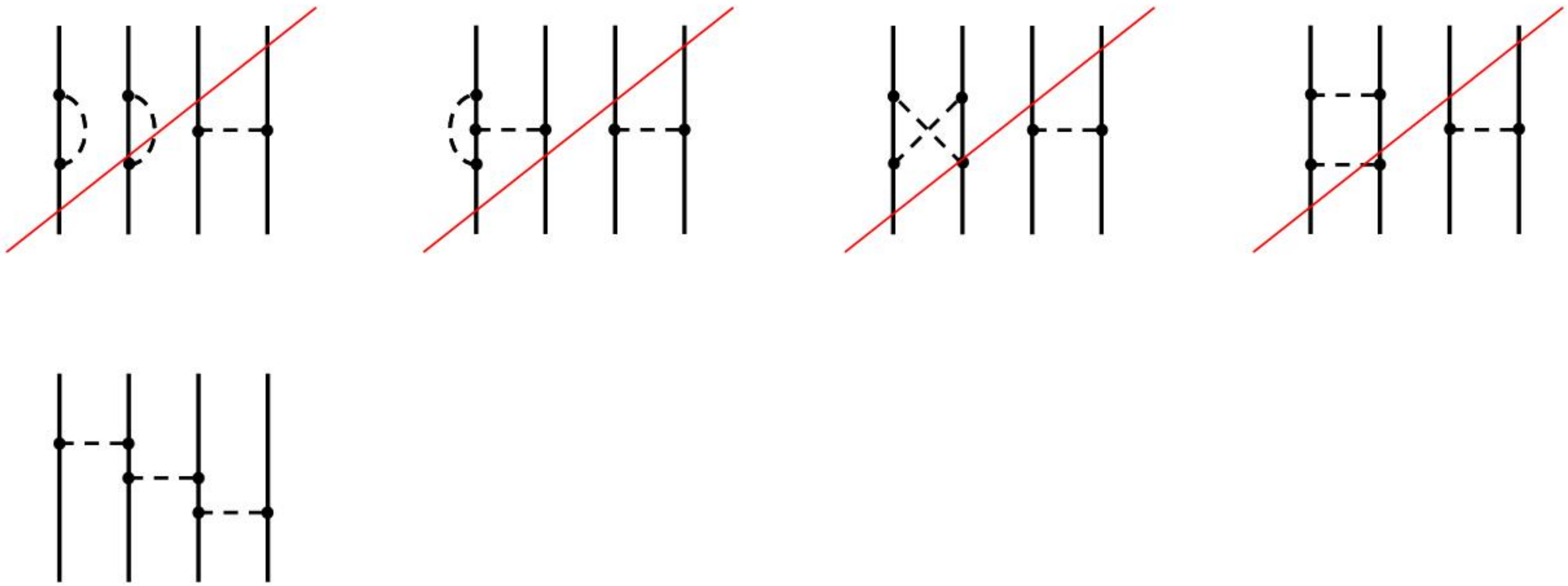
- hermitean and energy independent V_{eff},
- can be used to derive nuclear currents.

(Epelbaum et al. '98, '00)

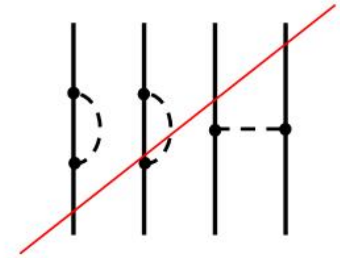
Class 1: graphs with 1 subleading vertex

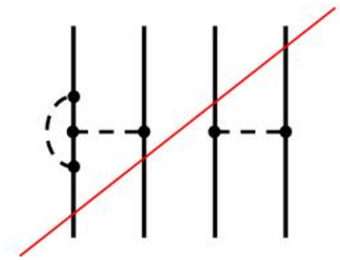


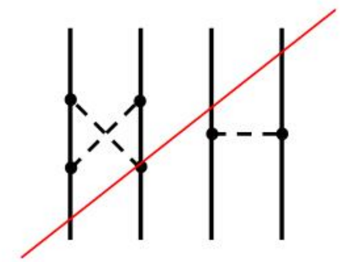
Class 2: graphs proportional to $g_A^{\,6}$

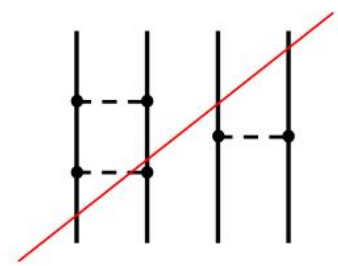


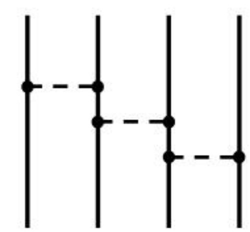
$$
\begin{aligned}
V \;=\; \eta\Big[& - H_I \frac{\lambda^1}{\omega} H_I \frac{\lambda^2}{(\omega_1+\omega_2)} H_I \frac{\lambda^1}{\omega} H_I \frac{\lambda^2}{(\omega_1+\omega_2)} H_I \frac{\lambda^1}{\omega} H_I \\
& - H_I \frac{\lambda^1}{\omega} H_I \frac{\lambda^2}{(\omega_1+\omega_2)} H_I \frac{\lambda^3}{(\omega_1+\omega_2+\omega_3)} H_I \frac{\lambda^2}{(\omega_1+\omega_2)} H_I \frac{\lambda^1}{\omega} H_I \\
& - H_I \frac{\lambda^1}{\omega} H_I \eta H_I \frac{\lambda^1}{\omega^3} H_I \eta H_I \frac{\lambda^1}{\omega} H_I \\
& - \frac{1}{4} H_I \frac{\lambda^1}{\omega^2} H_I \eta H_I \frac{\lambda^1}{\omega} H_I \eta H_I \frac{\lambda^1}{\omega^2} H_I \\
& - \frac{3}{8} H_I \frac{\lambda^1}{\omega} H_I \eta H_I \frac{\lambda^1}{\omega^2} H_I \eta H_I \frac{\lambda^1}{\omega^2} H_I + \text{h. c.} \\
& + \frac{1}{4} H_I \frac{\lambda^1}{\omega} H_I \eta H_I \frac{\lambda^1}{\omega} H_I \frac{\lambda^2}{(\omega_1+\omega_2)} H_I \frac{\lambda^1}{\omega^2} H_I + \text{h. c.} \\
& + \frac{1}{2} H_I \frac{\lambda^1}{\omega} H_I \eta H_I \frac{\lambda^1}{\omega} H_I \frac{\lambda^2}{(\omega_1+\omega_2)^2} H_I \frac{\lambda^1}{\omega} H_I + \text{h. c.} \\
& + \frac{3}{4} H_I \frac{\lambda^1}{\omega} H_I \eta H_I \frac{\lambda^1}{\omega^2} H_I \frac{\lambda^2}{(\omega_1+\omega_2)} H_I \frac{\lambda^1}{\omega} H_I + \text{h. c.} \\
& + \frac{1}{2} H_I \frac{\lambda^1}{\omega^2} H_I \eta H_I \frac{\lambda^1}{\omega} H_I \frac{\lambda^2}{(\omega_1+\omega_2)} H_I \frac{\lambda^1}{\omega} H_I + \text{h. c.}\Big]\eta
\end{aligned}
$$



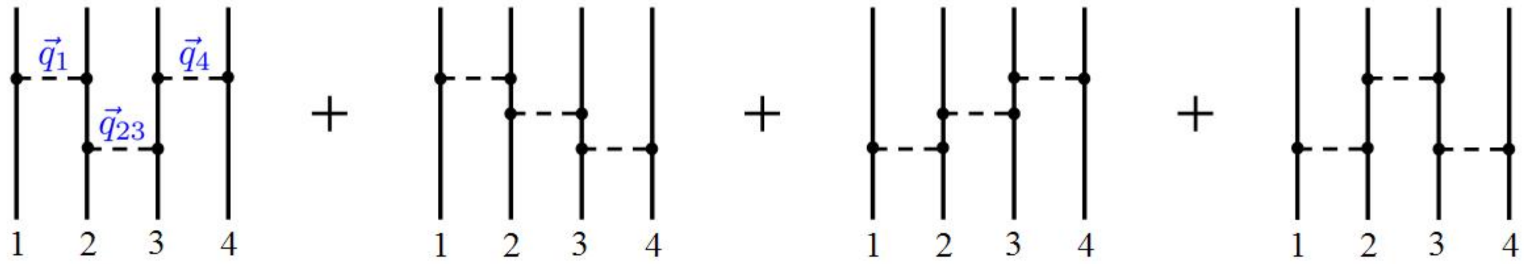






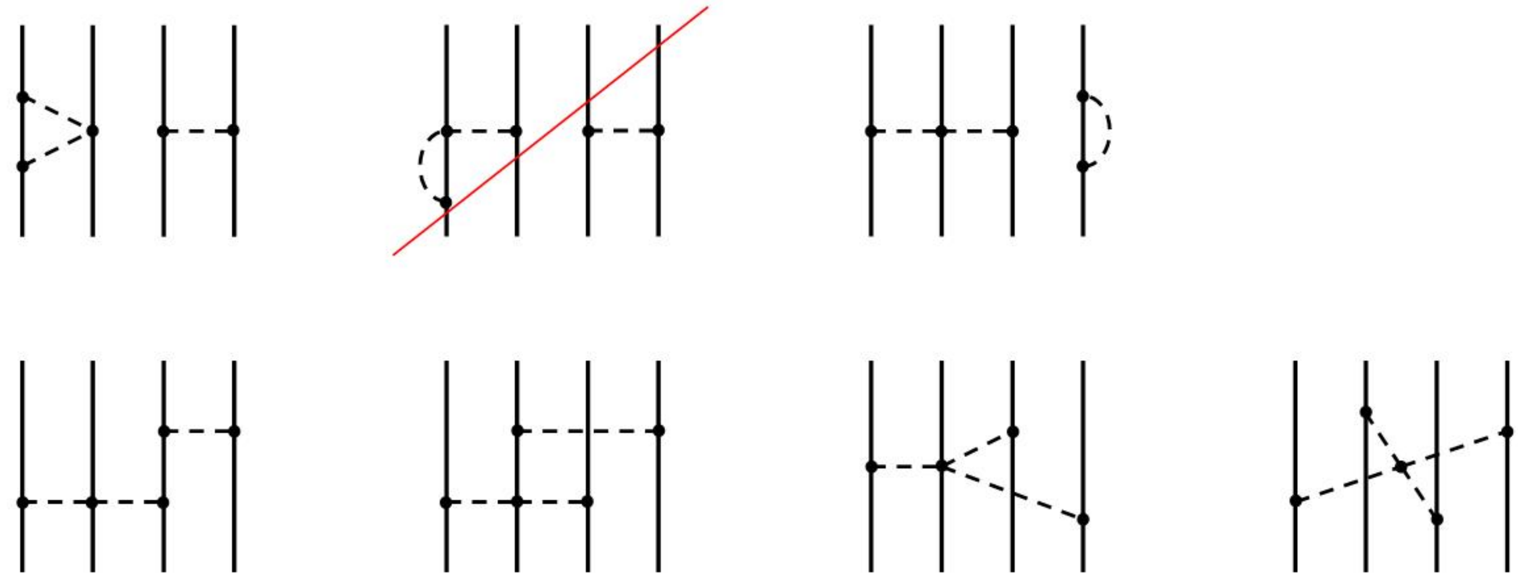


Notice: each graph contains ~ 100 (!) time orderings

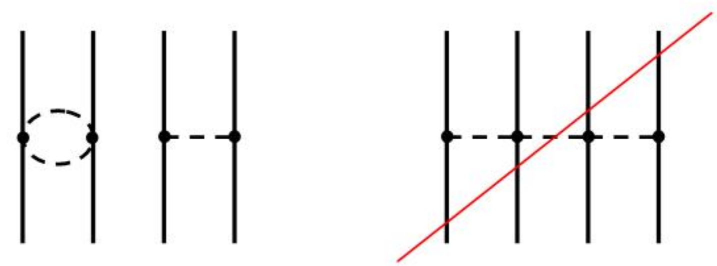


Preliminary results for these 4N forces obtained,
publication in preparation…

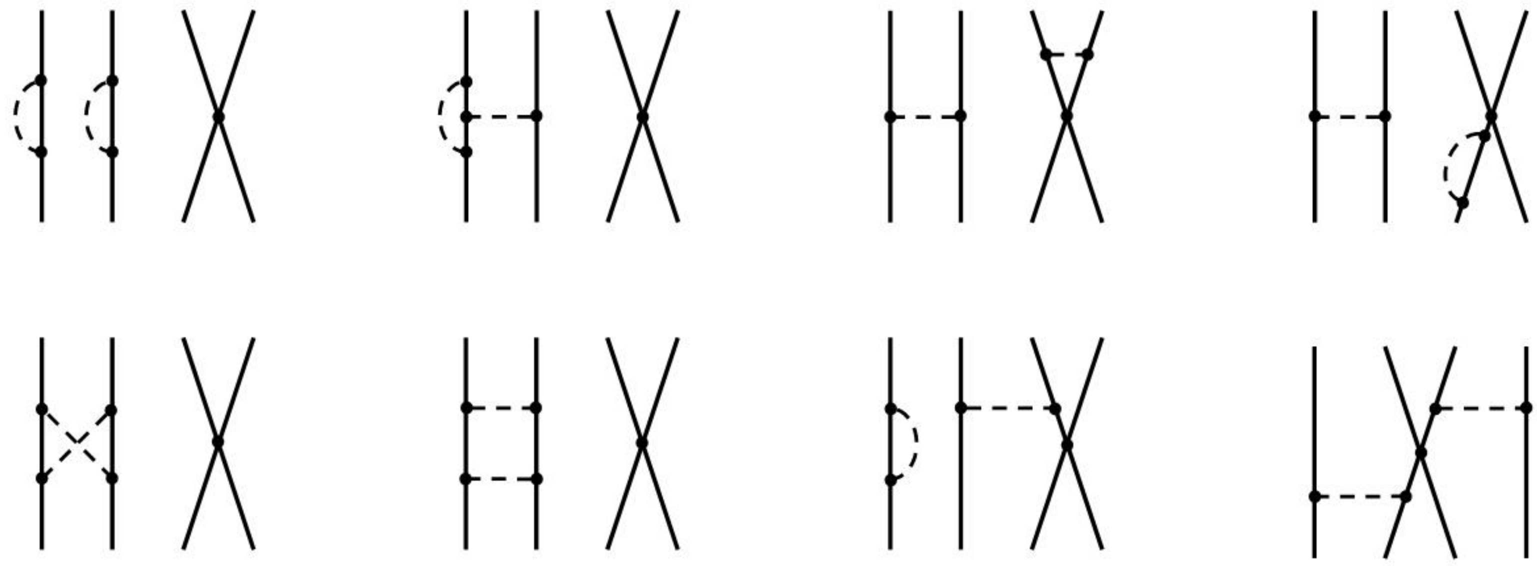
Class 3: graphs proportional to g_A^4



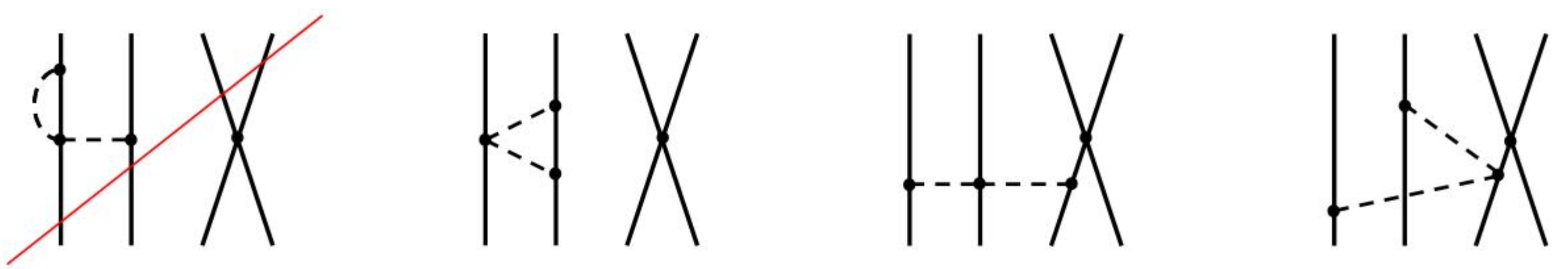
Class 4: graphs proportional to g_A^2



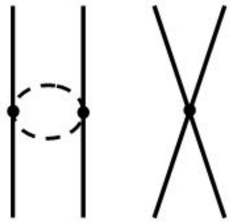
Class 5: graphs proportional to $C_{S,T}\ g_A^4$



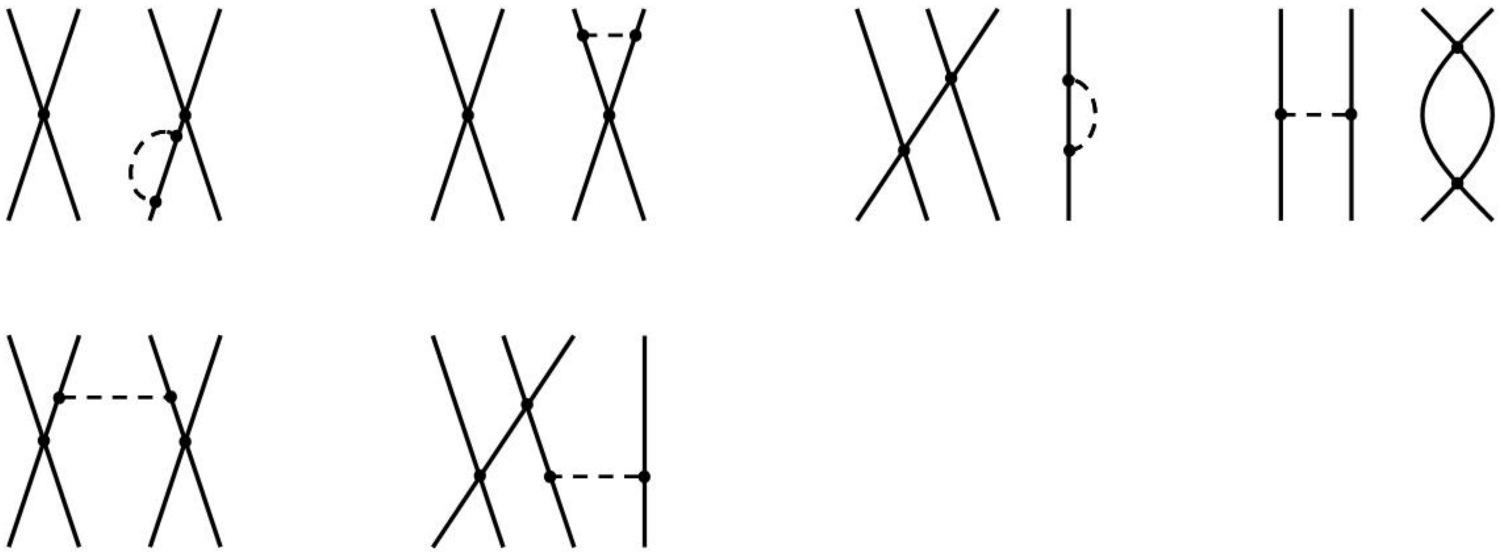
Class 6: graphs proportional to $C_{S,T}\ g_A^2$



Class 7: graphs proportional to $C_{S,T}$



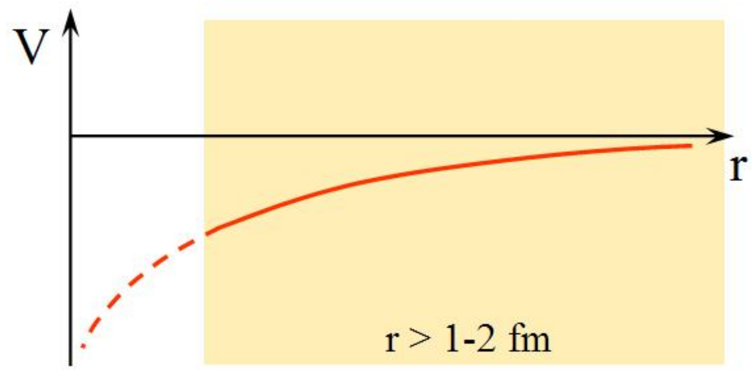
Class 8: graphs proportional to $C_{S,T}^2 \; g_A^2$



Summary & outlook

Part I. NN force & the cutoff issue

- Need **more physics** in order to improve (higher orders, explicit Δ's, large N_c, …).
- Simply increasing the cutoff range seems to be too naïve…



Part II. Few-nucleon systems

- Promising results.
- Need to go to N^3LO.

Part III. Few-nucleon forces at N^3LO

- Work in progress…

Why care about what Lepage is saying?

Answer Nr. 1:

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Well-known papers (50-99) :	16	14
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Less known papers (1-9) :	29	10
Unknown papers (0) :	3	0
Total eligible papers analyzed :	104	66
Total number of citations :	11113	10468
Average citations per paper :	106	158

Answer Nr. 2:

sit down and make calculations by yourself…