Two-pion exchange and peripheral NN scattering

Renato Higa, Jefferson Lab

Nuclear Forces and QCD: Never the Twain Shall Meet?

ECT Trento workshop

- C.A. da Rocha and M.R. Robilotta, PRC49, 1818 (1994), PRC52, 531 (1995), NPA615, 391 (1997),
- R.H. and M.R. Robilotta, PRC68, 024004 (2003),
- R.H., C.A. da Rocha, M.R. Robilotta, PRC69, 034009 (2004), arXiv: nucl-th/0501076
- R.H., arXiv: nucl-th/0411046.

Two-pion exchange and peripheral NN scattering

Renato Higa, Jefferson Lab

Nuclear Forces and QCD: Never the Twain Shall Meet?

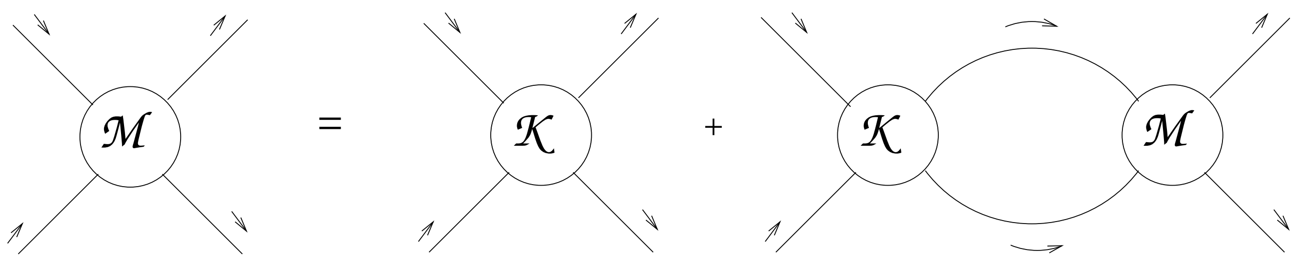
ECT Trento workshop

- Potential
- TPEP
- ChPT with baryons
- LECs, covariant formalism
- Results and comments

Definition of potential

- Bethe-Salpeter (relativistic amplitude)

$$\mathcal{M}(l', l \,|\, W) = \mathcal{K}(l', l \,|\, W) + \int \frac{d^4\xi}{(2\pi)^4} \mathcal{K}(l', \xi \,|\, W)\, G(\xi \,|\, W)\, \mathcal{M}(\xi, l \,|\, W)\,,$$



- Lippmann-Schwinger (non-relativistic)

$$T_{fi}(\boldsymbol{l}', \boldsymbol{l} \,|\, W) = V_{fi}(\boldsymbol{l}', \boldsymbol{l} \,|\, W) + \int \frac{d^3\xi}{(2\pi)^3} V_{fi}(\boldsymbol{l}', \boldsymbol{\xi} \,|\, W)\, \frac{m_N}{\boldsymbol{p}^2 - \boldsymbol{\xi}^2 + i\epsilon}\, T_{fi}(\boldsymbol{\xi}, \boldsymbol{l} \,|\, W)\,.$$

- quasi-potential $\mathcal{V}$ (not unique): $\mathcal{M} = \mathcal{V} + \mathcal{V}\, g\, \mathcal{M}$

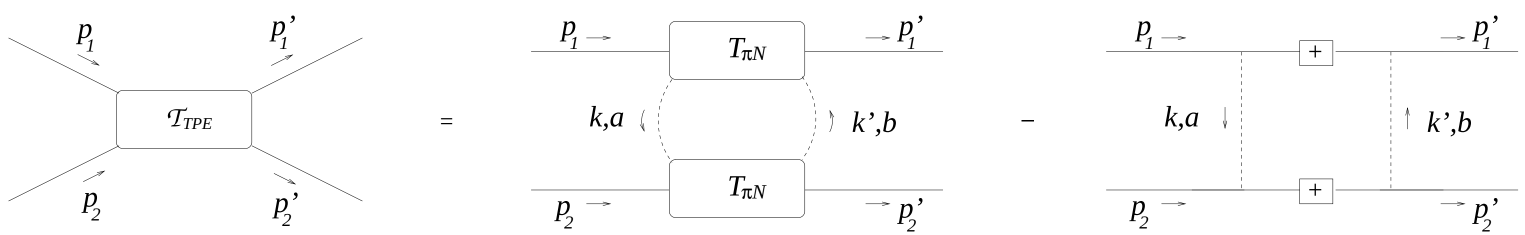
- BS: $V = \frac{1}{4Em_N}\mathcal{V}$

$$(\mathcal{K},\, \mathcal{M},\, V) = (\mathcal{K},\, \mathcal{M},\, V)^{(2)} + (\mathcal{K},\, \mathcal{M},\, V)^{(4)} + \cdots$$

$$V^{(2)} = \frac{1}{4Em_N}\mathcal{M}^{(2)} = \frac{1}{4Em_N}\mathcal{K}^{(2)} \qquad \text{(OPEP)}$$

$$V^{(4)} = \frac{1}{4Em_N}\left[\mathcal{M}^{(4)} - \mathcal{M}^{(2)}\, g\, \mathcal{M}^{(2)}\right]$$

$$= \frac{1}{4Em_N}\left[\mathcal{K}^{(4)} + \mathcal{K}^{(2)}\, G\, \mathcal{K}^{(2)} - \mathcal{K}^{(2)}\, g\, \mathcal{K}^{(2)}\right] \qquad \text{(TPEP)}$$



baryon ChPT

- Gasser, Sainio e Švarc: loops with dimensional regularization
  $\longrightarrow$ violation of power counting: $\Delta\mathcal{L}_N = \mathcal{L}_N^{(2)} + \mathcal{L}_N^{(3)} + \Delta\mathcal{L}_N^{(0)} + \Delta\mathcal{L}_N^{(1)}$

- Jenkins e Manohar: heavy baryon expansion (HBChPT)
  recovers power counting

- Ellis and Tang, Becher and Leutwyler: relativistic formalism

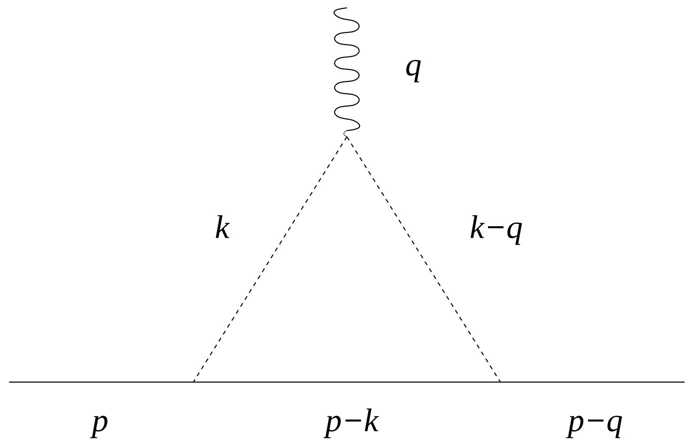
  ⋆ I (low-energy contributions)
  ⋆ R (high momenta contributions) Replace

  - R is analytic in all low-energy region $\longrightarrow$ Taylor expansion (local operators)

  - I exhibit a power counting rule

- q/m_N expansion of I $\longrightarrow$ HBChPT

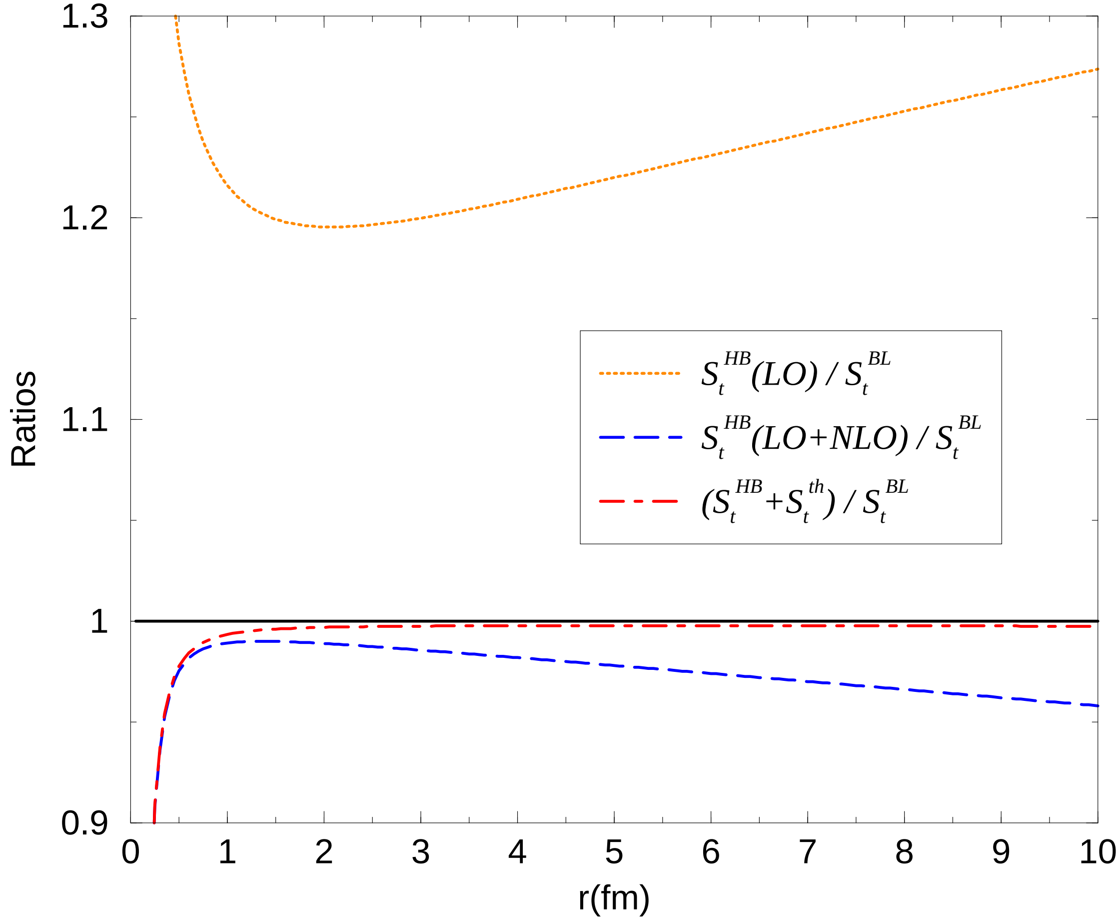
- triangle integral: q/m_N expansion fails near $t = 4m_\pi^2$



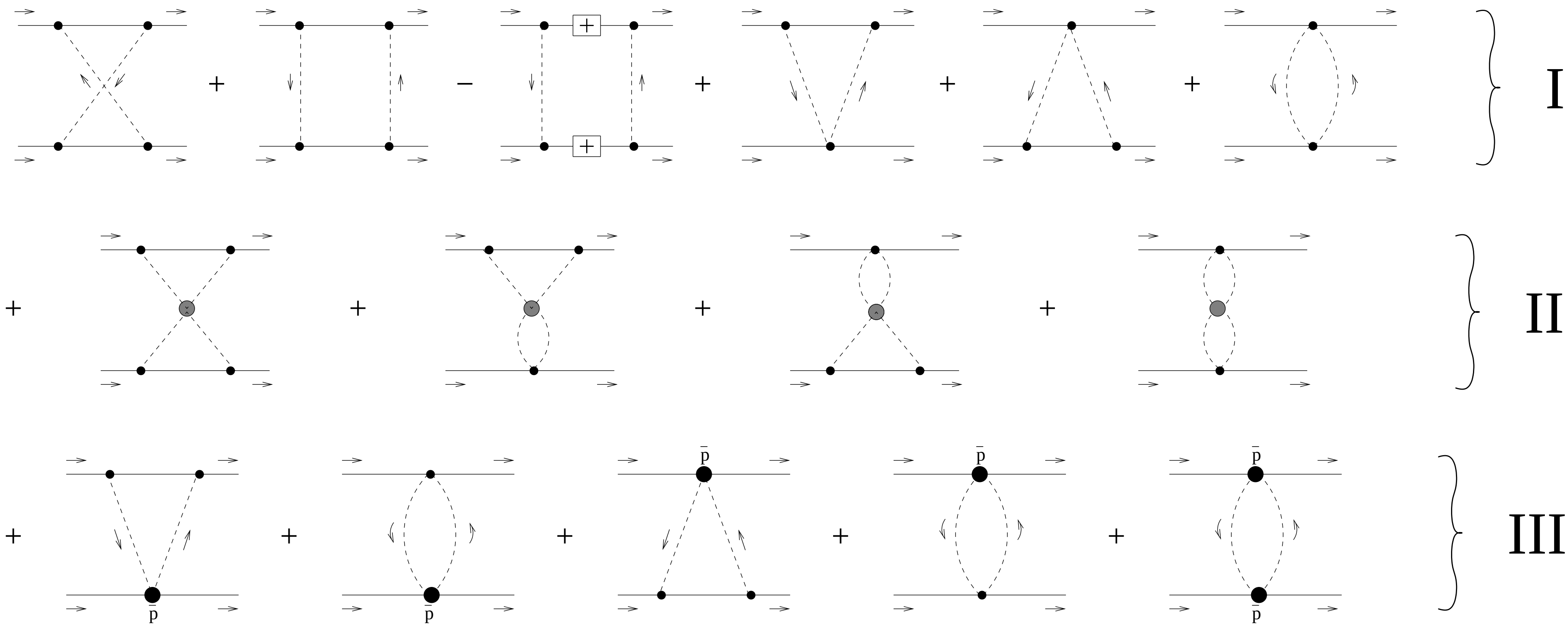
$$\gamma(t) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{dt'}{(t'-t)} \frac{1}{16\pi m_N \sqrt{t'}} \arctan \frac{2m_N\sqrt{t'-4m_\pi^2}}{t-2m_\pi^2}$$

$$\approx \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{dt'}{(t'-t)} \frac{1}{16\pi m_N \sqrt{t'}} \left\{ \left[\frac{\pi}{2} - \frac{(t'-2m_\pi^2)}{2m_N\sqrt{t'-4m_\pi^2}} \right]_{HB} + \left[\frac{m_\pi\sqrt{t'}}{2m_N\sqrt{t'-4m_\pi^2}} - \frac{\sqrt{t'}}{2m_\pi} \arctan \frac{m_\pi^2}{m_N\sqrt{t'-4m_\pi^2}} \right]_{th} \right\}.$$

R. H., M. R. Robilotta and C. A. da Rocha



TPEP



$$
\begin{aligned}
V_C = V_C^+ = \frac{3g_A^2}{16\pi f_\pi^4} & \left\{ -\frac{g_A^2\,\mu^5}{16m(4\mu^2+\boldsymbol{q}^2)} + \left[2\mu^2(2c_1-c_3) - \boldsymbol{q}^2 c_3\right](2\mu^2+\boldsymbol{q}^2)\,A(q) + \frac{g_A^2(2\mu^2+\boldsymbol{q}^2)\,A(q)}{16m}\left[-3\boldsymbol{q}^2 \;+(4\mu^2+\boldsymbol{q}^2)^\dagger\,\right]\right\} \\
& + \frac{g_A^2\,L(q)}{32\pi^2 f_\pi^4 m}\left\{\frac{24\mu^6}{4\mu^2+\boldsymbol{q}^2}(2c_1+c_3) + 6\mu^4(c_2-2c_3) + 4\mu^2\boldsymbol{q}^2(6c_1+c_2-3c_3) + \boldsymbol{q}^4(c_2-6c_3)\right\} \\
& - \frac{3L(q)}{16\pi^2 f_\pi^4}\left\{\left[-4\mu^2 c_1 + c_3(2\mu^2+\boldsymbol{q}^2) + c_2(4\mu^2+\boldsymbol{q}^2)/6\right]^2 + \frac{1}{45}(c_2)^2(4\mu^2+\boldsymbol{q}^2)^2\right\} \\
& + \frac{g_A^4}{32\pi^2 f_\pi^4 m^2}\left\{L(q)\left[\frac{2\mu^8}{(4\mu^2+\boldsymbol{q}^2)^2} + \frac{8\mu^6}{(4\mu^2+\boldsymbol{q}^2)} - 2\mu^4 - \boldsymbol{q}^4\right] + \frac{\mu^6/2}{(4\mu^2+\boldsymbol{q}^2)}\right\} \\
& - \frac{3g_A^4[A(q)]^2}{1024\pi^2 f_\pi^6}(\mu^2+2\boldsymbol{q}^2)(2\mu^2+\boldsymbol{q}^2)^2 - \frac{3g_A^4(2\mu^2+\boldsymbol{q}^2)\,A(q)}{1024\pi^2 f_\pi^6}\left\{4\mu\, g_A^2\,(2\mu^2+\boldsymbol{q}^2) + 2\mu\,(\mu^2+2\boldsymbol{q}^2)]\right\} ,
\end{aligned}
$$

$$
\begin{aligned}
V_T = -\frac{3\,V_T^+}{m^2} = \frac{3g_A^4\,L(q)}{64\pi^2 f_\pi^4} & - \frac{g_A^4\,A(q)}{512\pi f_\pi^4 m}\left[9(2\mu^2+\boldsymbol{q}^2) \;+3(4\mu^2+\boldsymbol{q}^2)^\dagger\,\right] \\
& - \frac{g_A^4\,L(q)}{32\pi^2 f_\pi^4 m^2}\left[\boldsymbol{z}^2/4 + 5\boldsymbol{q}^2/8 + \frac{\mu^4}{4\mu^2+\boldsymbol{q}^2}\right] + \frac{g_A^2\,(4\mu^2+\boldsymbol{q}^2)\,L(q)}{32\pi^2 f_\pi^4}\left[(\tilde{d}_{14}-\tilde{d}_{15}) \;-\left(g_A^4/32\,\pi^2 f_\pi^2\right)^*\,\right] ,
\end{aligned}
$$

$$
V_{LS} = -\frac{V_{LS}^+}{m^2} = -\frac{3g_A^4\,A(q)}{32\pi f_\pi^4 m}\left[(2\mu^2+\boldsymbol{q}^2) \;+(\mu^2+3\boldsymbol{q}^2/8)^\dagger\,\right] - \frac{g_A^4\,L(q)}{4\pi^2 f_\pi^4 m^2}\left[\frac{\mu^4}{4\mu^2+\boldsymbol{q}^2} + \frac{11}{32}\boldsymbol{q}^2\right] - \frac{g_A^2\,c_2\,L(q)}{8\pi^2 f_\pi^4 m}(4\mu^2+\boldsymbol{q}^2) ,
$$

$$
V_{\sigma L} = \frac{4\,V_Q^+}{m^4} = -\frac{g_A^4\,L(q)}{32\pi^2 f_\pi^4 m^2} ,
$$

$$W_C = V_C^- = \frac{L(q)}{384\pi^2 f_\pi^4}\left[4\mu^2\left(5g_A^4-4g_A^2-1\right)+\boldsymbol{q}^2\left(23g_A^4-10g_A^2-1\right)+\frac{48g_A^4\mu^4}{4\mu^2+\boldsymbol{q}^2}\right]$$

$$-\frac{g_A^2}{128\pi f_\pi^4 m}\left\{\frac{3g_A^2\mu^5}{4\mu^2+\boldsymbol{q}^2}+A(q)\,(2\mu^2+\boldsymbol{q}^2)\left[g_A^2\,(4\mu^2+3\boldsymbol{q}^2)-2(2\mu^2+\boldsymbol{q}^2)\ \color{red}{+g_A^2(4\mu^2+\boldsymbol{q}^2)^\dagger}\ \right]\right\}+\frac{\boldsymbol{q}^2\,c_4\,L(q)}{192\pi^2 f_\pi^4 m}\left[g_A^2(8\mu^2+5\boldsymbol{q}^2)+(4\mu^2+\boldsymbol{q}^2)\right]$$

$$+\frac{16g_A^4\mu^6}{768\pi^2 f_\pi^4 m^2}\,\frac{1}{4\mu^2+\boldsymbol{q}^2}-\frac{L(q)}{768\pi^2 f_\pi^4 m^2}\left\{(4\mu^2+\boldsymbol{q}^2)\boldsymbol{z}^2+g_A^2\left[\frac{48\mu^6}{4\mu^2+\boldsymbol{q}^2}-24\mu^4-12\,(2\mu^2+\boldsymbol{q}^2)\,\boldsymbol{q}^2+(16\mu^2+10\boldsymbol{q}^2)\boldsymbol{z}^2\right]\right.$$

$$\left.+g_A^4\left[\boldsymbol{z}^2\left(\frac{16\mu^4}{4\mu^2+\boldsymbol{q}^2}-7\boldsymbol{q}^2-20\mu^2\right)-\frac{64\mu^8}{(4\mu^2+\boldsymbol{q}^2)^2}-\frac{48\mu^6}{4\mu^2+\boldsymbol{q}^2}+\frac{16\mu^4\boldsymbol{q}^2}{4\mu^2+\boldsymbol{q}^2}+20\boldsymbol{q}^4+24\mu^2\boldsymbol{q}^2+24\mu^4\right]\right\}$$

$$-\frac{L(q)}{18432\pi^4 f_\pi^6}\left\{\left[192\pi^2 f_\pi^2\tilde{d}_3\ \ \color{blue}{-\frac{(15+7g_A^4)^*}{5}}\ \right](4\mu^2+\boldsymbol{q}^2)\left[2g_A^2(2\mu^2+\boldsymbol{q}^2)-3/5(g_A^2-1)(4\mu^2+\boldsymbol{q}^2)\right]\right.$$

$$+\left[6g_A^2(2\mu^2+\boldsymbol{q}^2)-(g_A^2-1)\,(4\mu^2+\boldsymbol{q}^2)\right]\left[384\pi^2 f_\pi^2\left((2\mu^2+\boldsymbol{q}^2)\,(\tilde{d}_1+\tilde{d}_2)+4\mu^2\,\tilde{d}_5\right)+L(q)\left(4\mu^2(1+2g_A^2)+\boldsymbol{q}^2(1+5g_A^2)\right)\right.$$

$$\left.\left.-\left(\frac{\boldsymbol{q}^2}{3}\,(5+13g_A^2)+8\mu^2(1+2g_A^2)\right)\ \ \color{blue}{+\left(2g_A^4(2\mu^2+\boldsymbol{q}^2)+\frac{2}{3}\,\boldsymbol{q}^2(1+2g_A^2)\right)^*}\ \right]\right\},$$

$$W_T = -\frac{3}{m^2}\,V_T^- = \frac{g_A^2 A(q)}{32\pi f_\pi^4}\left\{\left(c_4+\frac{1}{4m}\right)(4\mu^2+\boldsymbol{q}^2)-\frac{g_A^2}{8m}\left[10\mu^2+3\boldsymbol{q}^2\ \ \color{red}{-(4\mu^2+\boldsymbol{q}^2)^\dagger}\ \right]\right\}-\frac{c_4^2\,L(q)}{96\pi^2 f_\pi^4}(4\mu^2+\boldsymbol{q}^2)$$

$$+\frac{c_4\,L(q)}{192\pi^2 f_\pi^4 m}\left[g_A^2(16\mu^2+7\boldsymbol{q}^2)-(4\mu^2+\boldsymbol{q}^2)\right]-\frac{L(q)}{1536\pi^2 f_\pi^4 m^2}\left[g_A^4\left(28\mu^2+17\boldsymbol{q}^2+\frac{16\,\mu^4}{4\mu^2+\boldsymbol{q}^2}\right)-g_A^2(32\mu^2+14\boldsymbol{q}^2)+(4\mu^2+\boldsymbol{q}^2)\right]$$

$$-\frac{[A(q)]^2 g_A^4\,(4\mu^2+\boldsymbol{q}^2)^2}{2048\pi^2 f_\pi^6}-\frac{A(q)g_A^4\,(4\mu^2+\boldsymbol{q}^2)}{1024\pi^2 f_\pi^6}\,\mu(1+2g_A^2)\,,$$

$$W_{LS} = -\frac{1}{m^2}\,V_{LS}^- = \frac{A(q)}{32\pi f_\pi^4 m}\left[g_A^2\,(g_A^2-1)\,(4\mu^2+\boldsymbol{q}^2)\ \ \color{red}{+g_A^4\,(2\mu^2+3\boldsymbol{q}^2/4)^\dagger}\ \right]+\frac{c_4\,L(q)}{48\pi^2 m f_\pi^4}\left[g_A^2(8\mu^2+5\boldsymbol{q}^2)+(4\mu^2+\boldsymbol{q}^2)\right]$$

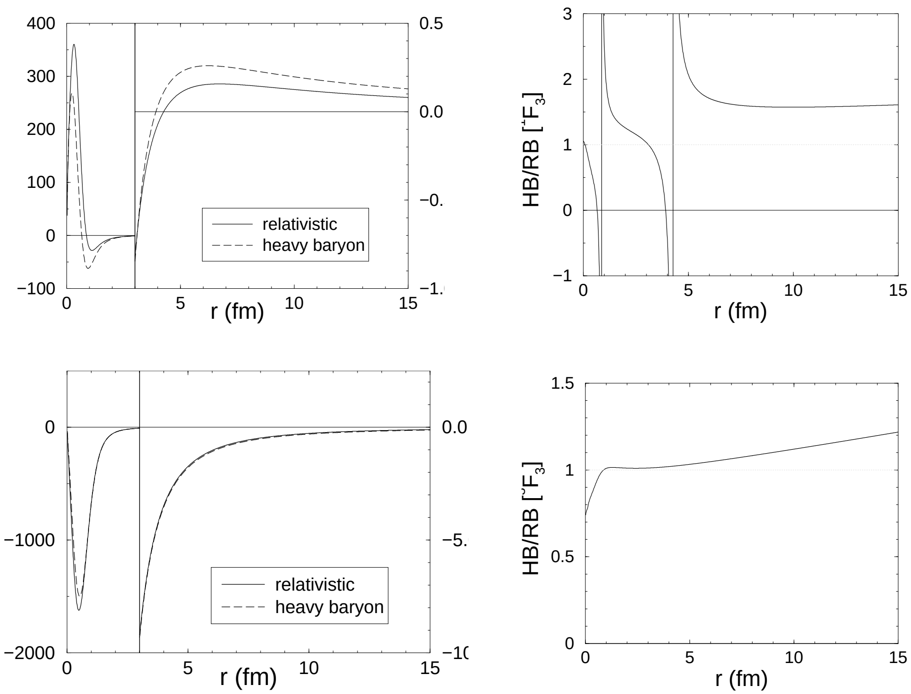
$$+\frac{L(q)}{256\pi^2 m^2 f_\pi^4}\left[(4\mu^2+\boldsymbol{q}^2)-16g_A^2\,(\mu^2+3\boldsymbol{q}^2/8)+\frac{4g_A^4}{3}\left(9\mu^2+11\boldsymbol{q}^2/4-\frac{4\mu^4}{4\mu^2+\boldsymbol{q}^2}\right)\right],$$

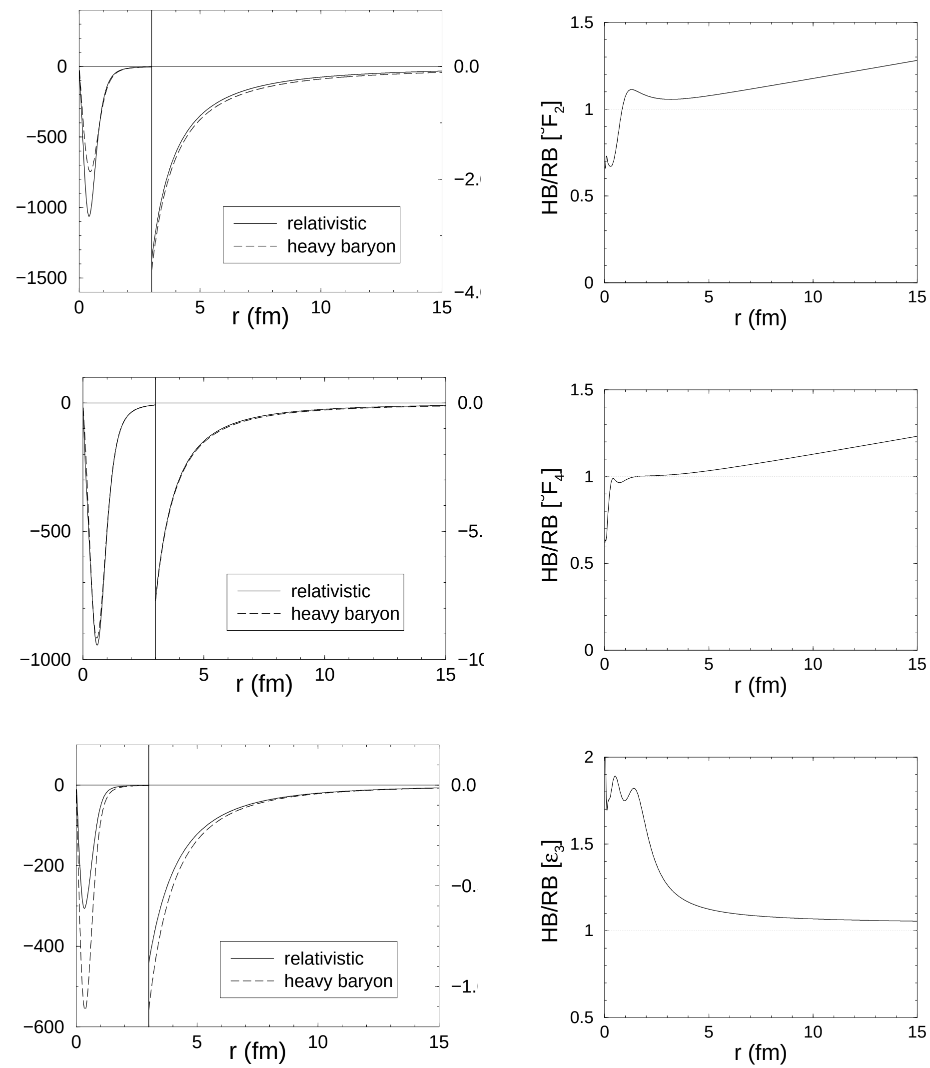
$$W_{\sigma L}\simeq 0\,,$$

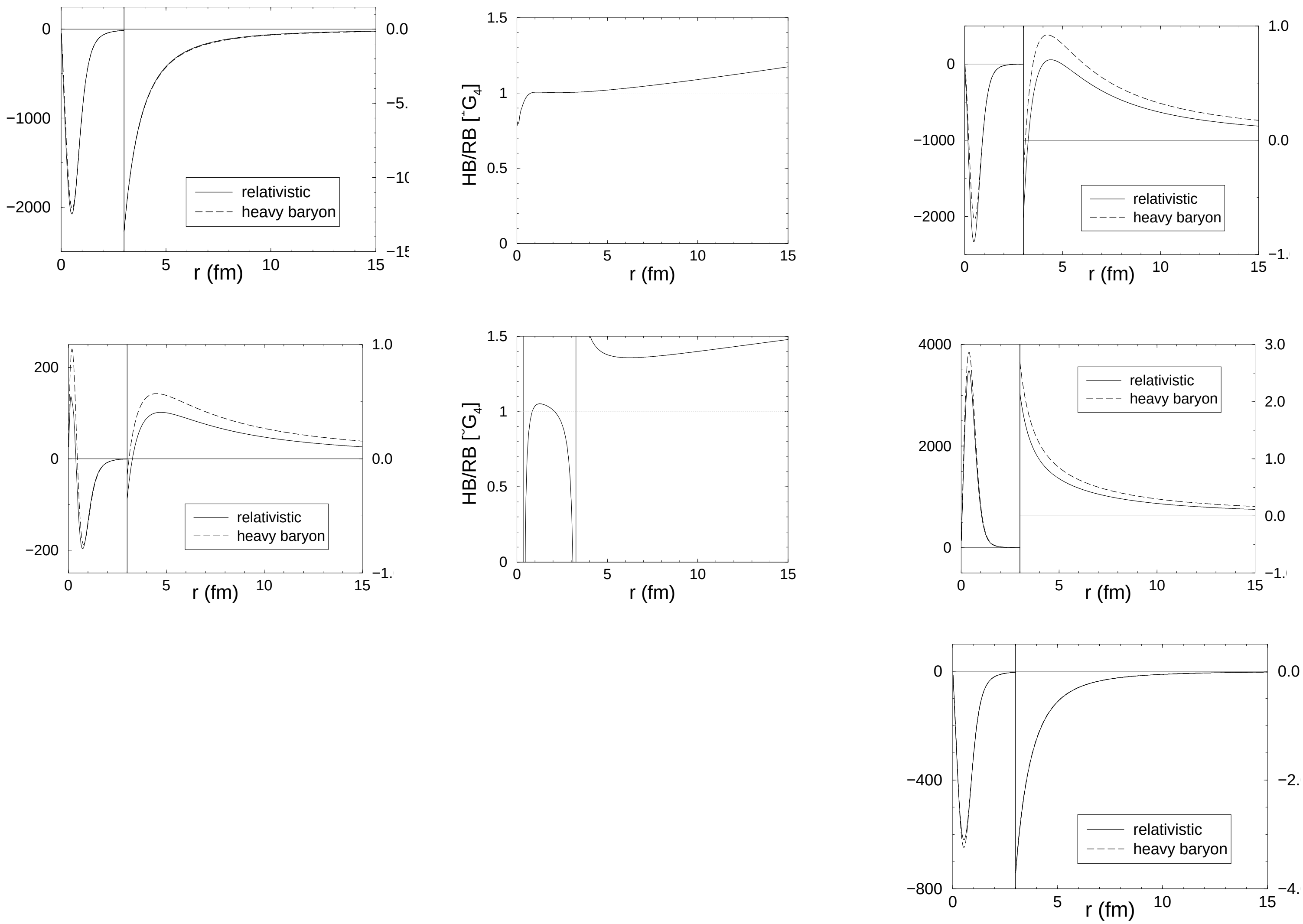
Comparison with HB expressions

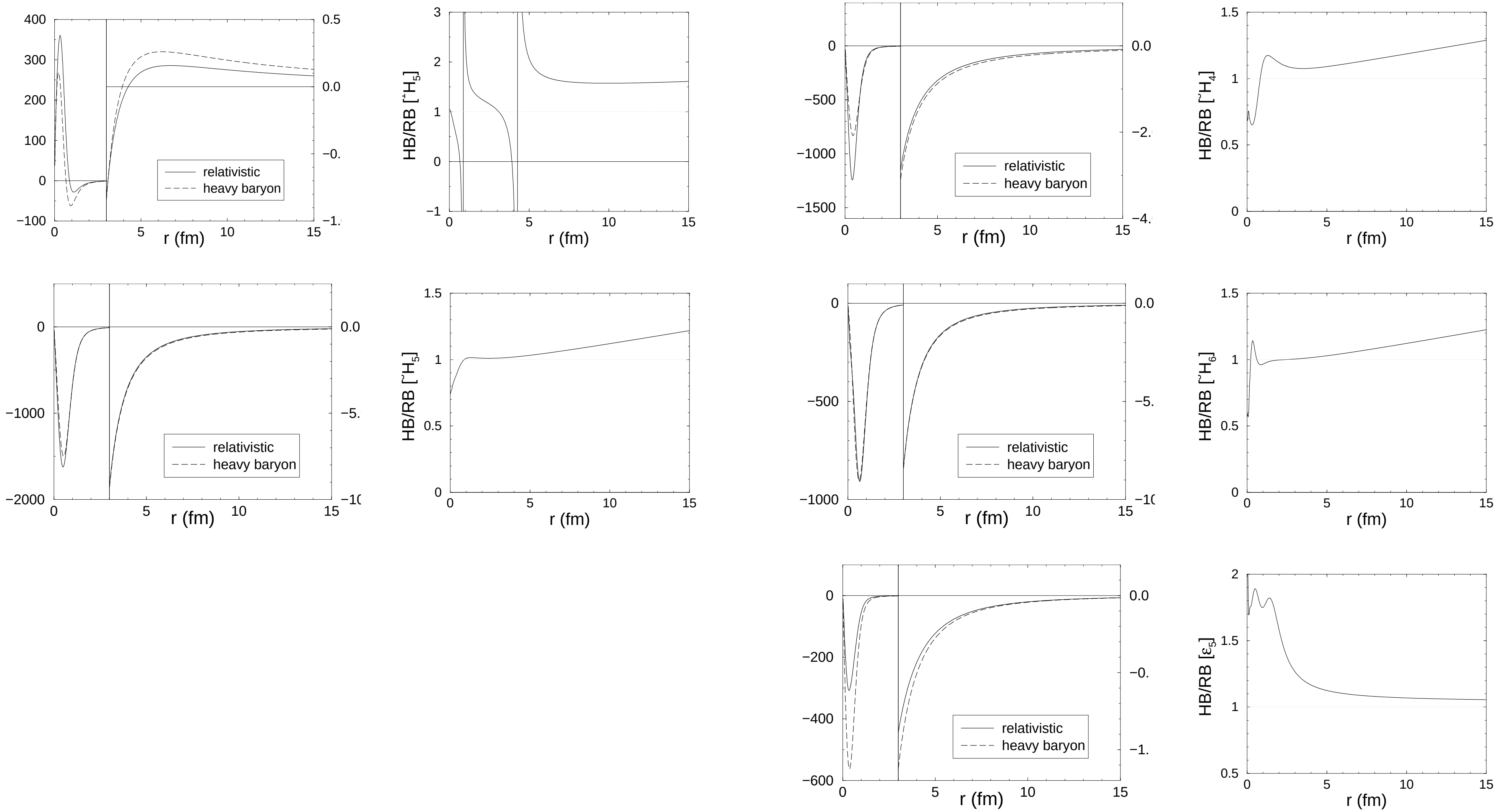
(thanks to N. Kaiser)

- relatively small number of differences
- origins:
  - ⋆ dynamical equations (our choice: Blankenbecler and Sugar)
  - ⋆ two loop calculations (not completely understood)
- to isolate the $1/m_N$ expansion effect: ignore the above differences
- cutoff: $\left[1 - \exp(-cr^2)\right]^4 \times$TPEP


relativistic
heavy baryon
r (fm)
HB/RB [˜F3]
relativistic
heavy baryon
r (fm)
HB/RB [˜F3]
r (fm)


relativistic
heavy baryon
r (fm)
HB/RB [˜F2]
r (fm)
relativistic
heavy baryon
r (fm)
HB/RB [˜F4]
r (fm)
relativistic
heavy baryon
r (fm)
HB/RB [ε3]
r (fm)


relativistic
heavy baryon
r (fm)
HB/RB [˜G4]
HB/RB [˜G3]
HB/RB [˜G5]
HB/RB [ε4]


relativistic
heavy baryon
r (fm)
HB/RB [˜H5]
HB/RB [˜H4]
HB/RB [˜H6]
HB/RB [ε5]

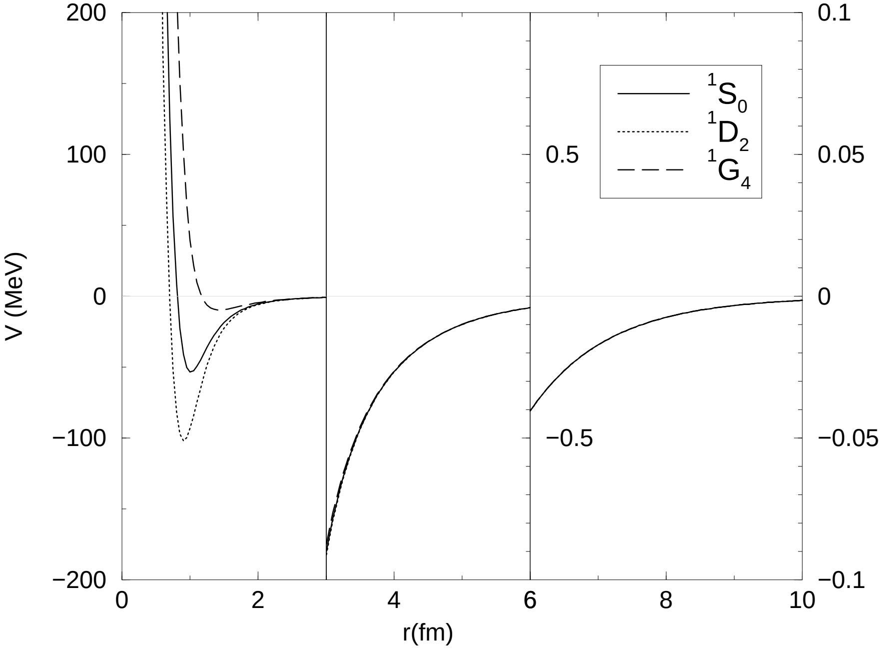
The phase function method

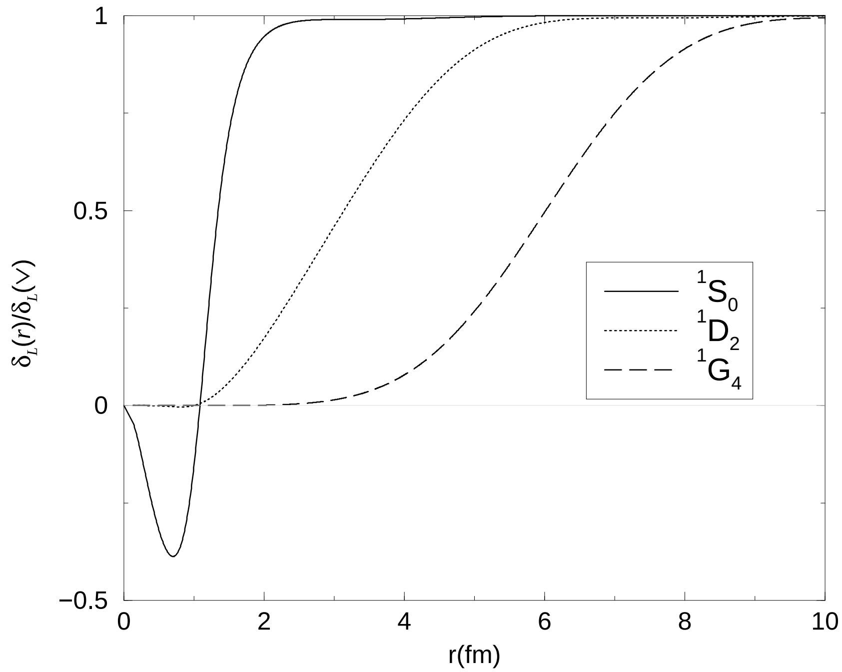
- Kynch, Cox and Perlmutter, Calogero, Babikov
  - ⋆ $u_L(r) = A_L(r)\big[\cos\delta_L(r)\,\hat{j}_L(kr) - \sin\delta_L(r)\,\hat{n}_L(kr)\big]$
  - ⋆ $u'_L(r) = A_L(r)\big[\cos\delta_L(r)\,\hat{j}'_L(kr) - \sin\delta_L(r)\,\hat{n}'_L(kr)\big]$

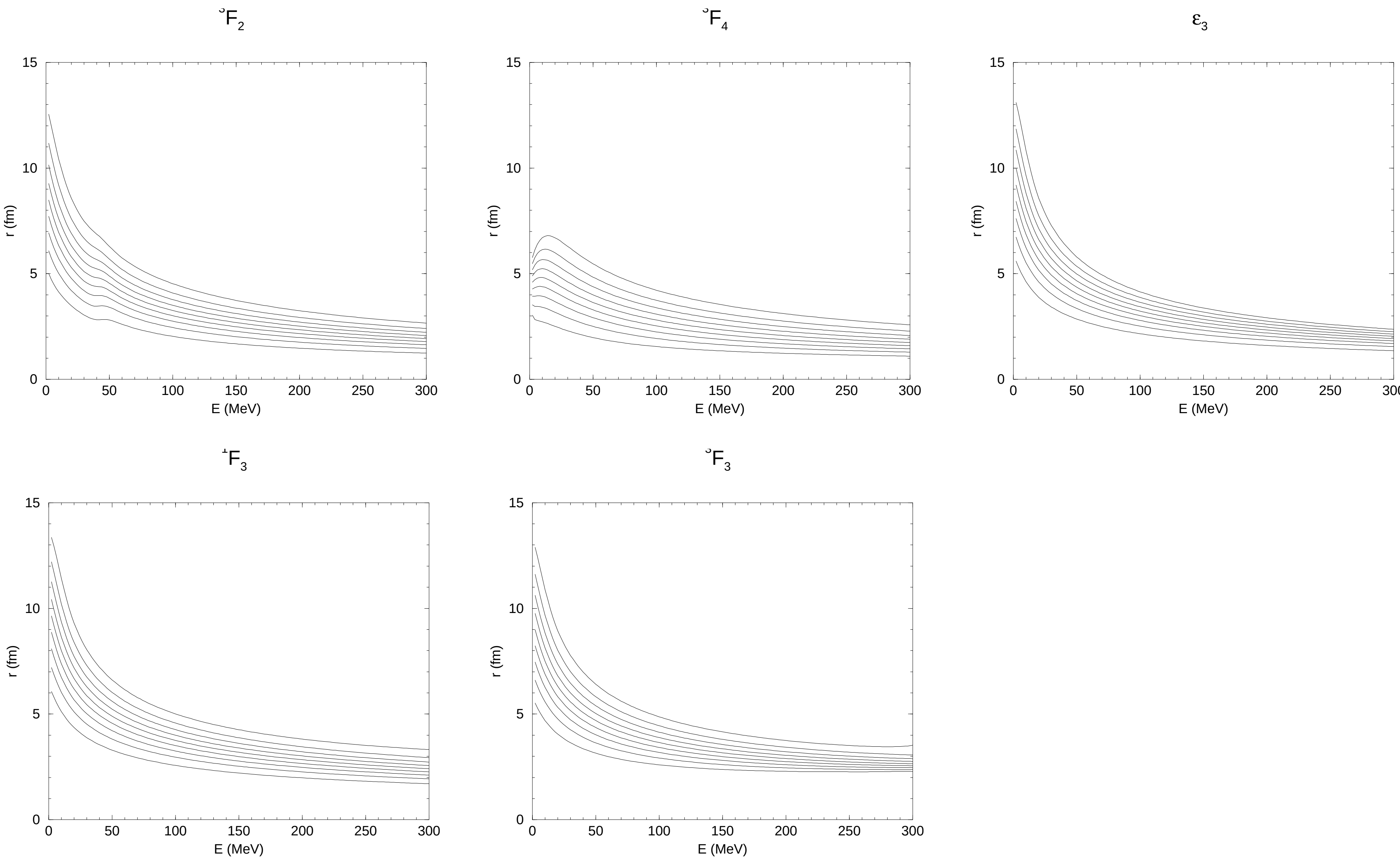
- $\delta'_L(r) = -\frac{U(r)}{k}\Big[\cos\delta_L(r)\,\hat{j}_L(kr) - \sin\delta_L(r)\,\hat{n}_L(kr)\Big]^2$ Replace

  where $U(r) = 2m_{red}\,V(r)$

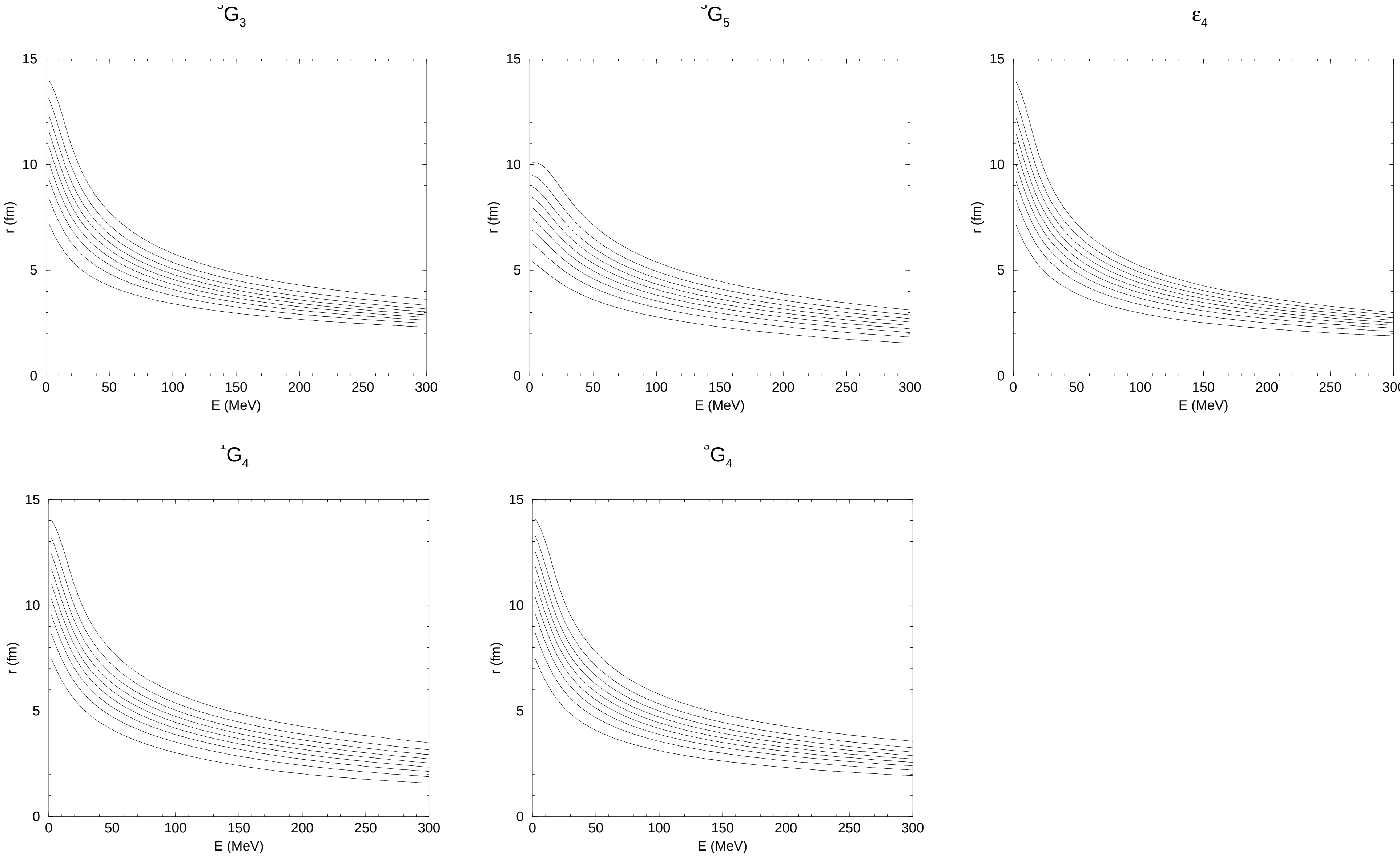
- $\delta_L(R)$: phase shift associated to the potential $V(r)\,\theta(R-r)$


V (MeV)
r(fm)
200
100
0
−100
−200
0
2
4
6
8
10
0.1
0.05
0
−0.05
−0.1
0.5
−0.5
$^1S_0$
$^1D_2$
$^1G_4$

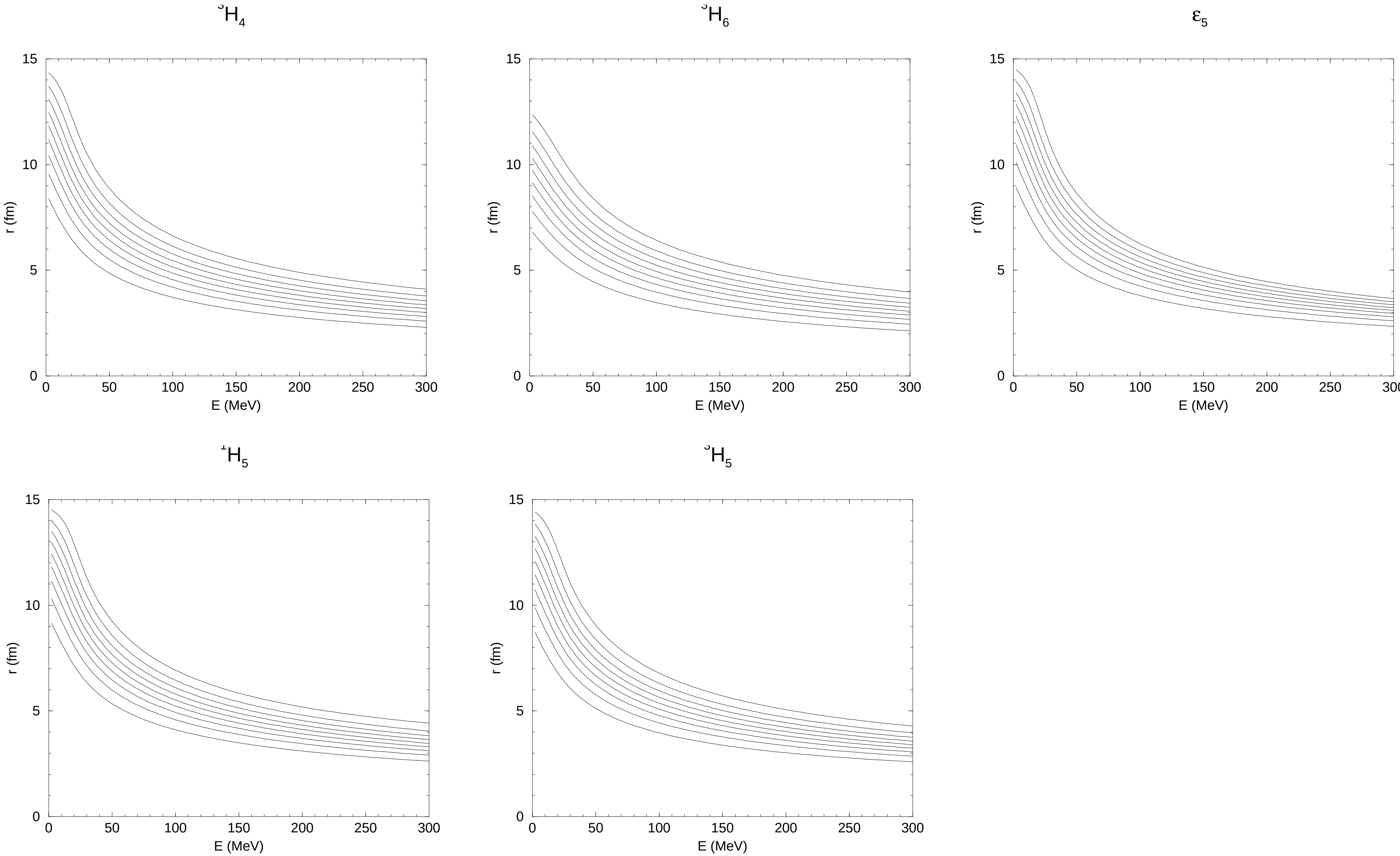

$\delta_L(r)/\delta_L(\infty)$
r(fm)
1
0.5
0
−0.5
0
2
4
6
8
10
$^1S_0$
$^1D_2$
$^1G_4$



3F2
3F4
ε3
1F3
3F3
r (fm)
E (MeV)
0
5
10
15
50
100
150
200
250
300



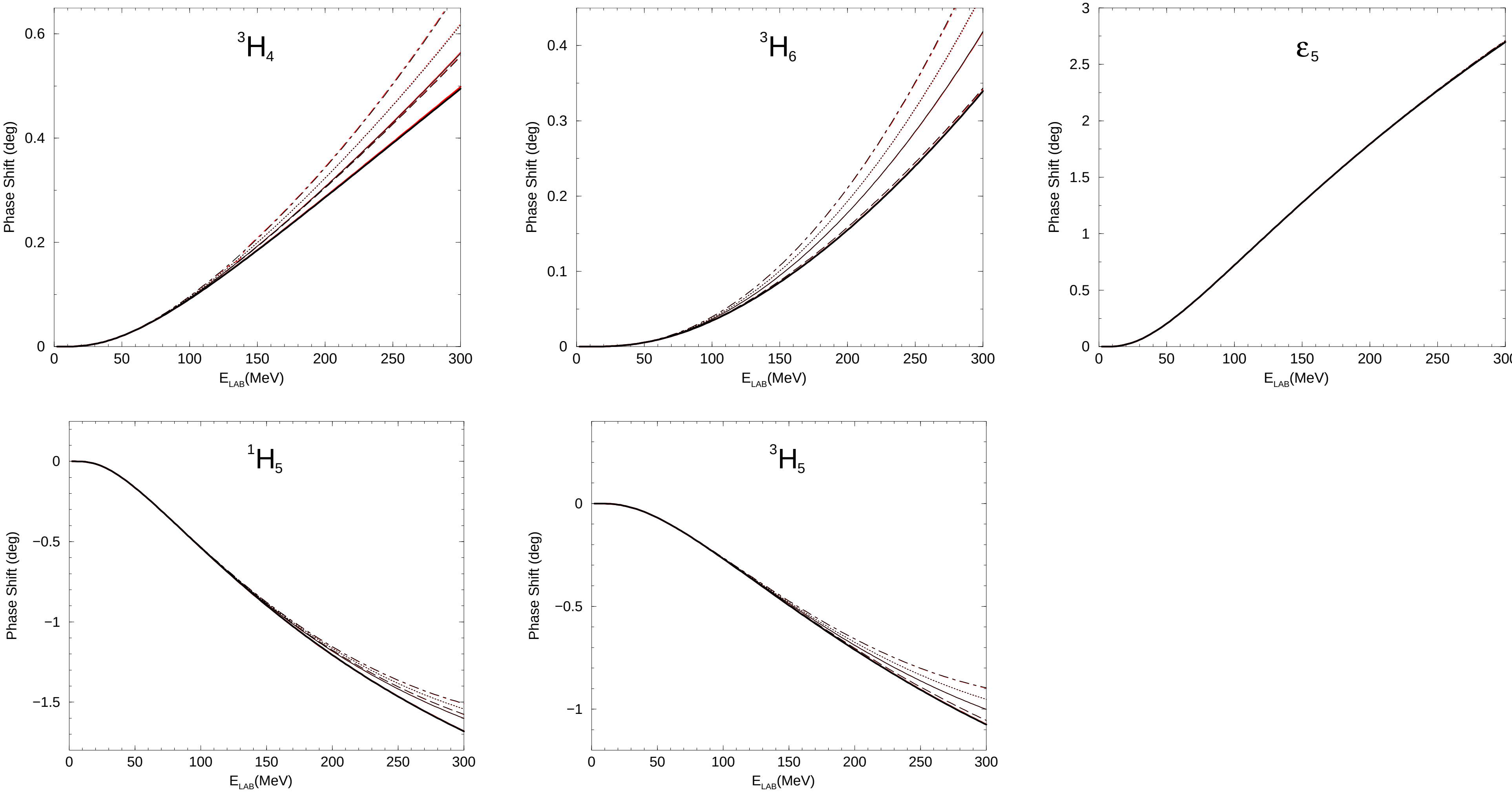
3G3
3G5
ε4
1G4
3G4
r (fm)
E (MeV)

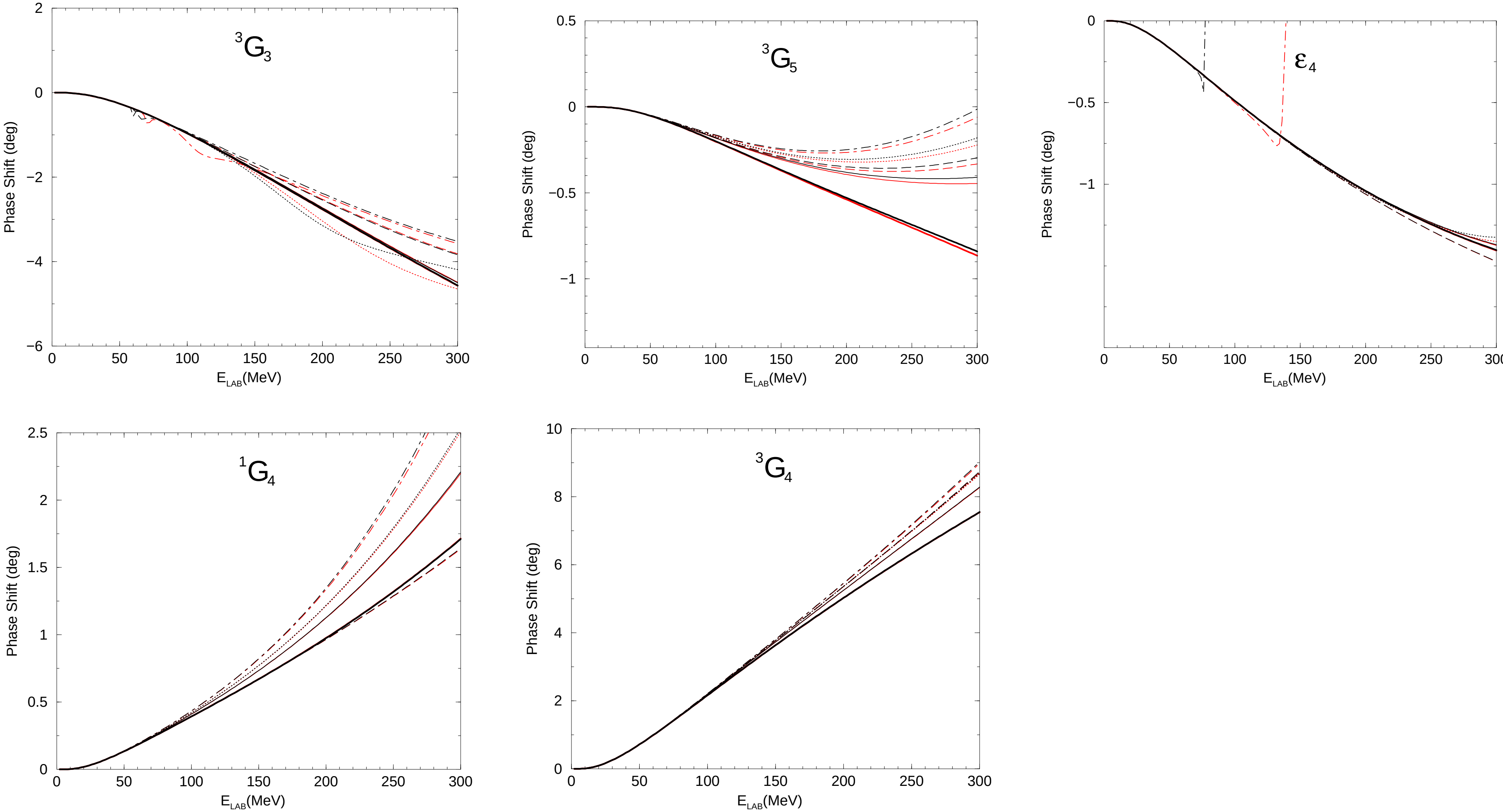


3H4
3H6
ε5
1H5
3H5
r (fm)
E (MeV)
0
5
10
15
0
50
100
150
200
250
300

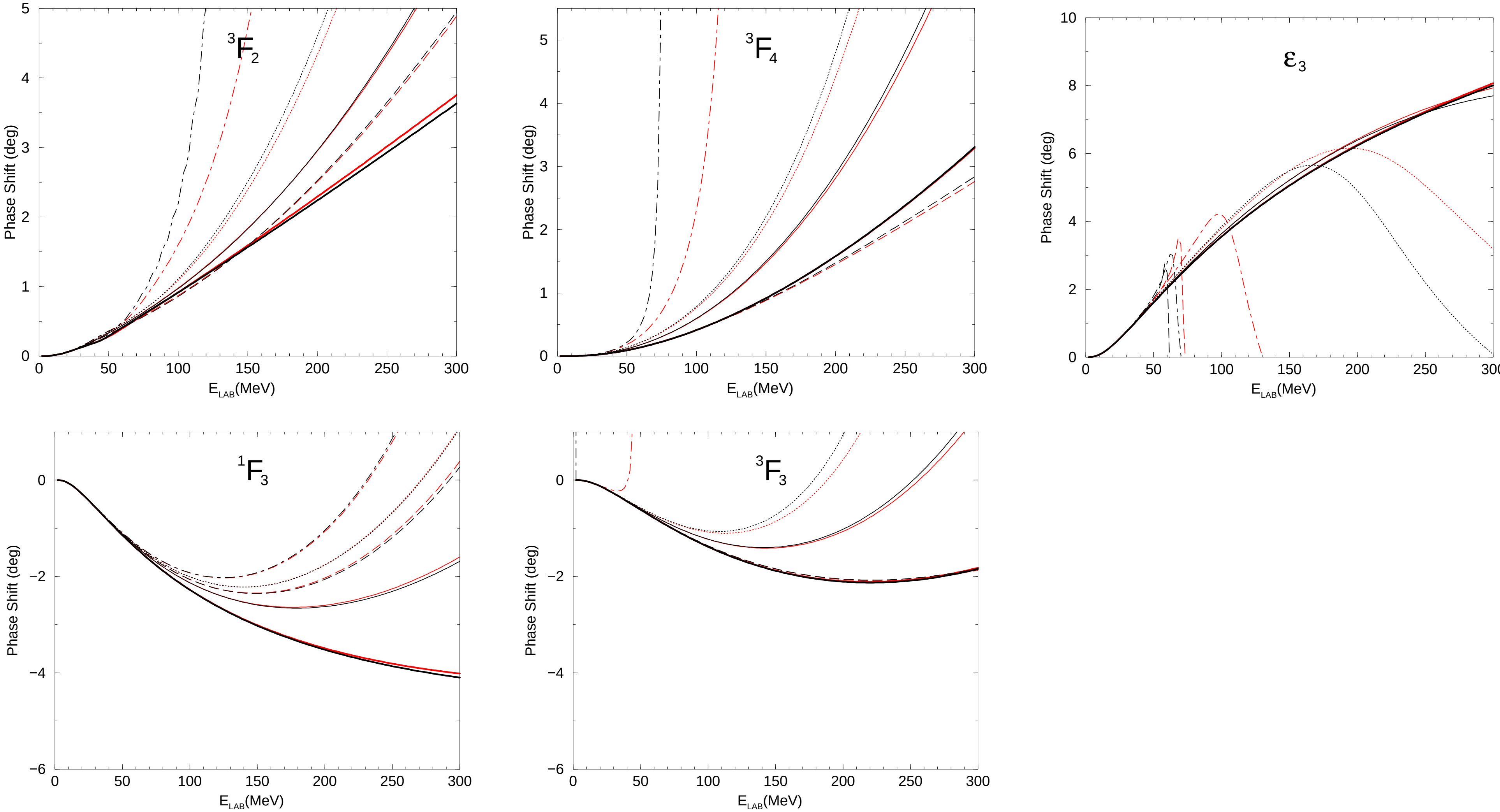
LECs

LEC	Mojžiš	BM	Fettes (fit 1)	Nijmegen	EM
c_1	-0.94	-0.81	-1.23	-0.76	-0.81
c_2	3.20	8.43	3.28	3.20	3.28
c_3	-5.40	-4.70	-5.94	-4.78	-3.40
c_4	3.47	3.40	3.47	3.96	3.40
$\bar{d}_1 + \bar{d}_2$	2.40	3.06	3.06	2.40	3.06
$\bar{d}_3$	-2.80	-3.27	-3.27	-2.80	-3.27
$\bar{d}_5$	1.40	0.45	0.45	1.40	0.45
$\bar{d}_{14} - \bar{d}_{15}$	-6.10	-5.65	-5.65	-6.10	-5.65

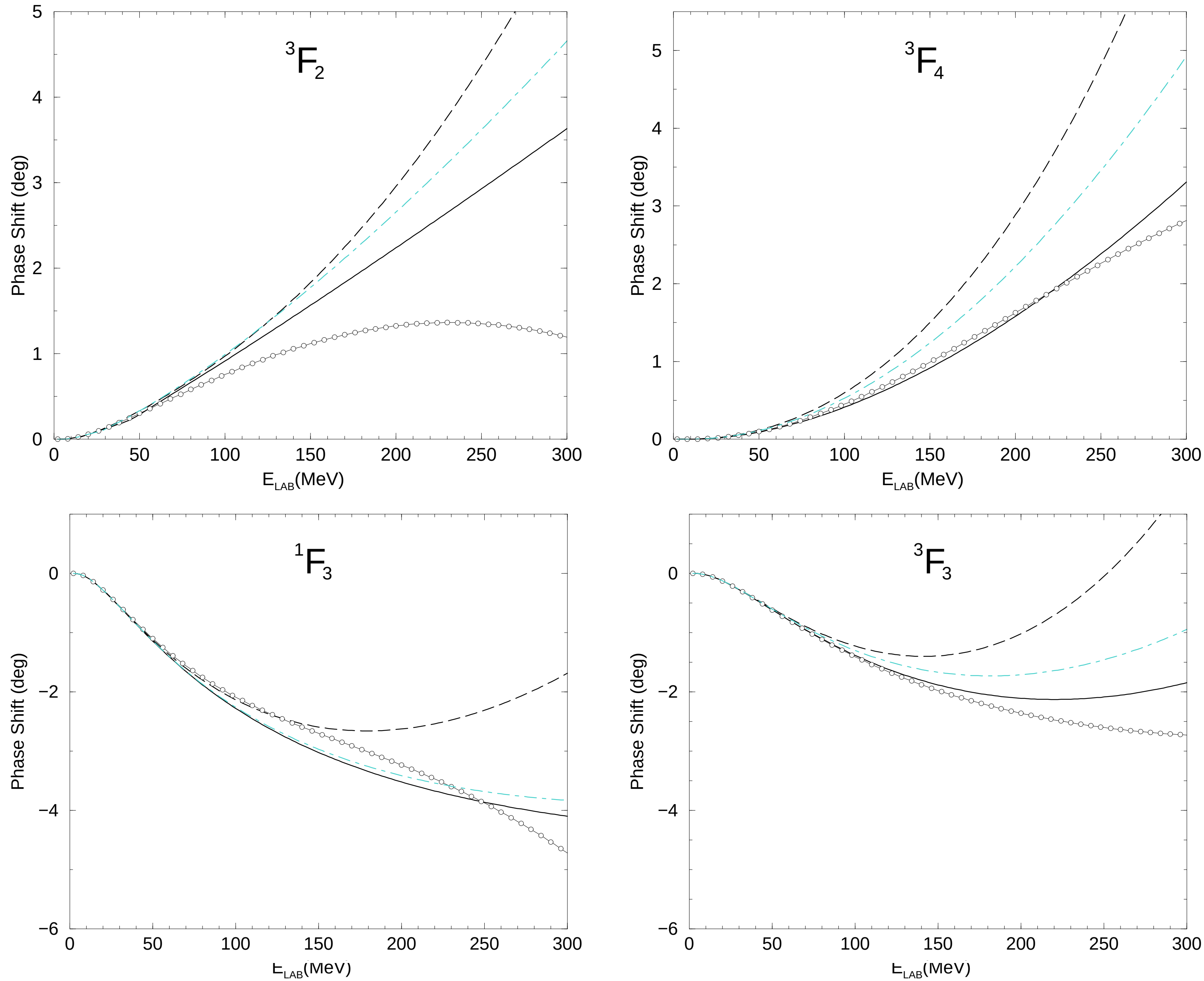

$^3H_4$
$^3H_6$
ε_5
$^1H_5$
$^3H_5$
Phase Shift (deg)
E_{LAB}(MeV)
0
50
100
150
200
250
300



3G3
3G5
ε4
1G4
3G4
Phase Shift (deg)
ELAB(MeV)
2
0
−2
−4
−6
0.5
0
−0.5
−1
−1
2.5
2
1.5
1
0.5
10
8
6
4
2
0
50
100
150
200
250
300



$^3F_2$
$^3F_4$
ε_3
$^1F_3$
$^3F_3$
Phase Shift (deg)
E_{LAB}(MeV)



$^3F_2$
$^3F_4$
$^1F_3$
$^3F_3$
Phase Shift (deg)
E_{LAB}(MeV)
0
50
100
150
200
250
300
1
2
3
4
5
−2
−4
−6

Summary

- revision of our expanded results: closer to HB, origin of differences identified
  - ⋆ dynamical equations (our choice: Blankenbecler and Sugar)
  - ⋆ two loop calculations (not completely understood)
- TPEP : RB and HB results differ
- phase function method: probing the tail of the potential
- OPEP+TPEP : RB and HB differences are strongly reduced
- LECS: phase shifts from $O(q^4)$ NN potential favor smaller values for c_3 (Δ?)