$np \to d\pi^0$ in chiral perturbation theory —a status report

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for the IOU collaboration

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Charge symmetry breaking

QCD Lagrangian almost symmetric under $u \leftrightarrow d$ exchange (Charge Symmetry, CS), $P_{CS} = \exp(i\pi\tau_2/2)$
broken by $m_u \neq m_d$, Charge Symmetry Breaking (CSB)
Special case of isospin violation

For hadrons and nuclei CS implies

$p \rightarrow -n$

$n \rightarrow p$

$d \leftrightarrow d$

$\pi^0 \leftrightarrow -\pi^0$, etc.

CSB reviews:

Miller, Nefkens, and Šlaus, PRt**194**, 1 (1990);

Miller and van Oers, in *Symmetries and Fundamental Interactions in Nuclei*,

Eds. Haxton and Henley, WS, Singapore (1995) p 127

Experimental evidence

- ρ^0-ω mixing ($e^+e^- \to \pi^+\pi^-$)
- n-p and other hadron mass differences
- mirror nuclei (e.g. $^3\mathrm{He}$-$^3\mathrm{H}$) binding energy, N-S anomaly
- $a_{nn} \neq a_{pp}$
- $np \to np$: $A_n(\theta_n) \neq A_p(\theta_p)$ analyzing powers
- $dd \to \alpha\pi^0$ (IUCF)
- $np \to d\pi^0$ forward-backward asymmetry (TRIUMF)

Neutron-proton mass difference

$m_n - m_p = \delta M + \bar{\delta} M = 1.29$ MeV, natural sizes:

$$\begin{aligned} \delta M &\sim \epsilon m_\pi^2/\Lambda_\chi \sim \text{few MeV — strong term} \\ \bar{\delta} M &\sim \alpha \Lambda_\chi/\pi \sim -2 \text{ MeV — EM term} \\ \epsilon &= (m_d - m_u)/(m_d + m_u) \sim 1/3 \end{aligned}$$

Models (e.g., Cottingham sum rule) typically $\bar{\delta} M \sim -1$ MeV

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χPT CSB $\Rightarrow$ syms diff for strong and EM:

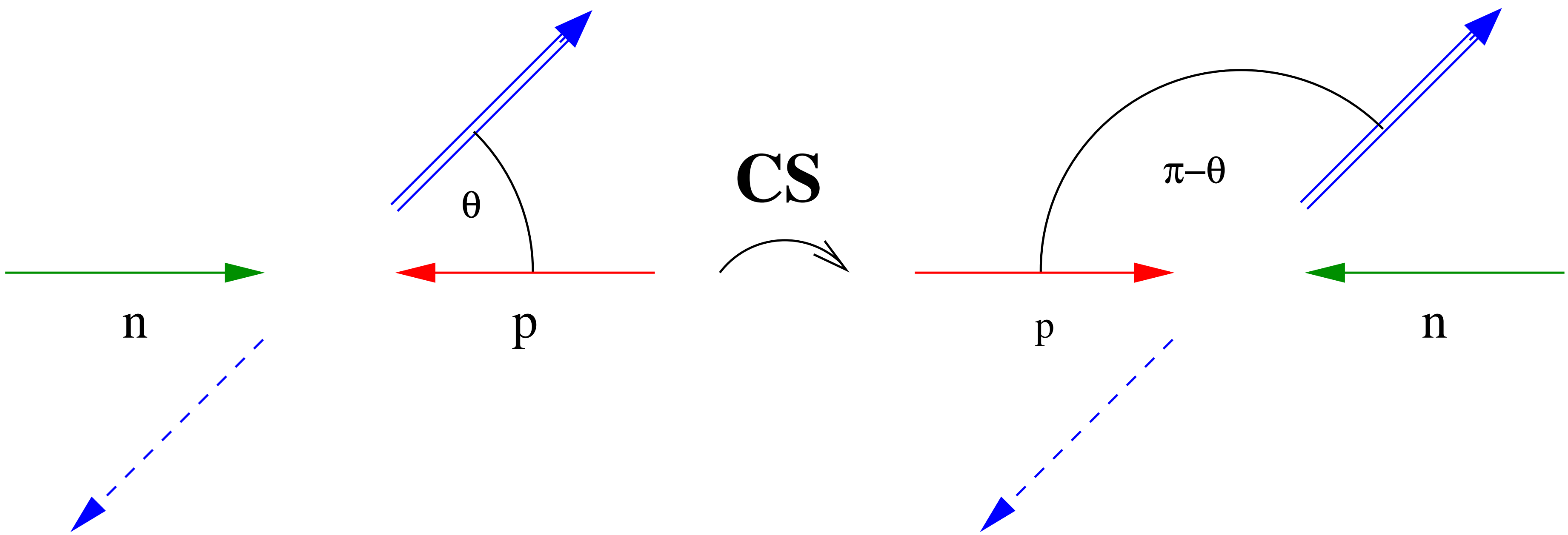
$$\mathcal{L}_{CSB} = \frac{\delta M}{2} N^\dagger \left(\tau_3 - \frac{\pi_3 \boldsymbol{\tau} \cdot \boldsymbol{\pi}}{2 f_\pi^2} \right) N + \frac{\bar{\delta} M}{2} N^\dagger \left(\tau_3 + \frac{\pi_3 \boldsymbol{\tau} \cdot \boldsymbol{\pi} - \boldsymbol{\pi}^2 \tau_3}{2 f_\pi^2} \right) N$$

different comb for δM and $\bar{\delta} M$ for $m_n - m_p$ and pions $\Rightarrow$

Pion reactions to extract $\bar{\delta} M$ and check $\mathcal{L}_{\rm CSB}$!

CSB in $np \to d\pi^0$ I

CS $\Rightarrow n \leftrightarrow p$:



$$A_{\rm fb}(\theta) \equiv \frac{\sigma(\theta) - \sigma(\pi - \theta)}{\sigma(\theta) + \sigma(\pi - \theta)}$$

$A_{\rm fb} = 0$ if charge symmetry conserved!

$np \to d\pi^0$ and $dd \to \alpha\pi^0$

Miller and van Kolck: For TRIUMF and IUCF χPT (small $\mathbf{p}$)

$$A_{\rm fb}(np \to d\pi^0) \propto \frac{\langle \pi^0 | H | \eta \rangle}{-0.0059\ {\rm GeV}^2} - \frac{0.87}{\rm MeV}\left(\delta M - \frac{\bar{\delta} M}{2}\right)$$

$$\sigma(dd \to \alpha\pi^0) \propto \left[A\langle \pi^0 | H | \eta \rangle + B\delta M + C\bar{\delta} M + D\langle \rho^0 | H | \omega \rangle\right]^2$$

where A, B, C, D are numbers and $m_n - m_p = \delta M + \bar{\delta} M$;

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$\delta M \sim$ few MeV — strong term
$-2 < \bar{\delta} M < -1$ MeV — EM term

$\delta M - \frac{1}{2}\bar{\delta} M \sim 2.1$–$3.6$ MeV for reasonable δM and $\bar{\delta} M$

$\langle \pi^0 | H | \eta \rangle = (-5900)$ to (-4300) MeV^2, $g_\eta = ?$

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IUCF+TRIUMF constrain δM, $\bar\delta M$ and $\langle\pi^0|H|\eta\rangle$ ($\leftrightarrow \beta_1$ in χPT)

Lessons from $dd \to \alpha\pi^0$

Executive summary of Tuesday last week:

- Effectively one (1) data point (s-wave XS) PRL**91**, 142302 (2003).
- Formally leading amplitude suppressed (spin-isospin)
- Short range amplitudes (higher orders) are significant
- Many diagrams remain to be calculated
- Slow convergence
- dd wave function very complicated and largely unknown
- Realistic wfs overshoot data

AG *et al.*, PRC**69**, 044606 (2004)

Talks last week, available on-line soon!

$np \rightarrow d\pi^0$ should be easier!

- Both CS and CSB data exists!
- Wave functions well known
- Leading amplitudes not suppressed (cf. $pp \rightarrow pp\pi^0$)
- Plenty of work on CS $NN \rightarrow NN\pi^0$
- Influence of short-range needs to be checked

Theory for $NN \to NN\pi$

Extensive literature after IUCF (and TSL) $pp \to pp\pi^0$ expt.

Meyer et al., PRL**65**, 2846 (1990); NP**A539**, 633 (1992)

Bondar et al., PLB**356**, 8 (1995)

Short-range correlations and HME

T-S H Lee and DO Riska, PRL **70**, 2237 (1992).

CJ Horowitz, DK Griegel, and HO Meyer, PRC **49**, 1337 (1994).

Off-shell amplitudes

E Hernandez and E Oset, PLB **350**, 158 (1995). + many more

Modified χPT power counting scheme ($\sqrt{Mm_\pi}$ vs. m_π)

TD Cohen, JL Friar, GA Miller, and U van Kolck, PRC **53**, 2661 (1996).

+ many, many more articles

Status summarized in C Hanhart, PRt**397**, 155 (2004).

Cf. higher orders promoted for NN (Andreas on Tuesday)

CSB in $np \to d\pi^0$ II

Theory:

Cheung, Henley and Miller, PRL**43**, 1215 (1979)

Niskanen, Sebestyen, and Thomas, PRC38, 838 (1988)

η–π^0 mixing; $A_{\rm fb} = -0.28\%$

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η–π^0 + χPT (δM and $\bar{\delta} M$)

$A_{\text{fb}} = 0.23\%$–0.60%: Change of sign!

van Kolck, Niskanen, and Miller, PLB**493**, 65 (2000)

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Recent TRIUMF result confirms sign and size:

$A_{\mathrm{fb}} = [0.172 \pm 0.080(\mathrm{stat}) \pm 0.055(\mathrm{syst})]\%$

Opper *et al.*, PRL**91**, 212302 (2003)

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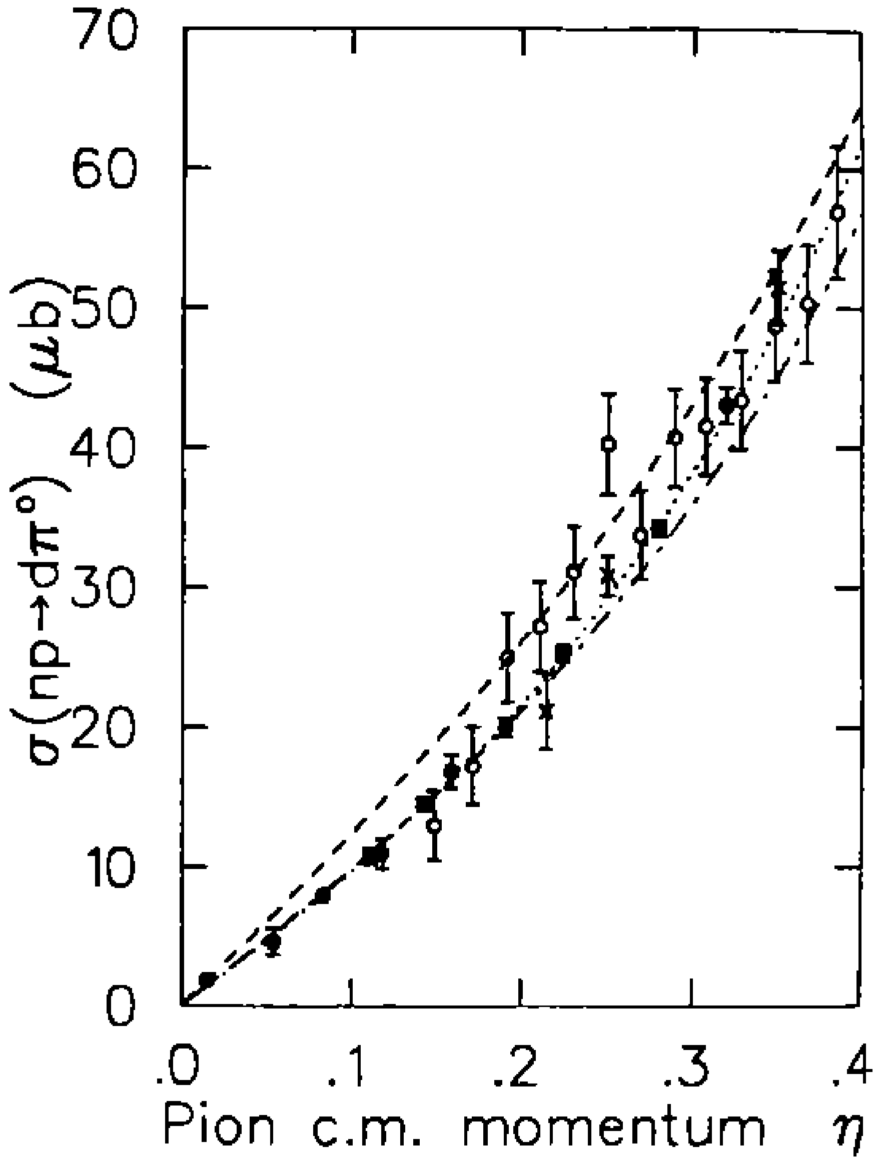
No consistent chiral calculation carried out yet!

TRIUMF CS data for $np \rightarrow d\pi^0$



$\eta = q/m_\pi$

$\eta < 0.32$ for TRIUMF expt (•)

other data: $\frac{1}{2}\sigma(pp \rightarrow d\pi^+)_{\rm CC}$

$\sigma(np \rightarrow d\pi^0) = \frac{1}{2}(\alpha\eta + \beta\eta^3)$

$\alpha = 184 \pm 5$ μb

$\beta = 781 \pm 79$ μb

Hutcheon et al., PRL**64**, 176 (1990); NP**A535**, 618 (1991).

Low-level analysis

CSB ($\alpha' \ll \alpha$, $\beta' \ll \beta$) $\Rightarrow$

$\sigma(np \to d\pi^0) = \frac{1}{2}(\alpha\eta + \beta\eta^3) + \frac{1}{2}(\alpha'\eta + \beta'\eta^3)$

Ignoring phases:

$$A_{\rm fb}(np \to d\pi^0) = \frac{\sqrt{\alpha\beta'}\eta^2 + \sqrt{\alpha'\beta}\eta^2}{\alpha\eta + \beta\eta^3} \approx \sqrt{\frac{\beta'}{\alpha}}\eta + \frac{\sqrt{\alpha'\beta}}{\alpha}\eta$$

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$A_{\rm fb}$ depends on understanding of CS XS!

$1^{\rm st}$ task: calculate $\sigma(np \to d\pi^0)$!

Partial wave analysis

Spin-isospin, parity and symmetry $\Rightarrow$ $({}^{2S+1}L_{np} l_\pi)$

${}^3P_1 s$ $(\leftrightarrow \sqrt{\alpha})$ CS ${}^1S_0 p$, ${}^1D_2 p$ $(\leftrightarrow \sqrt{\beta})$

${}^1P_1 s$ $(\leftrightarrow \sqrt{\alpha'})$ CSB ${}^3S_1 p$, ${}^3D_1 p$, ${}^3D_2 p$ $(\leftrightarrow \sqrt{\beta'})$

$A_{\rm fb} \leftrightarrow S$-$P$-wave singlet-singlet and triplet-triplet interferences

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Need to calculate:

- CS triplet pion s-wave
- CS singlet pion p-waves
- CSB singlet pion s-wave
- CSB triplet pion p-waves

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${}^3P_1s\ (\leftrightarrow \sqrt{\alpha})$	CS	${}^1S_0p,\ {}^1D_2p\ (\leftrightarrow \sqrt{\beta})$
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For now restrict to:

- CS triplet pion s-wave

Power counting (CS)

Energy/momentum scales:

$$\mathbf{q} \lesssim \gamma = \sqrt{MB} \ll m_\pi < \Delta < P = \sqrt{Mm_\pi} \ll \Lambda_\chi$$
$$24 \lesssim 46 \ll 140 < 290 < 360 \ll 800 \quad \text{MeV}$$

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Pion production: new scale $P = \sqrt{M m_\pi}$ (initial N momentum)

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Expand in $x \equiv \frac{P}{M} = \frac{m_\pi}{P} = \sqrt{\frac{m_\pi}{M}}$, not $\frac{m_\pi}{M}$

Also use η (mainly p-waves)

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With explicit Δ poorly separated scales. Looks really bad!

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Don't despair, I'll show later what saves the day…

CS Lagrangian

$$
\begin{aligned}
\mathcal{L}_{CS} &= -\frac{1}{4f_\pi^2} N^\dagger \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \dot{\boldsymbol{\pi}}) N \\
&+ \frac{g_A}{2f_\pi} \left\{ N^\dagger \boldsymbol{\tau} \cdot \vec{\sigma} \cdot \vec{\nabla} \boldsymbol{\pi} N - \frac{1}{2m_N} \left[i N^\dagger \boldsymbol{\tau} \cdot \dot{\boldsymbol{\pi}} \vec{\sigma} \cdot \vec{\nabla} N + \mathrm{H.c.} \right] \right\} \\
&+ \frac{h_A}{2f_\pi} \left\{ N^\dagger \boldsymbol{T} \cdot \vec{S} \cdot \Delta(\vec{\nabla} \boldsymbol{\pi}) + \mathrm{H.c.} \right. \\
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$\Rightarrow \pi$ prod operator $\mathcal{O}$

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$\Rightarrow \pi$ prod operator $\mathcal{O}$

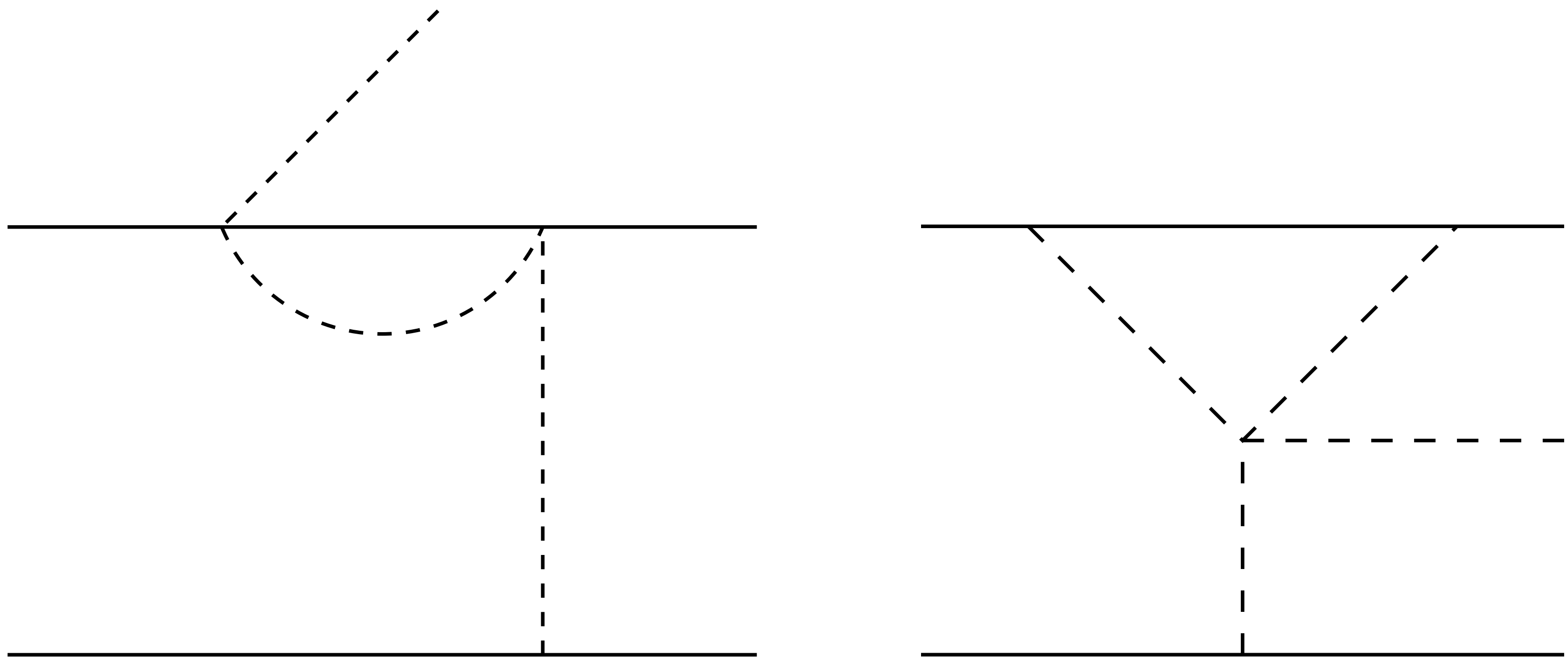
Sandwiched between wfs (we’ll use a menagerie of wfs):

$$\mathcal{M} \sim \langle f | \mathcal{O} | i \rangle$$

Power counting in loops

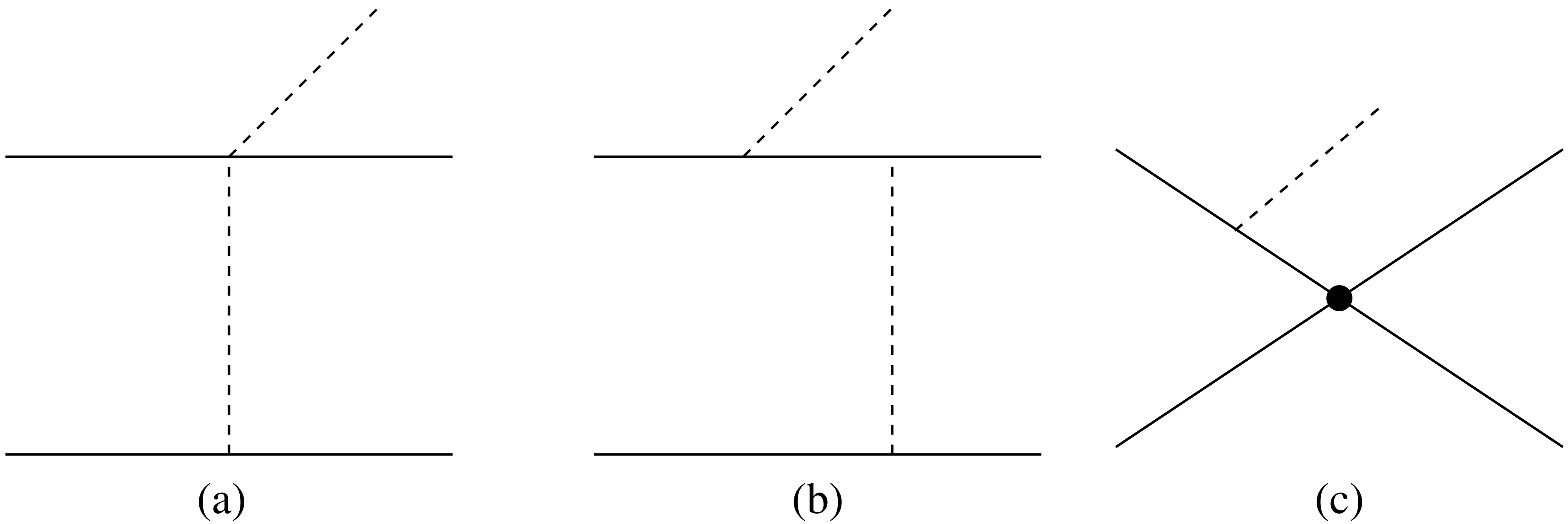
Two cases:

- P can be carried by nucleon: scales with m_π (left)
- P has to pass through a pion: scales with P (right)



P-reducible *P-irreducible*

CS s-wave LO, $O(x)$



Weinberg-Tomozawa (a) + single-nucleon π production (b,c)

Pion exchange and contact term $\rightarrow$ wave function

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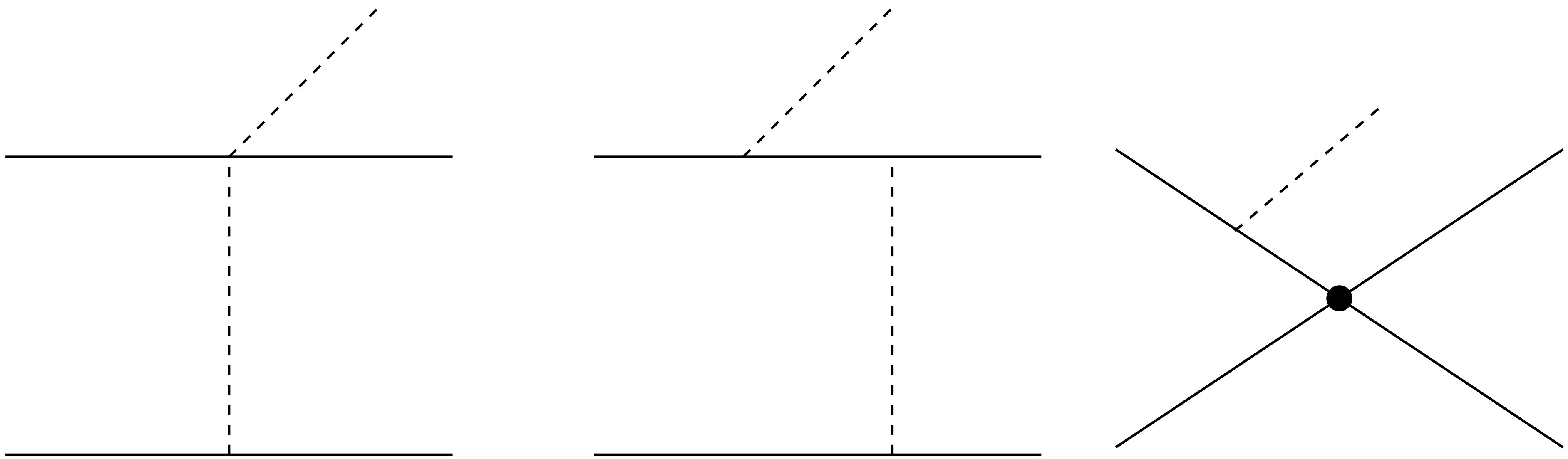
(a) (b) (c)

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Caveats for W-T and 1-body

CAUTION!

Remember the separation:

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$$\sigma = \frac{1}{2}[\alpha_0\eta + (\alpha' + \beta)\eta^3]$$

Leakage from α to β!
Experiments with ang dis or polarization needed!

What is a one-body term, really?

Power counted using OPE, leads to ‘off-shell’ nucleon

But: wfs (potential or chiral) have ‘on-shell’ nucleons and fixed kinematics (FK) pion exchanges

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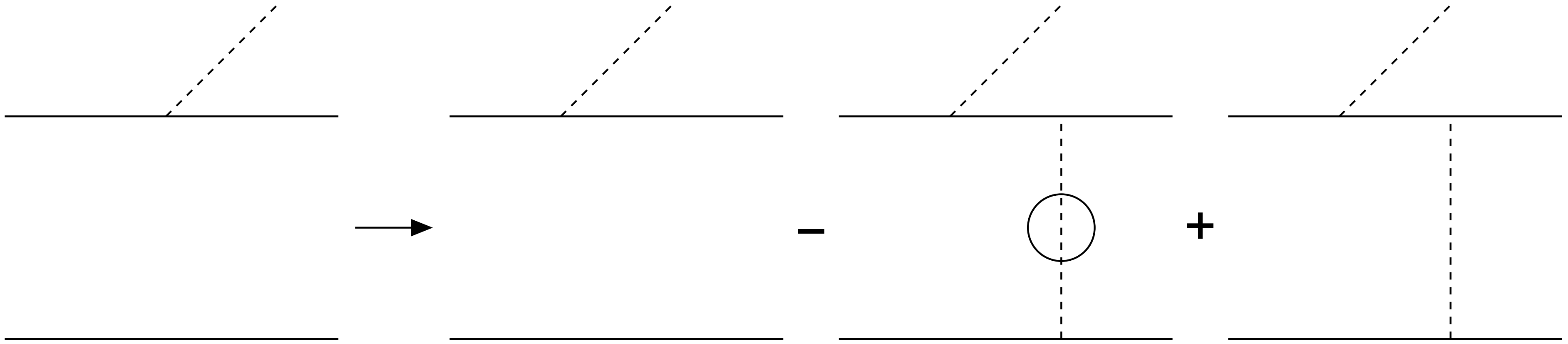
Correction: 1B - FKOPE + full OPE (TOPT), in ISI and FSI

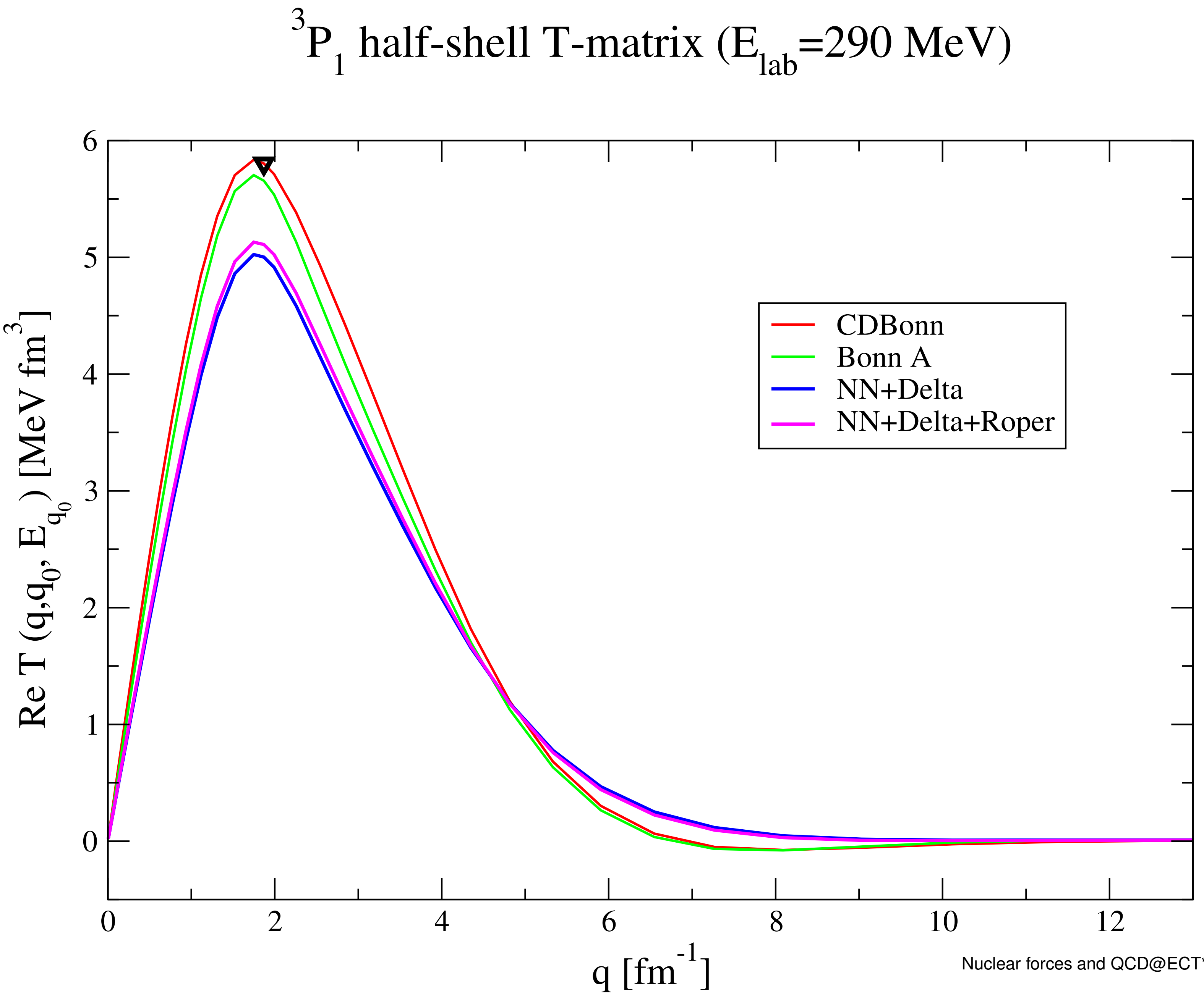
For FSI (first correction):



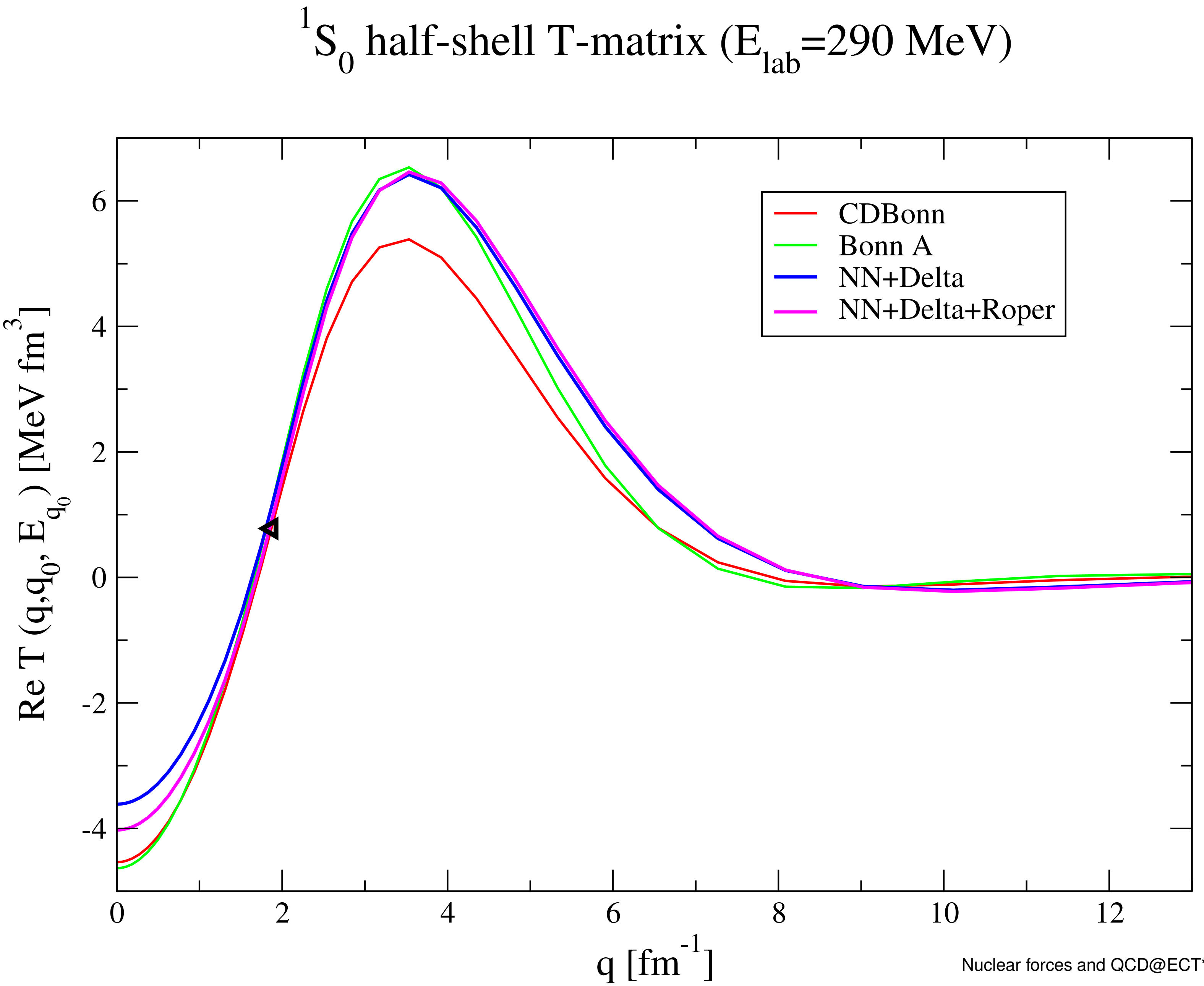
Numerical investigation needed!

Hopefully small effect $\Rightarrow$ further corrs not needed?

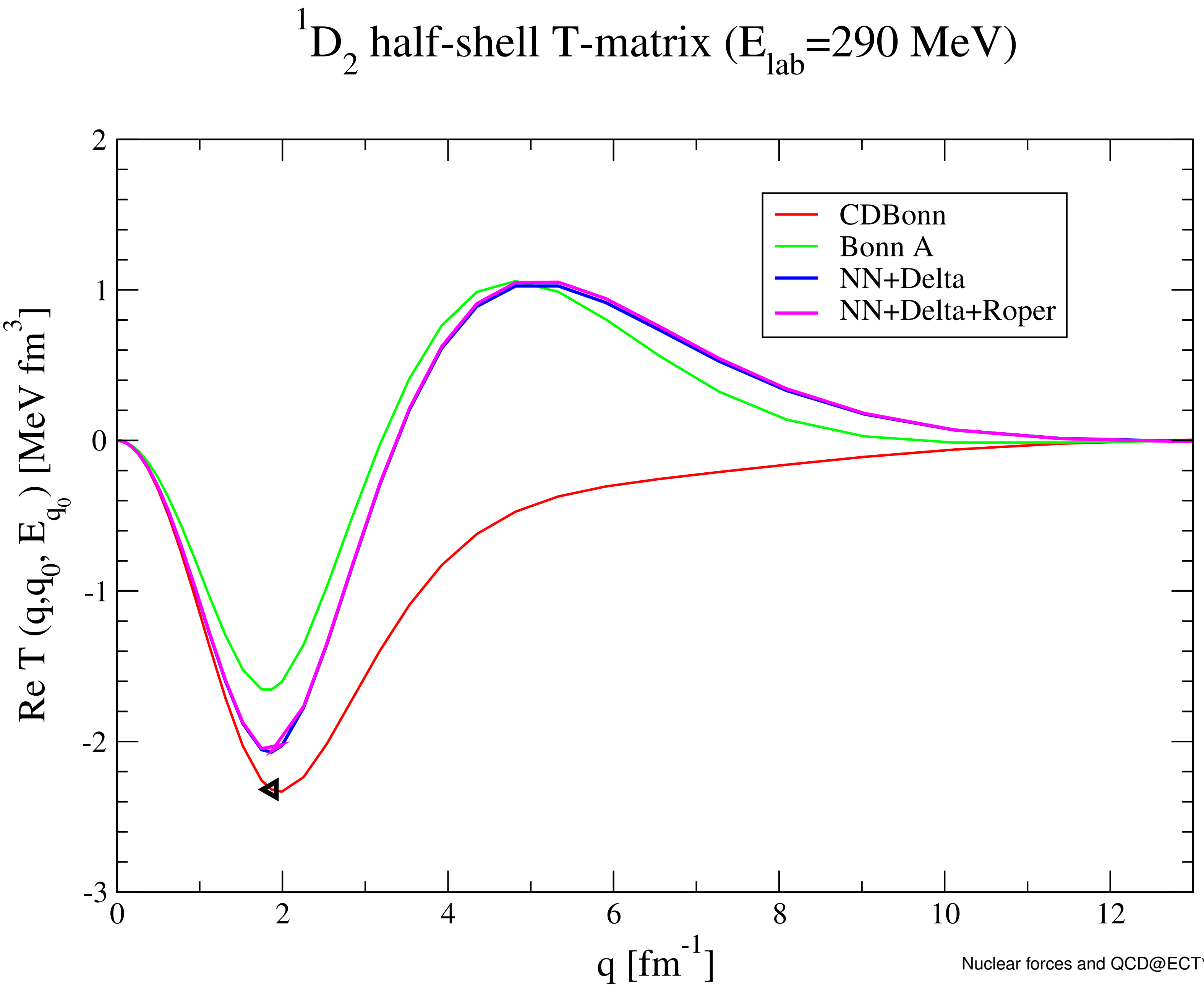
Half-off-shell $T(^3P_1)$



Half-off-shell $T(^1S_0)$

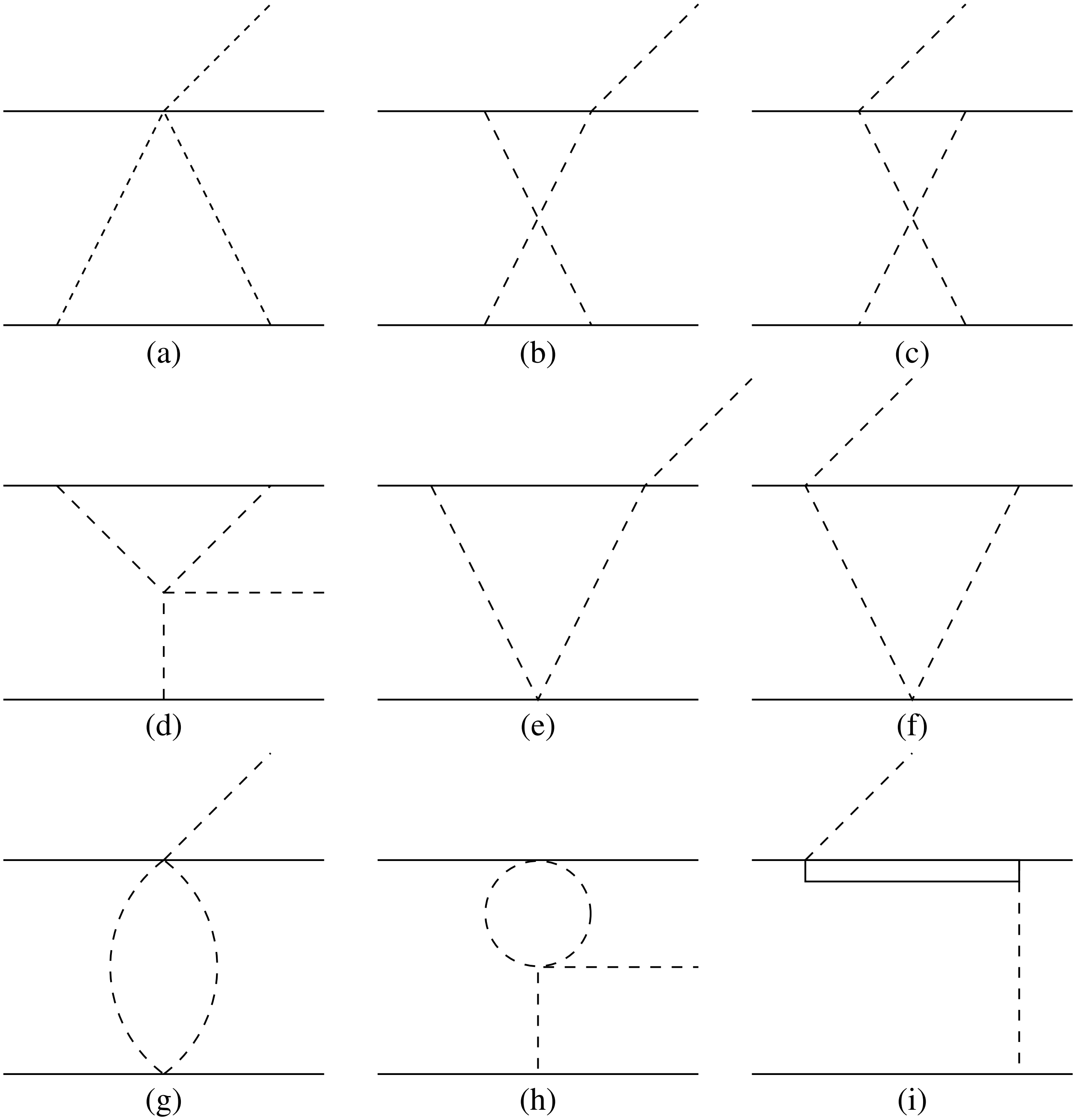


Half-off-shell $T(^1D_2)$



CS s-wave NLO, $O(x^2)$

Hanhart & Kaiser, [PRC**66**, 054005 (2002)]



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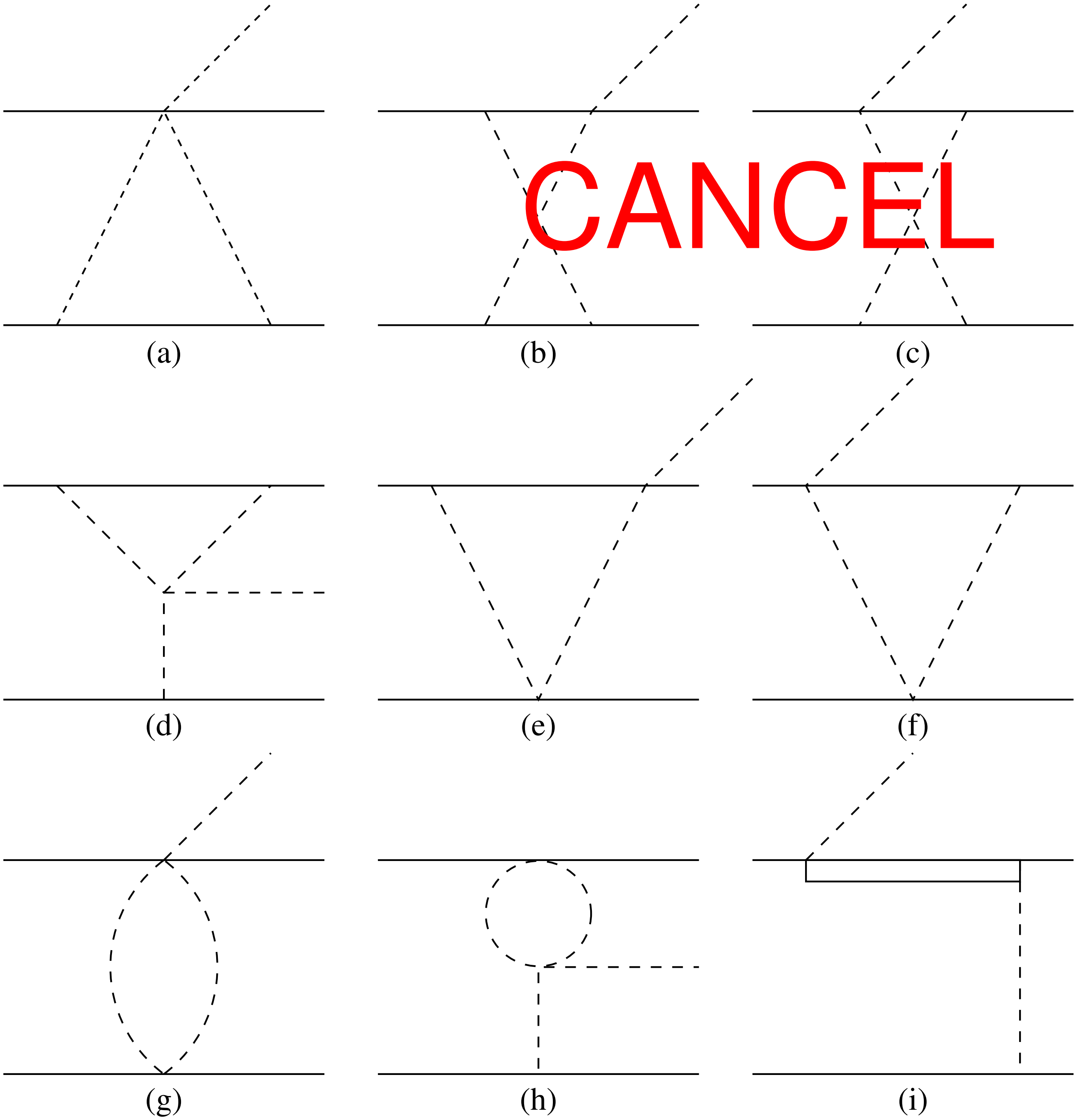

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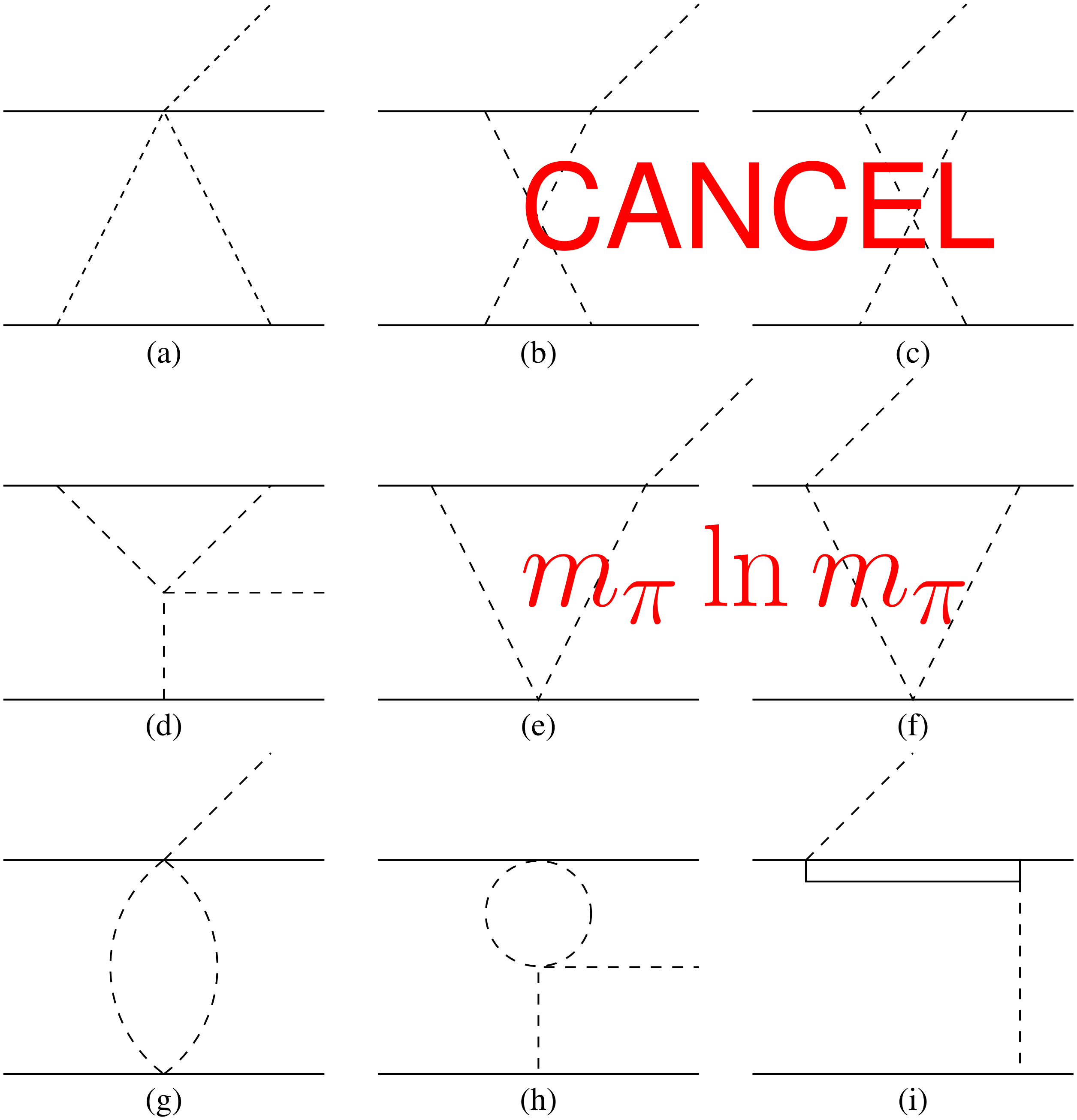

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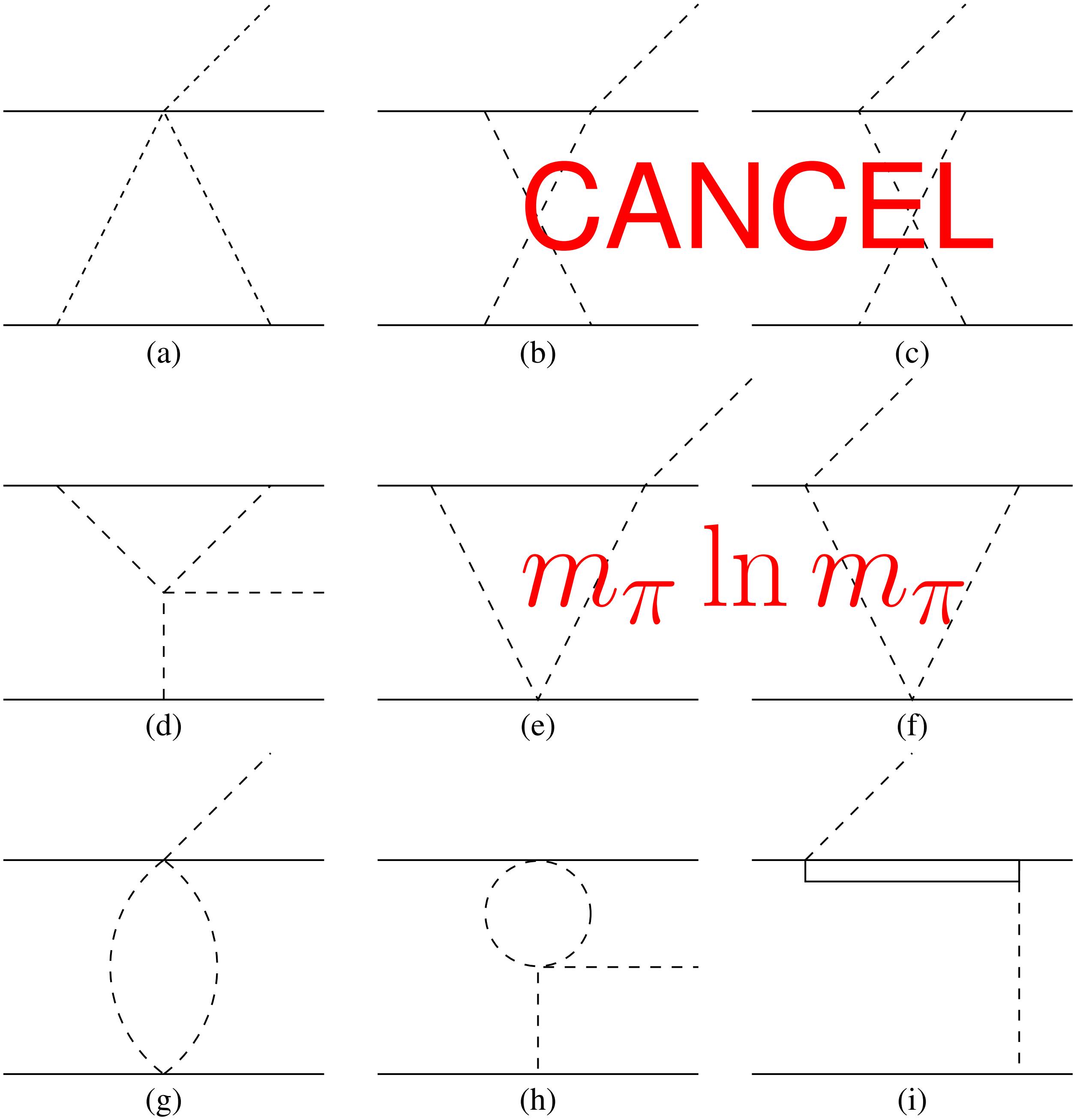

$$\frac{1}{m_\pi + \Delta} \sim \frac{1}{P}$$

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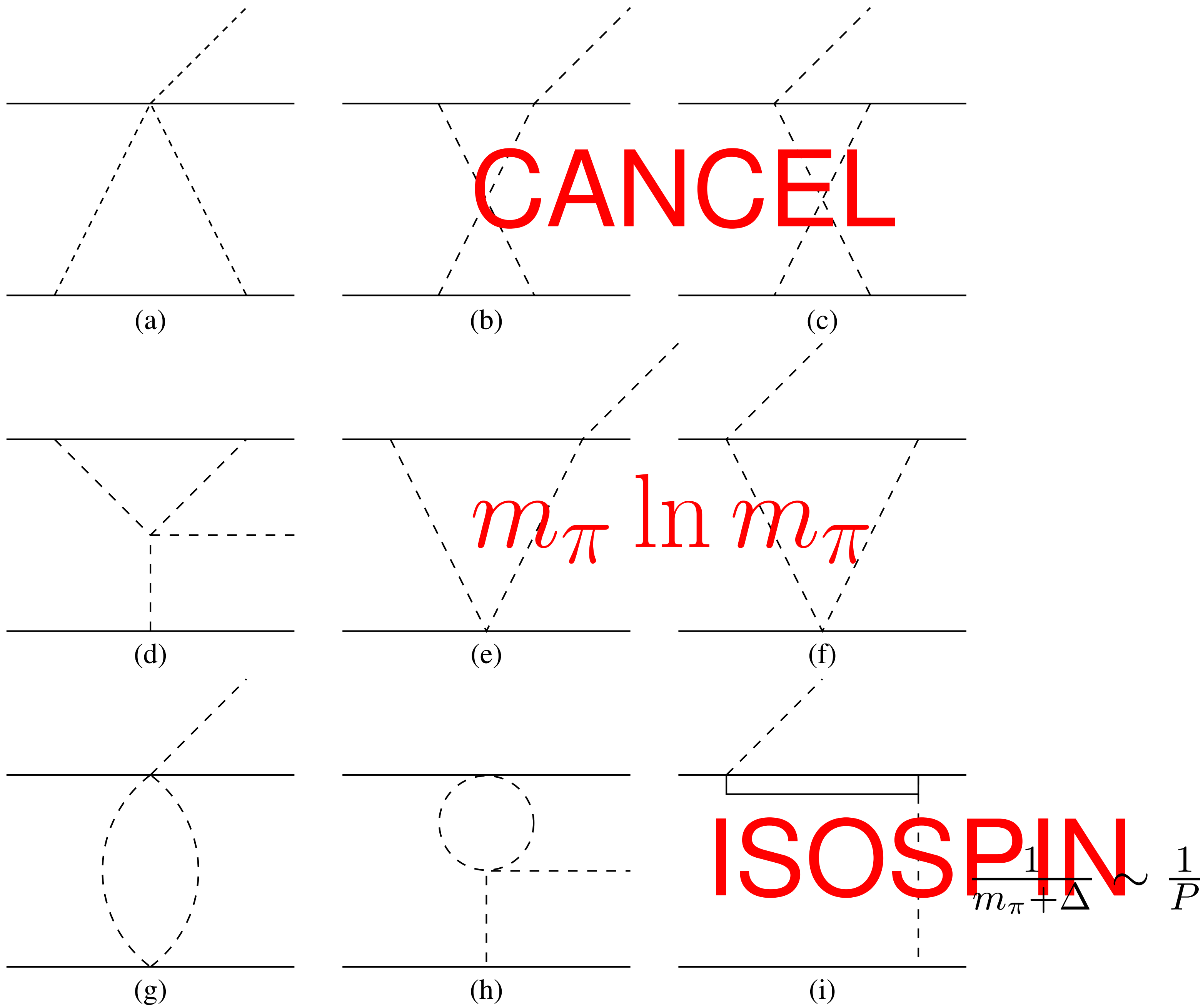

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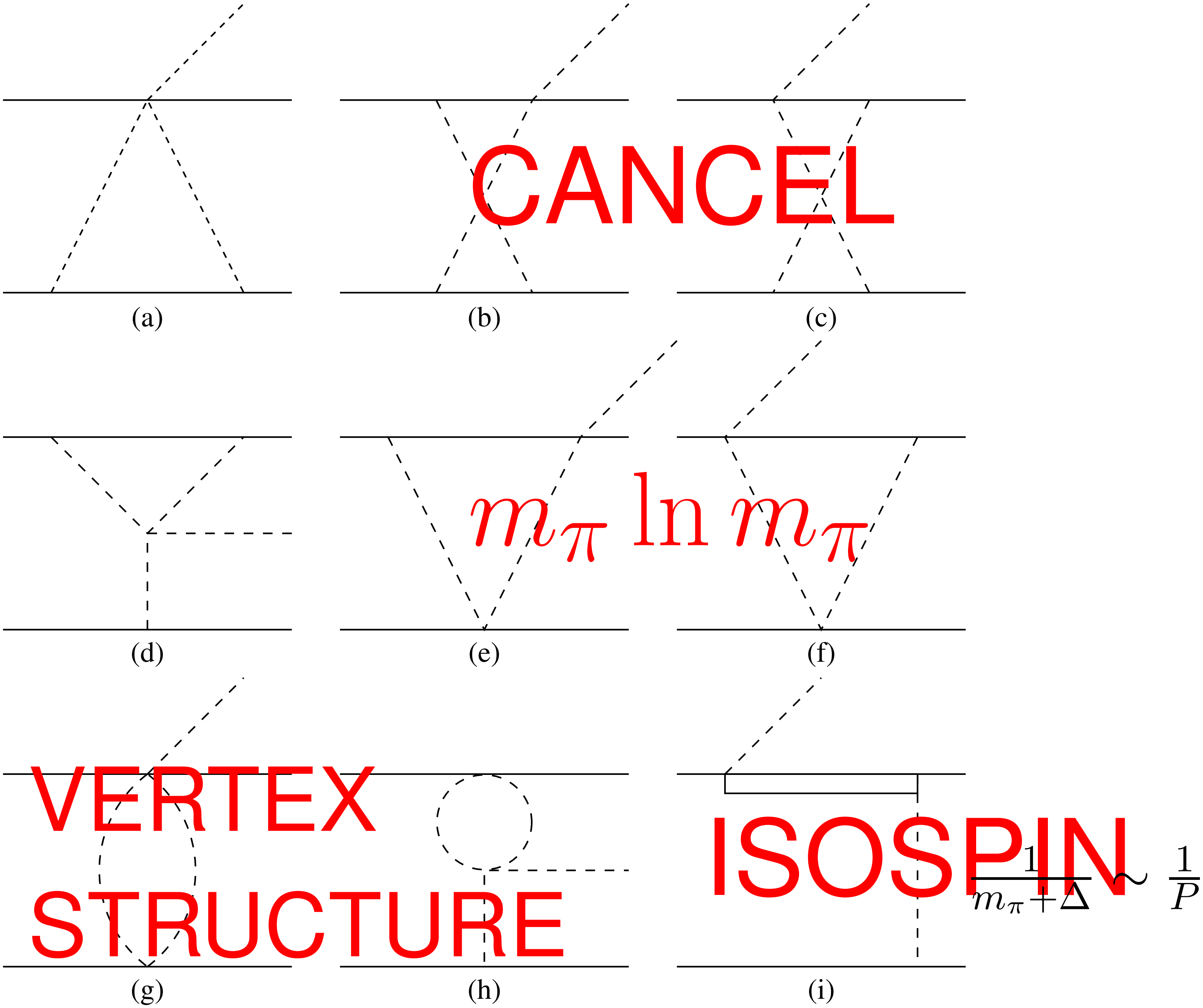
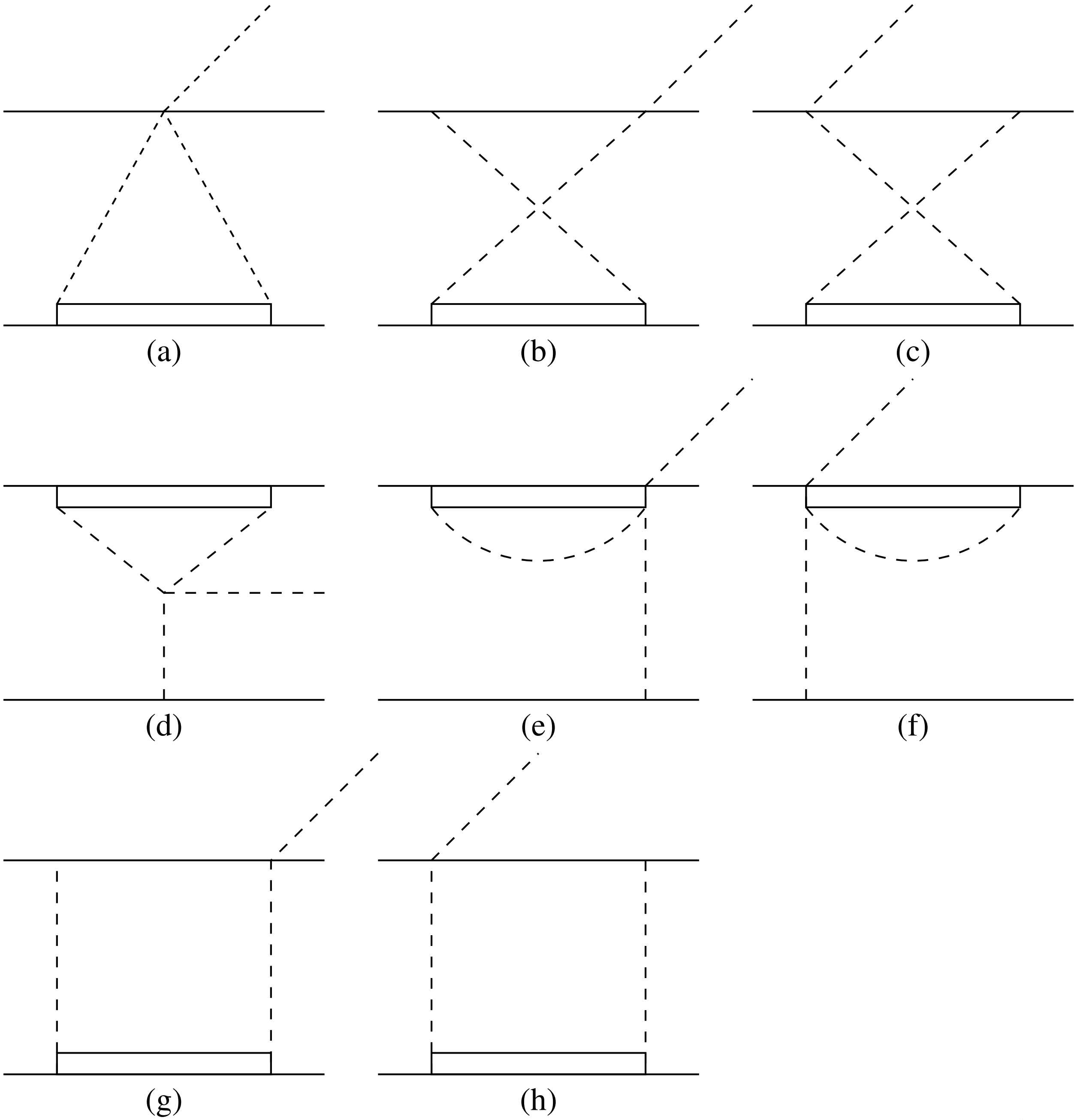
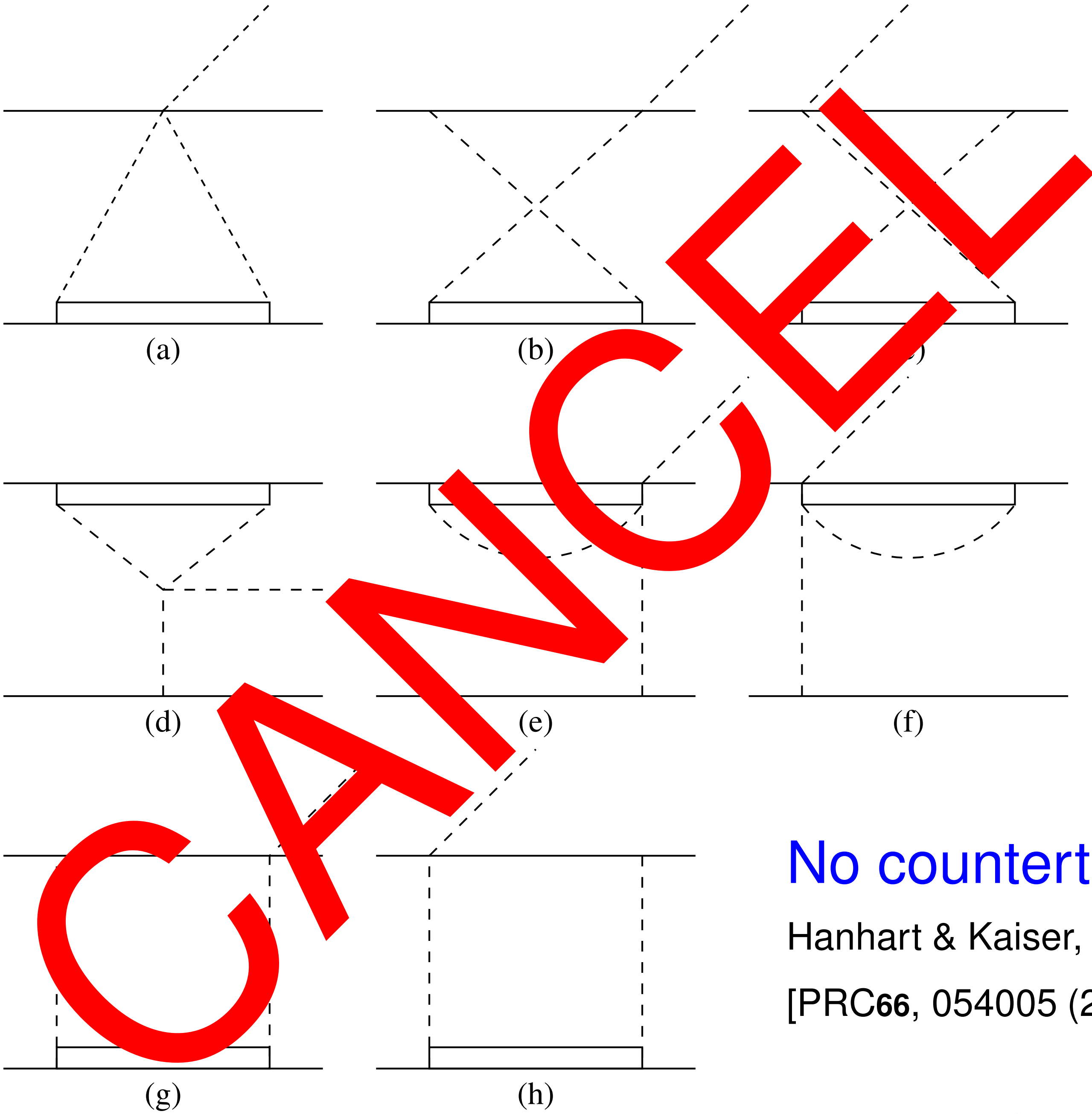


CS *s*-wave cont'd



CS *s*-wave cont'd

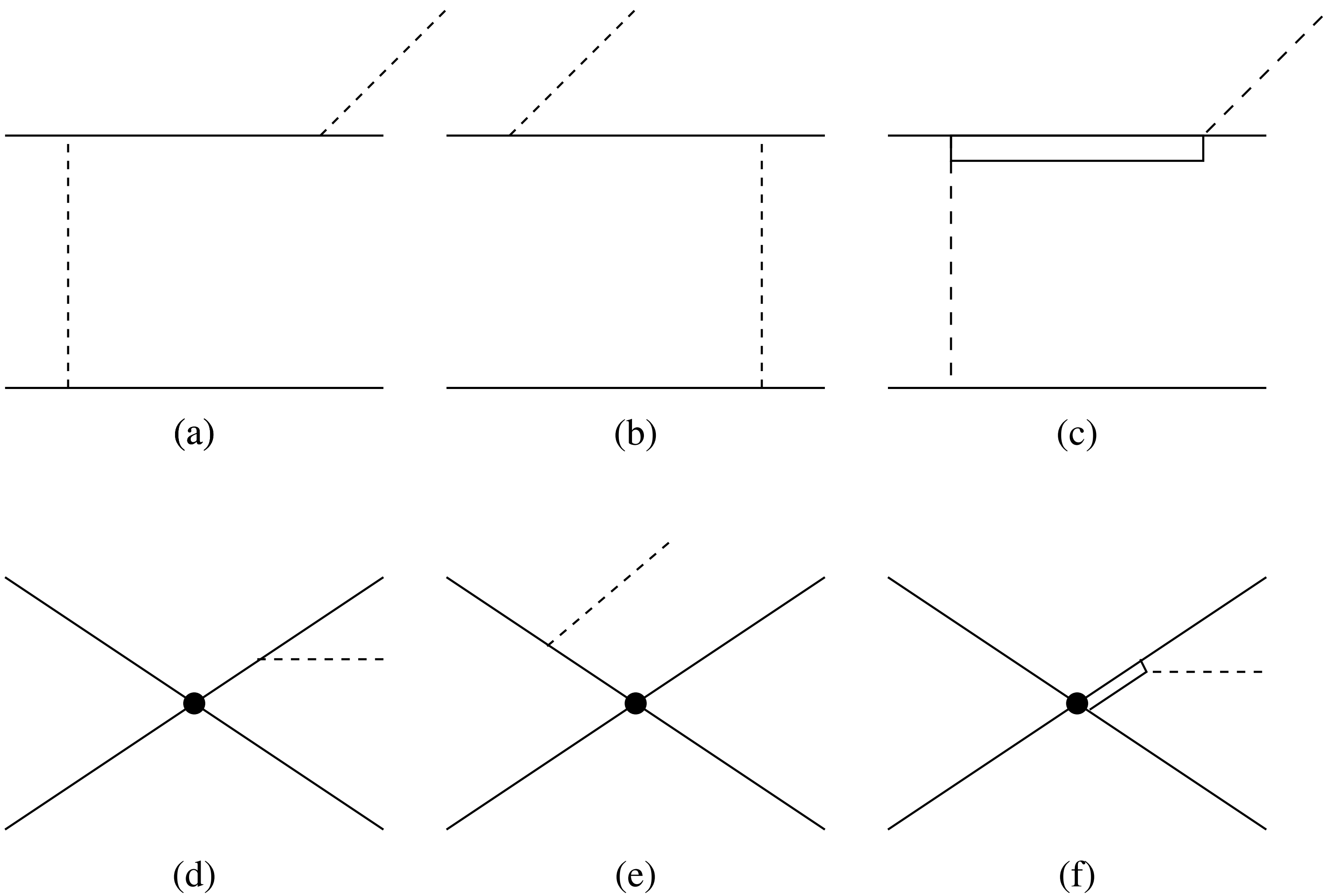


No counterterm

Hanhart & Kaiser,

[PRC**66**, 054005 (2002)]

CS p-wave LO, $O(\eta)$



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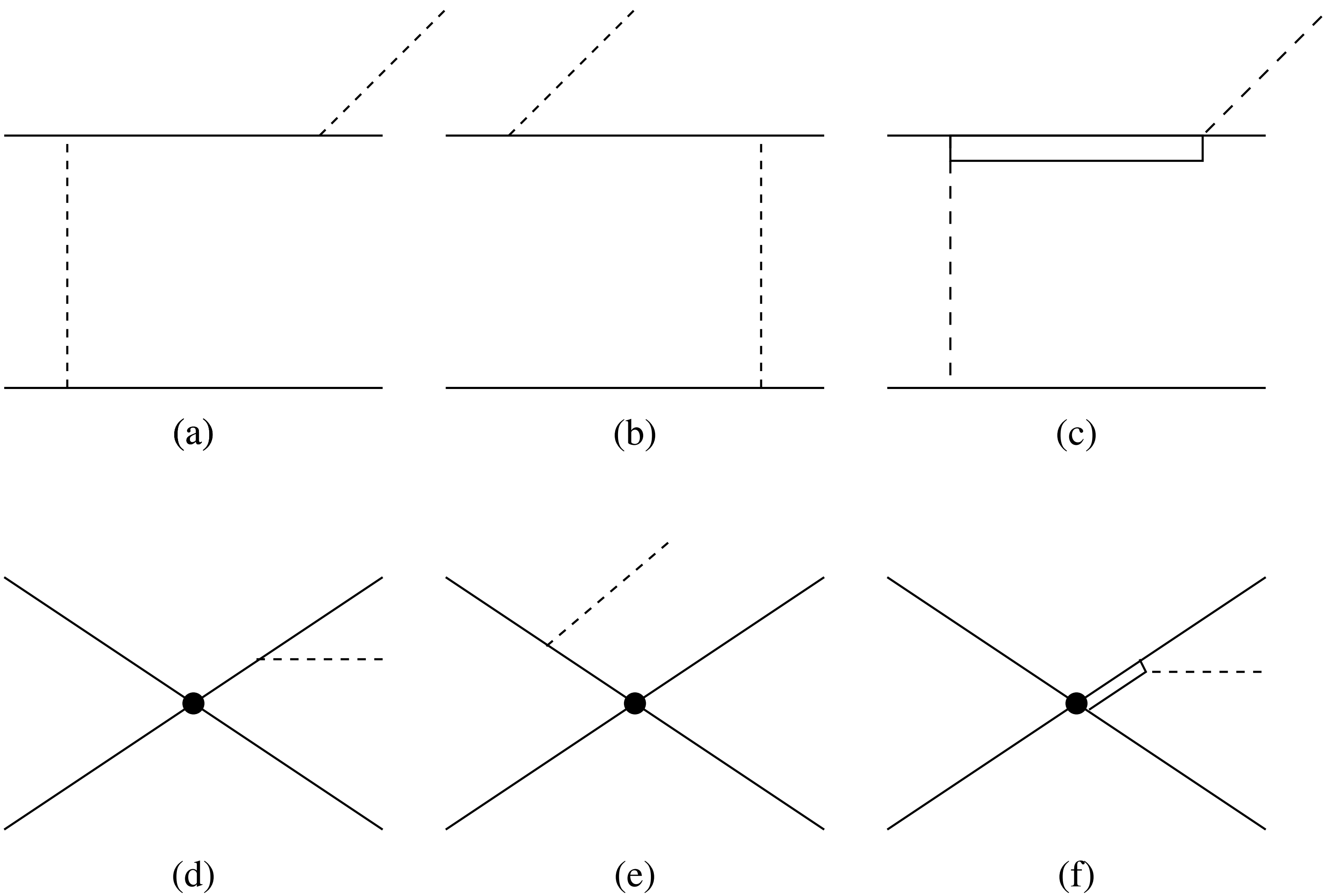

Δ propagator: $1/(\Delta - m_\pi) \sim 1/m_\pi$

No surviving Δ scale!

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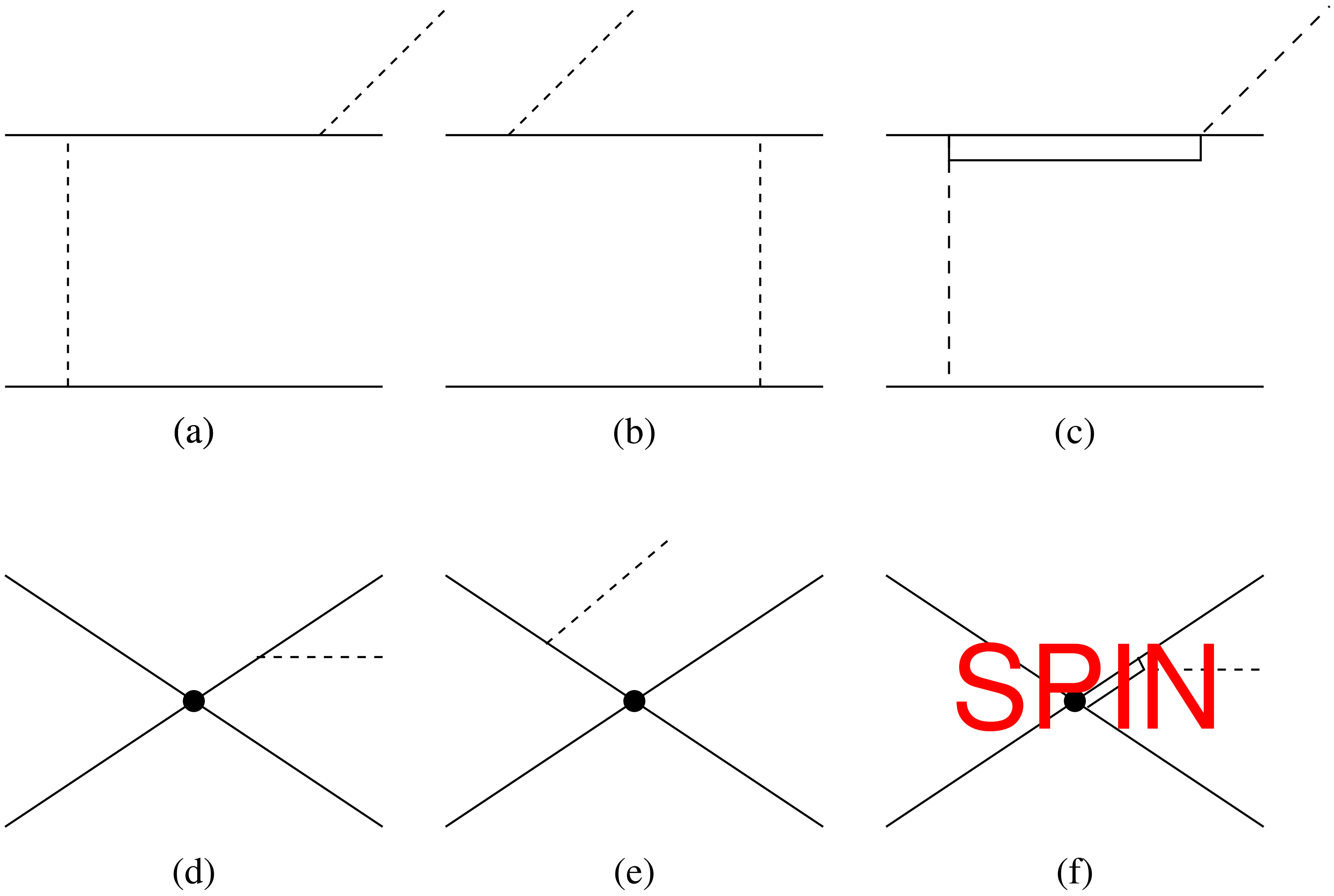

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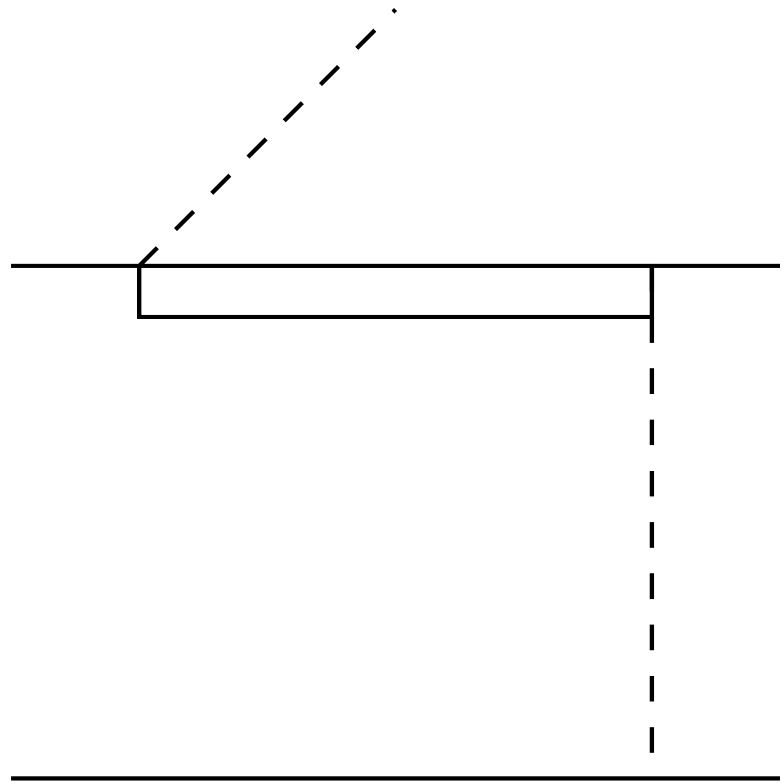
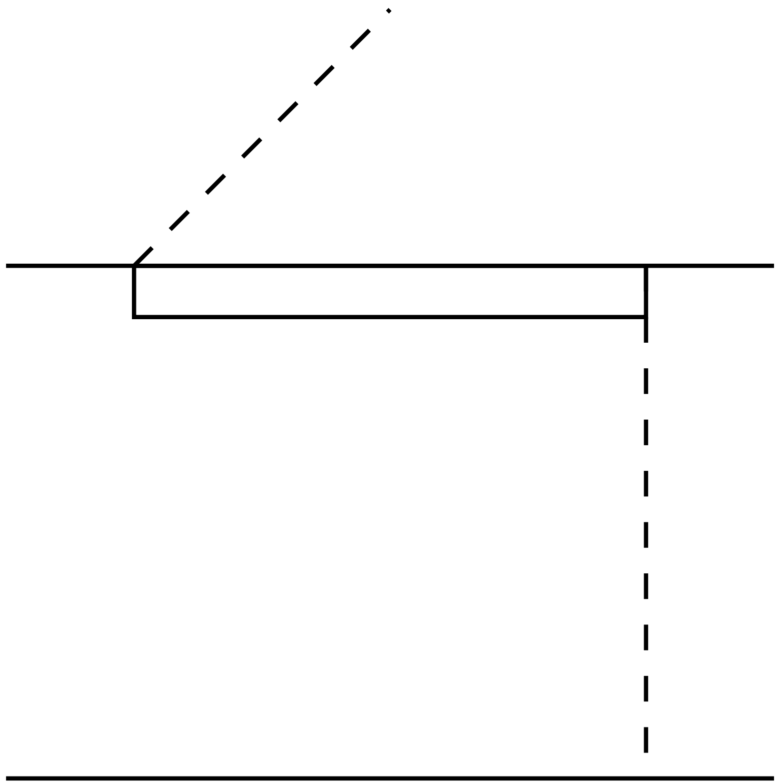
CS p-wave NLO, $O(x\eta)$



Δ propagator: $1/(\Delta + m_\pi) \sim 1/P$

Δ scale disappears again!

CS p-wave NLO, $O(x\eta)$



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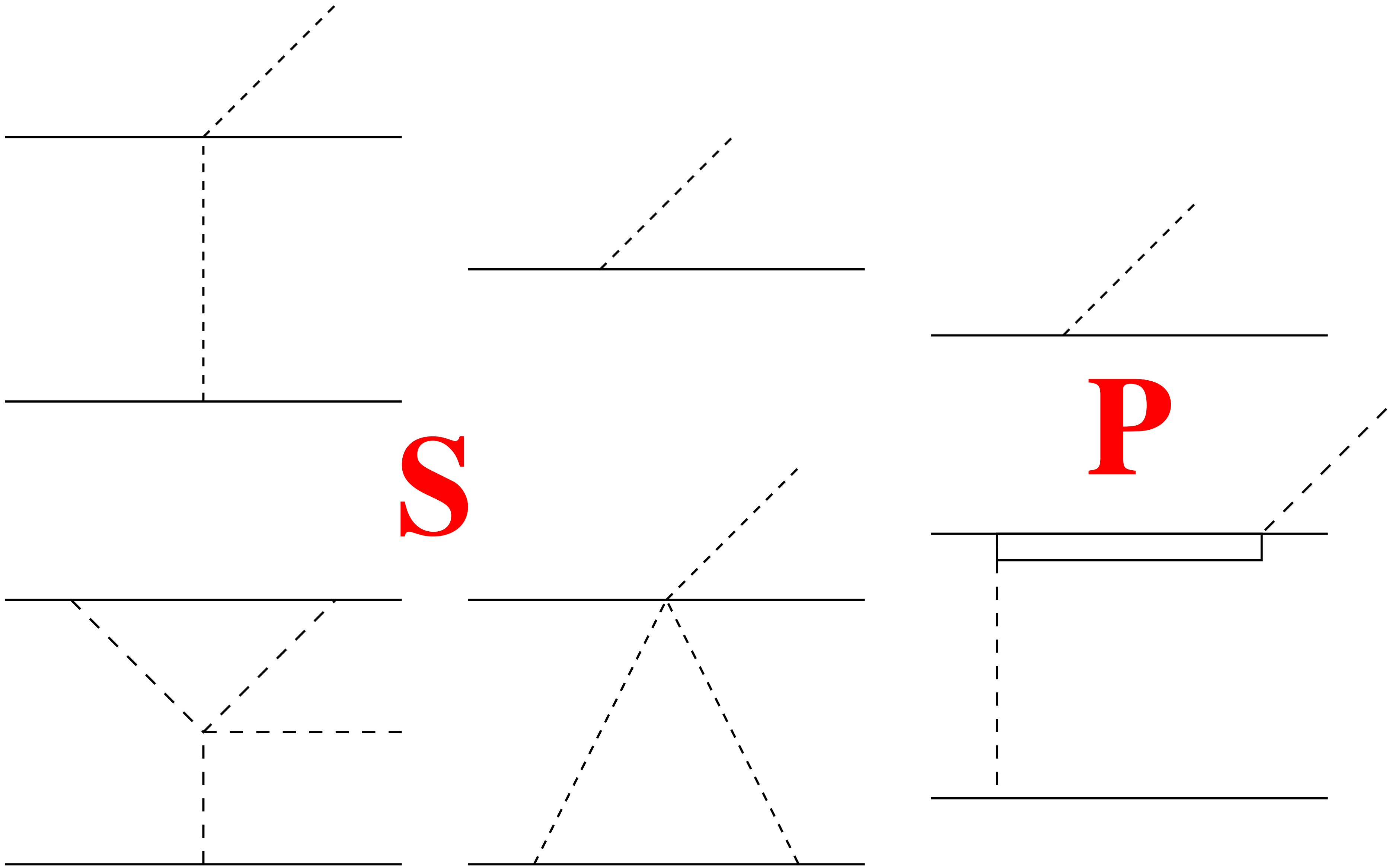
Deuteron isoscalar $\Rightarrow$ no NLO p-wave diagram!

Summary CS diagrams to NLO



Calculate α to NLO

Impl. of WT, one-body and Δ (p-wave) straight-forward

Hanhart & Kaiser, PRC **66**, 054005 (2002) surviving DR'd loops:

$$(\text{isospin}) \times \frac{g_A^3 p(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \mathbf{p}}{256 f_\pi^5}$$

In r-space, with gaussian cut-off Λ:

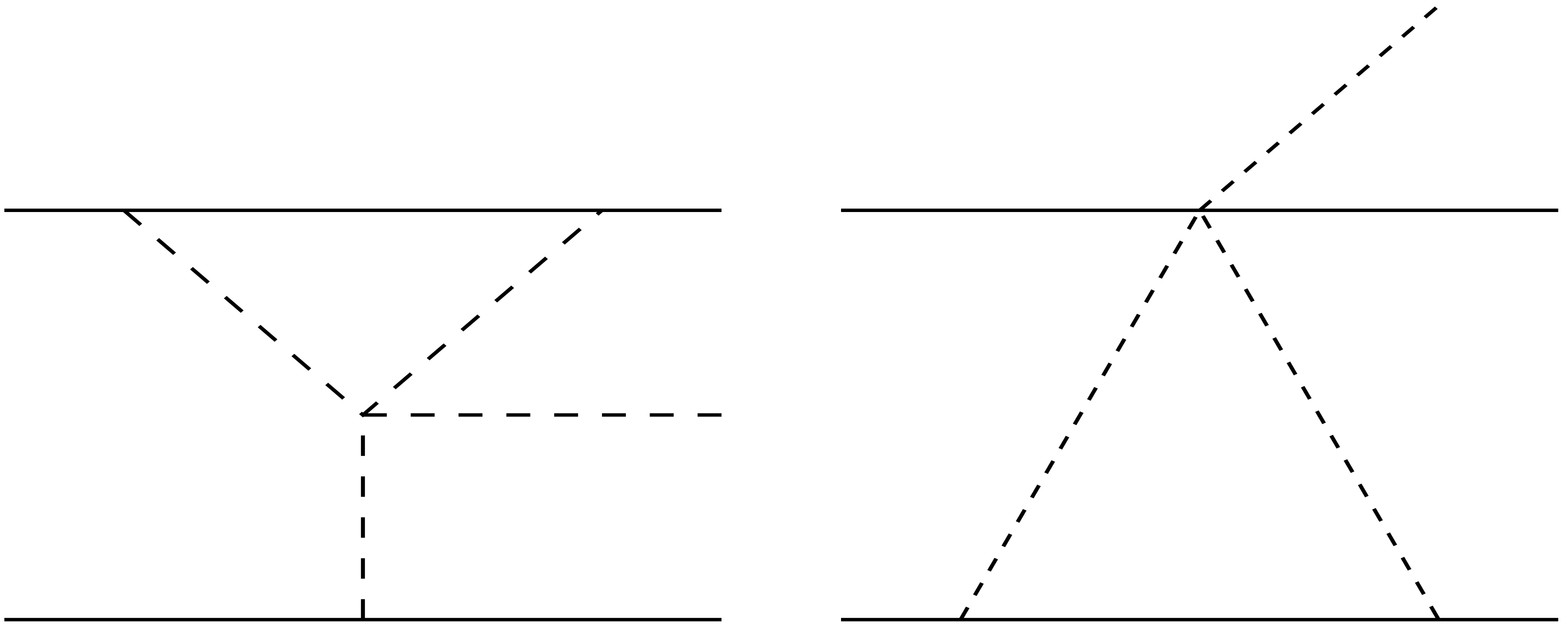
$$\mathcal{M} \propto \Lambda^5 \int dr u(r) F(\Lambda r) v(r)$$

$$F(x) = \frac{1}{x} - x + \sqrt{2}\left(\frac{1}{x^2} + 2 - x^2\right) e^{-\frac{x^2}{2}} \int_0^{\frac{x}{\sqrt{2}}} dt e^{t^2} \rightarrow \frac{8}{x^5} \text{ (for large } x)$$

$\Rightarrow \mathcal{M} \propto \Lambda$ (for large Λ)

Renormalization I (operators)

Renormalize div. loop diagrams (using DR)



DR $\Rightarrow$ divergencies removed from operators, excellent!

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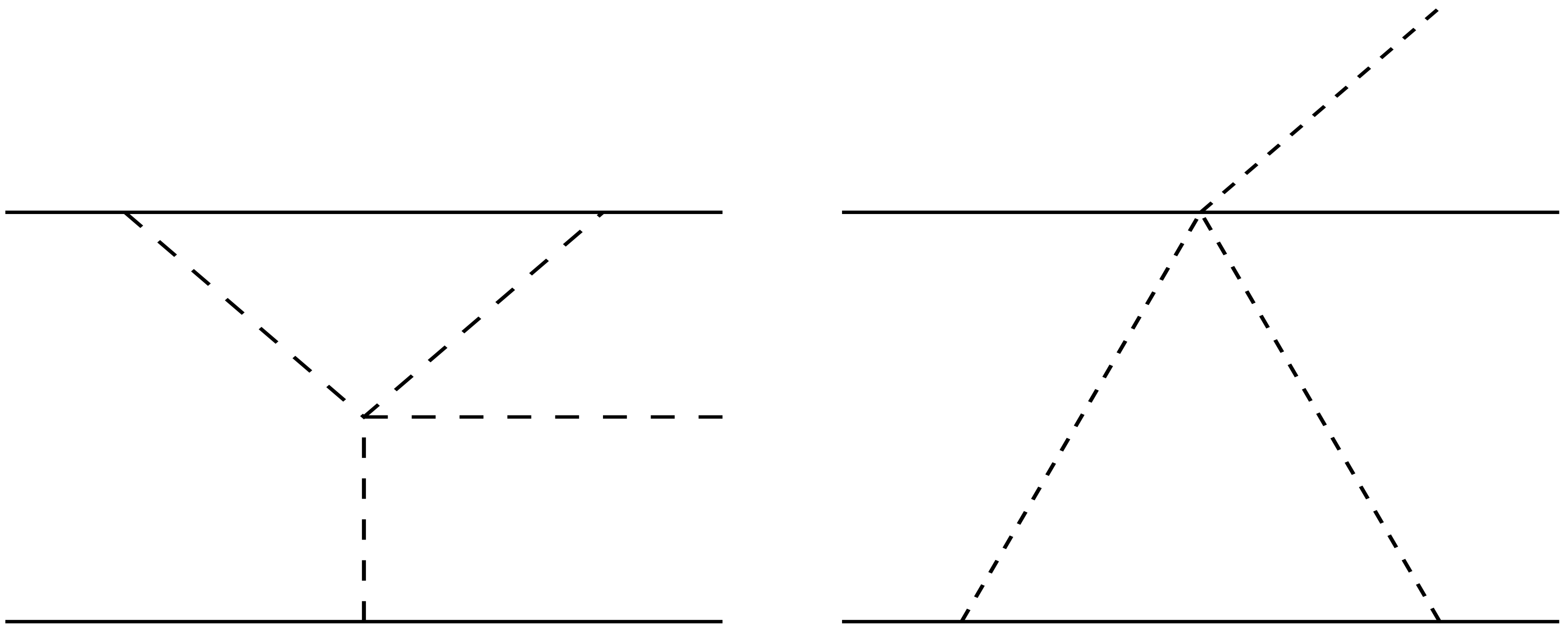


DR ⇒ divergencies removed from operators, excellent!

But we need MEs and XSs, and thus wfs

Might not be convergent if operator is singular
Breaks chiral sym! (linear div)

First results (preliminary)

Without Δ amplitudes:

	α [μb]			β [μb]
Λ [MeV]	500	800	1200	—
NijmI	333	293	259	723
exp't	184±5 μb			781±79 μb

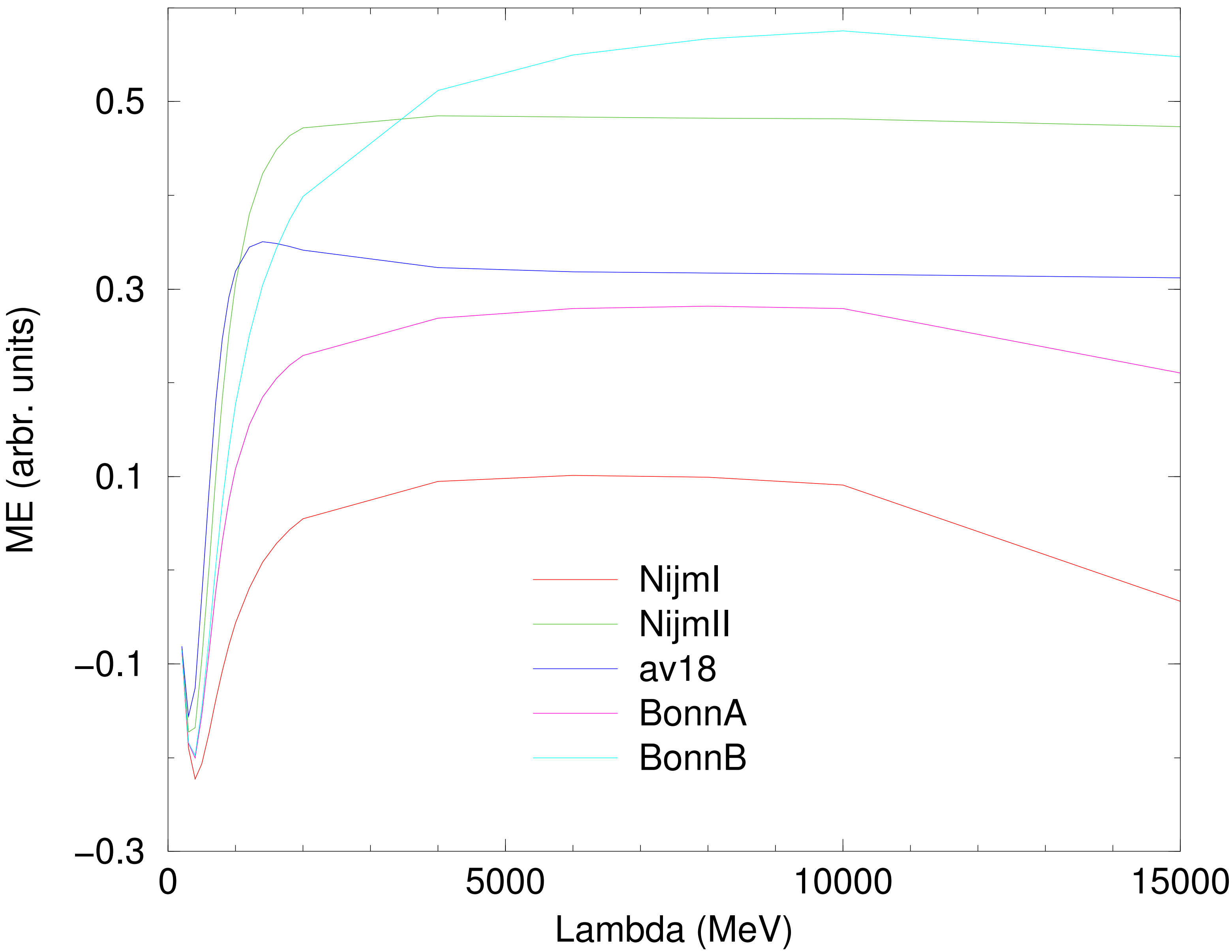
First results (preliminary)

Without Δ amplitudes:

	α [μb]			β [μb]
Λ [MeV]	500	800	1200	—
NijmI	333	293	259	723
NijmII	217	134	86.9	653
av18	146	80.0	61.2	662
BonnA	253	190	153	604
BonnB	265	190	137	653
exp't	184±5 μb			781±79 μb

What's going on here?

NLO loops



0.5
0.3
0.1
−0.1
−0.3
0
5000
10000
15000
Lambda (MeV)
ME (arbr. units)
NijmI
NijmII
av18
BonnA
BonnB

Renormalization II ($\mathcal{M}$)

Renormalize ME when ISI and FSI (non-pert wfs) included

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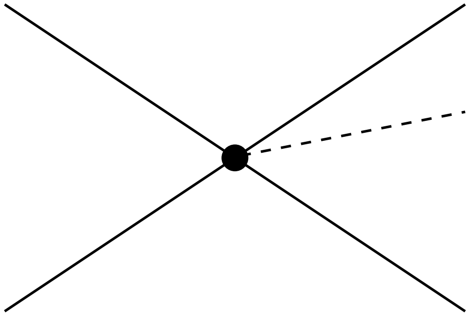
Regularize non-pert wfs???

2nd ren num, counter term to restore (?) χ symmetry:



$$(\text{isospin}) \times \frac{\tilde{d}_0(\Lambda)}{M f_\pi^3} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \mathbf{p}$$

$$\mathcal{M} \propto \Lambda^5 \int dr u(r) r e^{-\frac{r^2\Lambda^2}{2}} v(r) \to \tilde{d}_0(\Lambda) \text{ (for large } \Lambda)$$

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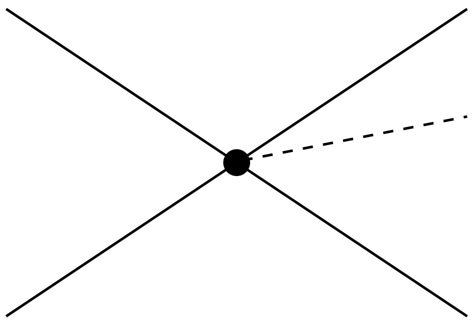

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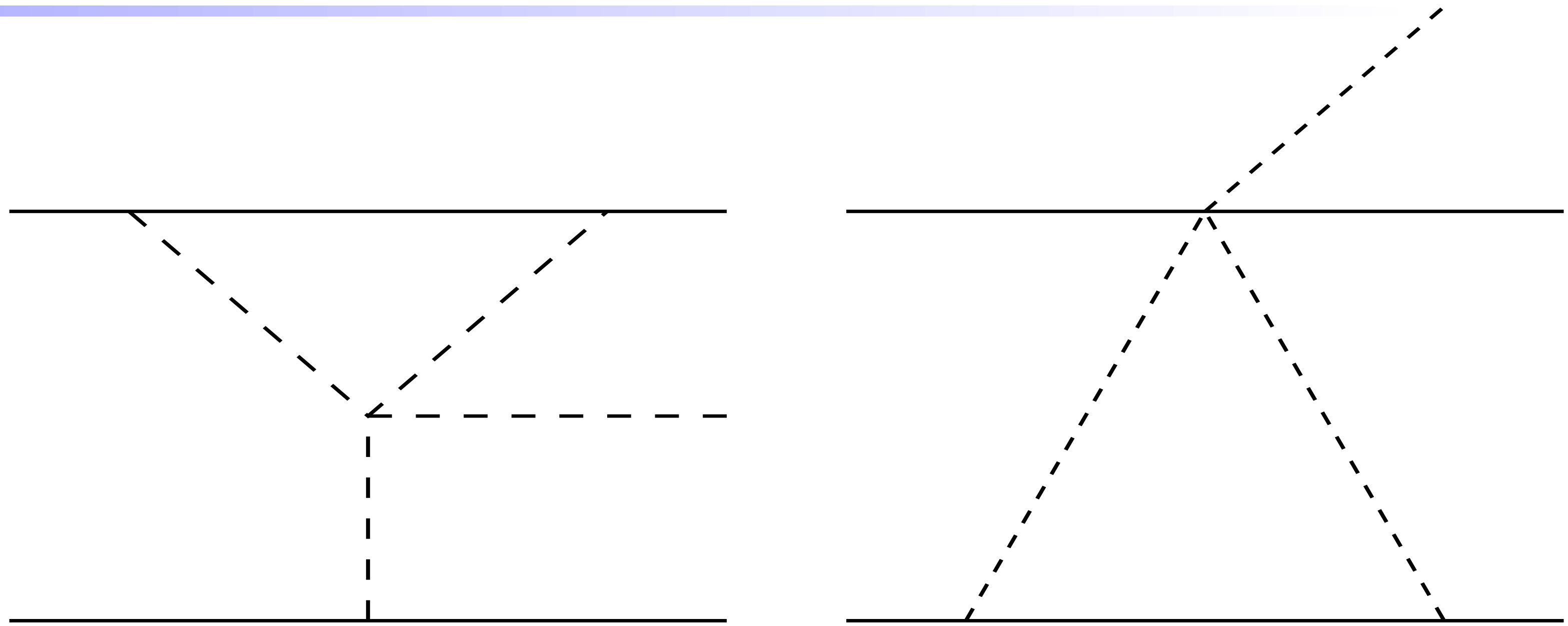
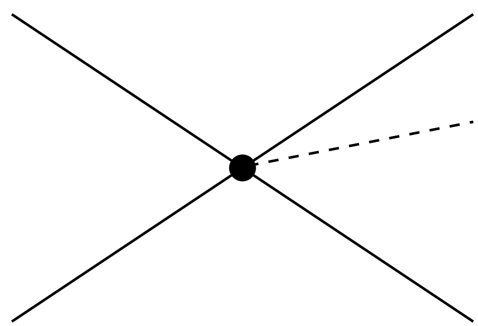
- mixing DR and cut-off?
- DR/cut-off for matrix elements? $V_{\mathrm{low}k}$ useful?
- Using cut-off+counter term to ren $\mathcal{O} \Rightarrow$ autoren of $\mathcal{M}$?

Surviving s-wave NLO



Need counter-term to ‘restore’ χS for $\mathcal{M}$:



Cf. Andreas’s talk on NN P-waves Tuesday this week

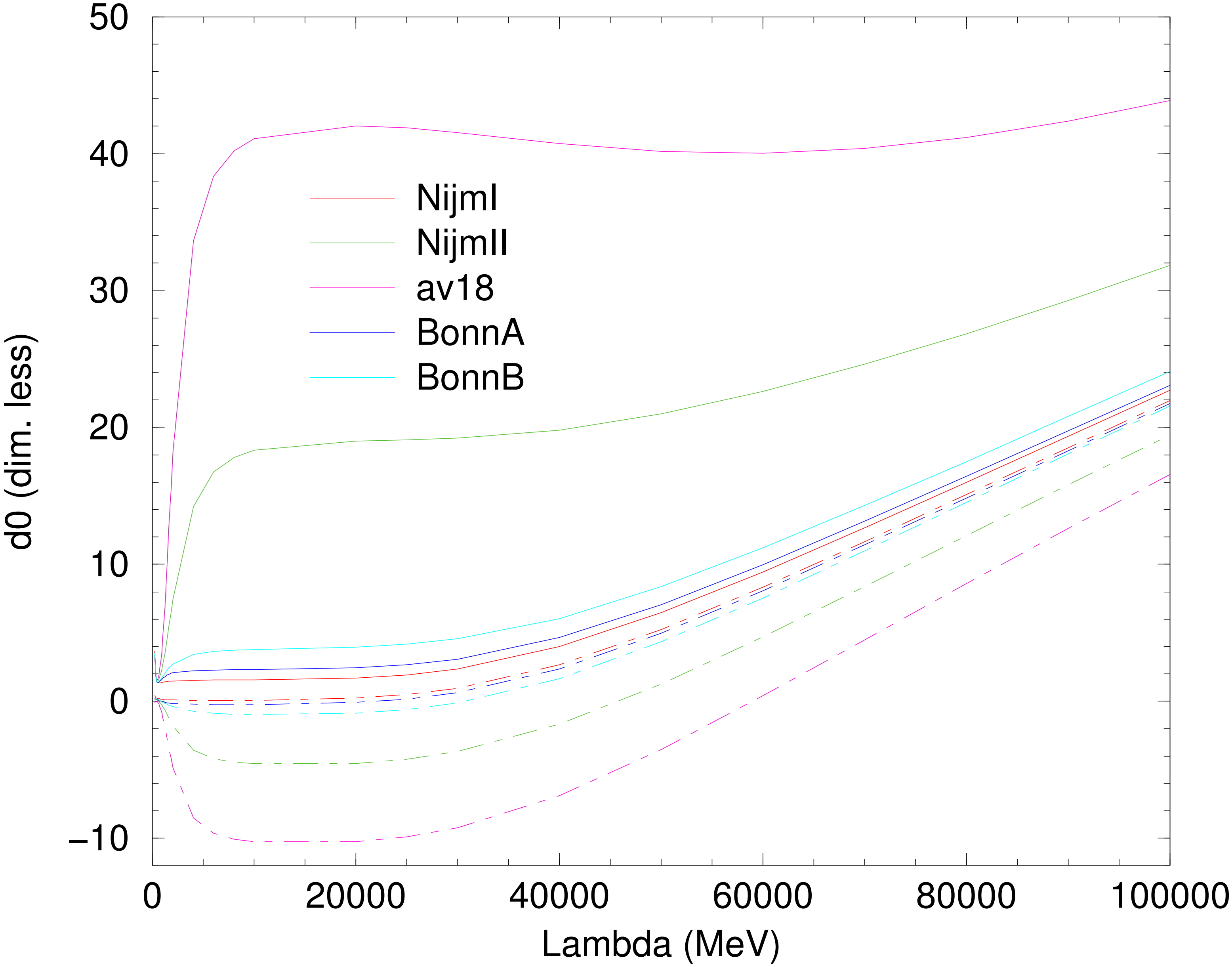
Nogga, Timmermans, and van Kolck, `nucl-th/0506005`

Results without Δ (prel.)

For each Λ there are 2 solutions for $\tilde{d}_0$:

pot	$\Lambda = 500$ MeV		$\Lambda = 800$ MeV		$\Lambda = 1200$ MeV	
NijmI	1.342	0.198	1.345	0.156	1.414	0.121
NijmII	1.480	0.0618	2.038	-0.162	3.495	-0.648
av18	1.626	-0.0952	2.931	-0.602	6.944	-1.865
Bonn A	1.355	0.108	1.525	0.0127	1.795	-0.0826
Bonn B	1.383	0.125	1.612	0.0122	2.053	-0.1498

$\tilde{d}_0(\Lambda)$ at REALLY high Λ



Conclusions and outlook

- Understand CS XS before tackling CSB!
- CS diagrams known to NLO
- Many s-wave diagrams, but most loops cancel
- Few p-wave diagrams, no NLO!
- No new scale with explicit Δ!
- Loops wf dep. NEED COUNTER TERM!
- Implementation of 1-body term
- Leakage $\alpha \to \beta$, new experiments?
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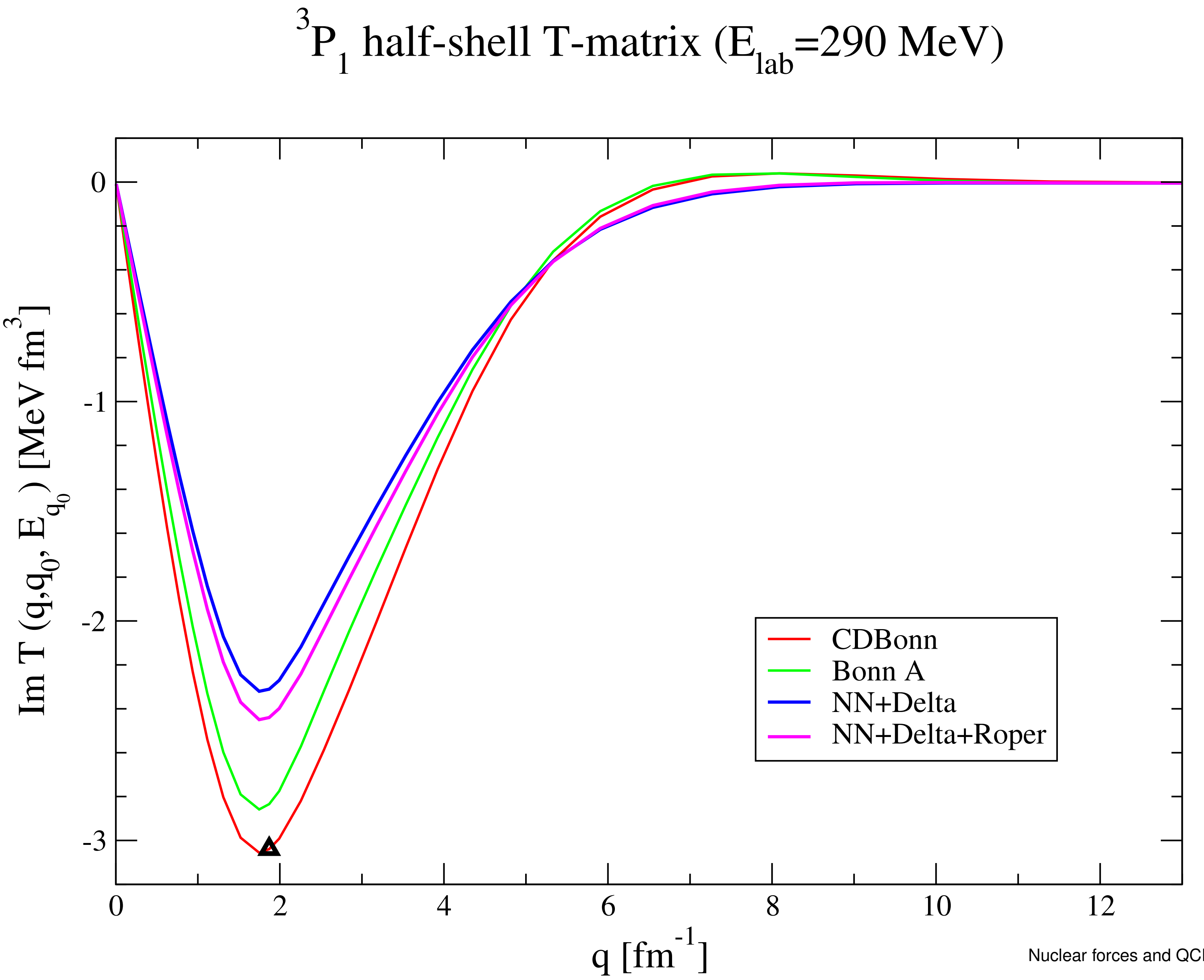
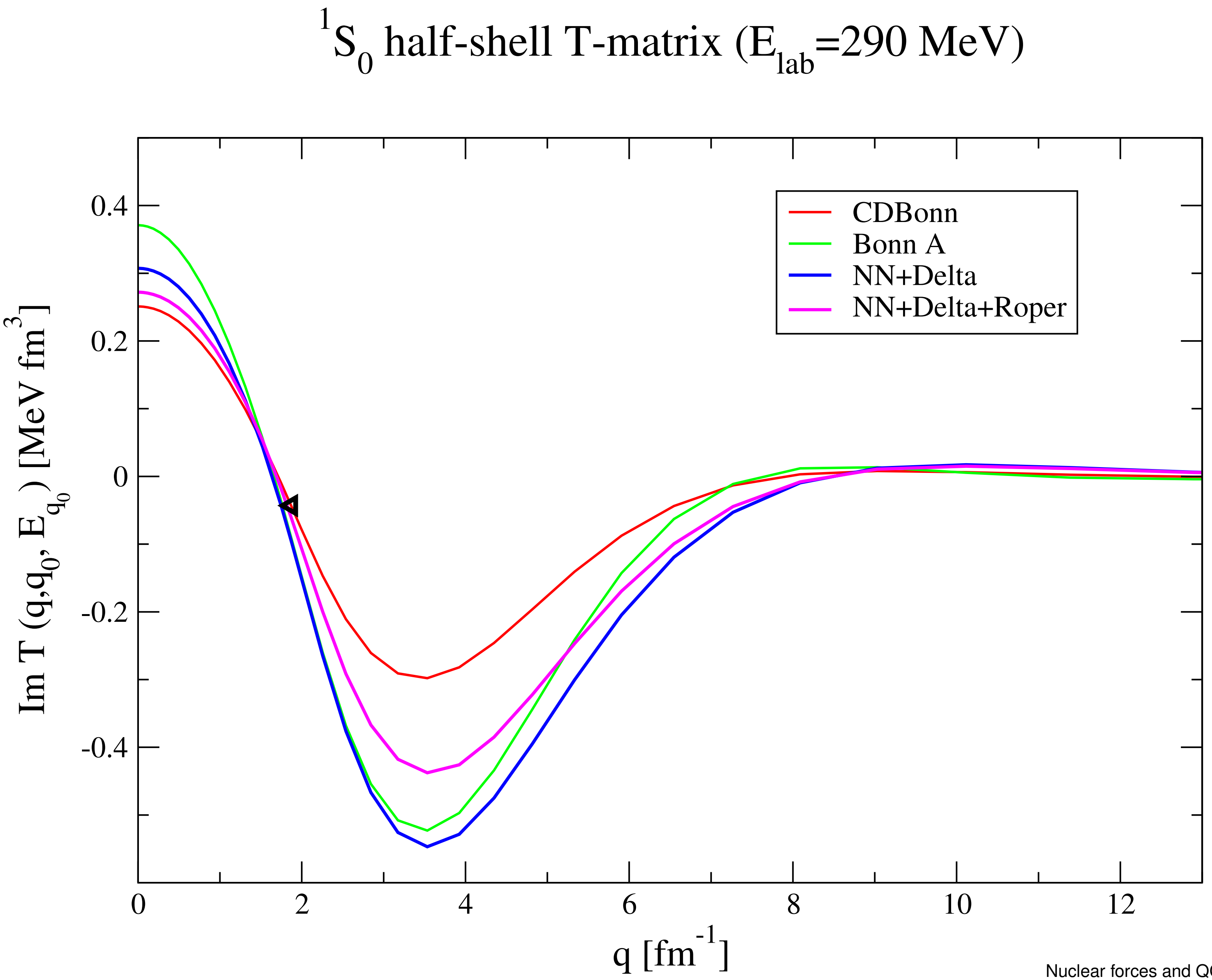
- Use chiral wfs, $V_{\mathrm{low}\,k}$
- Develop CSB s- and p-waves! Counter terms?
- CSB with explicit Δ?

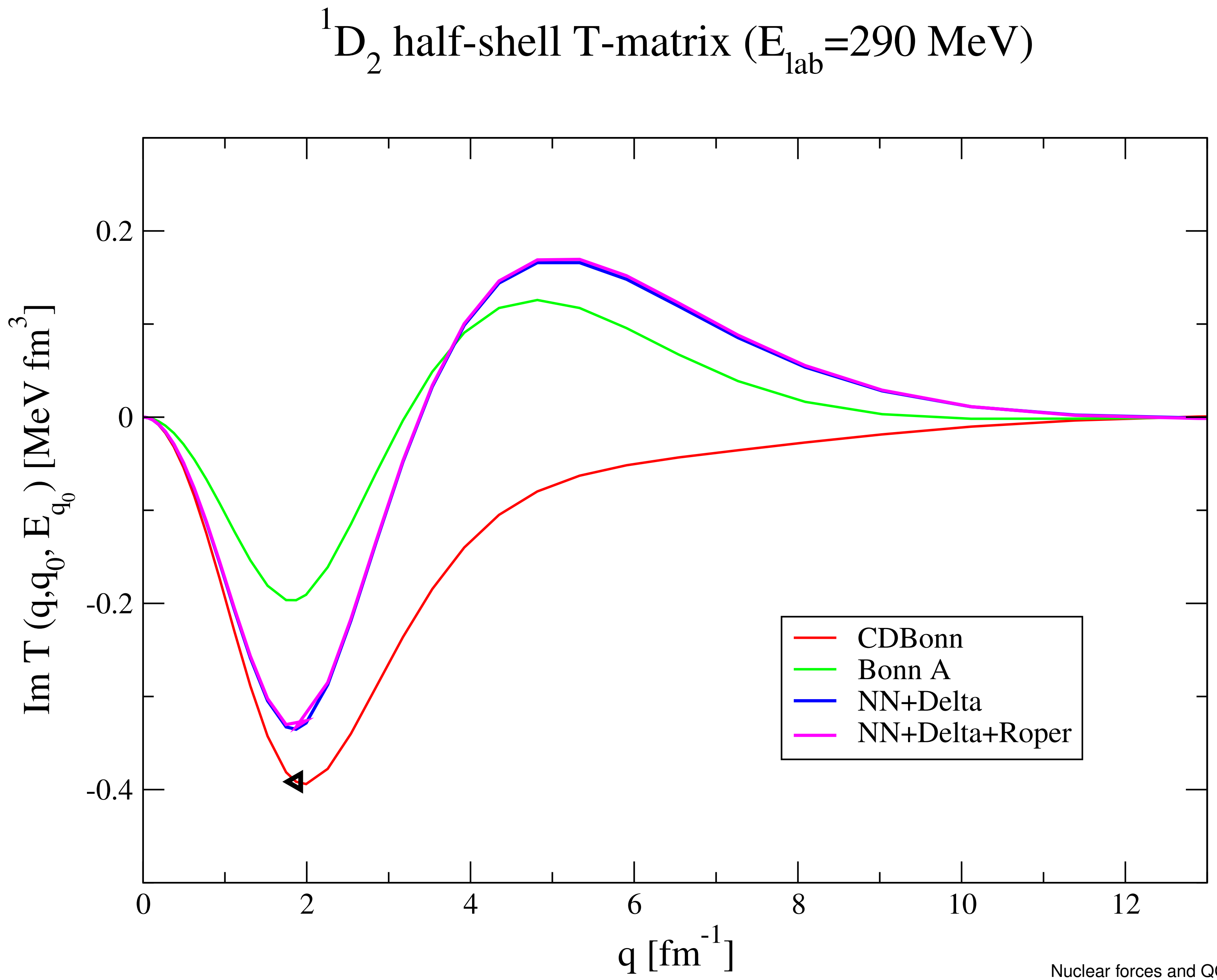
Imaginary $T(^3P_1)$



Imaginary $T(^1S_0)$



Imaginary $T(^1D_2)$



Power counting (CSB)

Relevant new scales:

- $(M_\Delta - M)/M$, Δ-nucleon mass difference
- $\epsilon = (m_d - m_u)/(m_d + m_u)$
- α from photon exchanges

Typical CSB parameters:

$$\epsilon \frac{m_\pi^2}{\Lambda_\chi^2} \sim 1\% \text{ — strong: } \delta M,\ \beta_1,\ \gamma_1$$

$$\frac{\alpha}{\pi} \sim 0.25\% \text{ — EM: } \bar{\delta} M,\ \bar{\beta}_3,\ \bar{\gamma}_3$$

Also soft photon exchanges contribute

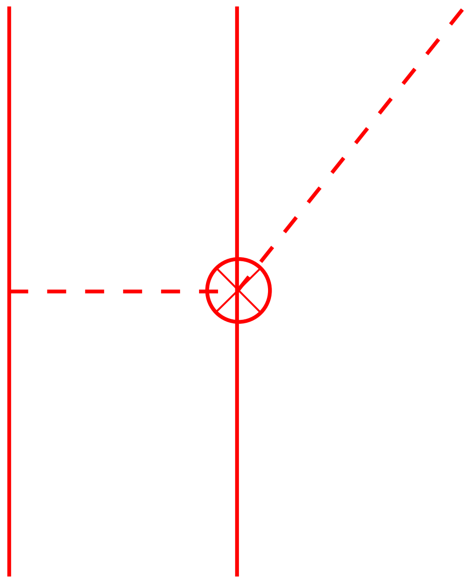
CSB Lagrangian

$$
\begin{aligned}
\mathcal{L}_{CSB} &= \frac{\delta M}{2} N^\dagger \left(\tau_3 - \frac{\pi_3 \boldsymbol{\tau} \cdot \boldsymbol{\pi}}{2 f_\pi^2} \right) N - \frac{\beta_1 + \bar{\beta}_3}{2 f_\pi} N^\dagger \vec{\sigma} \cdot \vec{\nabla} \pi_3 N \\
&+ \frac{\bar{\delta} M}{2} N^\dagger \left(\tau_3 + \frac{\pi_3 \boldsymbol{\tau} \cdot \boldsymbol{\pi} - \boldsymbol{\pi}^2 \tau_3}{2 f_\pi^2} \right) N \\
&- \frac{(\gamma_1 + \bar{\gamma}_3)}{2 f_\pi} N^\dagger N \left\{ N^\dagger \vec{\sigma} N \cdot \vec{\nabla} \pi_3 - \frac{1}{2M} \left[i N^\dagger \dot{\pi}_3 \vec{\sigma} \cdot \vec{\nabla} N + \text{h.c.} \right] \right\} \\
&+ \ldots
\end{aligned}
$$

Undetermined coefficients!

$$\delta M, \ \bar{\delta} M, \ \beta_1, \ \bar{\beta}_3, \ \gamma_1, \ \bar{\gamma}_3$$

CSB s-wave LO



$m_n - m_p = \delta M + \bar{\delta} M = 1.29$ MeV.

Natural sizes:

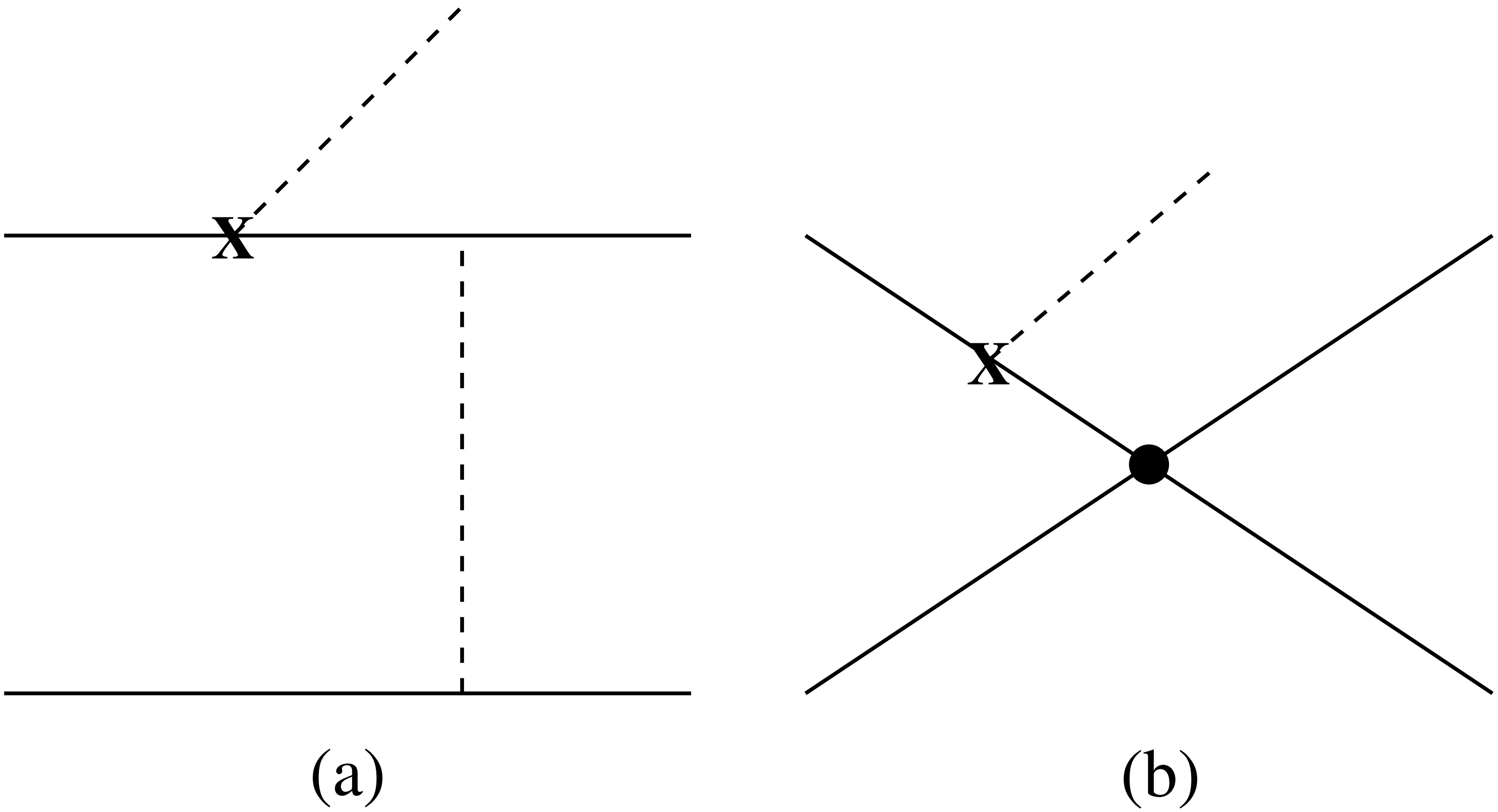
$$\begin{aligned} \delta M &\sim \epsilon m_\pi^2/\Lambda_{\rm QCD} \sim 7 \text{ MeV} \\ \bar{\delta} M &\sim -\alpha \Lambda_{\rm QCD}/\pi \sim -2 \text{ MeV} \\ \epsilon &= (m_d - m_u)/(m_d + m_u) \sim 1/3 \end{aligned}$$

CSB parameter $(\delta M - \frac{1}{2}\bar{\delta} M)/2 f_\pi \sim 22 \cdot 10^{-3}$

Soft photons gives other LO and NLO amplitudes

Short-range interactions (HME) at NNLO

CSB *p*-wave



Under development!