

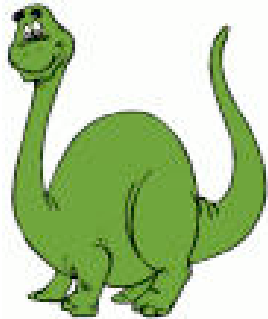
# Covariant effective field theory (EFT!)

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(or, can the dinosaur learn anything from the cockroach?)

Franz Gross

## Outline



Part I: The Covariant Spectator<sup>©</sup> approach for two and three nucleon interactions at JLab momentum transfers (*aside*)



Part II: Ideas for improvements -- toward a Covariant Effective field theory (EFT!) for GeV reactions

# Acknowledgements

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★ CS<sup>©</sup> collaborators (partial list):

- J. W. Van Orden and Karl Holinde (NN - 1990)
- D. O. Riska (2N currents -- 1987)
- Alfred Stadler (3N -- 1997)
- John Tjon and Cetin Savkli (Feynman-Schwinger studies -- 1999)
- Alfred Stadler and Teresa Pena (3N currents -- 2004)

★ CS<sup>©</sup> **teachers**: Dick Arndt, Ruprecht Machleidt, Nijmegen group

★ JLab EFT discussion group (E. Epelbaum, R. Higa, Jose Goity) and Vladimir Pascalutsa

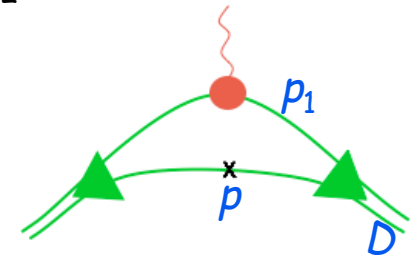
★ All the participants at this series of Workshops -- **many thanks for very enjoyable and stimulating discussions.**

★ Its fun to argue, BUT let's not degenerate into a "Fox News syndrome"  
let's argue about real issues and not fabricated ones

## Aside

★ Typical JLab process: deuteron form factor at  $Q^2 \geq 1 \text{ GeV}^2$

★ In relativistic physics momenta and energies are NOT correlated! We go off the mass-shell, but remain on the energy-shell. In this case (in the Breit system)



$$p^2 = m^2$$

$$p_1^2 = (D - p)^2 = M_d^2 + m^2 - 2\sqrt{M_d^2 + \frac{1}{4}Q^2} \sqrt{m^2 + \mathbf{p}^2} + \mathbf{Q} \cdot \mathbf{p} \leq (M_d - m)^2$$

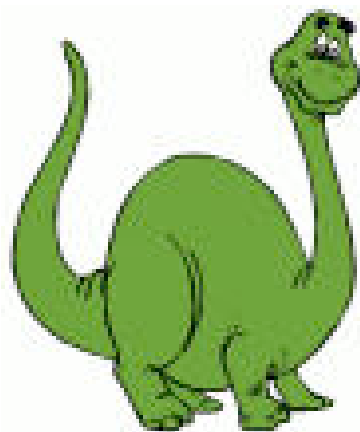
$$D^2 = M_d^2$$

If  $p$  is large, we are far off-shell and probe the short range structure (**the important relative momenta are of order  $Q/4$** ), but still the rest energy of the deuteron is FIXED at its mass; the only energy change is due to the Lorentz boost

★ In nonrelativistic physics we go off the energy shell but remain on the mass shell. In this case  $E + E_1 = \sqrt{m^2 + \mathbf{p}^2} + \sqrt{m^2 + (\frac{1}{2}\mathbf{Q} - \mathbf{p})^2} \geq 2m$  and energy and momentum are correlated

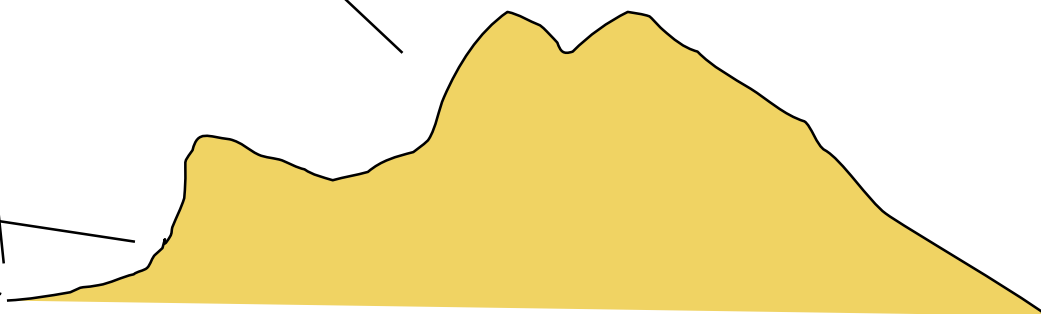
# *The true landscape #1*

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*what does the dinosaur see?*

*what does the cockroach see?*

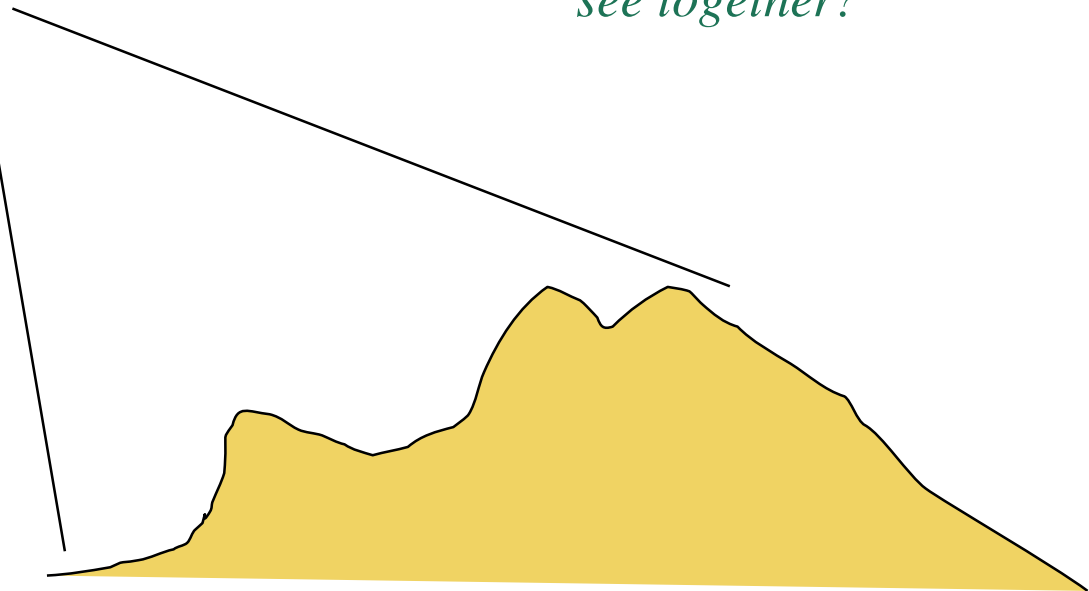


## *The true landscape #2*

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*what do they  
see together?*



## Part I

# Covariant Spectator<sup>©</sup> theory - philosophy

Few body nuclear physics at JLab (GeV) energies (conventional EFT **NOT** an option - *aside*).

What do we do?

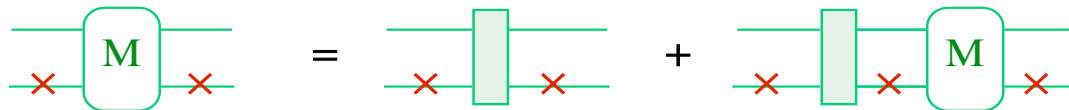
- Preserve all symmetries
  - ◆ *Poincare invariance essential -- manifest covariance useful*
  - ◆ unitarity (conservation of flux)
  - ◆ electromagnetic gauge invariance
  - ◆ chiral invariance
- Microscopic dynamics
  - ◆ OBE dynamics with point couplings, but form factors for the self energies of each hadron
  - ◆ Organizational principle -- include exchanges of all mesons and quantum numbers up to about 1 GeV. Cutoff at the nucleon mass scale.
  - ◆ *Mesons needed:  $\pi$ ,  $2\pi$  ( $\sigma_0$ ,  $\sigma_1$ ),  $\eta$ ,  $\rho$ ,  $\omega$  plus short distance counter terms.*
- Maintain consistency
  - ◆ electromagnetic currents constrained by WT identities (but still not unique)
  - ◆ three-body forces constrained by two-body forces

## Covariant Spectator<sup>©</sup> theory -- Definition

- ★ The spectator theory starts from the  $n$ -body Bethe-Salpeter equation and restricts  $n-1$  particles to their positive energy mass shells. The propagator for these particles is replaced by

$$S_{\alpha\beta}(p) = \frac{(m + \not{p})_{\alpha\beta}}{m^2 - p^2 - i\epsilon} \Rightarrow 2\pi i \delta_+(m^2 - p^2) \sum_s u_\alpha(\mathbf{p}, s) \bar{u}_\beta(\mathbf{p}, s)$$

- ★ Integration over the  $n-1$  internal energies ( $p_0$ ) places these particles on their *positive energy* mass-shell. All 4-d integrations reduce to 3-d integrations.
- ★ Remark: These on-shell particles do not *propagate* in intermediate states. The spinors are absorbed into matrix elements, and the on-shell particles becomes part of the "source" for the single propagating off-shell particle.
- ★ The two body scattering equation is, diagrammatically,

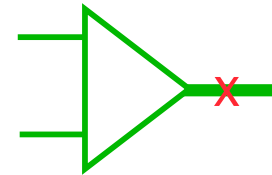


## Both the BS and the CS<sup>©</sup> theories have a close connection to field theory

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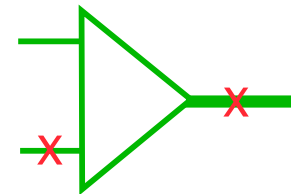
- ★ The Bethe-Salpeter amplitude is a well defined field theoretic matrix element:

$$\Psi(x_1, x_2) = \langle 0 | T(\psi(x_1)\psi(x_2)) | d \rangle$$



- ★ The Covariant Spectator<sup>©</sup> amplitude is *also* a well defined field theoretic amplitude:

$$\Psi(x_1) = \langle N | \psi(x_1) | d \rangle$$



- ★ Equations for the Bethe-Salpeter and the Spectator\* amplitudes can be derived from field theory

- Both are manifestly covariant under *all* Poincaré transformations (advantage)
- Both incorporate negative energy (antiparticle) states (disadvantage?)

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\*O. W. Greenberg's "n-quantum approximation"



# Properties of the two-body Spectator<sup>©</sup> amplitude

★ from translational invariance:

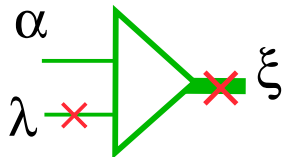
$$\int d^4x e^{-ip \cdot x} \langle n | \psi(x) | d \rangle = (2\pi)^4 \delta^4(p + n - d) \langle n | \psi(0) | d \rangle$$

*exact*

conservation of momentum  
and energy at the vertex

★ from rotational invariance

$$\begin{aligned} \langle n, \lambda | \psi_\alpha(0) | d, \xi \rangle &= \frac{1}{\sqrt{2M_d (2\pi)^3}} \left[ S(p) \Gamma^\mu(p) C \right]_{\alpha\beta} \bar{u}_\beta^T(-\mathbf{p}, \lambda) \xi_\mu \\ &= \left\{ \psi_{\lambda'\lambda}^{+\mu}(\mathbf{p}) u_\alpha(\mathbf{p}, \lambda') + \psi_{\lambda'\lambda}^{-\mu}(\mathbf{p}) v_\alpha(-\mathbf{p}, \lambda') \right\} \xi_\mu \end{aligned}$$



positive  
energy  
spinor

negative  
energy  
spinor

*exact*

the most general form  
possible for the coupling  
of a spin 1 particle to two  
spin 1/2 particles, one  
off-shell

★ from transformations under boosts

$$B(\Lambda) \langle n, \lambda | \psi_\alpha(0) | d, \xi \rangle = B_{\alpha\alpha'} \langle \Lambda n, \lambda' | \psi_{\alpha'}(0) | \Lambda d, \Lambda \xi \rangle D_{\lambda'\lambda}^{(1/2)}(\omega)$$

boost matrix for  
off-shell particle in  
Dirac space

Wigner  
rotation of the spin  
of the on-shell particle

*exact*

obtained from Wigner  
rotations and Dirac  
boost matrix

# Dynamics: phenomenological OBE

★ One boson exchange diagram:

$$\frac{\Lambda(p_1', p_1) \Lambda(p_2', p_2)}{m_m^2 - (p_1' - p_1)^2}$$

★ Scalar:  $\sigma_0$ NN (and  $\sigma_1$ NN) coupling

$$\Lambda(p', p) = g + \frac{v}{2m} [2m - p' - p]$$

← zero on-shell

★ Pseudoscalar:  $\pi$ NN (and  $\eta$ NN) coupling

$$\Lambda(p', p) = g \left\{ \gamma^5 - \frac{1-v}{2m} \left[ (m - \not{p}') \gamma^5 + \gamma^5 (m - \not{p}) \right] \right\}$$

★ Vector:  $\rho$ NN (and  $\omega$ NN) coupling

$$\Lambda(p', p) = g \left\{ \gamma^\mu + \frac{\kappa}{2m} i \sigma^{\mu\nu} (p' - p)_\nu + \frac{v}{2m} \left[ (m - \not{p}') \gamma^\mu + \gamma^\mu (m - \not{p}) \right] \right\}$$

# Family of OBE models for NN scattering based on 1993 calculations

★ Kernel of the integral equation was represented by OBE

$$\begin{aligned}
 & \text{Box Kernel} = \text{Sum of Meson Exchanges} \\
 & = \frac{g_\pi}{v_\pi} \pi + \frac{g_\eta}{v_\eta} \eta + \frac{g_\sigma}{v_\sigma} \sigma + \frac{g_\delta}{v_\delta} \delta + \frac{g_\omega}{\kappa_\omega} \omega + \frac{g_\rho}{\kappa_\rho} \rho
 \end{aligned}$$

$\lambda_\rho = 1 - \frac{v_\rho}{\kappa_\rho}$   
 $\lambda_\omega = 1$

★ 13 Parameters

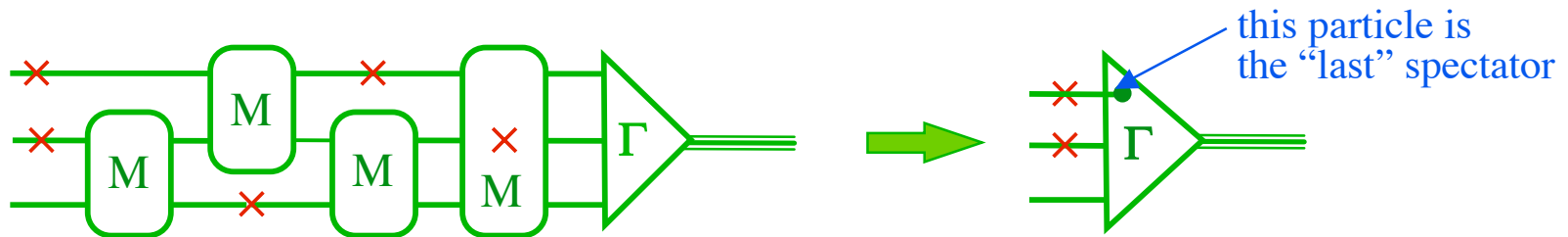
	spin parity	I-spin	mass	$g^2/4\pi$	$\kappa$	# of Para	cutoffs
$\pi$	$0^-$	1	134.98	13.34	--	0	$\Lambda_\pi \approx 2000$
$\eta$	$0^-$	0	548.8	$3.0 \pm 0.25$	--	1	$\Lambda_m \approx 1300$
$\sigma$	$0^+$	0	$\approx 500$	$5.0 \pm 0.5$	--	2	$\Lambda_N \approx 1800$
$\delta$	$0^+$	1	$\approx 500$	$0.6 \pm 0.4^*$	--	2	$\rho$ mixing
$\omega$	$1^-$	0	782.8	$15.0 \pm 1.0$	$\approx 0.2$	2	
$\rho$	$1^-$	1	760.0	$0.8 \pm 0.2$	$7.0 \pm 0.5$	3	$\lambda_\rho = 1.55 \pm 0.4$

We fixed the ratio of the  $v$ 's  $\left\{ \begin{array}{l} v_\sigma = -0.75 v g^2/4\pi \\ v_\delta = 2.60 v g^2/4\pi \end{array} \right.$

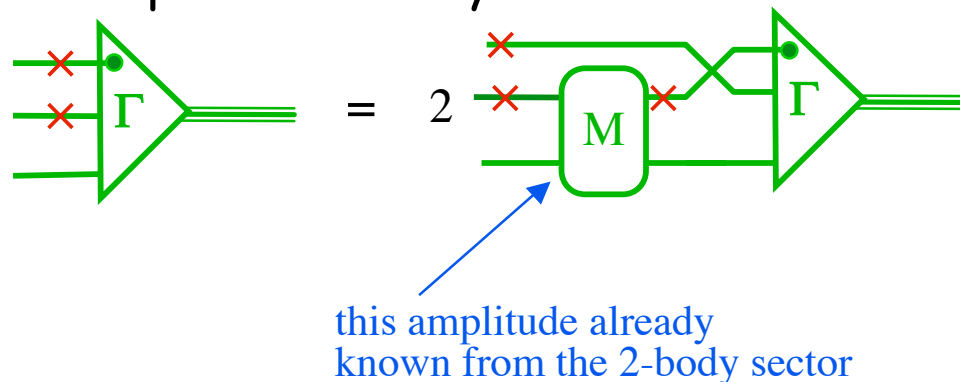
$\chi^2/\text{datum} \sim 2.2$

## Spectator equations for three-body systems\*

- ★ Define three-body vertex functions for each possibility



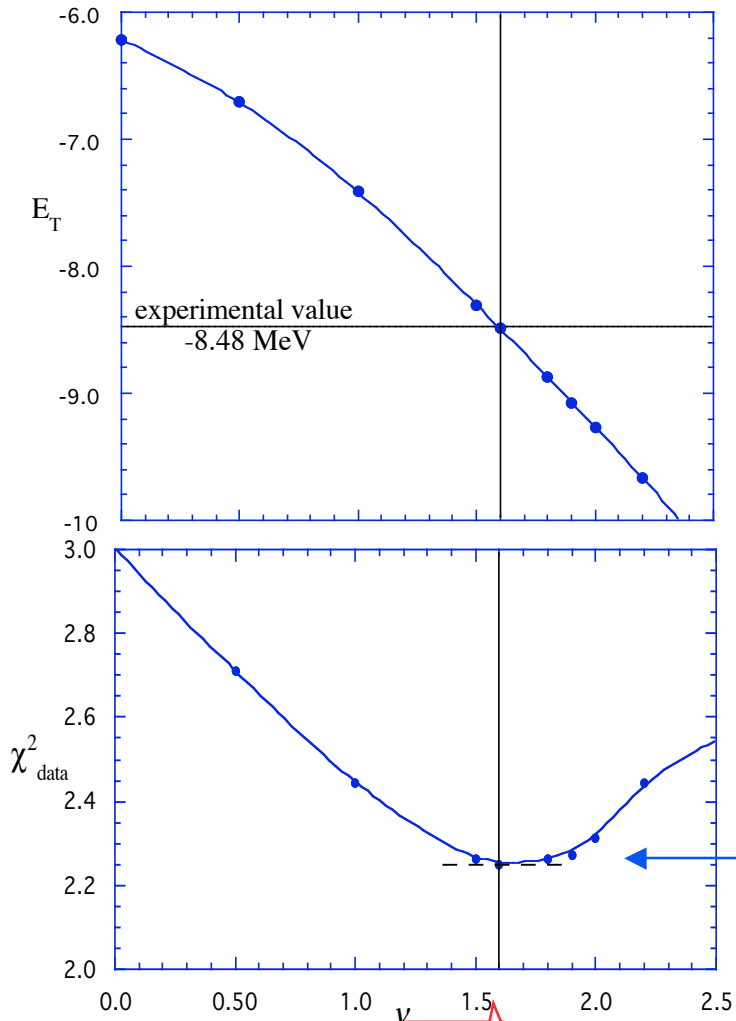
- ★ then three body Faddeev-like equations emerge automatically. For identical particles they are:



\*Alfred Stadler, FG, and Michael Frank, Phys. Rev. C **56**, 2396 (1997)

# 3N binding energy is very sensitive to $\nu$

(off shell coupling of the scalar mesons)\*



$\nu=1.6$  ( $\nu_\sigma = -1.2, \nu_\delta = 4.16$ ) is strongly favored, both by the 3N binding energy and the 2N data!

← experimental binding energy at  $\nu=1.6$ !

← best fit to the 2N data (minimum  $\chi^2$ ) at  $\nu=1.6$ !

\*three body calculations done with Alfred Stadler, Phys. Rev. Letters **78**, 26 (1997)

$\nu=1.6$

## Recent results (in progress): OBE model for NN scattering

★ Kernel of the integral equation is still represented by OBE

★ Recent fit (*still under development*) with 21 Parameters

Thanks to  
J. de Swart  
for helpful  
advice.

	spin parity	I- spin	mass	$g^2/4\pi$	$\kappa$	$\nu$	# of Para
$\gamma$	$1^-$	--	0.001				0
$\pi^\pm$	$0^-$	1	139.57	13.93	--	-0.098	2
$\pi^0$	$0^-$	1	134.98	13.93	--	-0.098	--
$\eta$	$0^-$	0	548.8	4.899	--	1.540	2
$\sigma_0$	$0^+$	0	447	2.597	--	-7.872	3
$\sigma_1$	$0^+$	1	534	1.165	--	3.400	3
$\omega$	$1^-$	0	717	9.409	0.222	0.313	4
$\rho$	$1^-$	1	912	2.270	5.383	-2.107	4

Cutoffs (3)

$\Lambda_\pi = 1786$

$\Lambda_m = 1192$

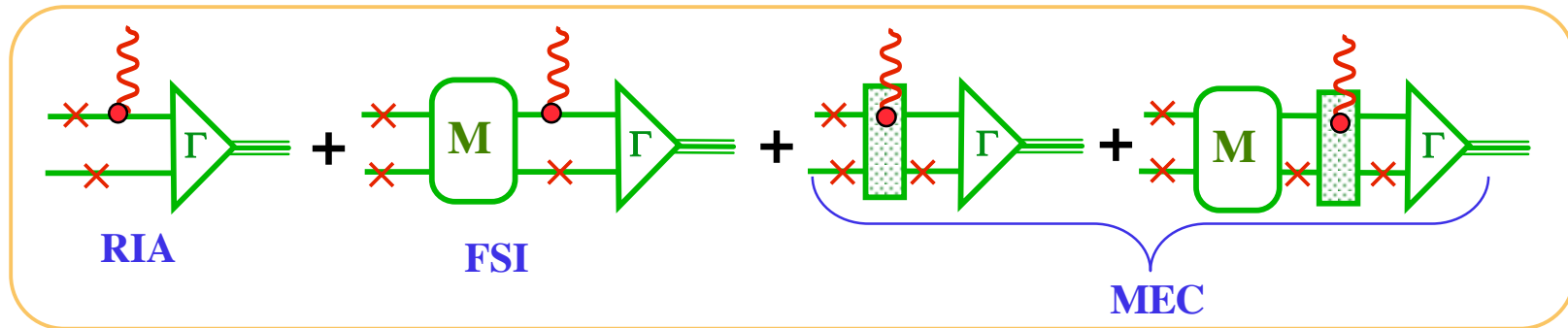
$\Lambda_N = 1861$

★  $\chi^2$  / datum = 1.26 (for the 2001 data set) !

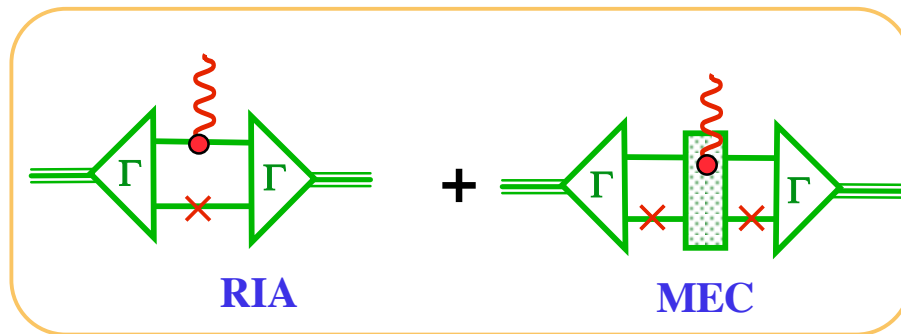
Thanks to Mart Rentmeester and Rob Timmermans for helpful discussions about data

# Two body current operator in the spectator theory<sup>©</sup>

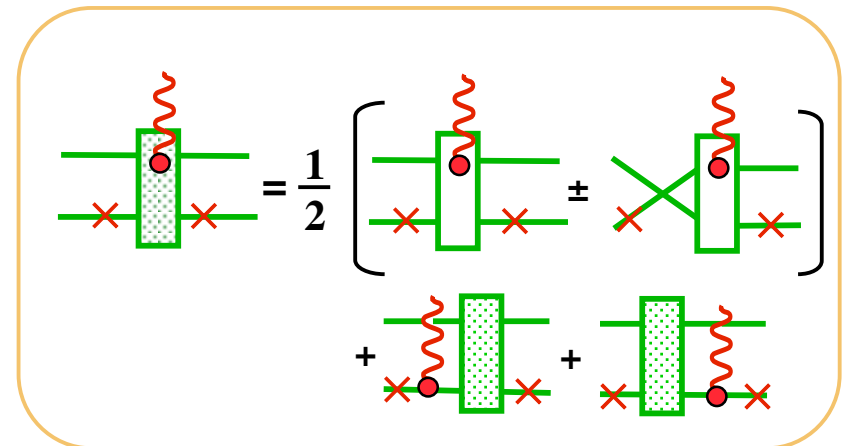
## ★ Inelastic Scattering



## ★ Elastic Scattering

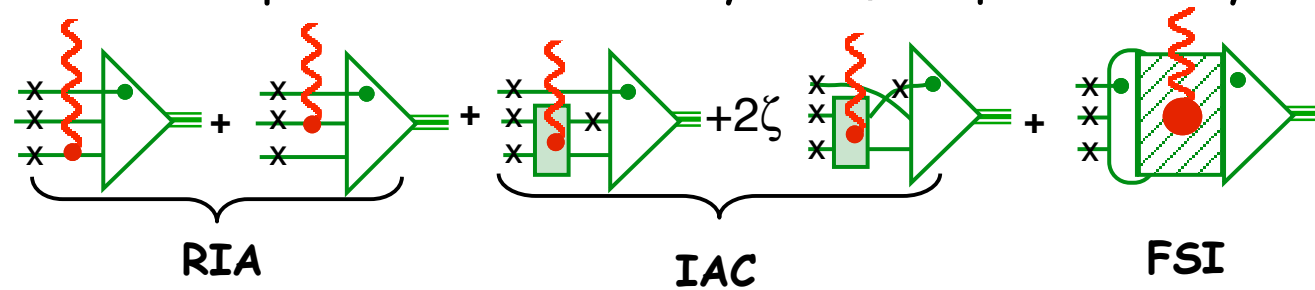


## ★ Interaction current

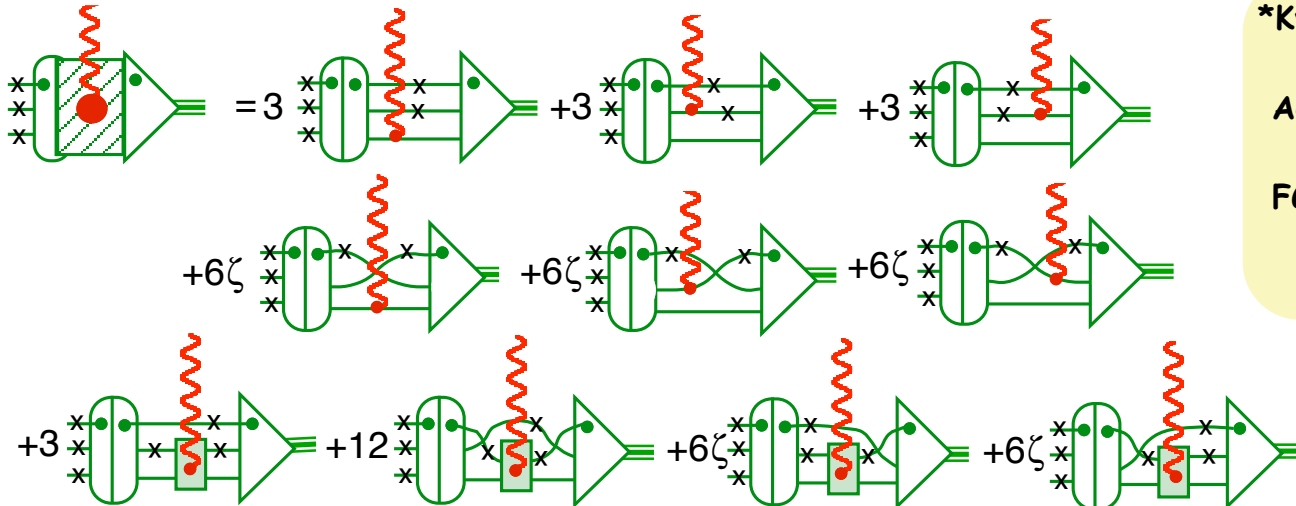


# Three body current operator in the spectator theory<sup>©\*</sup>

- ★ The gauge invariant three-body breakup current in the spectator theory (with on-shell particles labeled by an x) requires many diagrams



where the FSI term is



\*Kvinikhidze & Blankleider,  
 PRC 56, 2973 (1997)  
 Adam & Van Orden  
 PRC 71: 034003 (2005)  
 FG, A. Stadler, & T. Pena  
 PRC 69: 034007 (2004)



## Conclusions to part I

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- ★ We have a covariant theory ( $CS^{\odot}$  theory) suitable for the calculation of 2 and 3 body electromagnetic observables when the excitations are small but the momentum transfers are large.
- ★ It has been (and is being) applied to NN (and 3N?) scattering, deuteron form factors, electrodisintegration of the deuteron,  $^3\text{He}$  form factors, and 2 and 3 body electrodisintegration of  $^3\text{He}$ .
- ★ The goals are to
  - explain these interactions in terms of a consistent dynamics based on the  $CS^{\odot}$  theory using a covariant OBE model.
  - determine the parameters of the OBE model and the OBE interaction currents that emerge.
  - compare these effective interactions with QCD predictions!

## Part II

# Can the ideas of EFT improve the $CS^{\text{C}}$ theory?

- ★ At present, regularization and short range physics are both contained in the form factors
- ★ The most important of these is the nucleon form factor

$$S(p) = \frac{f(p)}{m - \not{p}}; \quad f(p) = \frac{2(\Lambda^2 - m^2)^2}{(\Lambda^2 - p^2)^2 + (\Lambda^2 - m^2)^2}$$

The fits are very sensitive to  $\Lambda$

- ★ Use the ideas of EFT to separate these two roles:
  - Regularize using the PDS of Kaplan, Savage, and Wise
  - Parameterize short range physics using constants
- ★ Assume that the physics is "known" up to exchange masses of about 1 GeV. Short range physics is above 1 GeV

# Overview -- Report on work in progress

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## ★ Assumptions:

- Ignore the “known” physics corresponding to exchanges of bosons with masses less than 1 GeV (add this later).
- Parameterize the short range physics with contact interactions of the  $|\bar{\psi}\psi|^2$  type.
- Chose the mass scale  $M$  for the  $|\bar{\psi}\psi|^2$  interaction to be  $\geq m$  (the nucleon mass)
- Regularize using power divergence subtraction (PDS)

## ★ Example: the $^1S_0$ partial wave

## Lagrangian for a $^1S_0$ state

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★ Introduce the  $NN \Rightarrow ^1S_0$  vertex function

$$\Gamma^{0+}(x) = (C\gamma^5)_{ab} \psi_a(x) \psi_b(x) = \psi^T(x) C\gamma^5 \psi(x)$$

★ Then, the Lagrangian density for a  $^1S_0$  state is

$$L(x) = \bar{\psi}(x) \left( i \overleftrightarrow{\not{D}} - m \right) \psi(x) - \lambda [\Gamma^{0+}(x)]^\dagger \Gamma^{0+}(x)$$

★ In  $d$  dimensions,  $\lambda$  has dimensions of  $\ell^{2-d}$ , so the coupling is

$$\lambda = \frac{\lambda_0}{M^{2-d}}$$

where  $\lambda_0$  is dimensionless.

## Power counting (naive)

---

- ★ Scales: there are only three scales in the problem:  $p$ ,  $m$ , and  $M$ .

First, assume  $p \sim m \ll M$ , so that  $\delta \approx \frac{p}{M} \sim \frac{m}{M} \ll 1$

- ★ Naive dimensional counting gives a superficial divergence  $D$  for any  $NN$  scattering Feynman diagram equal to

$$D = (d-2)(n-1)$$

where  $n$  is the number of vertices (or the order) of the diagram. Hence we expect the size of any diagram  $V^{(n)}$  to go like

$$V^{(n)} \approx \frac{\lambda_0}{M^{d-2}} (\lambda_0 \delta^{d-2})^{n-1}$$

- ★ *Conclusion:* if  $\lambda_0 \sim 1$ , and  $d > 2$ , perturbation theory applies.

## Power counting (nonrelativistic domain)

★ Assume,  $p \ll m$ . Introduce a new scale  $\alpha = \frac{p}{m} \sim \delta^2 \ll \delta$

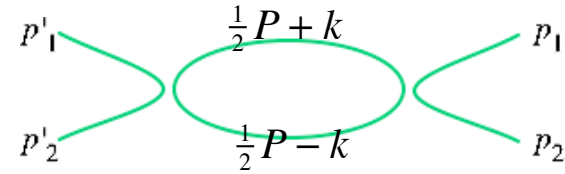
★ Then the  $n^{\text{th}}$  order diagram has the form:

$$V^{(n)} \approx \frac{\lambda_0}{M^{d-2}} \left( \lambda_0 \left( \frac{m}{M} \right)^{d-2} \left[ \underbrace{1}_{\text{large "constant" term}} + \underbrace{c_1 \alpha}_{\sim 1} + \underbrace{\sum_1^{\infty} c_n \alpha^n}_{\text{small remainder term}} \right] \right)^{n-1}$$

★ Assume that the term of order  $\alpha$  is of order 1. Then only the terms of order  $\alpha^2$  or smaller can be ignored. All other terms must be summed to all orders. Hence:

- All diagrams contribute to the constant term. This depends on the scale, and will be fixed phenomenologically.
- The task is to calculate the terms of order  $\alpha$ , which are independent of the scale.

## Example: the $s$ -channel bubble



The  $s$ -channel bubble  $B(s)$  is  $V^{(2)}(s) = \lambda^2 B(s) (C\gamma^5)_{a'b'} (C\gamma^5)_{ba}$

where  $\varepsilon_d = 2 - d/2$ ,  $\mathbf{p}^2 = m^2 - s/4$ , and

$$\begin{aligned} \lambda B(s) &= -i \frac{\lambda_0}{2M^{d-2}} \int \frac{d^d k}{(2\pi)^d} \frac{\text{tr} \left[ C\gamma^5 \left( m + \frac{1}{2} \not{\mathcal{R}} + \not{\mathcal{K}} \right) C\gamma^5 \left( m + \frac{1}{2} \not{\mathcal{R}} - \not{\mathcal{K}} \right)^T \right]}{\left[ m^2 - \left( \frac{1}{2} P + k \right)^2 - i\varepsilon \right] \left[ m^2 - \left( \frac{1}{2} P - k \right)^2 - i\varepsilon \right]} \\ &= \frac{\lambda_0 \Gamma(\varepsilon_d) M^{2\varepsilon_d - 2}}{(4\pi)^{2-\varepsilon_d}} \int_{-\frac{1}{2}}^{\frac{1}{2}} dz \left\{ \frac{4m^2}{\left( \mathbf{p}^2 + sz^2 \right)^{\varepsilon_d}} - \frac{6 - 4\varepsilon_d}{(1 - \varepsilon_d) \left( \mathbf{p}^2 + sz^2 \right)^{\varepsilon_d - 1}} \right\} \end{aligned}$$

$$\lim_{d \rightarrow 4} \lambda B(s) = \lambda_0 \left\{ \mu - \frac{\mathbf{p}\sqrt{s}}{8\pi M^2} + R(\mathbf{p}^2) \right\}$$

with the large constant  $\mu = \frac{1}{2\pi} + \frac{m^2}{8\pi^2 M^2} \left( \Gamma'(1) - \log \left( \frac{4\pi m^2}{M^2} \right) + 3 - \frac{10}{9} \log 2 \right)$

and the small remainder  $R(\mathbf{p}^2) = \frac{\mathbf{p}\sqrt{s}}{4\pi^2 M^2} \arctan \frac{2\mathbf{p}}{\sqrt{s}} - \frac{\mathbf{p}^2}{4\pi^2 M^2} \left( \Gamma'(1) - \log \left( \frac{4\pi m^2}{M^2} \right) + 2 - \frac{5}{9} \log 2 \right)$

## Summation of the leading bubble terms gives

---

$$\begin{aligned}
 M^{0+}(s) &= \frac{\lambda_0}{M^2} \left[ 1 + \lambda B(s) + (\lambda B(s))^2 + \dots \right] \\
 &= \frac{\lambda_0}{M^2 - \lambda_0 \left( \mu M^2 - \frac{\mathbf{p}\sqrt{s}}{8\pi} \right)} = \frac{1}{M^2 \left( \frac{1}{\lambda_0} - \mu \right) + \frac{\mathbf{p}\sqrt{s}}{8\pi}} \\
 &= \frac{8\pi}{m_0^2 + \mathbf{p}\sqrt{s}}
 \end{aligned}$$

where  $m_0$  is a parameter fixed by the effective range expansion.

Hence,  $\lambda_0$  runs with  $M$  according to:

$$\lambda_0 = \frac{1}{\mu + \frac{m_0^2}{8\pi M^2}}$$

and  $M$  has a pole (or resonance) at

$$s = 2m^2 \left( 1 \pm \sqrt{1 - \frac{m_0^4}{m^4}} \right)$$



## The $u$ - and $t$ -channel bubbles

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- ★ New feature of relativistic theory is the presence of  $u$ - and  $t$ -channel bubbles
- ★ We can show(?) that these are analytic in  $p^2$  near  $p^2 \sim 0$ . Hence they contribute only to the constant and  $p^2$  terms, and their effect can be absorbed into adjustable parameters
- ★ Conclusion: the presence of  $u$ - and  $t$ -channel bubbles does not change the conclusions drawn from study of  $s$ -channel bubbles

## Conclusion:

### Power counting for relativistic theory in a nonrelativistic domain

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- ★ Each diagram is of the form  $M = \text{Constant} + c_1\alpha + R(\alpha^2)$
- ★ All diagrams contribute a constant term, which sometimes violates power counting. This does not matter since the constant term is simply a parameter describing the short range physics that must be fit to the data.
- ★ Only diagrams with an elastic cut contribute non-analytic terms of order  $\alpha$ , and these can be calculated and summed.
- ★ The remainder terms  $R(\alpha^2)$  are analytic and can be absorbed into derivative terms in the Lagrangian. These are then fit to the data.
- ★ Not much predictive power, but divergences are handled without form factors. (Is this really an advantage?)

## Connection with the Covariant Spectator<sup>©</sup> theory

★ If  $A_{\pm} - i\varepsilon = m^2 - \left(\frac{1}{2}P \pm k\right)^2 - i\varepsilon = \left(E_k - \frac{1}{2}P_0 \mp k_0 - i\varepsilon\right)\left(E_k + \frac{1}{2}P_0 \pm k_0 - i\varepsilon\right)$

$$A_+ - A_- - i\varepsilon = -2P \cdot k - i\varepsilon$$

the CS<sup>©</sup> theory gives the following bubble contribution

$$\begin{aligned}
 B_{CS}(s) &= -\frac{i}{4} \int \frac{d^d k}{(2\pi)^d} N \left\{ \frac{1}{\underbrace{(A_- - A_+ - i\varepsilon)(A_+ - i\varepsilon)}_a} + \frac{1}{\underbrace{(A_+ - A_- - i\varepsilon)(A_- - i\varepsilon)}_b} \right\} \\
 &\quad \text{one pole LHP} \qquad \qquad \qquad \text{one pole UHP} \\
 &= -\frac{i}{4} \int \frac{d^d k}{(2\pi)^d} N \int_0^{\infty} dx \left\{ \frac{1}{\left[ (1-x)A_+ + xA_- - i(1+x)\varepsilon \right]^2} + \frac{1}{\left[ (1-x)A_- + xA_+ - i(1+x)\varepsilon \right]} \right\} \\
 &= -\frac{i}{4} \int \frac{d^d k}{(2\pi)^d} N \left\{ \int_{-\frac{1}{2}}^{\infty} dz \frac{1}{\left[ \left(\frac{1}{2} - z\right)A_+ + \left(\frac{1}{2} + z\right)A_- - i\left(\frac{3}{2} + z\right)\varepsilon \right]^2} + \int_{-\infty}^{\frac{1}{2}} dz \frac{1}{\left[ \left(\frac{1}{2} + z\right)A_- + \left(\frac{1}{2} - z\right)A_+ - i\left(\frac{3}{2} - z\right)\varepsilon \right]} \right\} \\
 &\quad \text{x=1/2+z} \qquad \qquad \qquad \text{x=1/2-z}
 \end{aligned}$$

## Connection with the Covariant Spectator<sup>©</sup> theory (2)

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★ Hence

$$B_{CS}(s) = \frac{i}{4} \int \frac{d^d k}{(2\pi)^d} \int_{-\infty}^{\infty} dz \frac{N}{\left[ \left(\frac{1}{2} - z\right)A_+ + \left(\frac{1}{2} + z\right)A_- - i\left(\frac{3}{2} + z\right)\epsilon \right]^2} + \frac{1}{2} B(s)$$

$$\mu' - \frac{\mathbf{p}\sqrt{s}}{8\pi M^2} + \frac{1}{2} R(\mathbf{p}^2)$$

Different constant and half the remainder term

THEREFORE, in an EFT sense, equivalent to the full bubble term

★ CS<sup>©</sup> theory is equivalent (from an EFT! point of view) to the full field theory. (Is this useful?)

## Extensions and reinterpretation: EFT! in a relativistic domain

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- ★ Suppose we are in the relativistic domain ( $p \sim m$ ). Then either:
  - power counting works, because  $\lambda_0 \ll (M/m)^2$  and the physics is perturbative, or
  - $\lambda_0 \sim (M/m)^2$ , power counting does not work, and all diagrams are of equal size, and all must be summed
- ★ Even if power counting does NOT work, we may
  - separate the terms nonanalytic in  $p^2$  from those which are analytic, and sum them using  $s$ -channel bubbles. These terms are predicted by the theory (up to the arbitrary constant!). Is this important??
  - adjust the size of the analytic terms by adding derivative terms to the Lagrangian. These cannot be predicted. There is also no longer an organizational principle for choosing derivatives -- all are of equal size.
- ★ In either case, this relativistic effective theory has no content (i.e.) is purely phenomenological (in common with its nonrelativistic counterpart). But we have found a way to regularize and handle the short range physics.

## Where do we go from here? One scenario:

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- ★ Step 1: Start with a kernel of the form

$$V = \mu - \frac{\mathbf{p}\sqrt{s}}{8\pi M^2} + c_2 \mathbf{p}^2 + c_4 \mathbf{p}^4 + \dots$$

- ★ Fit data

adjustable parameters

- ★ Add the pion (and other meson exchanges?) in an attempt to reduce the number of unknown short range parameters. For example, get  $c_2$  and  $c_4$  from meson exchange??
- ★ Calculate meson exchange without form factors or cutoffs using a "two potential" formalism and a Pade' series.

## Where do we go from here? One scenario (2)

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★ the “two potential” series is

$$M = M_m + (1 + M_m)|0\rangle \frac{\lambda}{1 + \lambda B(1 + M_m)} \langle 0|(1 + M_m)$$

★ the Pade series is used to calculate  $M_m$

$$M_m = \sum_{n=1}^{2p} c_n x^n = \frac{\sum_{n=1}^p d_n^1 x^n}{\sum_{n'=1}^p d_n^2 x^{n'}}$$

## Conclusions to Part II

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- ★ What has been learned from this exercise?
  - A full power counting scheme, with accompanying organizational principle, seems to exist only in nonrelativistic situations.
  - BUT it can be done with either a relativistic, or nonrelativistic formalism.
  - From the EFT! point of view, CS<sup>©</sup> theory is just as good as a full field theory (to be thought about some more)
- ★ Does this help justify our original approach and calculations?
- ★ Why do relativistic calculations?
  - justified if fewer parameters are needed to fit data and a greater unity between dynamics and interaction currents can be achieved



END

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