Separating Long- and Short-distance Physics in ChEFT

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Outline of the talk

- Introduction to the problem
- Simple example: Mass of the nucleon
- Example 2: Polarizabilities of the nucleon
- Example 3: Compton scattering on the deuteron
- Summary/Outlook

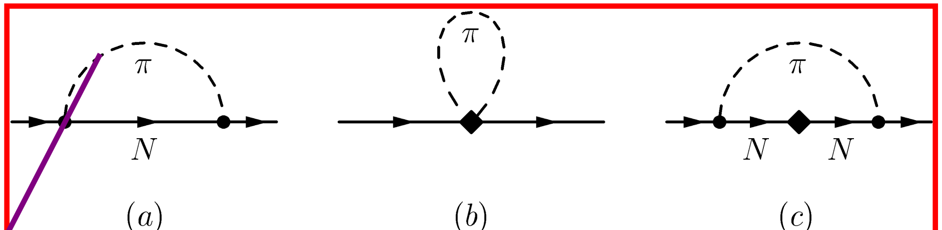
Introduction to the problem I

- Results in Field Theory are/should be cutoff/scale independent:
  - (Scale-dependent) counter terms make sure that we can even use λ=1 TeV to obtain the same finite result as we get for λ=500 MeV
- Effective field theories like ChEFT need a power-counting scheme to be utilized:
  - (Naive) power-counting utilized to identify the counter terms to be included in a calculation at a particular order
  - Power counting also needed for the calculation of the loop diagrams

Introduction to the problem II

- In order to calculate the loop graphs in the p-regime, we assume that the pion propagator counts as $1/p^2$, the nucleon propagator counts as $1/p$, ...

E.g. Nucleon Mass:
3 $O(p^4)$ diagrams



- Assumption behind the (naive) counting rule: $p < \Lambda\chi$
- Loop-Integrals:

Momentum p

$$\int_0^\infty p^2 dp \ldots = \int_0^{\Lambda_\chi} p^2 dp \ldots + \int_{\Lambda_\chi}^\infty p^2 dp \ldots$$

"static" regime

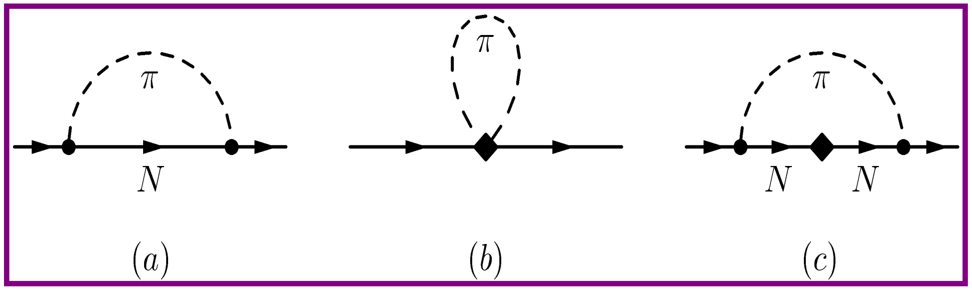
- Conclusion: Demand that **second term** in all (finite) loop-integrals is **negligible** !

Example 1: Nucleon Mass

- Nucleon Mass to O(p^4) in HBChPT

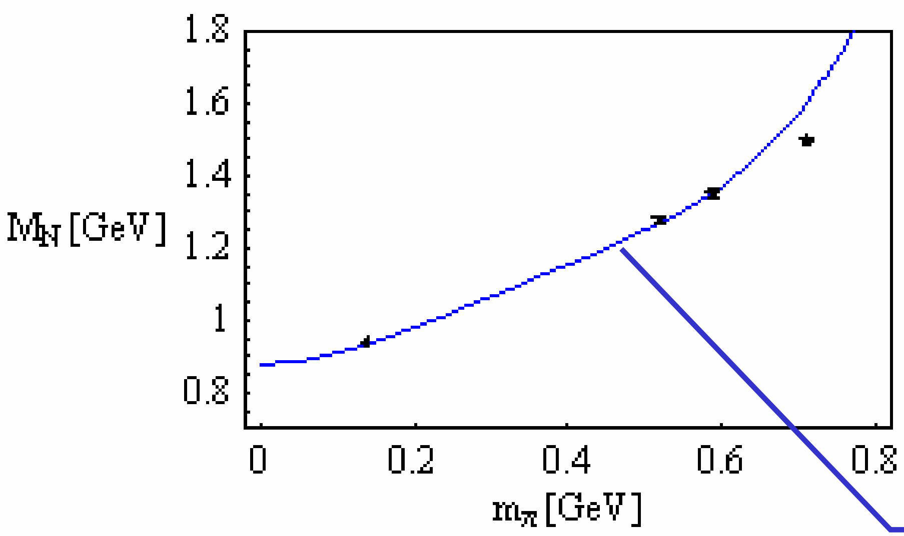
$$M_N = M_0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{32\pi F_\pi^2} m_\pi^3 + \frac{3}{128\pi^2 F_\pi^2}\left(c_2 - \frac{2g_A^2}{M_0}\right) m_\pi^4$$
$$- 4e_1(\lambda)\, m_\pi^4 - \frac{3}{32\pi^2 F_\pi^2}\left(-8c_1 + c_2 + 4c_3 + \frac{g_A^2}{M_0}\right) m_\pi^4 \ln\frac{m_\pi}{\lambda}$$

O(p^4) Diagrams:
(+ 3 c.t.s)



Example 1: Nucleon Mass

- Fit HBChPT $O(p^4)$ result to (CP-PACS) lattice data:



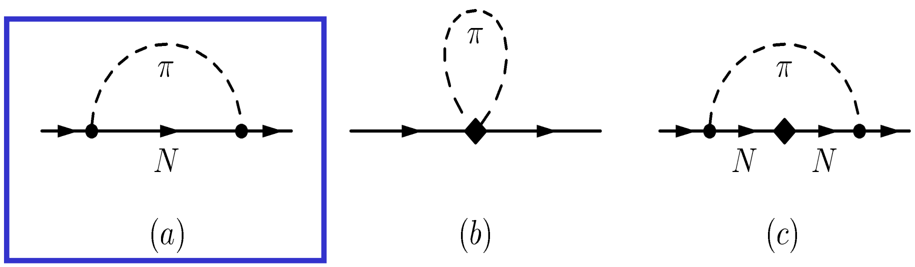
v. Bernard, TRH, U.-G. Meißner, Nucl. Phys. A732, 149 (2004)

Is the second Integral for all points on this curve really negligible ?

- Input: g_A, c_2, c_3, F_π
- Output: M_0, c_1, $e_1(\lambda)$ → see literature
- Same curve for any chosen value of λ

Example 1: Nucleon Mass

- E.g. pick diagram a (b+c behave “well”)



- Evaluate diagram a by integrating up to a sharp (spherical) cutoff in momentum-space.

Find:
$$\frac{g_A^2}{(4\pi F_\pi)^2}\Lambda^3 - \frac{3g_A^2 m_\pi^2}{(4\pi F_\pi)^2}\Lambda - \frac{3g_A^2 m_\pi^3}{32\pi F_\pi^2} + \frac{3g_A^2 m_\pi^3}{(4\pi F_\pi)^2}\arctan\frac{m_\pi}{\Lambda}$$

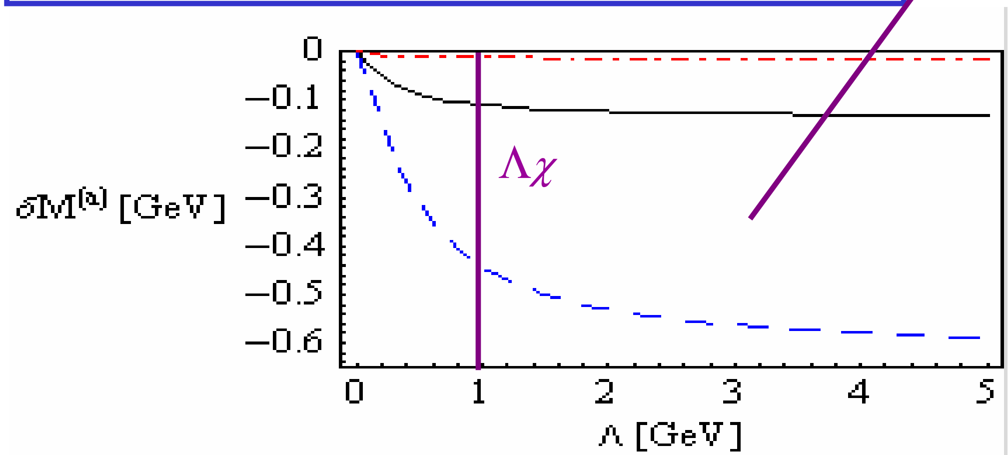
- Terms ~ Λ, Λ^3 must be absorbed in c_1, M_0
→ finite result: $M_N = M_0^{(r)} - 4\,c_1^{(r)}\, m_\pi^2 + \ldots$

Example 1: Nucleon Mass

- Study the finite contribution of diagram a to the nucleon mass for various values for m_π:

$$\delta M^{(a)} = -\frac{3g_A^2 m_\pi^3}{32\pi F_\pi^2} + \frac{3g_A^2 m_\pi^3}{(4\pi F_\pi)^2} \arctan\frac{m_\pi}{\Lambda}$$

Contribution of Integral 2



M_π=140 MeV→ o.k.
M_π=300 MeV→ o.k.
M_π=500 MeV→ †

- Breakdown of HBChPT for m_π> 400 MeV ???

(bad news for chiral SU(3) dynamics?) Not so simple …

Example 1: Nucleon Mass

- Careful analysis: A large part of the contributions from integral 2 actually cannot be distinguished from a counter term contribution present in the O(p^4) formula !

$$\delta M^{(a)} = -\frac{3g_A^2 m_\pi^3}{32\pi F_\pi^2} + \frac{3g_A^2 m_\pi^3}{(4\pi F_\pi)^2} \arctan\frac{m_\pi}{\Lambda}$$

$$= -\frac{3g_A^2 m_\pi^3}{32\pi F_\pi^2} + \boxed{\frac{3g_A^2 m_\pi^3}{(4\pi F_\pi)^2}\left(\frac{m_\pi}{\Lambda}\right.} - \frac{1}{3}\frac{m_\pi^3}{\Lambda^3} + \ldots\left.\right)$$

Compare with counterterm contributions

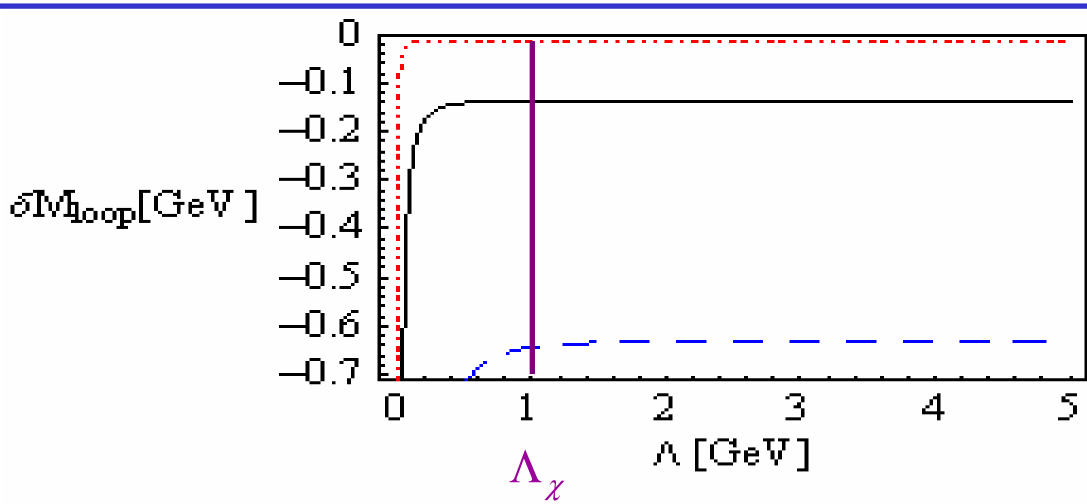
$$\delta M^{(c.t.)} = M_0 - 4c_1 m_\pi^2 - \boxed{4e_1 m_\pi^4}$$

$$\Rightarrow \text{split up } e_1: \quad e_1 = e_1^{fin} - \frac{3g_A^2}{64\pi^2 F_\pi^2}\frac{1}{\Lambda}$$

Example 1: Nucleon Mass

- Absorb all scale dependence that cannot be distinguished from given counter term contributions at the order we are working (here: c.t. Required uniquely identified via m_π dependence)
- Remaining contribution of diagram a: (e_1^{fin} not included)

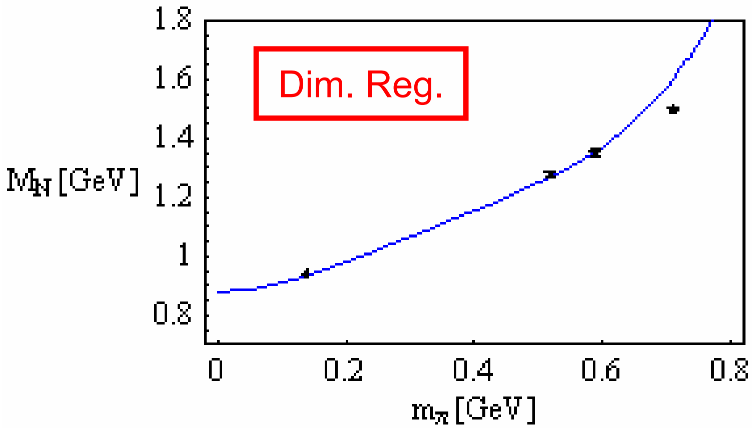
$$\delta M^{(a)}_{loop} = -\frac{3g_A^2 m_\pi^3}{32\pi F_\pi^2} - \frac{3g_A^2 m_\pi^4}{(4\pi F_\pi)^2 \Lambda} + \frac{3g_A^2 m_\pi^3}{(4\pi F_\pi)^2} \arctan\frac{m_\pi}{\Lambda}$$

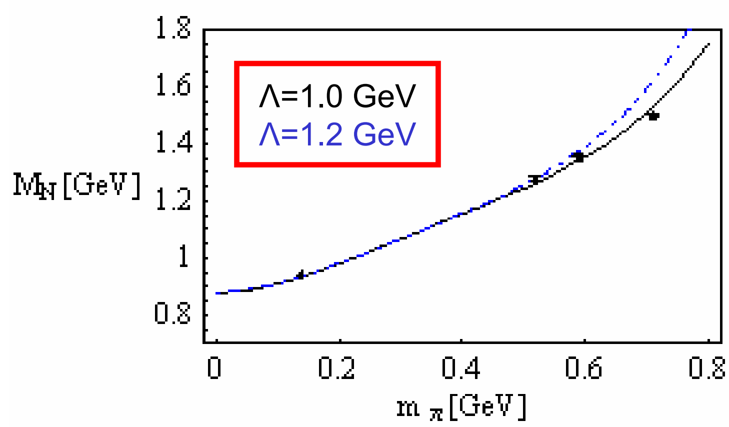


M_π=140 MeV→ o.k.
M_π=300 MeV→ o.k.
M_π=500 MeV→ o.k
Good news for SU(3) ?

Example 1: Nucleon Mass

- In example 1 we have not/marginally changed the final $O(p^4)$ HBChPT result





- We have learned that even the HBChPT $O(p^4)$ result for the nucleon mass is consistent with the condition that integral 2 is negligible up to $m_\pi \approx m_K$
- In ChEFT one can systematically absorb **even finite** "high"-momentum modes into counter terms

v. Bernard, TRH, U.-G. Meißner, Nucl. Phys. A732, 149 (2004)

Interlude

- Q: Why did the HBChPT $O(p^4)$ result in example 1 not change?
- A: Because the counter term e_1 responsible for the scale dependence was part of the $O(p^4)$ HBChPT result
- Q: Are there examples where this is not the case ?
- A: Many. E.g. Leading HBChPT result for polarizabilities of the nucleon

Dipole Polarizabilities

- Nucleon of mass M with internal structure in an external electric/magnetic field
- Effective Hamiltonian:

$$H_{eff}^{(2)} = -\frac{1}{2} 4\pi \alpha_E \vec{E}^2 - \frac{1}{2} 4\pi \beta_M \vec{H}^2$$

- Proton:

$$\alpha_E^{\text{exp.}} = (12.4 \pm 0.6 \mu 0.5 \pm 0.1) \times 10^{-4} fm^3,$$
$$\beta_M^{\text{exp.}} = (1.4 \pm 0.7 \pm 0.4 \pm 0.1) \times 10^{-4} fm^3.$$

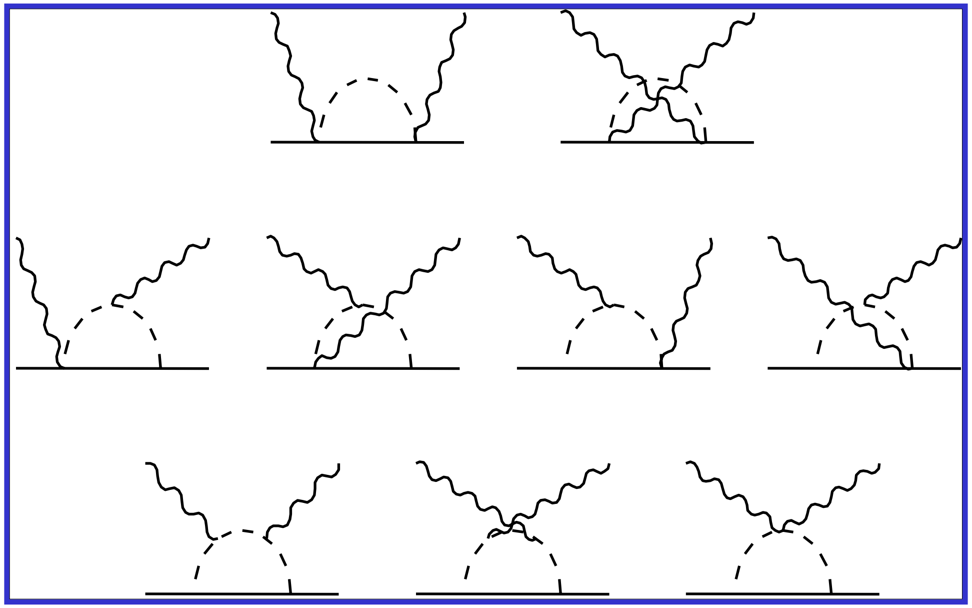
Real Compton-Scattering

- 2 “numbers”, physics ?

α_E in $O(p^3)$ HBChPT

- Find

$$\alpha_E = \frac{5e^2 g_A^2}{384\pi^2 F_\pi^2}\frac{1}{m_\pi} + O(p^4)$$
$$\approx 12 \times 10^{-4}\, fm^3$$



- Good agreement with experiment !?
- No physics beyond πN at leading order ?

O(p3) HBChPT result is finite and has no counter terms !
→ Study finite "high"-energy modes

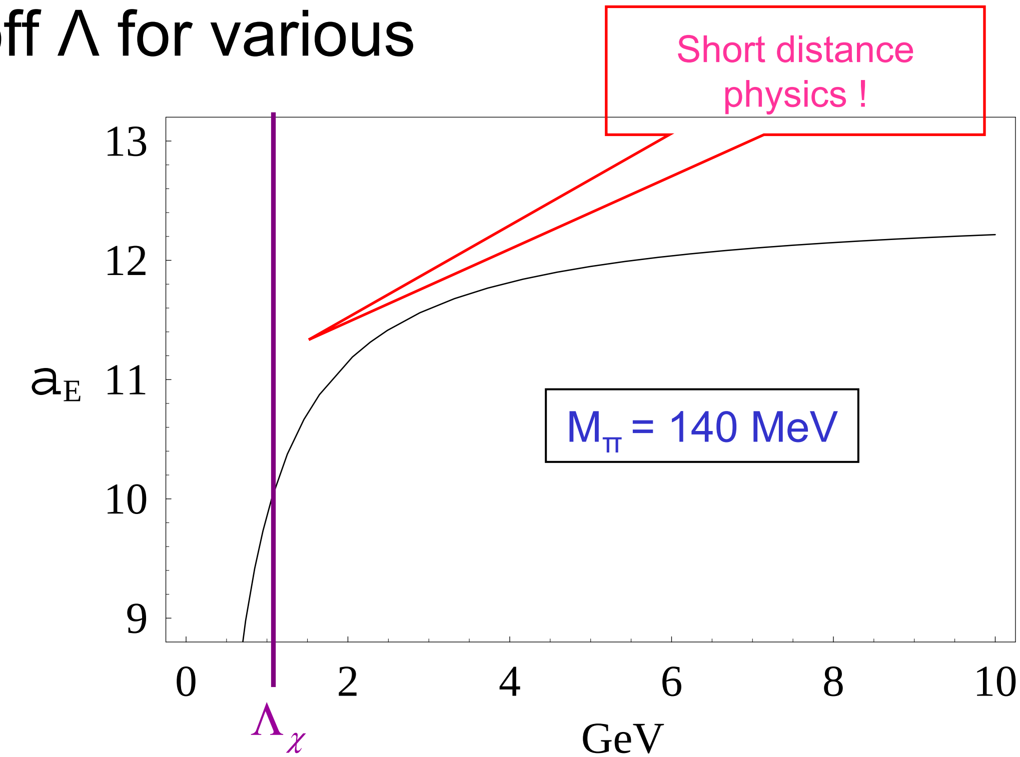
Contributions to α_E

- Same procedure as in example 1
- Study the **finite** loop contribution at $O(p^3)$ as a function of a radial cutoff Λ for various values of m_π :

$$\int_{-\infty}^{+\infty} d^3 l \quad \Rightarrow \quad 4\pi \int_0^{\Lambda} d\, l$$

$$l << \Lambda_\chi \approx 1\,\mathrm{GeV}\,!$$

(Ultraviolet terms ~ Λ^n subtracted !)



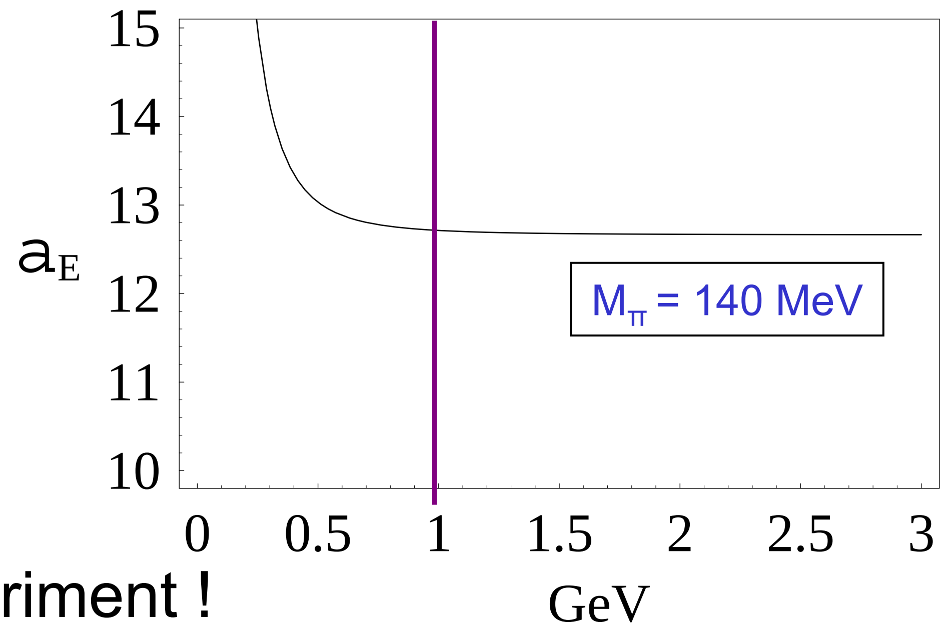
Short Distance Physics

- Couplings from the O(p^4) Lagrangian can be utilized to properly account for the short-distance dynamics in the O(p^3) result

TRH, B.R. Holstein, forthcoming

$$\alpha_E = \frac{5\,e^2 g_A^2}{384\pi^2 F_\pi^2}\frac{1}{m_\pi} + \delta\alpha + O\left(\bar{p}^4\right)$$

1/Λ effects !

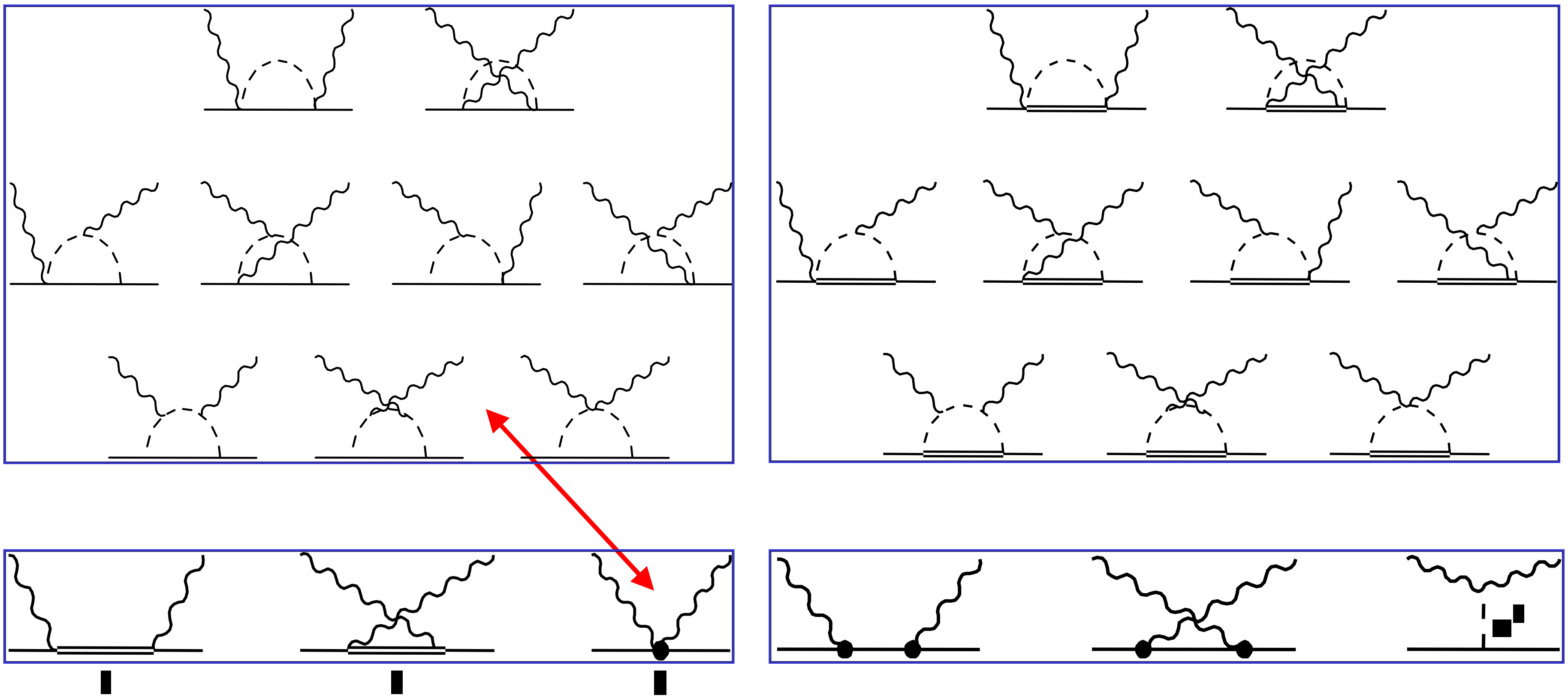


- Fit δα, δβ to experiment !

Compton Scattering to $O(\varepsilon^3)$

TRH, B.R. Holstein and J. Kambor, PRD 55, 2630 1997
R. Hildebrandt et al., EPJA 20, 293 (2004)

- Diagrams



Example 2: Polarizabilities

- In the case of α_E, β_M even the physical pion mass m_π=140 MeV is too large for a vanishing contribution from Integral 2 in $O(p^3)$ HBChPT

  (m_π=50 MeV would be ok!)

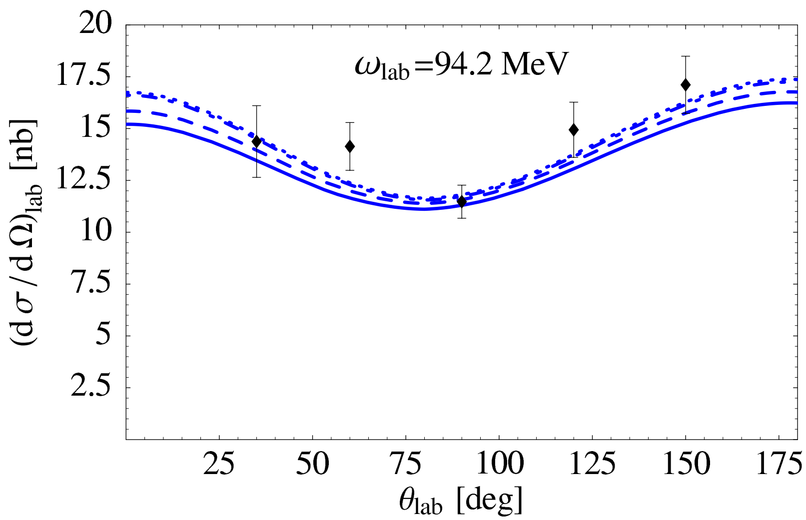
  Conclusion: This effect is channel dependent and has to be studied separately for each observable
- One can easily correct for that, at the cost of introducing 2 extra counter terms $\delta\alpha,\delta\beta$ in the $O(p^3)$ HBChPT result. (naively part of $O(p^4)$ Lagrange)

  → the finite parts $\delta\alpha^{fin},\delta\beta^{fin}$ of these 2 c.t.s have to be fit to data

  (= short distance contribution to polarizabilities)
- Consistency requirement can (**sometimes**) lead to the promotion of c.t.s into lower orders than estimated by naive dimensional analysis (Not required to know the actual numerical size of the finite parts of the c.t.s for this promotion!)
- Same story for $O(\varepsilon^3)$ SSE … ($O(p^3)/O(\varepsilon^3)$ Spin-polas are ok!)

Example 3: Compton on D

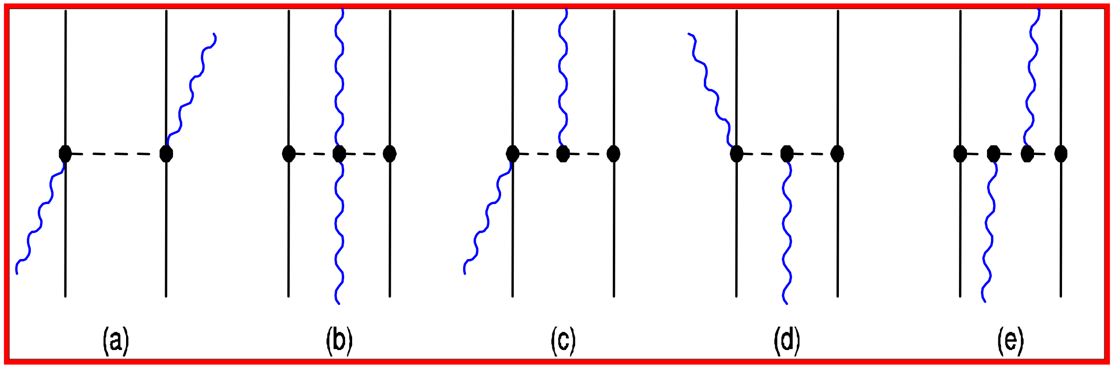
- $O(\varepsilon^3)$ SSE calculation: R.P. Hildebrandt et al., Nucl. Phys. A748, 573 (2005)
  - includes $\delta\alpha^{fin}$,$\delta\beta^{fin}$, fit to Compton proton data (justified phenomenologically)

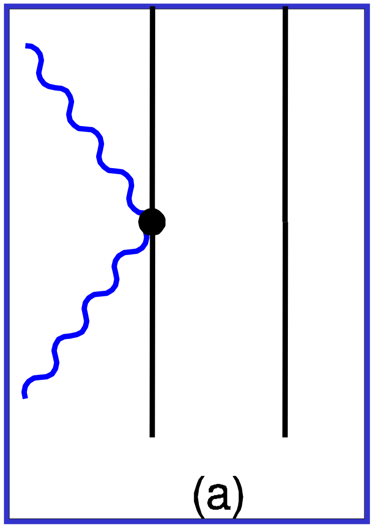


- Problem: Wave function dependence

Example 3: Compton on D

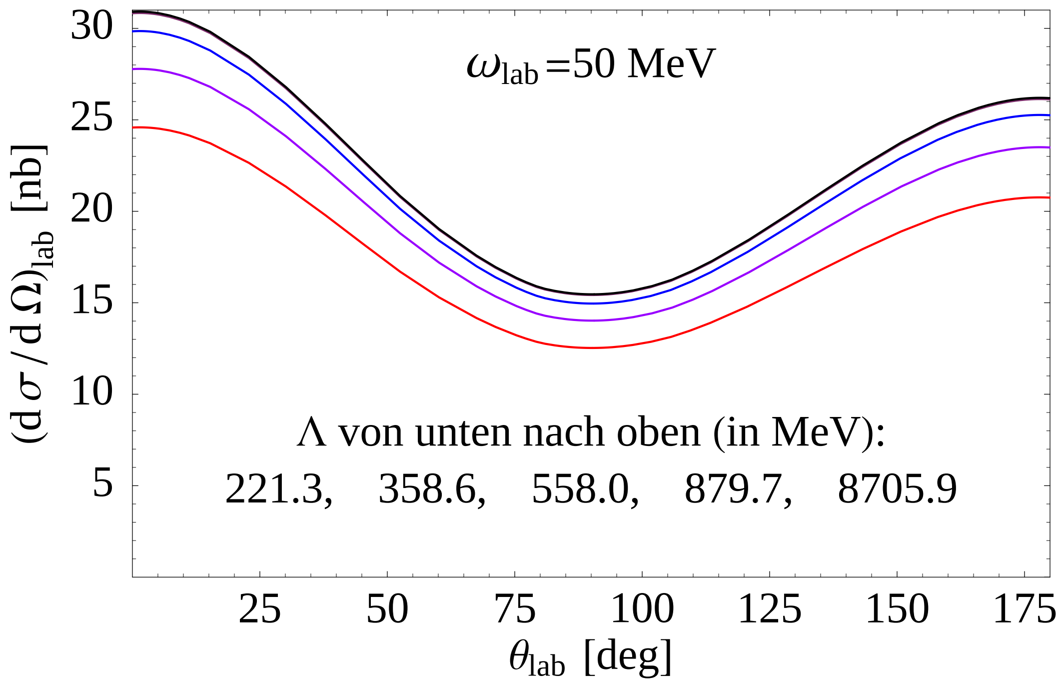
- Finite 2-loop and 1-loop contributions folded with deuteron wavefunctions
- vary the upper integral limits !





Example 3: Compton on D

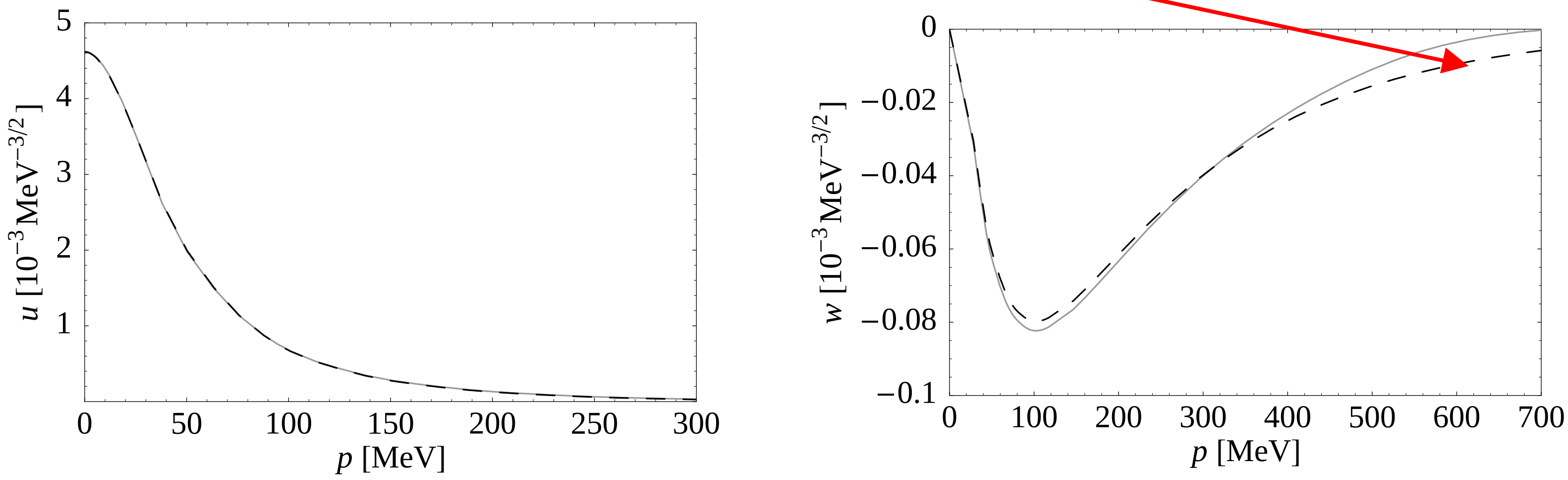
- The effect does not arise from momentum modes above Λχ (here scale-dependence of AV18)



R.P. Hildebrandt, Ph.D. Thesis
(see also work by D. Phillips et al.)

Example 3: Compton on D

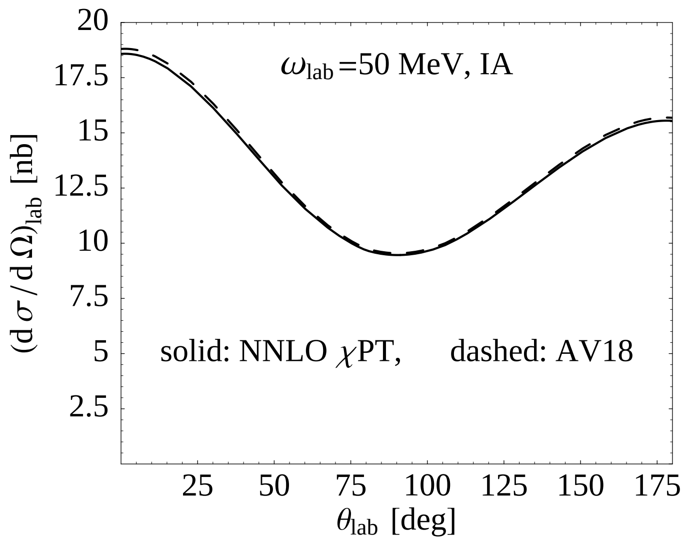
- AV18 is more sensitive to large momenta compared to chiral NNLO

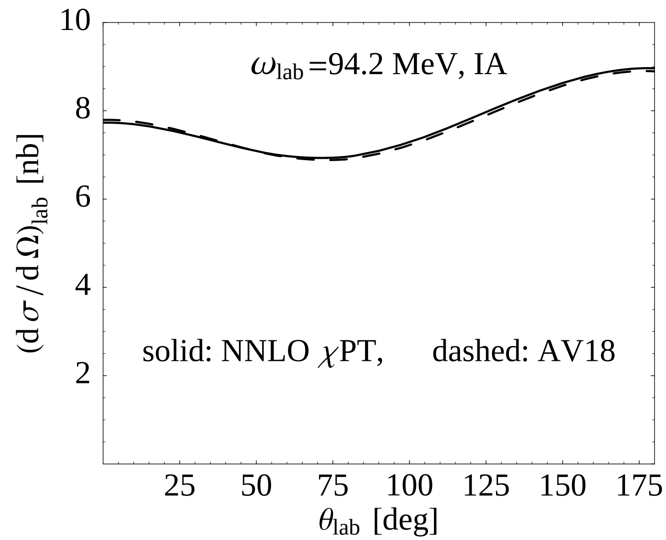


Example 3: Compton on D

- Where does the effect come from?
- Study impulse approximation (= only 1-loop diagrams)

R.P. Hildebrandt, Ph.D. thesis

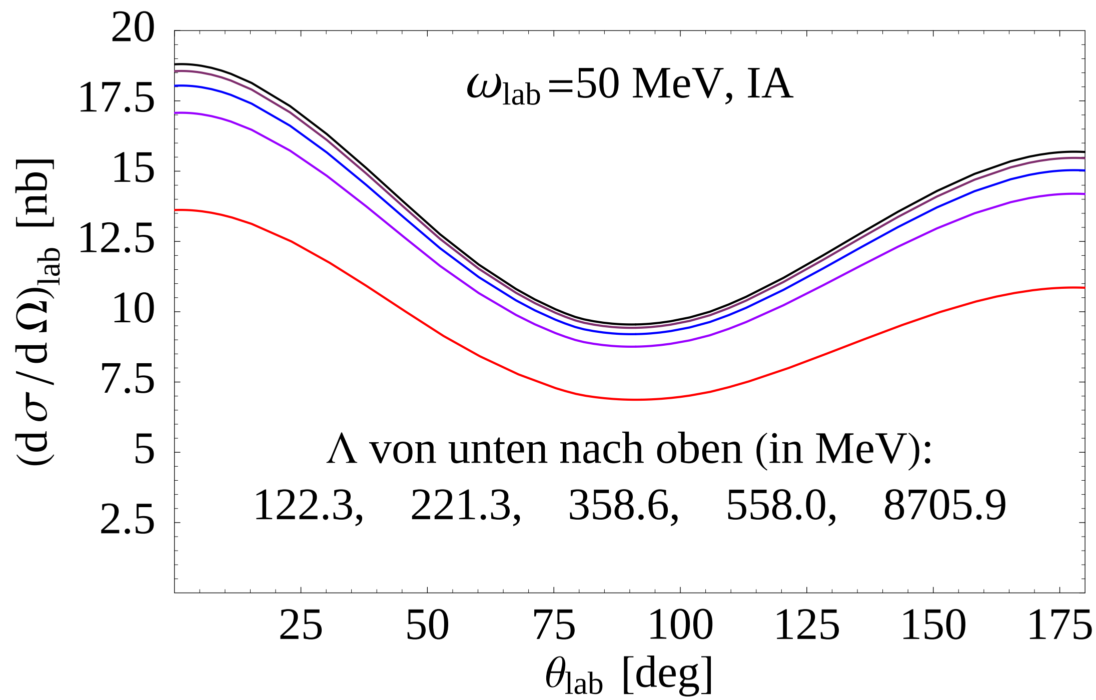




Example 3: Compton on D

- IA shows much less sensitivity on high-momentum modes

R.P. Hildebrandt,
Ph.D. Thesis
(see also work by
D. Phillips et al)



Summary

- Contribution from second integral ($\Lambda_\chi < p < \infty$) should be negligible to have consistency with the (naive) counting rules of the effective field theory

  → non-trivial **extra** check on any ChEFT calculation !
- Example 1: M_N behaves better than expected even for large pion masses
- Example 2: Leading HBChPT result for α_E, β_M is inconsistent with consistency demand even for m_π=140 MeV

  → 2 short distance correction terms (also in SSE)
- Example 3: Wave function dependence in Deuteron Compton scattering is not arising from contributions beyond Λ_χ