1999

There once was a workshop at Trento
to Effectives' and Potentials' memento.
Discussions galore
spilt blood on the floor –
and Bira told me 't was meant to.

Three-Nucleon Forces in Effective Field Theory without Pions

H. W. Grießhammer

Technische Universität München, Germany

1. A Few Nucleons at Very Low Energies
2. The Three-Nucleon System to All Orders
3. Implications and Applications
4. Concluding Questions

How to root Nuclear Physics in QCD?

How to systematise an EFT with fine-tuning?

Bedaque/hg/Hammer/Rupak: *Nucl. Phys.* **A714** (2003), 589 [nucl-th/0207034]

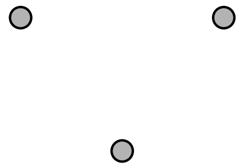
hg: *Nucl. Phys.* **A744** (2004), 192 [nucl-th/0404073]; *Nucl. Phys.* **A** in press [nucl-th/0502039]

Mathematica code: `http://www.ph.tum.de/~hgrie`

1. A Few Nucleons at Very Low Energies

Pictures of the nucleon:

$$p_{\text{typ.}} \lesssim 100\ \text{MeV} \quad , \quad \Delta x \gtrsim 2\ \text{fm}$$

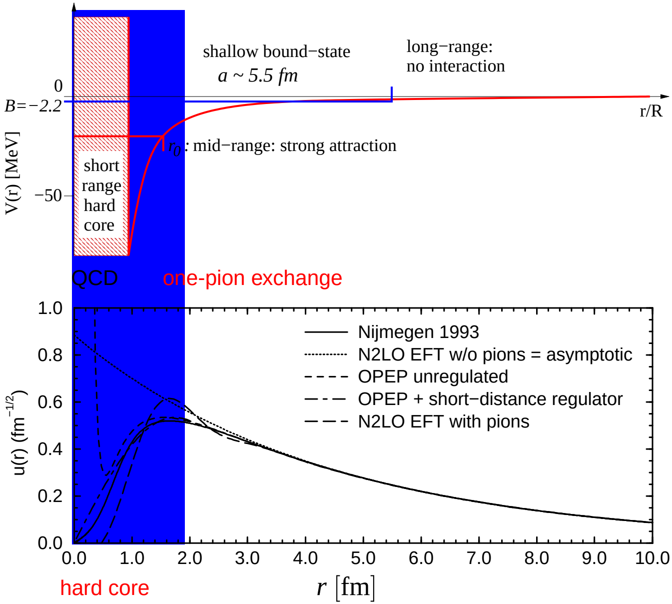


A microscope with very bad resolution!

Non-trivial: Few-nucleon systems with size $a \sim 5\ \text{fm} \gg 1\ \text{fm} \sim R_N$ consitituent size!

(a) It’s Natural to Have Unnatural Scales

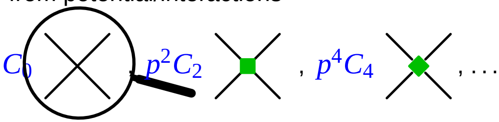
Deuteron bound-state fine-tuned: $B = 2.225\ \text{MeV}$, size $a \approx 5\ \text{fm} \gg R \approx 1\ \text{fm}$



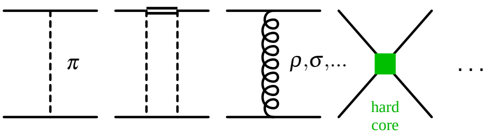
Wave-function at large distances $r \gg R$:

$$\Psi \to Z\,\frac{e^{-r/a}}{r}\left[1 + c_1\,\frac{R}{r} + c_2\left(\frac{R}{r}\right)^2 + \dots\right]$$

from potential/interactions



encode QCD: minimal number of parameters



$\Longrightarrow$ systematic expansion in $Q = \dfrac{\text{target size } R}{\text{resolution } a} \approx \dfrac{1}{3 \text{ to } 5} \ll 1$

“EFT without pions” EFT($\not\pi$)

conceptually well-understood $\Longrightarrow$ predictive power, error estimate

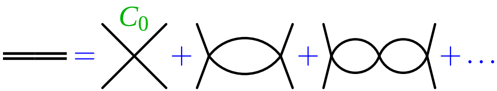
(a) It's Natural to Have Unnatural Scales

Bethe 1949, Kaplan/Savage/Wise & van Kolck 1997

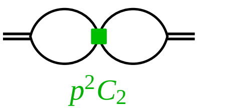
Deuteron bound-state fine-tuned: $B = 2.225$ MeV, size $a \approx 5$ fm $\gg R \approx 1$ fm

Coefficients C_{2n} from simple observables:

LO (30%) :



NLO (10%) :



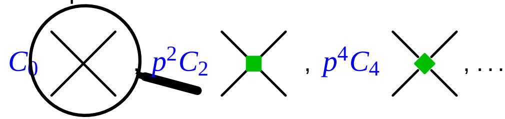
effective ranges ρ_0

NNLO (3%) : $\longrightarrow$ d-wave contributions etc.

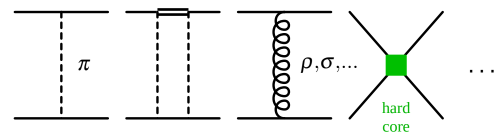
Wave-function at large distances $r \gg R$:

$$\Psi \to Z\,\frac{e^{-r/a}}{r}\left[1 + c_1\,\frac{R}{r} + c_2\left(\frac{R}{r}\right)^2 + \ldots\right]$$

from potential/interactions



encode QCD: minimal number of parameters



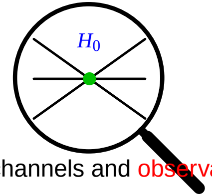
$\Longrightarrow$ systematic expansion in $Q = \dfrac{\text{target size } R}{\text{resolution } a} \approx \dfrac{1}{3 \text{ to } 5} \ll 1$

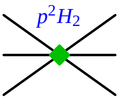
"EFT without pions" EFT($\not\pi$)

conceptually well-understood $\Longrightarrow$ predictive power, error estimate

(b) Three-Body Forces in EFT($\not\pi$)

EFT: All interactions permitted by the symmetries of QCD $\Longrightarrow$ 3-body forces

$H_0\,(N^\dagger N)^3$: 

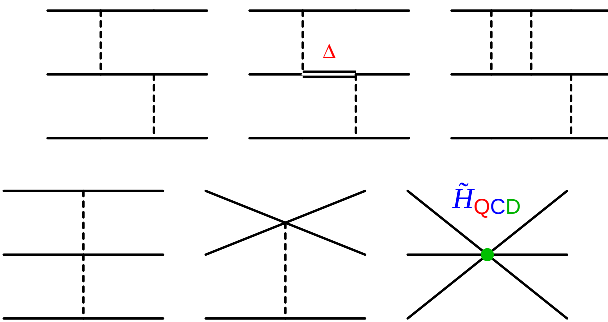
, $p^2\,H_2\,(N^\dagger N)^3$: 

, etc.

$\Longrightarrow$ Which channels and observables most sensitive?

How important? $\Longleftrightarrow$ **At which order in Q do they start to contribute?**

What are they?



etc.

${}^2S_{\frac{1}{2}}$-Channel: nd Scattering Length vs. Triton Binding Energy

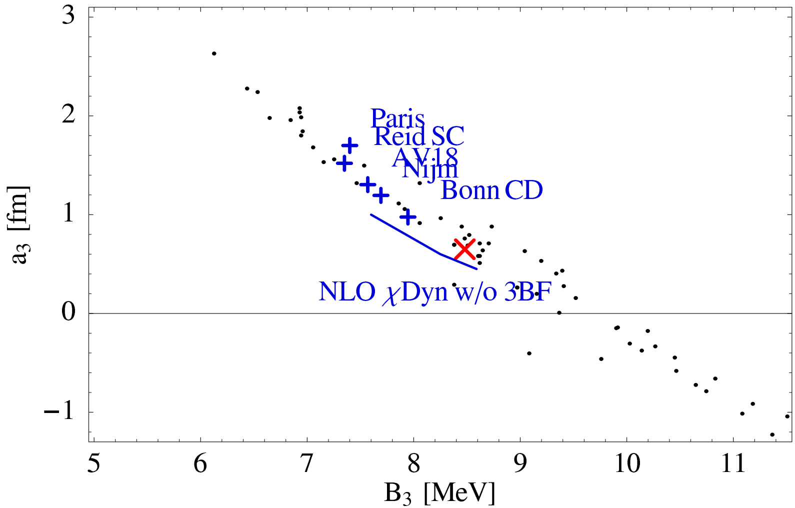
Bedaque et al . 2000, Bedaque/hg/. . . 2003, hg 2004

Generic problem: Predictions by modern, high-precision NN-potentials differ vastly, but all on Phillips line (1969).

Ad-hoc three-body forces make up for difference.

How to predict 3-body forces?

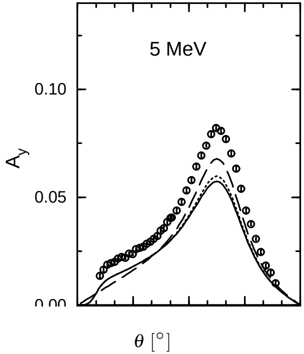
EFT($\not\pi$): universal, a priori one free parameter $H_0(\Lambda)$.



×: exp. +: modern NN potentials ·: more potentials

More Problems:

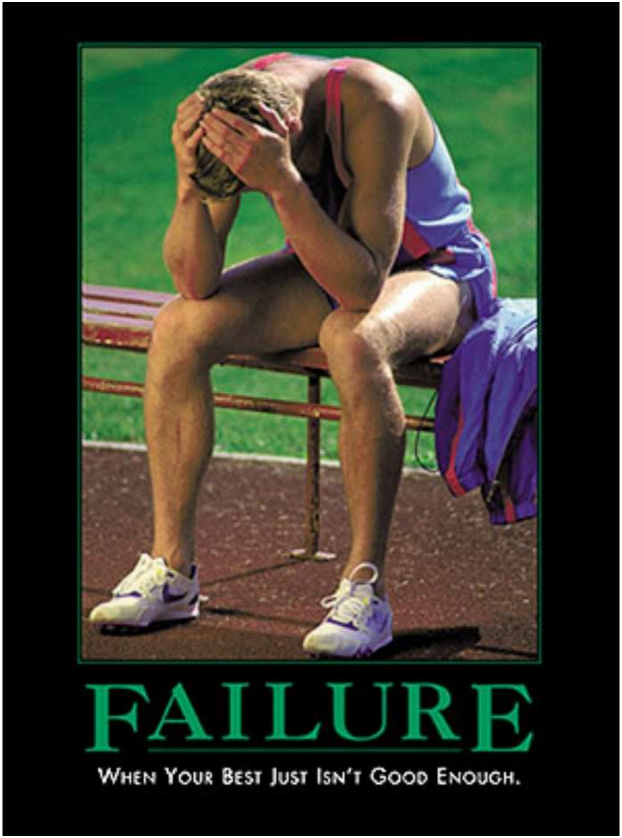
A_y-problem: Nd spin-observable



$nd \to t\gamma$ at thermal energies:

Pot. Mod.'s: $\sigma = [0.49\ldots0.66]$ mb

exp.: $\sigma = [0.508 \pm 0.015]$ mb

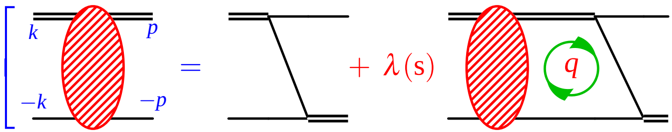

FAILURE
WHEN YOUR BEST JUST ISN'T GOOD ENOUGH.

(a) Leading Three-Body Forces from Naïve Dimensional Analysis

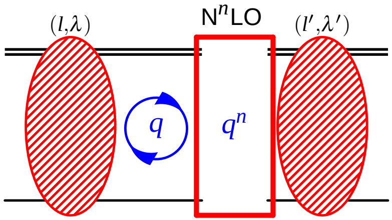
Include 3BF if and only if needed as counter-term to cancel cut-off dependence of low-energy observables.

LO: Faddeev integral equation for half off-shell amplitude $\mathcal{A}(k,q)$: (Skorniakov/Ter-Martirosian 1957)

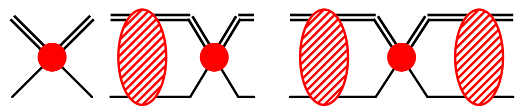
lth partial wave

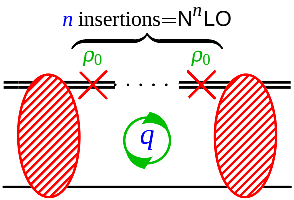


Higher Orders: Perturbation around LO



UV divergent $\Longrightarrow$ 3-body force





e.g.: from

$$\rightarrow \frac{1}{\gamma + \mathrm{i}\,p}\left[1 + \frac{\rho_0}{2}\,\frac{\gamma^2+p^2}{\gamma+\mathrm{i}\,p} + \dots\right] \xrightarrow{\text{UV}} \frac{1 + \rho_0\, p + \dots}{p}$$

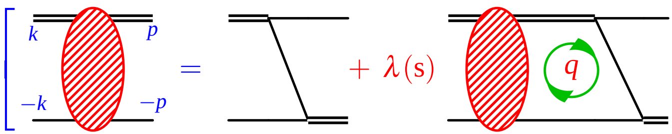
2. The Three-Nucleon System to All Orders

(a) Leading Three-Body Forces from Naïve Dimensional Analysis

Include 3BF if and only if needed as counter-term to cancel cut-off dependence of low-energy observables.

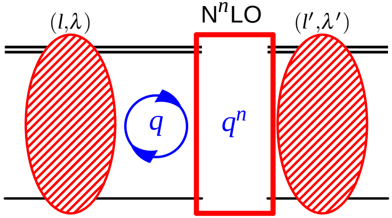
LO: Faddeev integral equation for half off-shell amplitude $\mathcal{A}(k,q)$: (Skorniakov/Ter-Martirosian 1957)

lth partial wave



$$\propto \frac{1}{p^{s_l(\lambda)+1}}$$

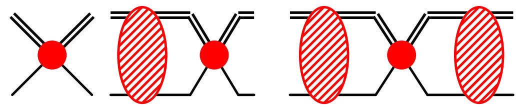
Higher Orders: Perturbation around LO



UV divergent

$\Longrightarrow$

3-body force



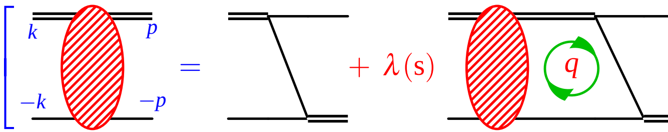
$$\xrightarrow{\text{UV}} \frac{q^5}{q\,q^2}\,\frac{1}{q^{s_l(\lambda)+1}}\,\frac{1}{q^{s_l'(\lambda')+1}}\,q^n = q^{n-s_l(\lambda)-s_l'(\lambda')}$$

3-body force counter-term $\Longleftrightarrow n \geq \mathrm{Re}[s_l(\lambda)+s_l'(\lambda')]$

(b) Asymptotics of the LO Amplitude

3-body force counter-term $\Longleftrightarrow n \geq \mathrm{Re}[s_l(\lambda) + s_l'(\lambda')]$

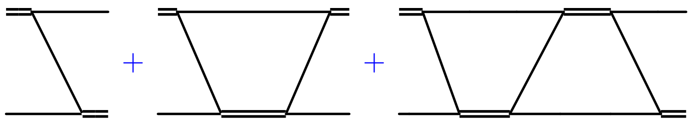
lth partial wave



$\propto \frac{1}{p^{s_l(\lambda)+1}}$; simplistic: $\frac{1}{p^2}\,\frac{k^l}{p^l}$

Simplistic argument: Amplitude in UV like potential/Born series (e.g. Weinberg's proposal)

Example S-wave:

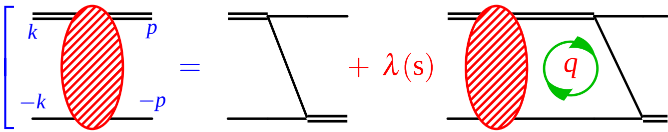


$\frac{1}{p^2}$ $\qquad \frac{q^5}{q^{2\times 3}\,q} \sim \frac{1}{q^2}$ $\qquad \frac{q^{5\times 2}}{q^{2\times 5}\,q\,q} \sim \frac{1}{q^2}$

(b) Asymptotics of the LO Amplitude

3-body force counter-term $\Longleftrightarrow n \geq \mathrm{Re}[s_l(\lambda) + s'_l(\lambda')]$

lth partial wave

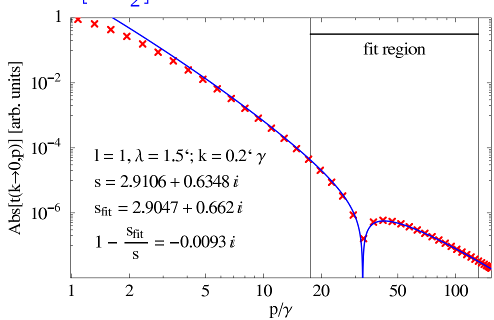


$\propto \frac{1}{p^{s_l(\lambda)+1}}$; simplistic: $\frac{1}{p^2}\,\frac{k^l}{p^l}$

UV $(k, \gamma \ll p, q)$ decoupled by Wigner's $\mathrm{SU}(4)$–symmetry

$\Longrightarrow$ resolvent $a_{(l,s)}(p) = (-)^l \frac{4\,\lambda(s)}{\sqrt{3}\,\pi} \int\limits_0^\infty \frac{\mathrm{d}q}{p}\, a_{(l,s)}(q)\, Q_l\left[\frac{p}{q} + \frac{q}{p}\right]$ by Mellin trafo: $a_{(l,s)}(p) \propto \frac{1}{p^{s_l(\lambda)+1}}$

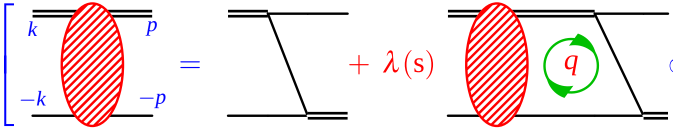
$$1 = (-)^l \frac{2^{1-l}\,\lambda}{\sqrt{3\pi}} \frac{\Gamma[\frac{l+1+s_l(\lambda)}{2}]\,\Gamma[\frac{l+1-s_l(\lambda)}{2}]}{\Gamma[l+\frac{3}{2}]}\ {}_2F_1[\frac{l+1+s_l(\lambda)}{2}, \frac{l+1-s_l(\lambda)}{2}; l+\frac{3}{2}; \frac{1}{4}]$$



(b) Asymptotics of the LO Amplitude

3-body force counter-term $\Longleftrightarrow n \geq \mathrm{Re}[s_l(\lambda) + s'_l(\lambda')]$

lth partial wave

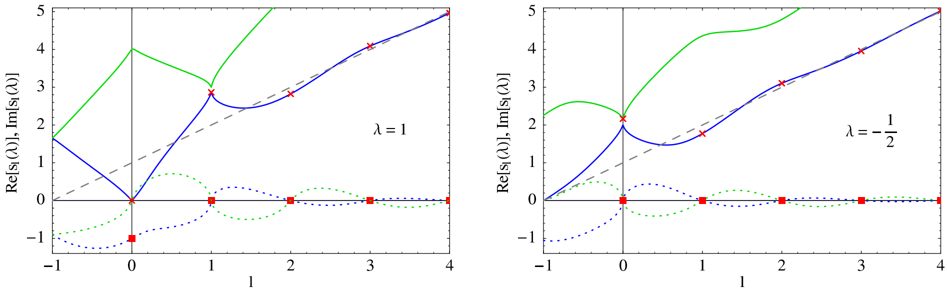


$\propto \dfrac{1}{p^{s_l(\lambda)+1}}$; simplistic: $\dfrac{1}{p^2}\,\dfrac{k^l}{p^l}$

UV $(k, \gamma \ll p, q)$ decoupled by Wigner's $\mathrm{SU}(4)$–symmetry

$\Longrightarrow$ resolvent $a_{(l,s)}(p) = (-)^l \dfrac{4\,\lambda(s)}{\sqrt{3}\,\pi} \displaystyle\int_0^\infty \dfrac{dq}{p}\, a_{(l,s)}(q)\, Q_l\left[\dfrac{p}{q} + \dfrac{q}{p}\right]$ by Mellin trafo: $a_{(l,s)}(p) \propto \dfrac{1}{p^{s_l(\lambda)+1}}$

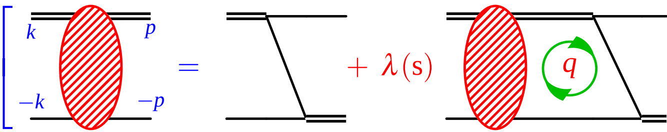
$$1 = (-)^l \frac{2^{1-l}\,\lambda}{\sqrt{3\pi}} \frac{\Gamma[\frac{l+1+s_l(\lambda)}{2}]\,\Gamma[\frac{l+1-s_l(\lambda)}{2}]}{\Gamma[l+\frac{3}{2}]}\ {}_2F_1[\frac{l+1+s_l(\lambda)}{2}, \frac{l+1-s_l(\lambda)}{2}; l+\frac{3}{2}; \frac{1}{4}]$$



(b) Asymptotics of the LO Amplitude

3-body force counter-term $\Longleftrightarrow n \geq \mathrm{Re}[s_l(\lambda) + s_l'(\lambda')]$

lth partial wave



$\propto \frac{1}{p^{s_l(\lambda)+1}}$; simplistic: $\frac{1}{p^2}\,\frac{k^l}{p^l}$

UV $(k, \gamma \ll p, q)$ decoupled by Wigner's $\mathrm{SU}(4)$–symmetry

$\Longrightarrow$ resolvent $a_{(l,s)}(p) = (-)^l \frac{4\,\lambda(\mathrm{s})}{\sqrt{3}\,\pi} \int\limits_0^\infty \frac{\mathrm{d}q}{p}\, a_{(l,s)}(q)\, Q_l\left[\frac{p}{q} + \frac{q}{p}\right]$ by Mellin trafo: $a_{(l,s)}(p) \propto \frac{1}{p^{s_l(\lambda)+1}}$

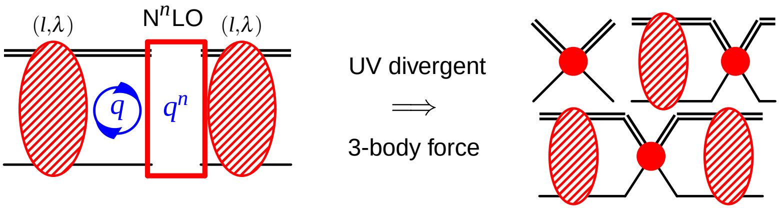
$$1 = (-)^l \frac{2^{1-l}\,\lambda}{\sqrt{3\pi}} \frac{\Gamma[\frac{l+1+s_l(\lambda)}{2}]\,\Gamma[\frac{l+1-s_l(\lambda)}{2}]}{\Gamma[l+\frac{3}{2}]}\; {}_2F_1[\frac{l+1+s_l(\lambda)}{2}, \frac{l+1-s_l(\lambda)}{2}; l+\frac{3}{2}; \frac{1}{4}]$$

	$s_l(\lambda)$	$l=0$	$l=1$	$l=2$	$l=3$
simplistic:	$s_l = l+1$	1	2	3	4
Quartet/Wigner-antisym. Doublet ($s=\frac{3}{2}$):	$\lambda = -\frac{1}{2}$	2.16	1.77	3.10	4.04
Wigner-sym. Doublet ($s=\frac{1}{2}$):	$\lambda = 1$	$\pm 1.0062\,\mathrm{i}$	2.86	2.82	3.92

$\Longrightarrow$ UV-scaling not as guessed.

(c) Leading Three-Body Forces in the 3-Nucleon Channels

Include 3BF if and only if needed as counter-term to cancel cut-off dependence of low-energy observables.



channel partial waves	naïve dim. an. $\mathrm{Re}[s_l(\lambda)+s_{l'}(\lambda')]$	simplistic $l+l'+2$		typ. size $1/3^n$
$^2S_{Ws}$-$^2S_{Ws}$ $^2S_{Ws}$-$^2S_{Wa}$ $^2S_{Wa}$-$^2S_{Wa}$	LO N$^{2.2+2}$LO N$^{4.3+2}$LO	N^{2}LO N^{2+2}LO	promoted demoted	100% 1% 0.1%
$^2S_{Ws}$-^4D $^2S_{Wa}$-^4D	N$^{3.1}$LO N$^{5.3}$LO	N^{4}LO	promoted demoted	3% 0.3%
$^2P_{Ws}$-$^2P_{Ws}$ $^2P_{Ws}$-$^2P_{Wa}$, $^2P_{Ws}$-^4P $^2P_{Wa}$-$^2P_{Wa}$, ^4P-^4P	N$^{5.7}$LO N$^{4.6}$LO N$^{3.5}$LO	N^{4}LO	demoted demoted	0.2% 0.6% 2%
^4S-^4S ^4S-$^2D_{Ws}$ ^4S-$^2D_{Wa}$, ^4S-^4D	N$^{4.3+2}$LO N$^{5.0}$LO N$^{5.3}$LO	N^{2+2}LO N^{4}LO	demoted demoted demoted	0.1% 0.4% 0.3%
higher	$\sim$ as simplistic	N$^{l+l'+2}$LO		

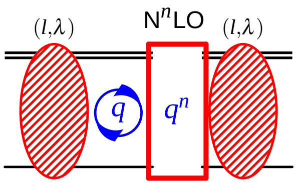
The Three Big Lies of Nuclear Physics

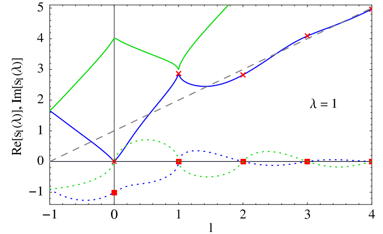
Nuclear Power is Safe.

They have Weapons of Mass Distruction.

My Power-Counting is Systematic.

(d) Context and Caveats





Context

– Extended Naïve Dimensional Analysis to non-perturbative field theories.

– Criterion: Independence of short-distance details at each order. Renormalisability.

– Only phenomenological input: shallow 2-body bound states $\Longrightarrow$ LO non-perturbative.

– Higher partial waves close to perturbative estimate.

– Higher orders: 3body-forces with $2m$ more derivatives at $n \geq \text{Re}[s_l(\lambda) + s'_l(\lambda')] - 2m$;

– External currents straightforward: replace kernel.

Caveats

– No more re-summations after LO. No contrary cases known.

– 3-body force $\Longleftrightarrow$ counter-term: minimal number. Assumed natural size set by running with cut-off.

– Only superficial degree of divergence. $\Longrightarrow$ Future: Construct 3body forces!

– Cancellations between diagrams? Overlapping divergences? $\Longrightarrow$ Demote 3body forces.

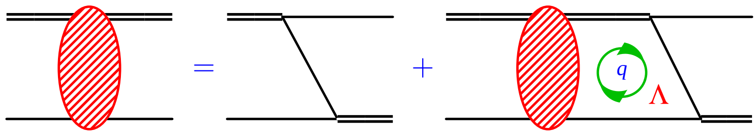
– Generically fractional orders: $\Longrightarrow$ Clear criterion?

$^4\text{S}_{\frac{3}{2}}$: $s = 2.16$ $\Longrightarrow$ 3BF at N$^{6.3}$LO. Diverges at N^{7}LO; converges at N^{6}LO very slowly.

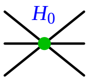
^4F: $s = 5.02$ $\Longrightarrow$ 3BF at N$^{10.04}$LO. Diverges at N^{11}LO; converges at N^{10}LO very slowly.

That's nice, dear.

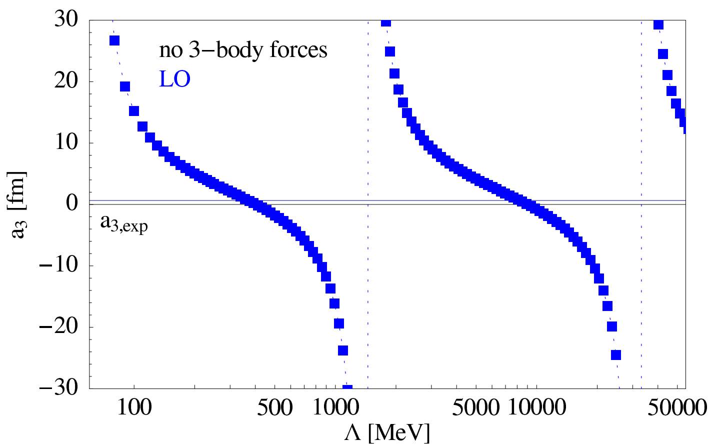
(a) The Problem: nd-Scattering, ${}^2\mathrm{S}_{\frac{1}{2}}$ Wave ("triton channel")

LO: 

$\sim Q^{-2}$

$\overset{n}{\Downarrow} \;\rightarrow\; \overset{n\;\;p}{\Uparrow\Uparrow}$: No Pauli principle, no centrifugal barrier $\Longrightarrow$ 3-body forces at N2LO? 

$\sim Q^0$



Slight cut-off variation has dramatic effect on low-energy observables.

Danilov, Minlos/Faddeev 1961

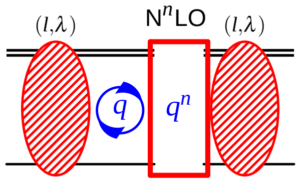
$s_0({}^2\mathrm{S}_{\frac{1}{2}}) = \pm 1.006\ldots\mathrm{i} \Longrightarrow$

On-shell depends on UV-phase δ:

$$\mathcal{A}_{{}^2\mathrm{S}_{\frac{1}{2}}}(k \to 0, p) \propto \frac{\cos[1.0062\ln p + \delta]}{p}$$

(a) The Problem: nd-Scattering, ${}^2\mathrm{S}_{\frac{1}{2}}$ Wave ("triton channel")

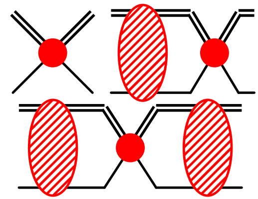
Include 3BF if and only if needed as counter-term to cancel cut-off dependence of low-energy observables.



UV divergent

$\Longrightarrow$

3-body force

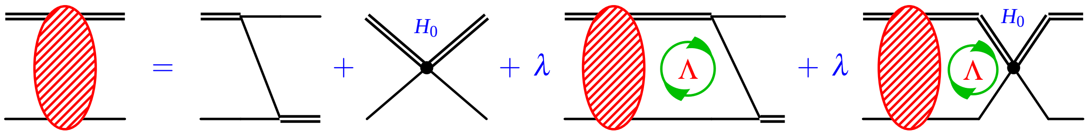


channel partial waves	naïve dim. an. $\mathrm{Re}[s_l(\lambda)+s_{l'}(\lambda')]$	simplistic $l+l'+2$		typ. size $1/3^n$
${}^2\mathrm{S}_{\mathrm{Ws}}$-${}^2\mathrm{S}_{\mathrm{Ws}}$ ${}^2\mathrm{S}_{\mathrm{Ws}}$-${}^2\mathrm{S}_{\mathrm{Wa}}$ ${}^2\mathrm{S}_{\mathrm{Wa}}$-${}^2\mathrm{S}_{\mathrm{Wa}}$	LO N$^{2.2+2}$LO N$^{4.3+2}$LO	N^{2}LO N^{2+2}LO	promoted demoted	100% 1% 0.1%
${}^2\mathrm{S}_{\mathrm{Ws}}$-${}^4\mathrm{D}$ ${}^2\mathrm{S}_{\mathrm{Wa}}$-${}^4\mathrm{D}$	N$^{3.1}$LO N$^{5.3}$LO	N^{4}LO	promoted demoted	3% 0.3%
${}^2\mathrm{P}_{\mathrm{Ws}}$-${}^2\mathrm{P}_{\mathrm{Ws}}$ ${}^2\mathrm{P}_{\mathrm{Ws}}$-${}^2\mathrm{P}_{\mathrm{Wa}}$, ${}^2\mathrm{P}_{\mathrm{Ws}}$-${}^4\mathrm{P}$ ${}^2\mathrm{P}_{\mathrm{Wa}}$-${}^2\mathrm{P}_{\mathrm{Wa}}$, ${}^4\mathrm{P}$-${}^4\mathrm{P}$	N$^{5.7}$LO N$^{4.6}$LO N$^{3.5}$LO	N^{4}LO	demoted demoted	0.2% 0.6% 2%
${}^4\mathrm{S}$-${}^4\mathrm{S}$ ${}^4\mathrm{S}$-${}^2\mathrm{D}_{\mathrm{Ws}}$ ${}^4\mathrm{S}$-${}^2\mathrm{D}_{\mathrm{Wa}}$, ${}^4\mathrm{S}$-${}^4\mathrm{D}$	N$^{4.3+2}$LO N$^{5.0}$LO N$^{5.3}$LO	N^{2+2}LO N^{4}LO	demoted demoted demoted	0.1% 0.4% 0.3%
higher	$\sim$ as simplistic	N$^{l+l'+2}$LO		

Bedaque/Hammer/van Kolck 1998, Bedaque/hg/Hammer/Rupak 2004, hg 2005

Tenet: Include specific 3BF if and only if needed to cancel off-shell dependence of observables.

Momentum-independent three-body force $H_0(\Lambda)$ must be LO to absorb cut-off dependence.



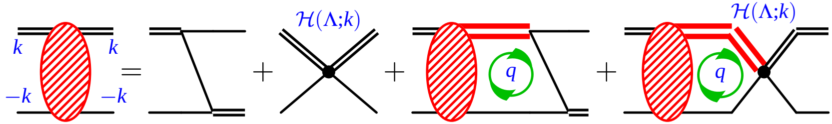
Tune such that $\mathcal{A}$ cut-off independent in UV.

Classify Three-Body Forces with EFT Tenet:

Bedaque/Hammer/van Kolck 2000, Bedaque/hg/Hammer/Rupak 2002

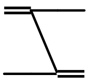
Include specific 3-body force $H_{2n}(\Lambda)$ if and only if needed as counter-term to cancel cut-off dependence of low-energy observables.

UV limit: $k \sim \gamma \ll \frac{1}{\rho_0} \ll \Lambda, q$ (off-shell mom.) $\Longrightarrow$ Lowest 3-body forces Wigner-$\mathrm{SU}(4)$-symmetric.

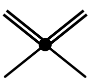


LO NLO N2LO

$$\Psi_d(q \to \infty) \rightarrow \sqrt{\frac{4}{3}}\frac{1}{q} + \frac{4\gamma}{3q^2} + \frac{\rho_0}{2} + \frac{3k^2+4\gamma^2}{3\sqrt{3}\,q^3} + \frac{2\gamma\rho_0}{\sqrt{3}\,q} + \frac{\sqrt{3}}{8}\,q\rho_0^2$$



$$V_{Nd}(q \to \infty) \rightarrow \frac{1}{q^2} + \frac{k^2 - 12\gamma^2}{12q^4}$$



$$\Longrightarrow \mathcal{H} = H_0^{\mathrm{LO}}(\Lambda) + H_0^{\mathrm{NLO}}(\Lambda) + \frac{k^2}{\Lambda^2} H_2(\Lambda) + H_0^{\mathrm{N2LO}}(\Lambda)$$

LO and NLO ($<10\%$ accuracy):

One free parameter H_0,

fixed e.g. by triton binding energy.

N2LO and N3LO ($<1\%$ accuracy):

One more free parameter H_2,

fixed best by scattering length.

$^2S_{\frac{1}{2}}$-Channel: nd Scattering Length vs. Triton Binding Energy

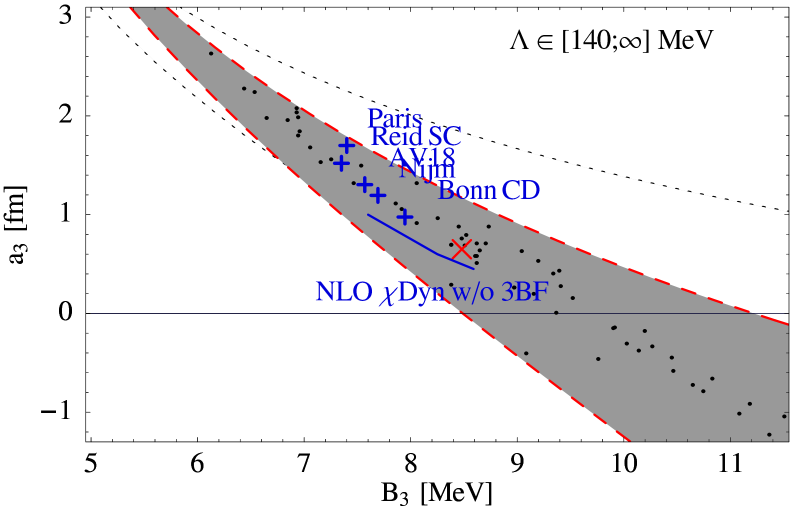
Bedaque et al . 2000, Bedaque/hg/. . . 2003, hg 2004

Generic problem: Predictions by modern, high-precision NN-potentials differ vastly, but all on Phillips line (1969).

Ad-hoc three-body forces make up for difference.

How to predict 3-body forces?

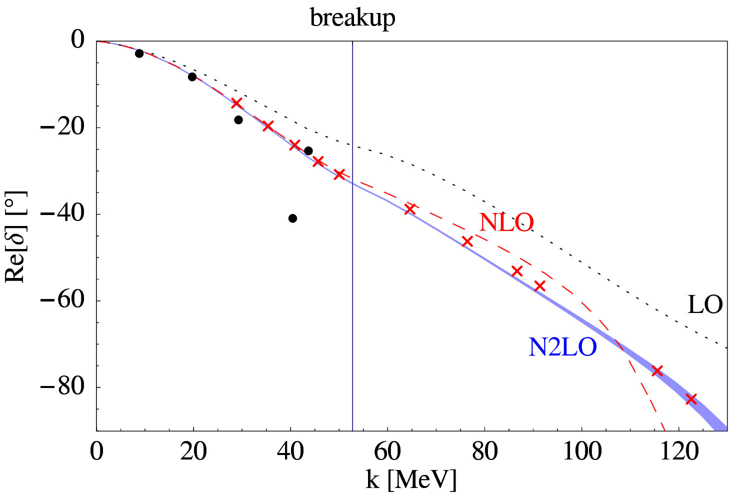
EFT($\not\pi$): universal, a priori one free parameter $H_0(\Lambda)$.



×: exp. +: modern NN potentials ·: more potentials

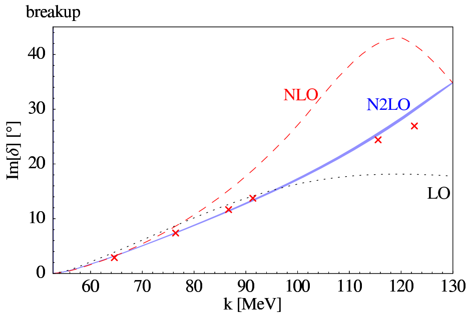
(c) Doublet-S Wave nd Phase Shift

×: AV18+U IX (Kievski 2002) ●: PWA 1967 (Seagrave/van Oers) ▬: N2LO, $\Lambda \in [200; \infty]$ MeV



Fix H_0 to one observable (a_3).

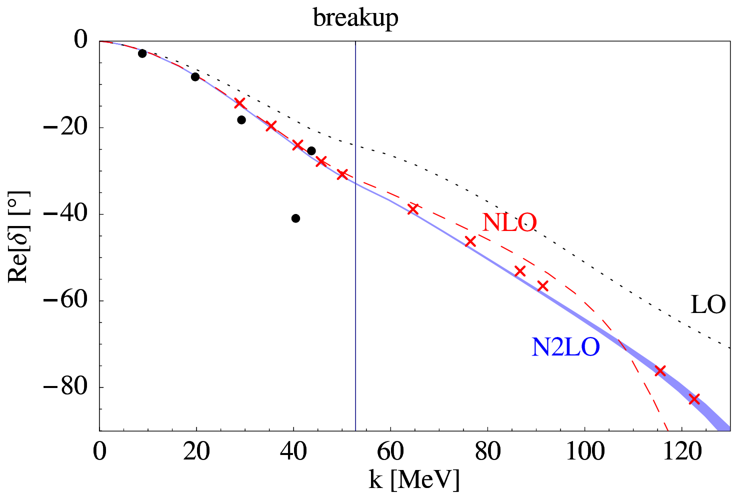
N2LO: H_0&H_2 to (B_3, a_3)



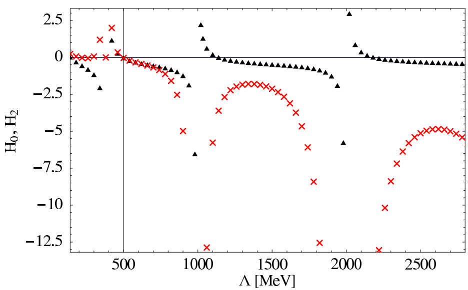
Agrees well with sophisticated, modern potential model calculations.

(c) Doublet-S Wave nd Phase Shift

×: AV18+U IX (Kievski 2002) ●: PWA 1967 (Seagrave/van Oers) ■: N2LO, $\Lambda \in [200; \infty]$ MeV



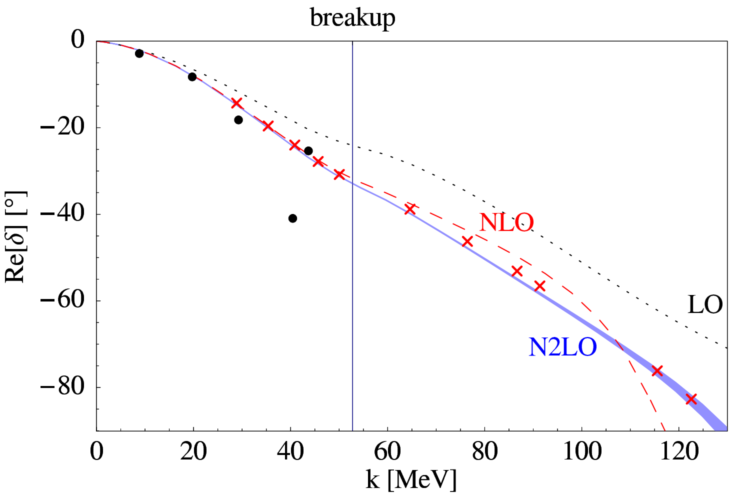
Variation of 3body force



Agrees well with sophisticated, modern potential model calculations.

(c) Doublet-S Wave nd Phase Shift

×: AV18+U IX (Kievski 2002) ●: PWA 1967 (Seagrave/van Oers) ▬: N2LO, $\Lambda \in [200; \infty]$ MeV



Agrees well with sophisticated, modern potential model calculations.

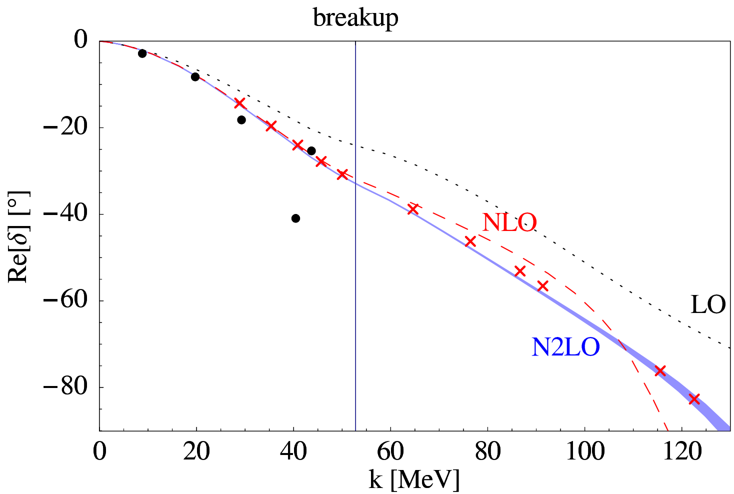
– Convergence to Nature. No Nature here.

– Order by order smaller corrections. O.k.

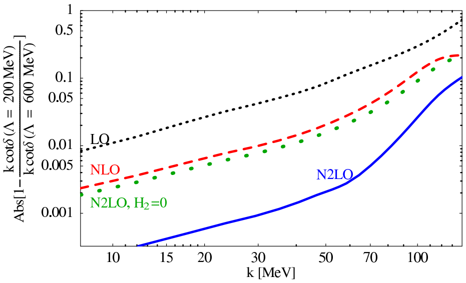
– Order by order smaller cut-off dependence. “Lepage plot”.

(c) Doublet-S Wave nd Phase Shift

×: AV18+U IX (Kievski 2002) ●: PWA 1967 (Seagrave/van Oers) ▬: N2LO, $\Lambda \in [200; \infty]$ MeV



"Lepage-plot": relative, log. cut-off dep.



Agrees well with sophisticated, modern potential model calculations.

$$\left|1 - \frac{k\cot\delta(\Lambda = 200\ \text{MeV})}{k\cot\delta(\Lambda = \infty)}\right| \sim \underbrace{\left(\frac{p_{\text{typ.}}}{\Lambda_{\not\pi}}\right)^n}_{Q^n}$$

$\Longrightarrow$ Fit to $k \in [70; 100 \ldots 130]$ MeV

	LO	NLO	N^{2}LO	N^{2}LO without H_2
n fitted	~ 1.9	2.9	4.8	3.1
n expected	2	3	4	4!!

(d) Simpler Picture: The Three-Nucleon System in Coordinate Space

Fourier: $$\left[-\frac{1}{R}\frac{\partial}{\partial R}R\frac{\partial}{\partial R} + \#\frac{\#\, l(l+1)+s_l{}^2}{R^2} - ME\right]F(R) = 0.$$

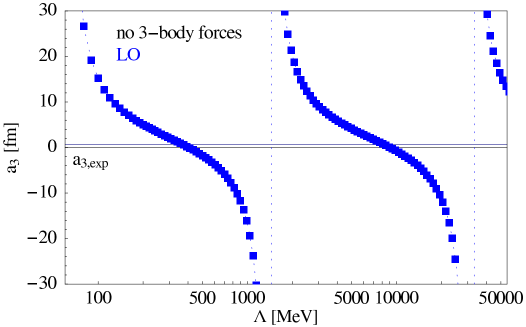
$s_l(\lambda)$ **imaginary,** $|s_l(\lambda)| \gg l$ (e.g. $s_0(^2\mathrm{S}_{\frac{1}{2}}) = \pm 1.006\ldots\mathrm{i}$)

$\Longrightarrow$ attractive potential, infinitely many, deeply bound states.

$\Longrightarrow$ Thomas & Efimov Effects (1935, 1971)

$\Longrightarrow$ too much sensitivity to short-distance

$\Longrightarrow$ 3body forces promoted



$s_l(\lambda) \gg l+1$ (e.g. $s_0(^4\mathrm{S}_{\frac{3}{2}}) = 2.16\cdots \gg 1$)

$\Longrightarrow$ more repulsive than estimated

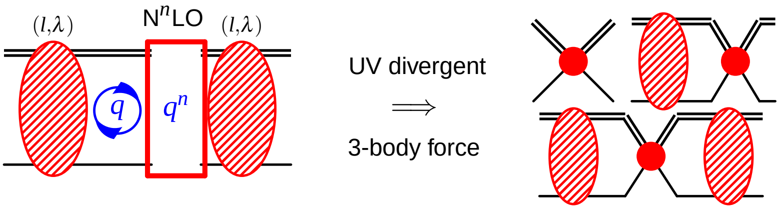
$\Longrightarrow$ less sensitivity to short-distance

$\Longrightarrow$ 3body forces demoted

(d) Simpler Picture: The Three-Nucleon System in Coordinate Space

nucl-th/0502039

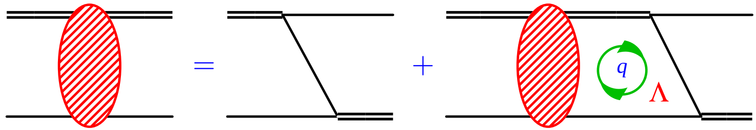
Include 3BF if and only if needed as counter-term to cancel cut-off dependence of low-energy observables.



channel partial waves	naïve dim. an. $\mathrm{Re}[s_l(\lambda)+s_{l'}(\lambda')]$	simplistic $l+l'+2$		typ. size $1/3^n$
$^2S_{Ws}$-$^2S_{Ws}$	LO	N^{2}LO	promoted	100%
$^2S_{Ws}$-$^2S_{Wa}$	N$^{2.2+2}$LO	N^{2+2}LO		1%
$^2S_{Wa}$-$^2S_{Wa}$	N$^{4.3+2}$LO		demoted	0.1%
$^2S_{Ws}$-^4D	N$^{3.1}$LO	N^{4}LO	promoted	3%
$^2S_{Wa}$-^4D	N$^{5.3}$LO		demoted	0.3%
$^2P_{Ws}$-$^2P_{Ws}$	N$^{5.7}$LO	N^{4}LO	demoted	0.2%
$^2P_{Ws}$-$^2P_{Wa}$, $^2P_{Ws}$-^4P	N$^{4.6}$LO		demoted	0.6%
$^2P_{Wa}$-$^2P_{Wa}$, ^4P-^4P	N$^{3.5}$LO			2%
^4S-^4S	N$^{4.3+2}$LO	N^{2+2}LO	demoted	0.1%
^4S-$^2D_{Ws}$	N$^{5.0}$LO	N^{4}LO	demoted	0.4%
^4S-$^2D_{Wa}$, ^4S-^4D	N$^{5.3}$LO		demoted	0.3%
higher	$\sim$ as simplistic	N$^{l+l'+2}$LO		

(e) nd scattering length, Quartet-S Wave

LO:



N^{2}LO EFT($\not\pi$): $a = [6.35 \pm 0.02]$ fm

experiment: $[6.35 \pm 0.02]$ fm

predict first 3BF at N$^{\sim 6}$LO

$$\approx \pm \left(\frac{1}{3}\right)^3 \times 0.02 \text{ fm} = \pm 0.001 \text{ fm}$$

simplistic: N^{4}LO

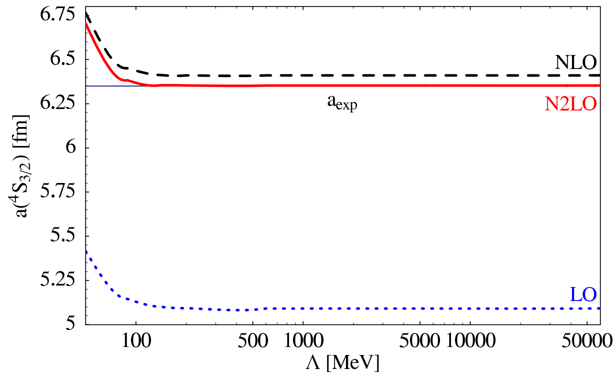
$$\approx \pm \left(\frac{1}{3}\right)^1 \times 0.02 \text{ fm} = \pm 0.010 \text{ fm}$$

pot. models: $[6.345 \pm 0.002]$ fm

$\Longrightarrow$ input for better $a(^2\mathrm{S})$ from

bound coherent scatt. length: Black et al 2003

$$a(^2\mathrm{S}) = [0.645 \pm 0.003 \pm 0.007_{\mathrm{th}}] \text{ fm}$$

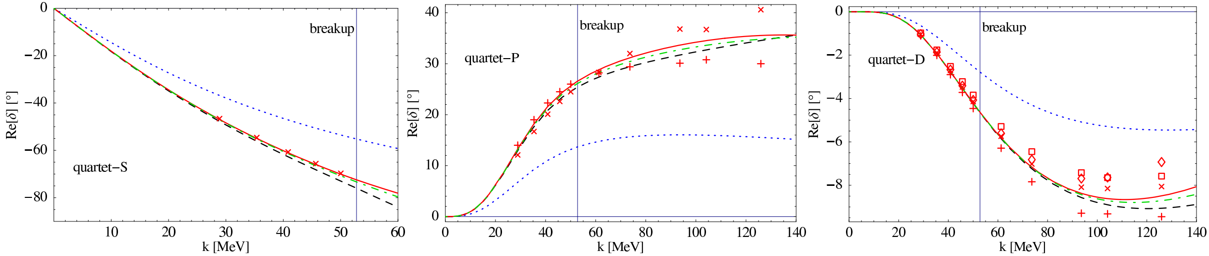


(LO: Skorniakov/Ter-Martirosian 1957, NLO: Efi mov 1991, N2LO: Bedaque/van Kolck 1998, hg 2005)

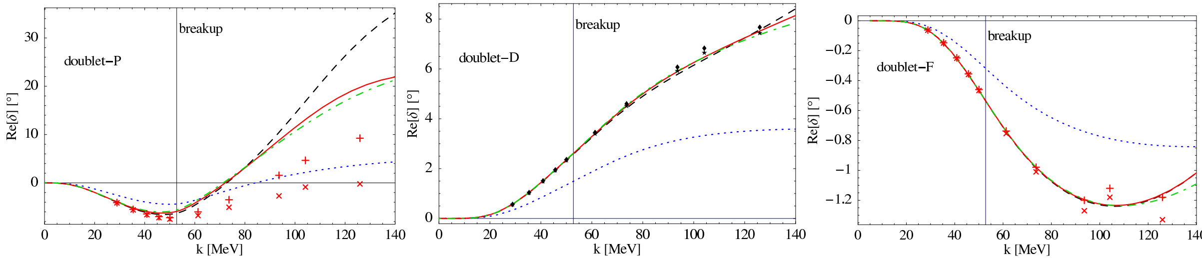
$\Longrightarrow$ Alleviates hunt for 3BFs in observables.

(f) nd Phase Shifts to 3% Accuracy

Quartet Channel ($s = \frac{3}{2}$)



Doublet Channel ($s = \frac{1}{2}$)



Numerically simple: N2LO code runs within a minute on PC

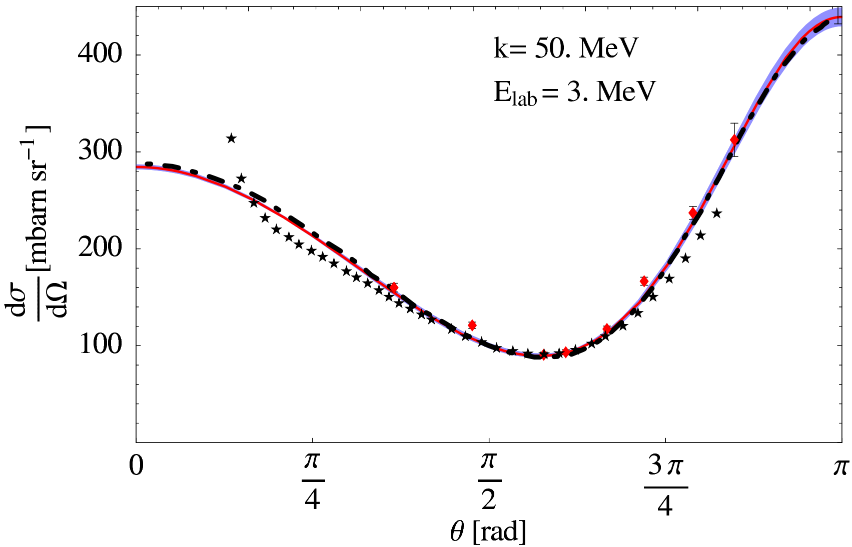
Agrees well with sophisticated, modern potential model calculations.

N3LO (3-body force!): Splitting/mixing of partial waves $\Longrightarrow A_y$

(g) *nd* Cross-section to 3% Accuracy

Three-body forces in all other channels even more suppressed than naïvely estimated from centrifugal barrier and Pauli principle.

EFT at N2LO with error bars: [band]; *nd*-data: ◆; *pd*-data: ⋆; potential models: - · - · -



Numerically simple: N2LO code runs within 5 minutes on PC.

Agrees well with sophisticated, modern potential model calculations.

(h) Triton Radiative Capture

hg/Sadeghi/Bayegan 2005

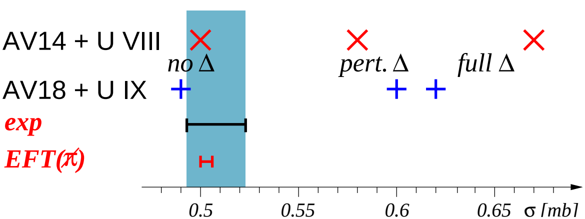
A Problem Solved: $nd \longrightarrow t\gamma$ at thermal energies

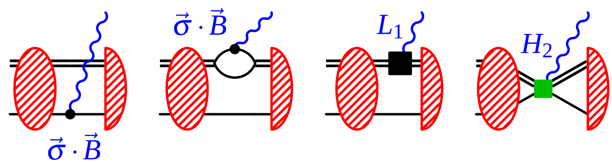
AV 14 $+$ Urbana VIII	no $\Delta(1232)$	0.50 mb	?
AV 14 $+$ Urbana VIII	$+\Delta(1232)$	0.58 mb	
AV 14 $+$ Urbana VIII	perturbative Δ	0.66 mb	
AV 18 $+$ Urbana IX	no $\Delta(1232)$	0.49 mb	
AV 18 $+$ Urbana IX	$+\Delta(1232)$	0.60 mb	
AV 18 $+$ Urbana IX	perturbative Δ	0.62 mb	

Kievsky/Schiavilla/Viviani 1996, 2004

experiment $[0.508 \pm 0.015]$ mb

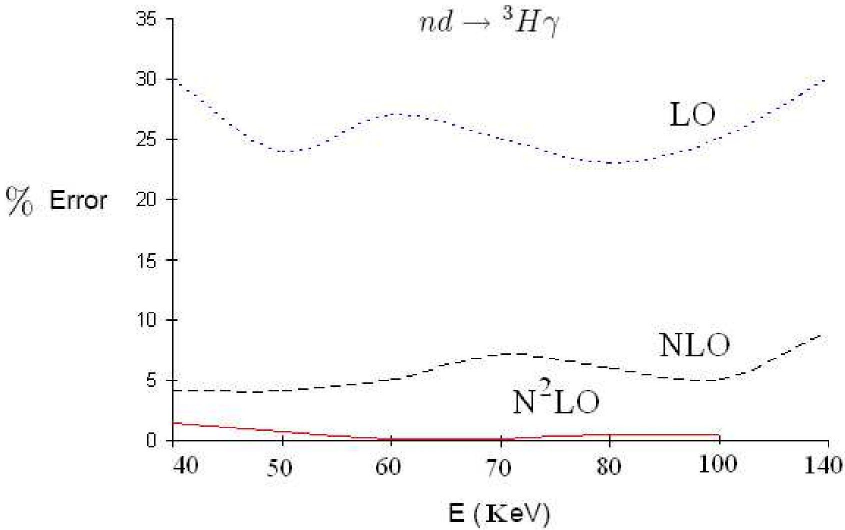
N^{2}LO EFT($\pi\!\!\!/$) no new 3BF $[0.503 \pm 0.003]$ mb $= [\underbrace{0.485}_{\text{LO}} + \underbrace{0.013}_{\text{NLO}} + \underbrace{0.007}_{\text{N}^2\text{LO}}]$ mb





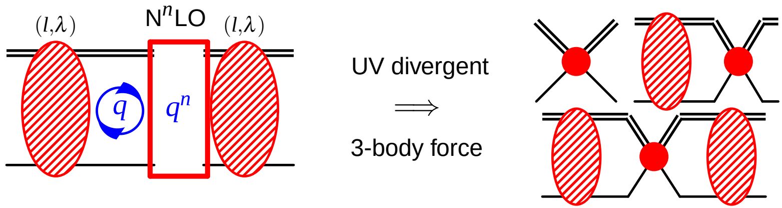
etc.

Factor 200 smaller cross-sections.



(i) $nd \to t\gamma$ for Big-Bang Nucleo-Synthesis

Include 3BF if and only if needed as counter-term to cancel cut-off dependence of low-energy observables.



channel partial waves	naïve dim. an. $\mathrm{Re}[s_l(\lambda)+s_{l'}(\lambda')]$	simplistic $l+l'+2$		typ. size $1/3^n$
$^2S_{Ws}$-$^2S_{Ws}$	LO	N^{2}LO	promoted	100%
$^2S_{Ws}$-$^2S_{Wa}$	N$^{2.2+2}$LO	N^{2+2}LO		1%
$^2S_{Wa}$-$^2S_{Wa}$	N$^{4.3+2}$LO		demoted	0.1%
$^2S_{Ws}$-^4D	N$^{3.1}$LO	N^{4}LO	promoted	3%
$^2S_{Wa}$-^4D	N$^{5.3}$LO		demoted	0.3%
$^2P_{Ws}$-$^2P_{Ws}$	N$^{5.7}$LO		demoted	0.2%
$^2P_{Ws}$-$^2P_{Wa}$, $^2P_{Ws}$-^4P	N$^{4.6}$LO	N^{4}LO	demoted	0.6%
$^2P_{Wa}$-$^2P_{Wa}$, ^4P-^4P	N$^{3.5}$LO			2%
^4S-^4S	N$^{4.3+2}$LO	N^{2+2}LO	demoted	0.1%
^4S-$^2D_{Ws}$	N$^{5.0}$LO		demoted	0.4%
^4S-$^2D_{Wa}$, ^4S-^4D	N$^{5.3}$LO	N^{4}LO	demoted	0.3%
higher	$\sim$ as simplistic	N$^{l+l'+2}$LO		

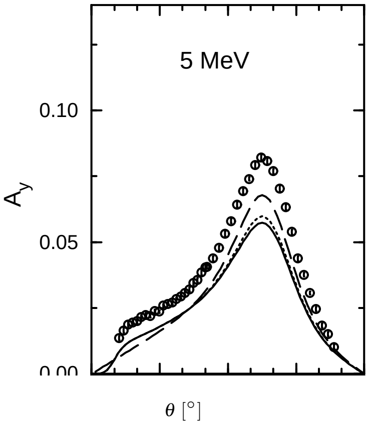
(j) The Future in Few-Nucleon Systems

Classified three-body forces in EFT($\not\pi$): counter-terms to cancel cut-off dependence.

hg et al. ,. . . 2000-05

An Unsolved Puzzle

A_y-problem: Nd spin-observable



30% discrepancy between all potential models and experiment

$\Longrightarrow$ EFT($\not\pi$) has to deliver.

The Future:

- ^{3}H and ^{3,4}He, light nuclei $\Longrightarrow$ exotic nuclei, hyper-nuclei
- nuclear & neutrino Astro-physics:

  big-bang nucleo-synthesis, stellar evolution, Standard Model,. . .
- Universality: molecular systems, Bose-Einstein Condensates,. . .
- Partial-wave regularisation?: answer for any l.
- Apply methodology to χEFT: cf. Nogga/Timmermans/van Kolck 2005

  (i) LO non-perturbative.

  (ii) Higher orders perturbative.

  (iii) Only those 2-, 3-, N-body forces to cancel cut-off dependence.

  (iv) Check by convergence: Nature, orders, “Lepage plots”.

CTs **promoted or demoted** against Weinberg’s proposal.

4. Concluding Questions

EFT($\not\pi$) is ***the*** EFT of QCD at very low energies: local interactions between nucleons only. $\Delta x \gtrsim 2.5\ \mathrm{fm}$

Systematic classification of all three-body forces:

Tenet: 3BF only as counter-term to cancel cut-off dependence in observables.

3BFs mostly suppressed or enhanced against simplistic estimate.

Simplistic power-counting is simplistic.

$\Longrightarrow$ Minimal number of free parameters at given accuracy.

$\Longrightarrow$ Model-independent, systematic, simple, fast, universal, error-estimates.

Successful extension of Naïve Dimensional Analysis to non-perturbative EFTs.

Plethora of pivotal physical processes for prediction & extraction of fundamental nucleon properties, e.g.:

$nd \to t\gamma$, big-bang nucleo-synthesis, stellar evolution ($E_{\mathrm{typ.}} = 30 - 300\mathrm{keV}$) with Sadeghi
properties of $^3\mathrm{H}$ and $^3\mathrm{He}$; neutrinos and light nuclei (calibrating SNO)
A_y-problem, hypernuclei, radioactive beams, atomic trimers, BEC, etc.

$\Longrightarrow$ ***Well on the way to a description of Nuclear Physics deeply rooted in QCD.***