Cluster Formation and The Virial Equation of State of Low Density Nuclear Matter

INT Workshop October ‘03

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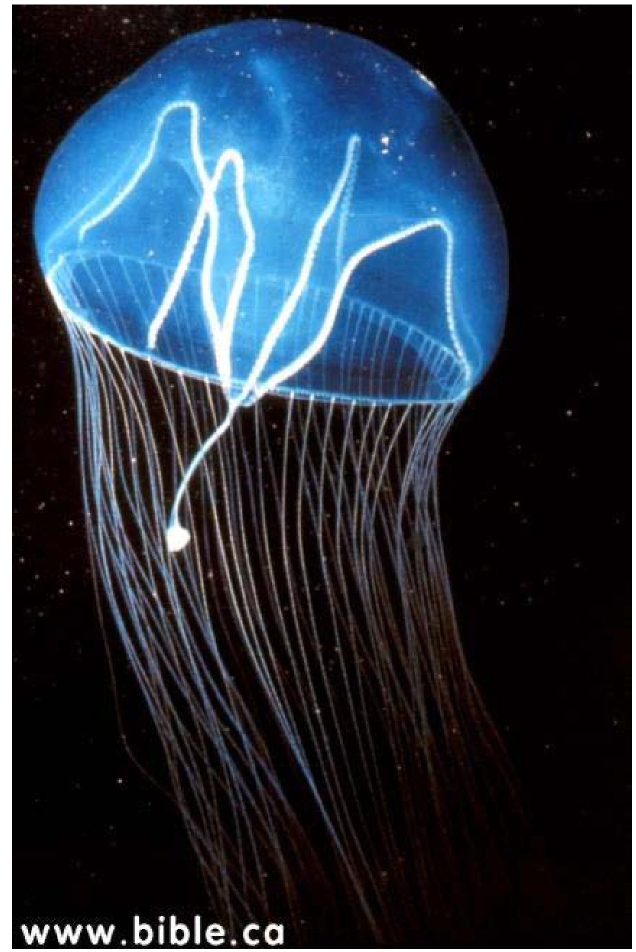
The Virial Equation of State of Low Density Nuclear Matter

- Is just data (phase shifts)
- Minimal framework to use scattering data (no potentials or wavefunctions)
- Transparent (what you see is what you get)
- Is a jellyfish.



Dinosaurs versus Jellyfishes

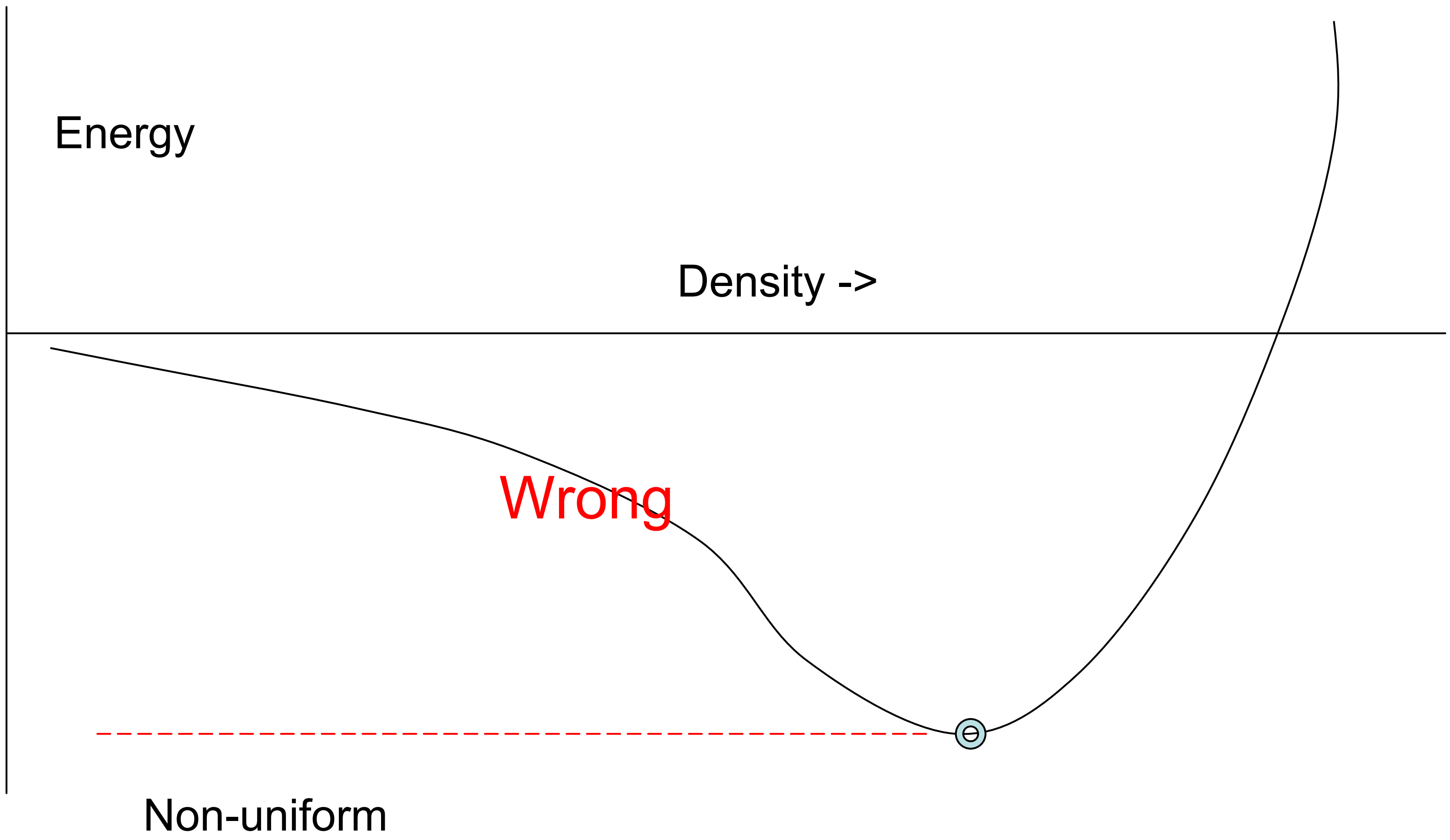
- 2NF fit to NN phase shifts.
- 3 nucleon force fit to properties of few body systems.
- Complex wave functions and many body calculations needed for nuclear matter.
- Model dependent.
- Applicable at low and high densities

- 2^{nd} virial from NN phase shifts
- Use N-α and α-α phase shifts to describe interactions of few nucleon systems.
- Nuclear matter properties follow directly.
- Model independent! No unobserved short ranged wave functions or potentials.
- Applicable only at low densities.
- Can't argue with a jellyfish!

Virial Equation of State

1. Introduction (low density matter is interesting)
   - Nuclear matter at low density and cluster formation.
   - Hadron gas and hadron resonances at RHIC or on Lattice at T~ 100 MeV.
   - Neutron matter at low densities and universal behavior of large scattering length problem.
   - Need low density EOS for Supernova neutrinosphere.
2. Virial Expansion 101 (basic virial formalism).
3. Results for neutron matter and universal behavior.
4. Results for nuclear matter and cluster formation.
5. Conclusions

Nuclear matter at low density



Low density matter is formed in heavy ion collisions and in astrophysics

Nuclear Pasta

- At subnuclear density, nuclear attraction and coulomb repulsion compete to give complex shapes.
- This nuclear pasta is expected in nuetron star crusts, supernovae.
- Semiclassical model

$$v(r)=a\, e^{-r^2/\Lambda} + b_{ij}\, e^{-r^2/2\Lambda} + e_i e_j\, e^{-r/\lambda}/r$$

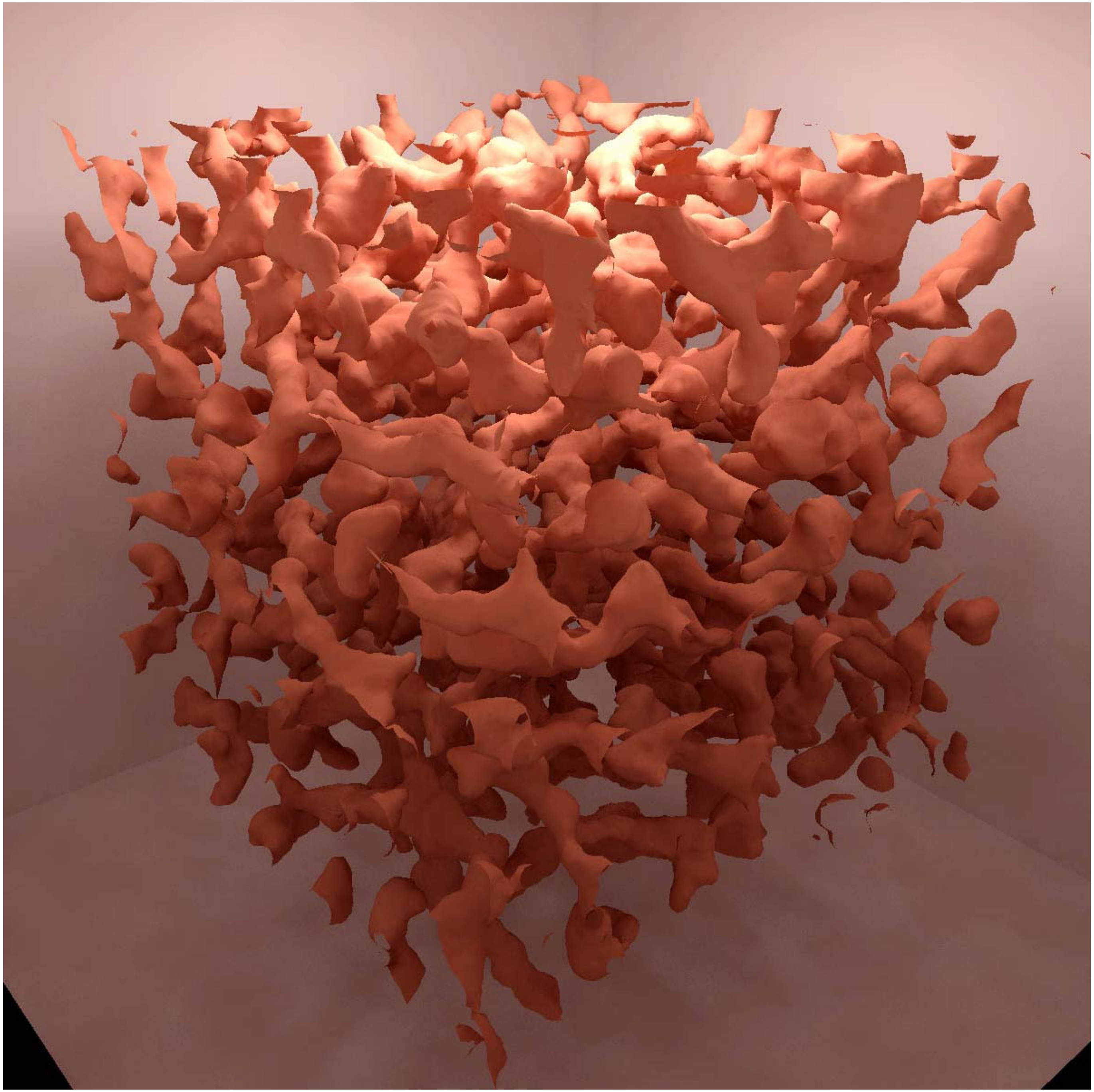
- Charge neutral system of n, p, and e. [e provide screening length λ.]
- Parameters fit to E and ρ_0 of nuclear matter.
- Molecular dynamics simulations.

Isosurface of proton density

$\rho = 0.05\ fm^{-3}$, T=1 MeV, Y_p=0.2

Simulation with 20,000 p and 80,000 n

Not shown is uniform e gas and low density n gas between pasta

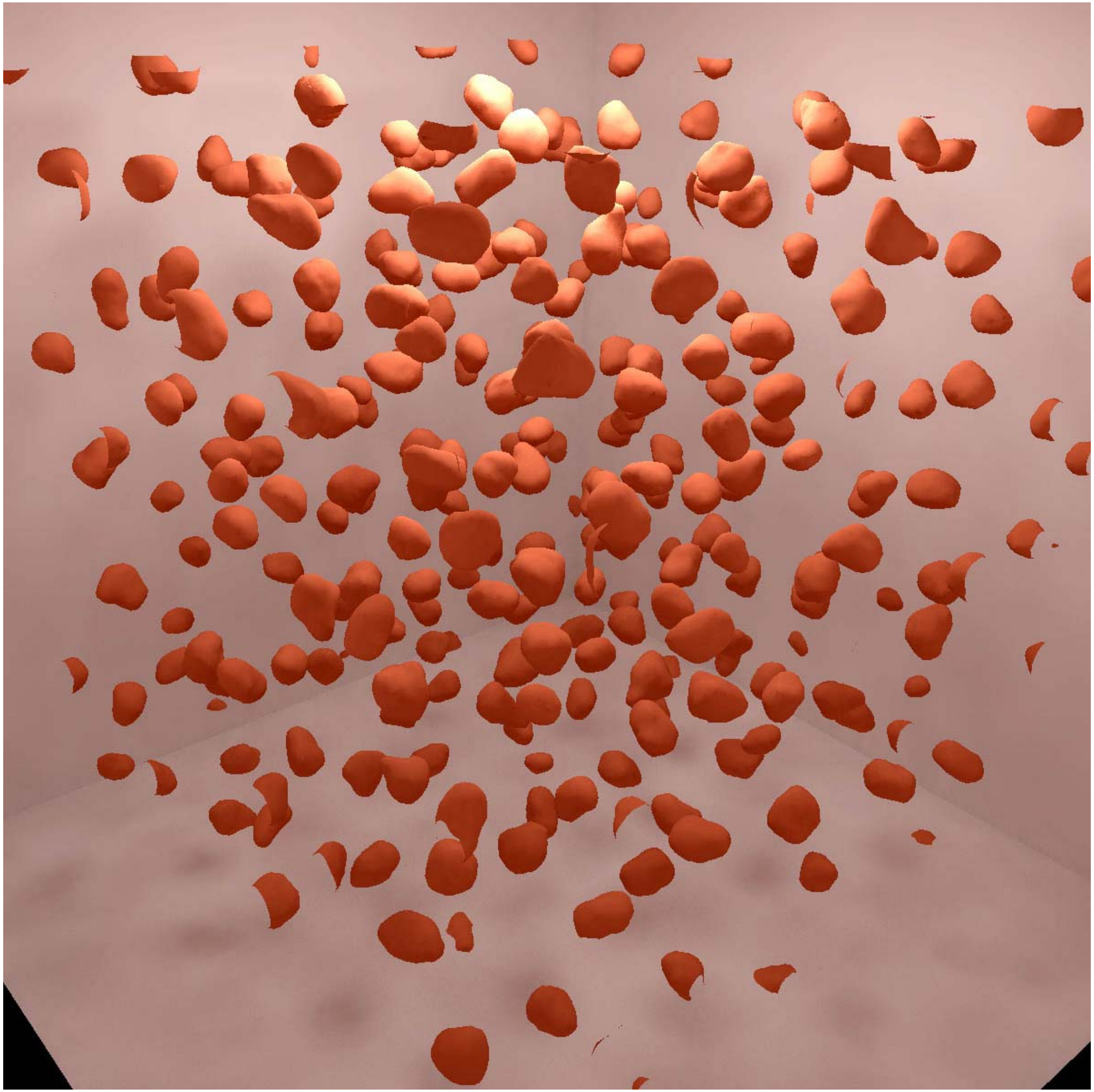


Graphics by Brad Futch FSU

What is distribution of clusters with different Z, N and how do they interact?

Simulation with 40,000 nucleons at T=1 MeV, ρ=0.01 fm^{-3}, Y_p=0.2

0.03 fm^{-3} surface of the proton density



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Hadron gas at High T

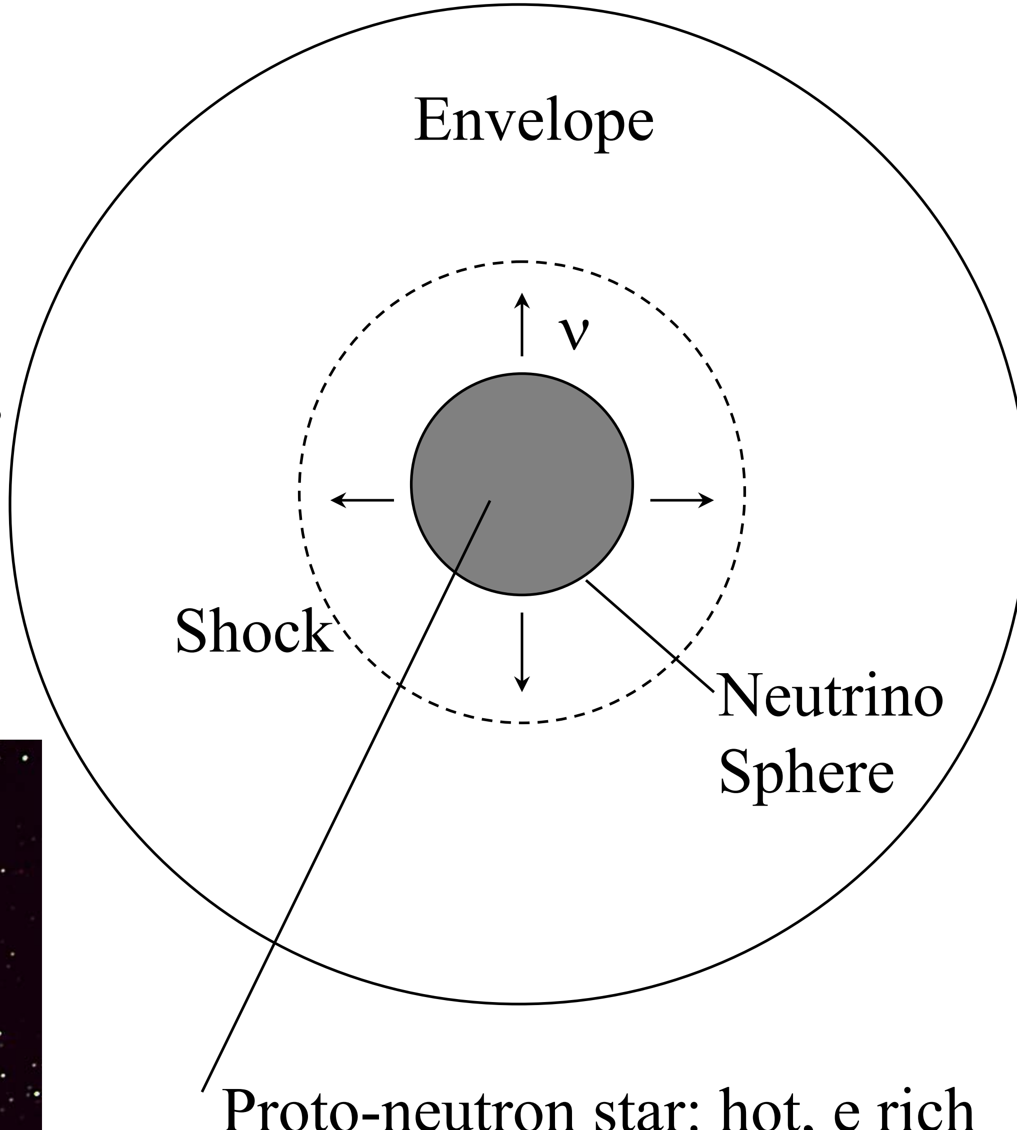
- Now have clusters of quarks instead of clusters of nucleons.
- Virial expansion for pion gas describes interactions in terms of pi-pi scattering phase shifts.
- Application to hot hadronic matter formed at RHIC.
- Extend formalism to describe gas at small but nonzero baryon density with nucleons and NN and N-pi scattering phase shifts plus Deltas…
- Compare to first lattice calculations with small chemical potentials.

Neutron Matter at Low Density and Universal Behavior

- G. Bertsch problem: consider a low density fermi system with large scattering length $a \rightarrow \infty$, and effective range $r \rightarrow 0$ much less then inter-particle spacing.
- There are no length scales associated with interaction. Therefore system will exhibit universal behavior independent of details.
- Example $E = \xi E_{FG}$ with E_{FG} energy of free Fermi gas and GFMC gives $\xi \approx 0.44$.
- To what extent does real neutron matter at low density approach this "unitary limit"? Real a=-19 fm, r=2.7 fm.
- Not many results know for universal behavior at finite T.
- Use Virial expansion to simply relate energy of neutron matter to nn scattering properties.
- Ho used virial expansion to describe cold atom systems.

Core Collapse Supernovae

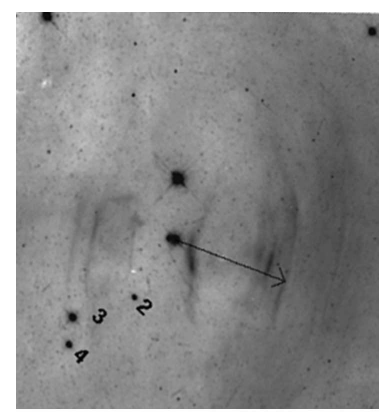
Core of massive star collapses to form proto-neutron star.
νs form neutron star energizes shock that ejects outer 90% of star.



July 5, 1054

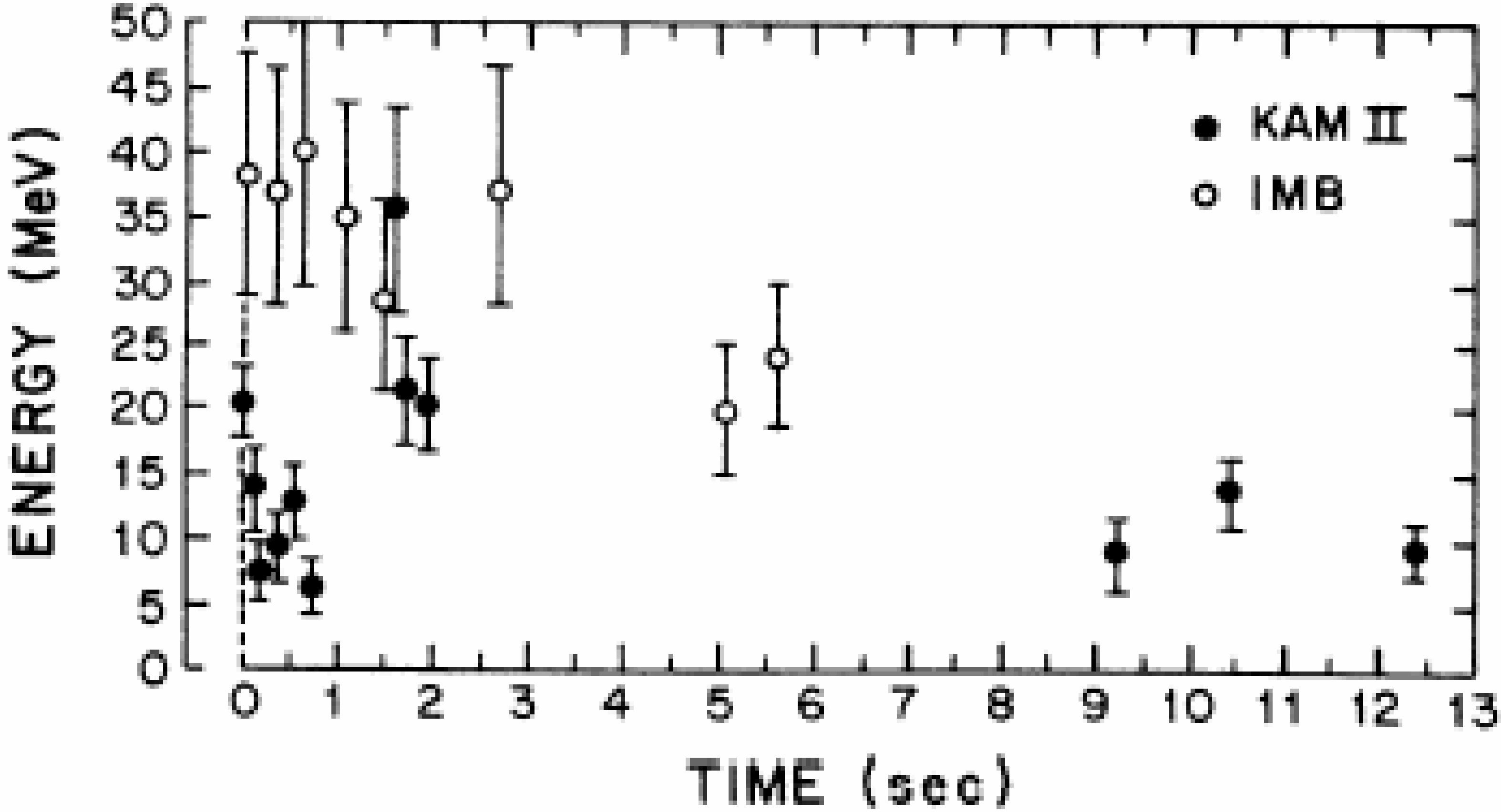
Crab nebula

Crab Pulsar



Hubble ST

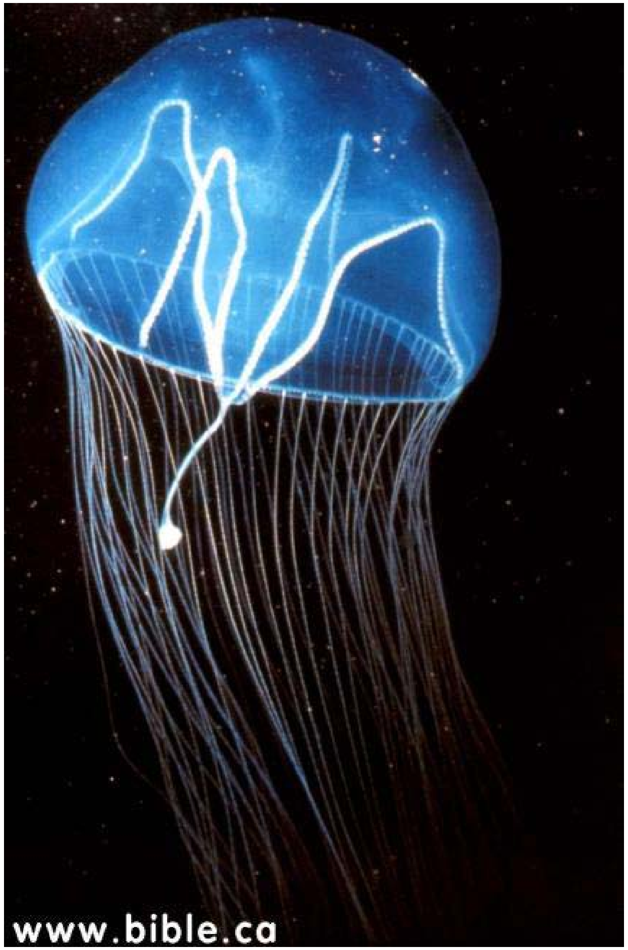
Detected Neutrinos from SN1987A



Neutrino-sphere in a Supernova

- View sun in viable light and see photosphere where γs last scattered.
- View supernova in neutrinos and see neutrino-sphere. Mean free path $\lambda \sim 1/\sigma\rho \sim R \sim 10$ km is size of system.
- Conditions at neutrino-sphere:
  - Temperature ~ 4 MeV crudely observed with 20 SN1987a events.
  - $\sigma \sim G_F^2 E_\nu^2$ and $E_\nu \sim 3T$
  - $\rho \sim 10^{11}$ to 10^{12} g/cm^3 [$\sim 10^{-4}$ fm^{-3}]
  - Proton fraction starts near ½ and drops to small values.
- What is the composition, equation of state, and neutrino response of nuclear matter under these conditions?
- *Virial expansion gives model independent answers!*

Virial Expansion Formalism



Virial 101

- Assume gas phase at low density $\rightarrow$ where fugacity $z=e^{\mu/T} \ll 1$ with μ the chemical pot.
- Expand grand canon. partition function Q in powers of z: P=T lnQ/V, n=z d/dz lnQ/V

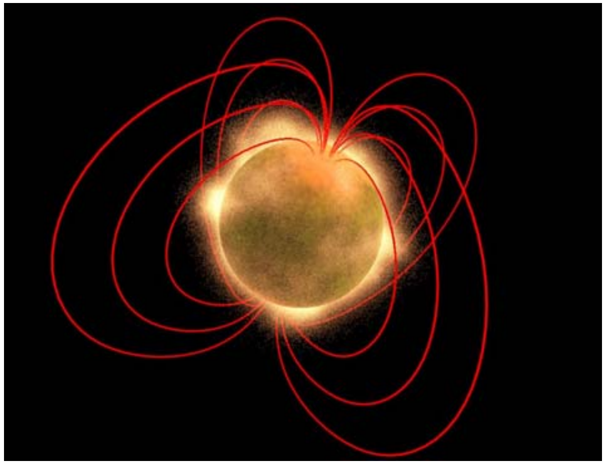
  $P=2T/\lambda^3[z+b_2z^2+b_3z^3+\ldots]$, $n=2/\lambda^3[z+2b_2z^2+3b_3z^3+\ldots]$

  Here λ=thermal wavelength=$(2\pi/mT)^{1/2}$
- 2^{nd} virial coef. $b_2(T)$ calculated from 2 particle partition function: $Q_2=\sum_{states} Exp[-E_2/T]$

  E_2 is energy of 2 particle state. Thus b_2 depends on density of states.

Density of states

- Put system in big spherical box of radius R
- Relative mom. k from $E_2=k^2/2m_{reduced}$.
- $\psi(r_1-r_2=R\rightarrow\infty)=0=\sin[kR+l\pi/2+\delta_l(k)]$ or
  $kR+l\pi/2+\delta_l(k)=n\pi$.
- Distance between states $\Delta k=\pi/(R+d\delta/dk)$ so $dn/dE \propto 1/\Delta k \propto R + d\delta/dk$
- $b_2=2^{1/2}\sum_B e^{E_B/T} +2^{1/2}/\pi \int_0^\infty dk\, e^{-E_k/2T} \sum_l{}'$
  $(2l+1)\, d\delta_l(k)/dk \pm 2^{-5/2}$
  with + for bose and – for fermions.
- b_2 Includes both bound states and scattering resonances on equal footing.

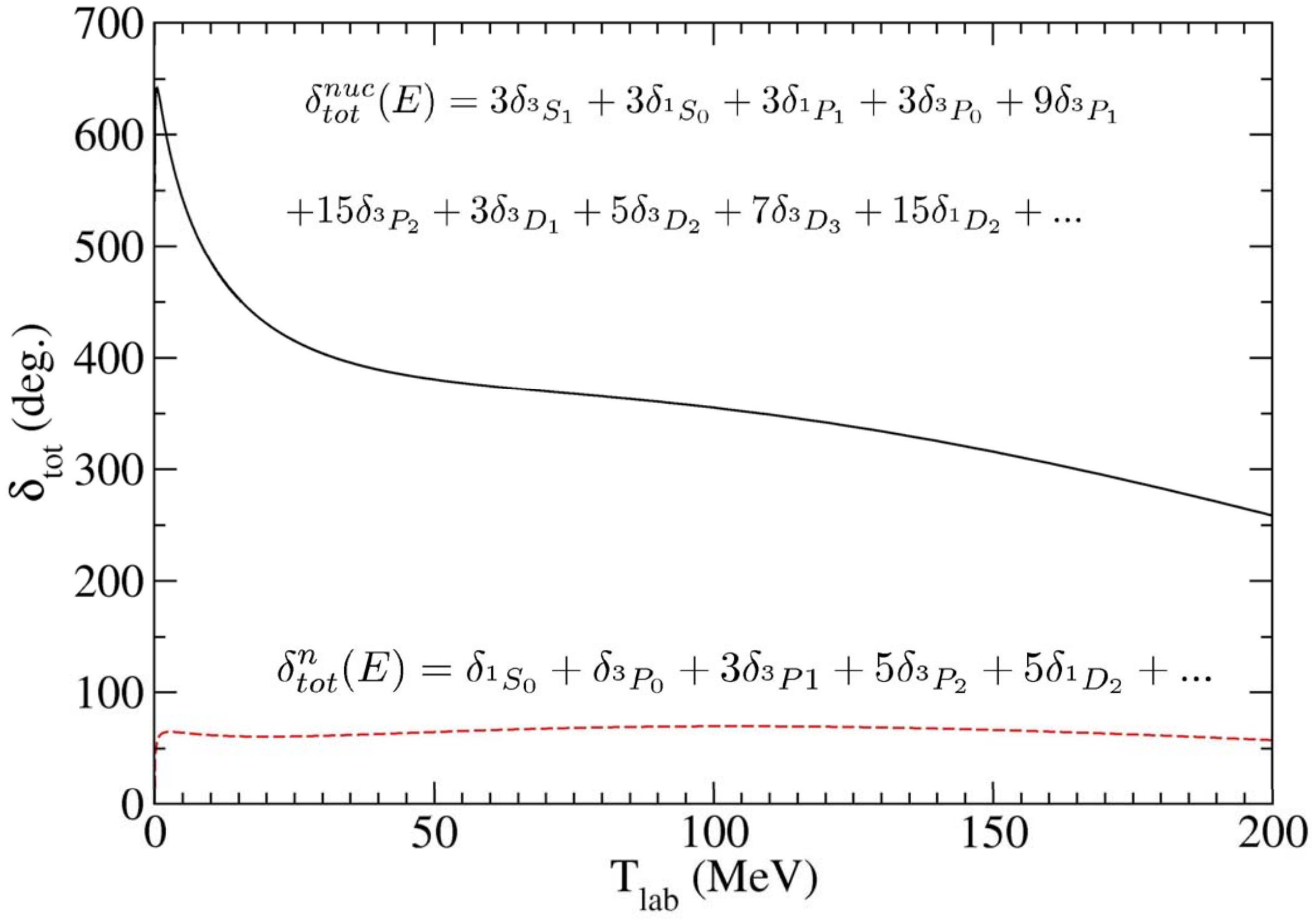


Neutron Matter

Neutron Matter

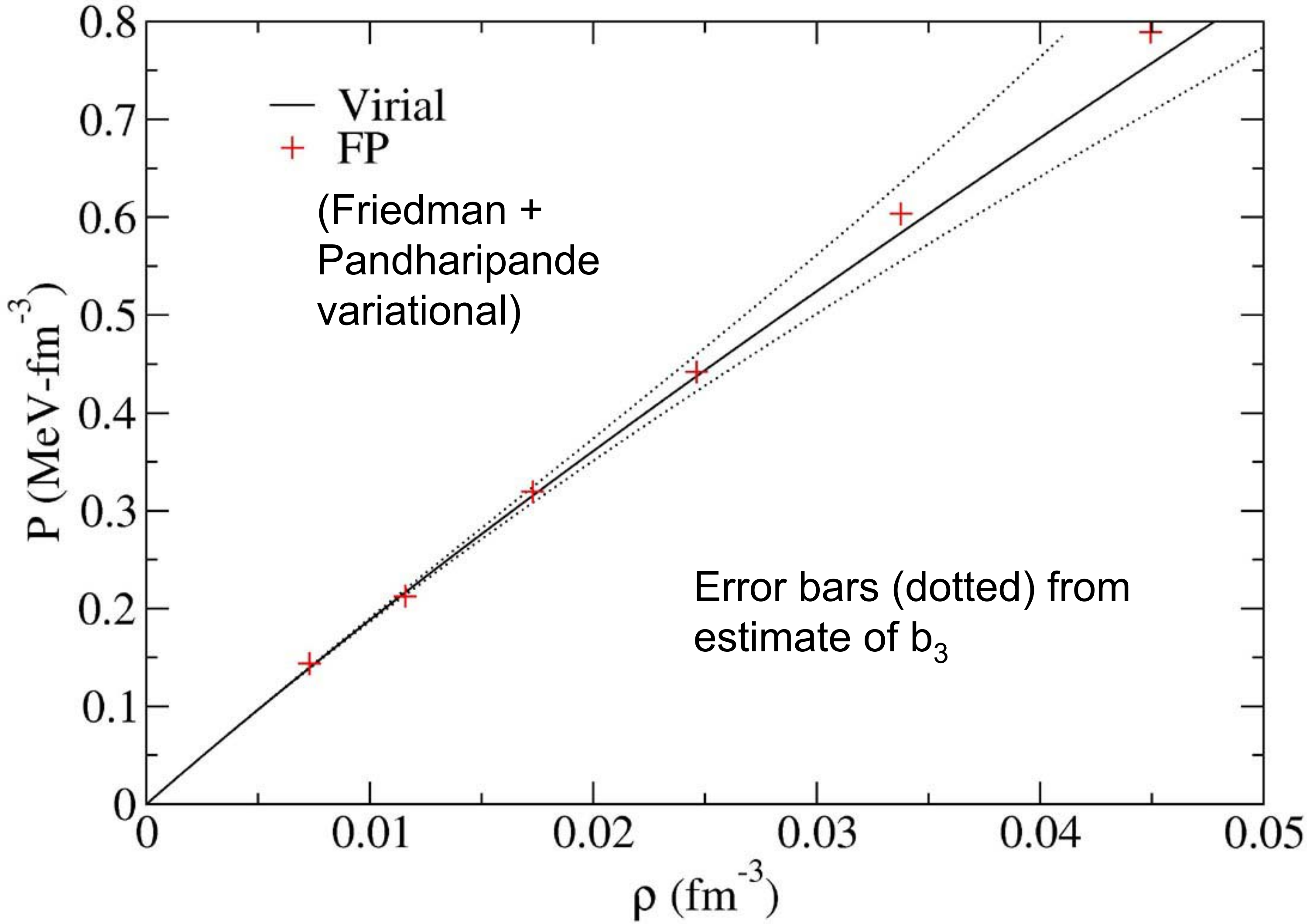
- Integrate by parts and include spin,
- $b_n(T)=1/(2^{1/2}\pi T)\int dE\ e^{-E/2T}\ \delta_{tot}(E)\ -2^{-5/2}$
- $\delta_{tot}=\delta(^1S_0)+\delta(^3P_0)+3\delta(^3P_1)+5\delta(^3P_2)+5\delta(^1D_2)+\ldots$
- Use isospin 1 pn phase shifts.
- $b_n(T)=0.301, 0.306, 0.309$ at T=2, 4, and 8 MeV
- b_n almost T independent, as s-wave phase falls with increasing energy, higher l contributions rise to almost cancel.
- Use b_3 for error estimate. 3 n can't be in s state so expect b_3 to be small. Use $|b_3|\leq b_2/2$.

Total Phase Shift for Nuclear Matter (top) and Neutron Matter



NN-OnLine

Neutron matter Equation of State at T=20 MeV

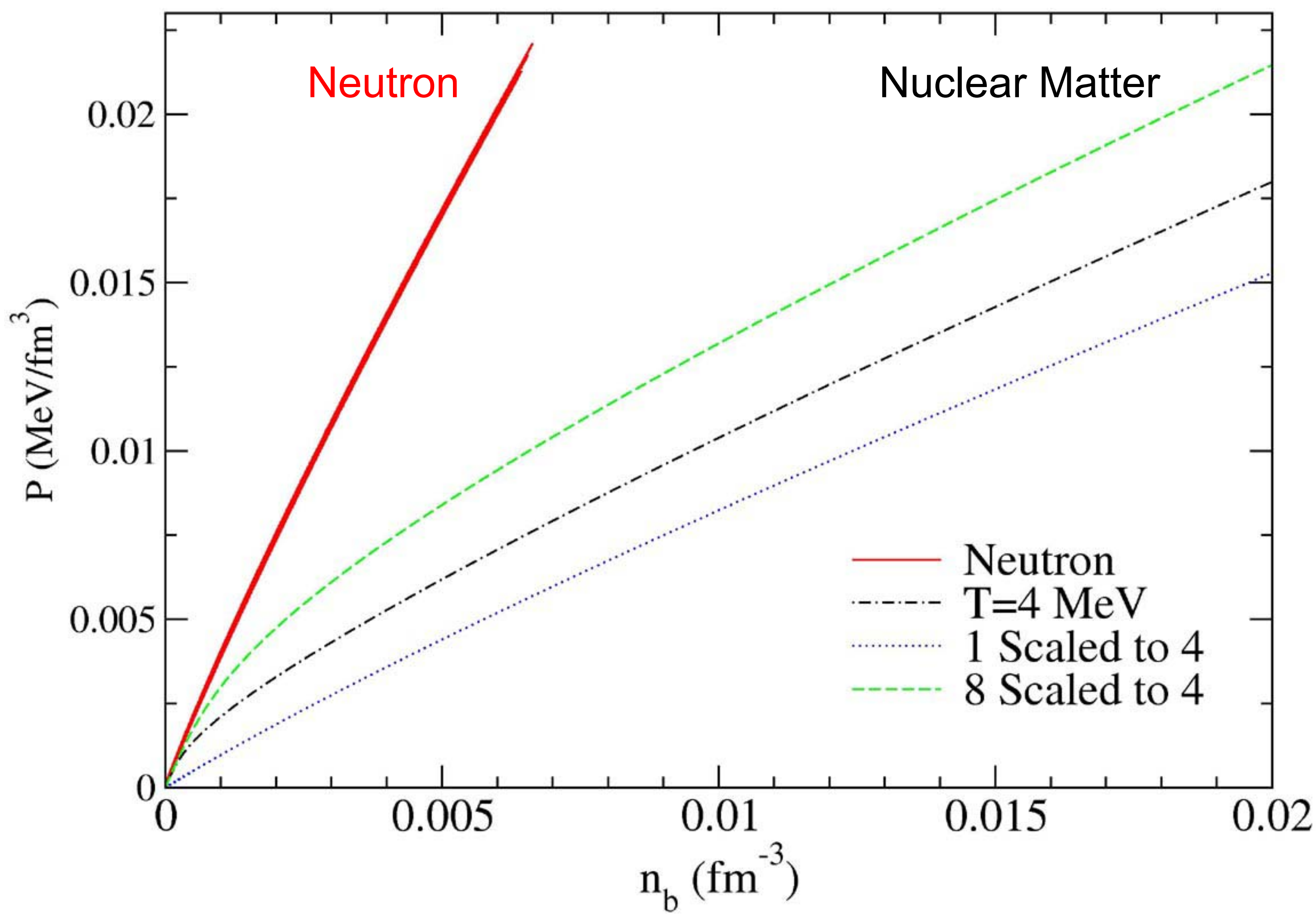


Scaling of Neutron Matter EOS

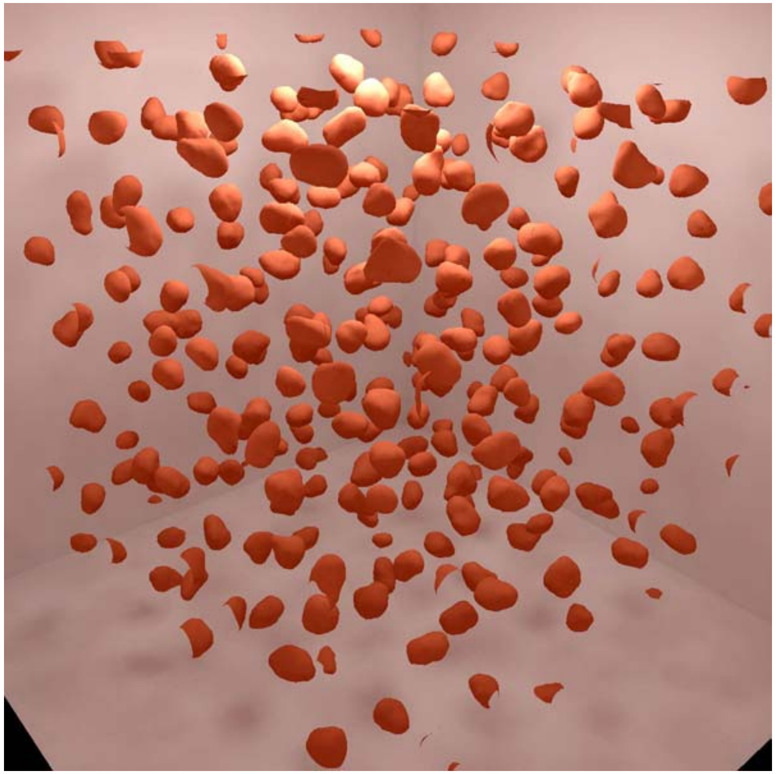
- If $b_i(T)$ are independent of T the EOS will scale $P/T^{5/2} = f(n/T^{3/2})$. From

  $P/T=g/\lambda^3[z+b_n z^2+\ldots]$ and
  $n=g/\lambda^3[z+2b_n z^2+\ldots]$ with $\lambda \propto T^{-1/2}$.
- Unitary Limit: calculate b_n with only s-wave and $a=-\infty$, $r=0$. $\delta(^1S_0)=\pi/2$

  $b_n(T)=3/2^{5/2}=0.5303$ independent of T.
- In unitary limit system clearly scales.
- Real neutron matter scales, to a very good approx., but with a $b_n \approx 0.3$ that is 40% smaller then unitary limit.



Scaled Pressure at T=4 MeV from equation of state at T=1, 4, and 8 MeV



Nuclear Matter

and cluster formation

Nuclear Matter

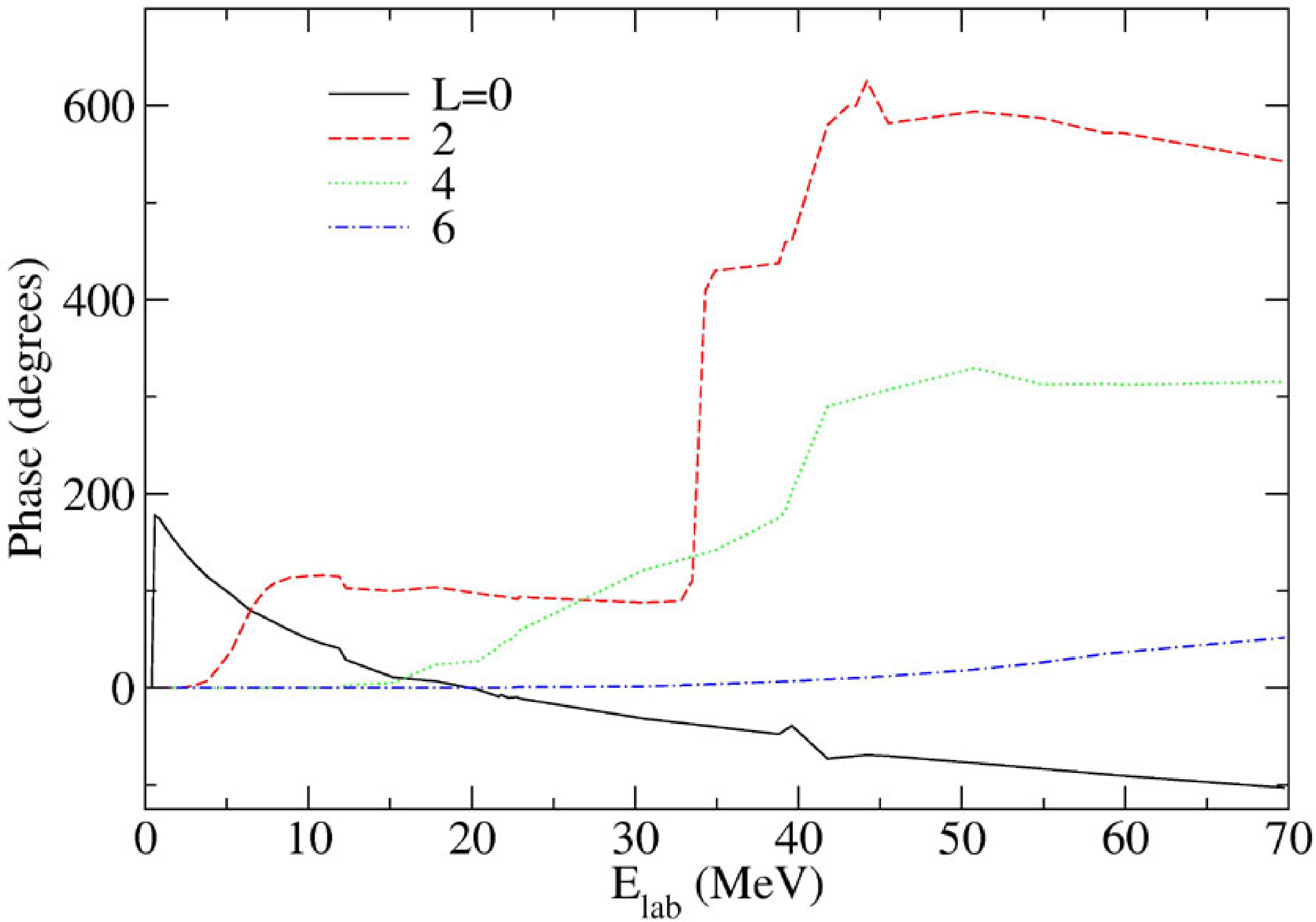
- Is very different from neutron matter because clusters can form.
- Deuterons appear as bound state in b_2.
- α particles will appear as bound state in b_4 (if you calculate this high).
- Large α binding E_α=28.3 MeV gives large $e^{+E\alpha/T}$ contribution to b_4.
- Nucleon only virial expansion may be accurate over a reduced density range because of the abnormally large b_4 (and higher) v. coefficients.
- Solution: include α explicitly and work with system of p, n, and α s. Chemical equilibrium $2\mu_p+2\mu_n=\mu_\alpha$ gives $z_\alpha=z_p{}^2 z_n{}^2\ e^{E\alpha/T}$.
- Work to 2nd order in z_p, z_n, z_α. Can include heavier nuclei at even higher densities.

n, *p*, α system

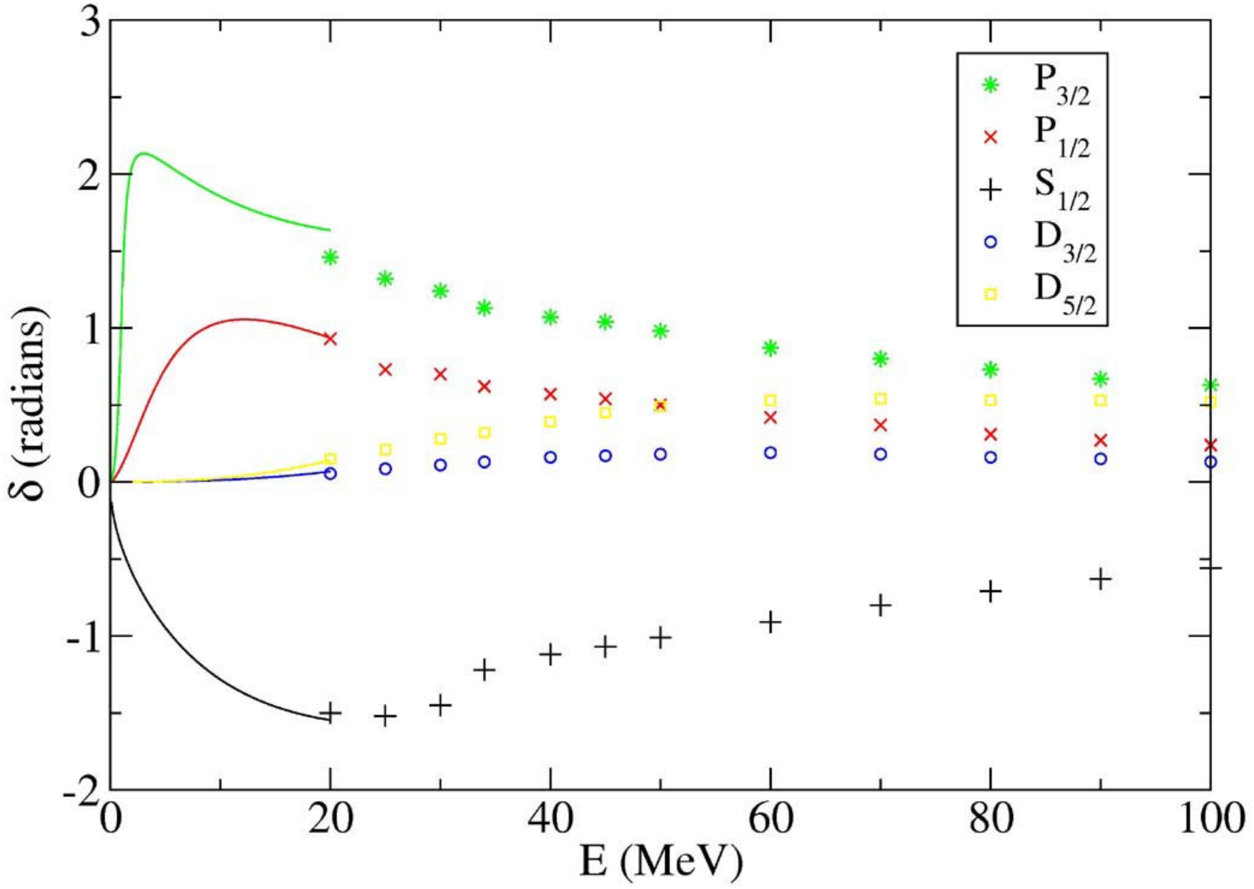
- $P/T = 2/\lambda^3[z_p + z_n + (z_p^2 + z_n^2)b_n + 2z_p z_n(b_{nuc} - b_n)]$
  $+ 1/\lambda_\alpha^3[z_\alpha + z_\alpha^2 b_\alpha + z_\alpha(z_p + z_n)\, b_{\alpha\, n}]$
- Need four 2nd virial coef: b_n for neutron matter, b_{nuc} for symmetric nuclear matter, b_α for interaction between two alphas, and $b_{\alpha\, n}$ for interaction between an α and a *n* or *p*.
- Guess z_p, z_n, and calculate z_α for chem. equilibrium $z_\alpha = z_p^2 z_n^2\, e^{E\alpha/T}$. Next calculate n_p, n_n, and n_α. Adjust z_p, z_n to reproduce desired proton fraction and total baryon density.

  $n_b = n_p + n_n + 4n_\alpha, \quad Y_p = (n_p + 2n_\alpha)/n_b$

α−α Elastic Scattering Phase Shifts

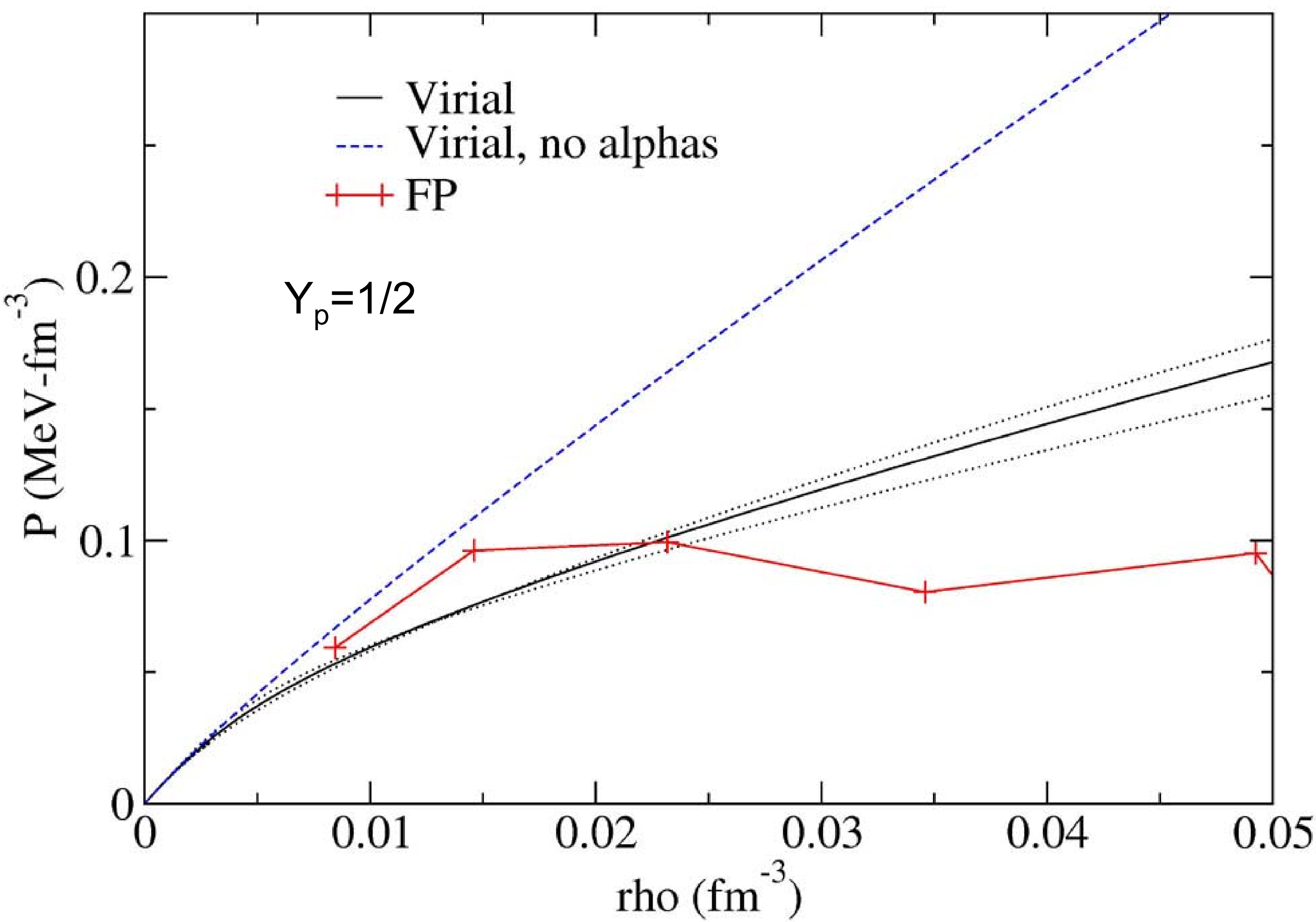


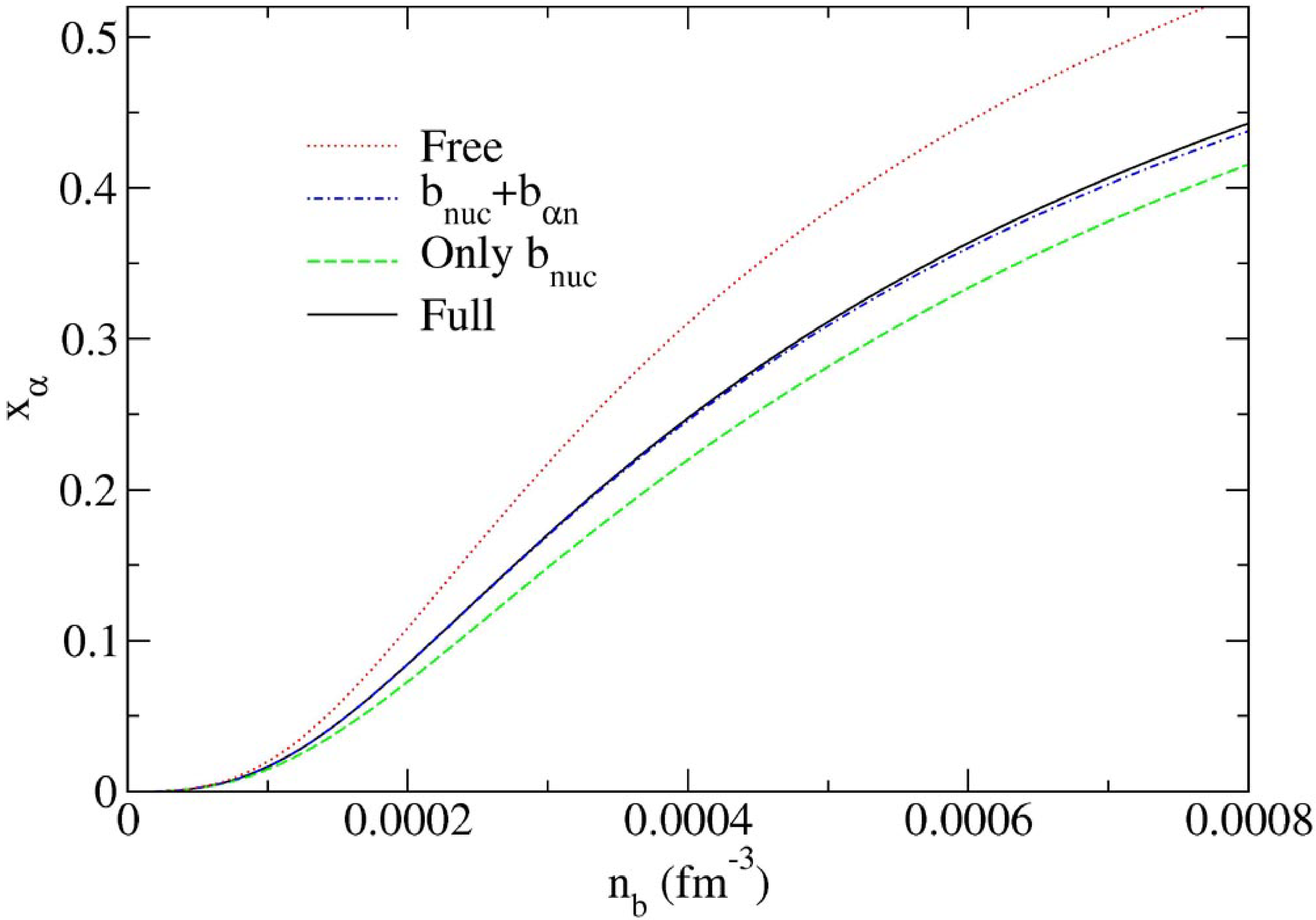
n-α Elastic Scattering Phase Shifts



Data for 0-20 MeV, calculation of K. Amos + S. Karataglidis for 20-100 MeV

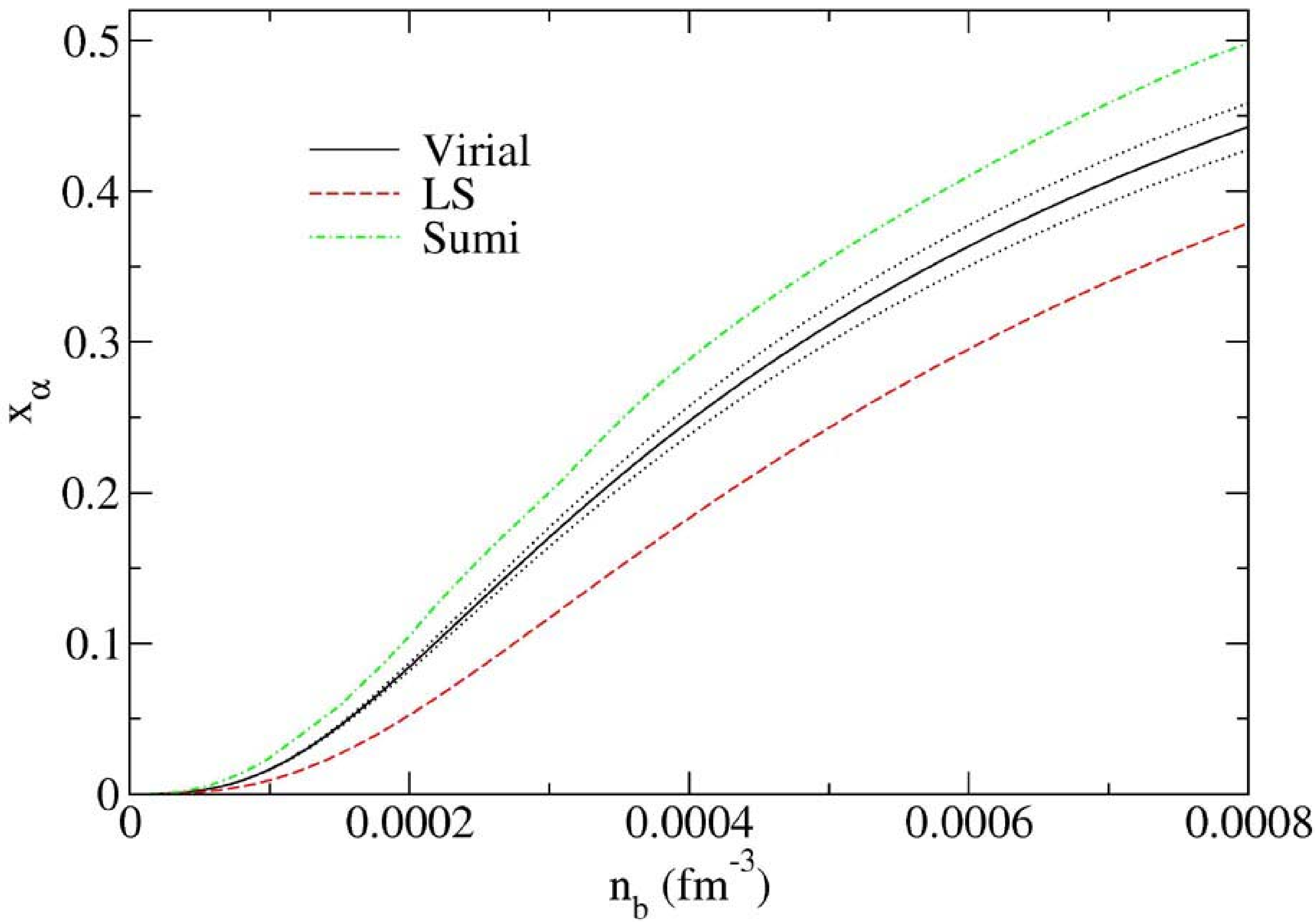
Nuclear matter EOS at T=10 MeV



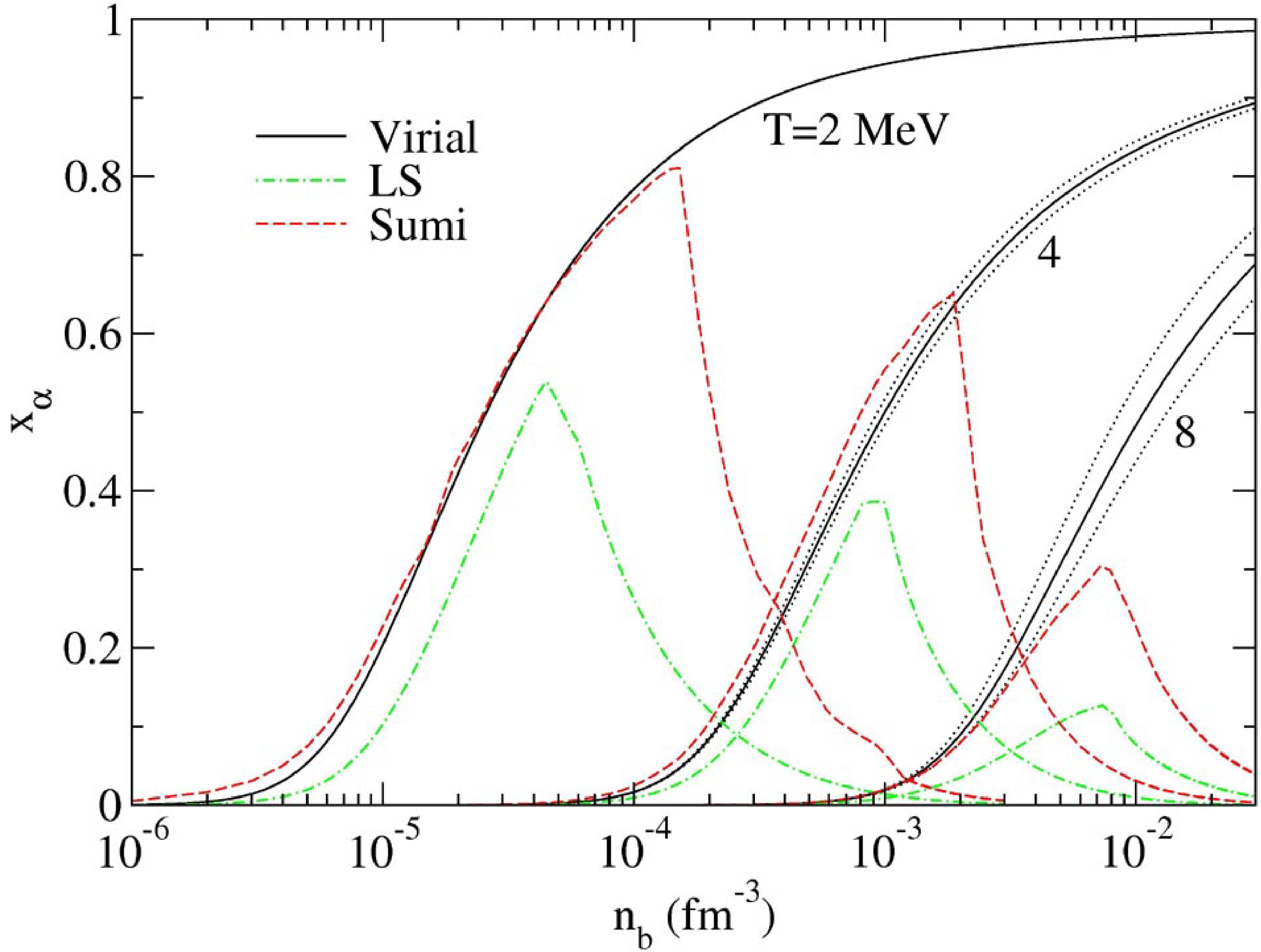


Alpha mass fraction for symmetric nuclear matter at T=4 MeV. The free curve has all b_i=0 while the b_{nuc} curve has $b_{\alpha\ n}$=b_α=0.

α Mass Fraction at T=4 MeV

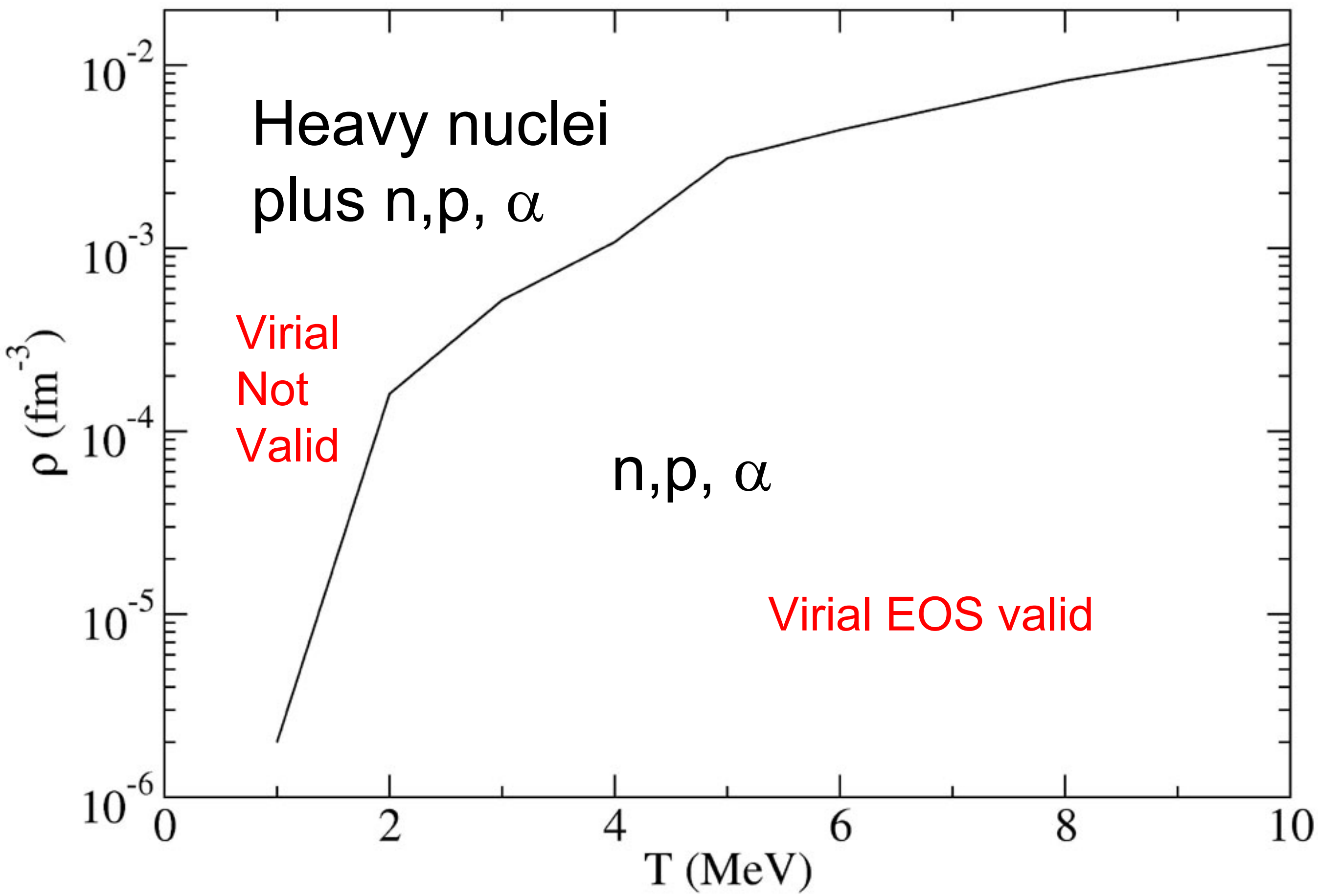


α particle mass fraction in symmetric nuclear matter vs density. The widely used phenomenalogical EOS by Lattimer Swesty is dashed while Sumi is an EOS based on a rel mean field interaction (dot-dashed).



Alpha mass fraction vs density for T=2, 4 and 8 MeV. Also shown are predictions for Lattime Swesty and Sumioshi EOS models. These have alpha fractions that drop at high density because of the formation of heavy nuclei.

Nuclear Matter Composition



Density above which Sumioshi EOS has 10% or more heavy nuclei for Y_p=1/2. Thus n, p, α EOS is only valid at lower densities.

Entropy, Energy

- From thermodynamics, entropy density is

$$s = (\frac{\partial P}{\partial T})_{\mu_i} \quad = \frac{1}{T}[\frac{5}{2}P - n_p\mu_p - n_n\mu_n - n_\alpha(\mu_\alpha + B_\alpha)]$$

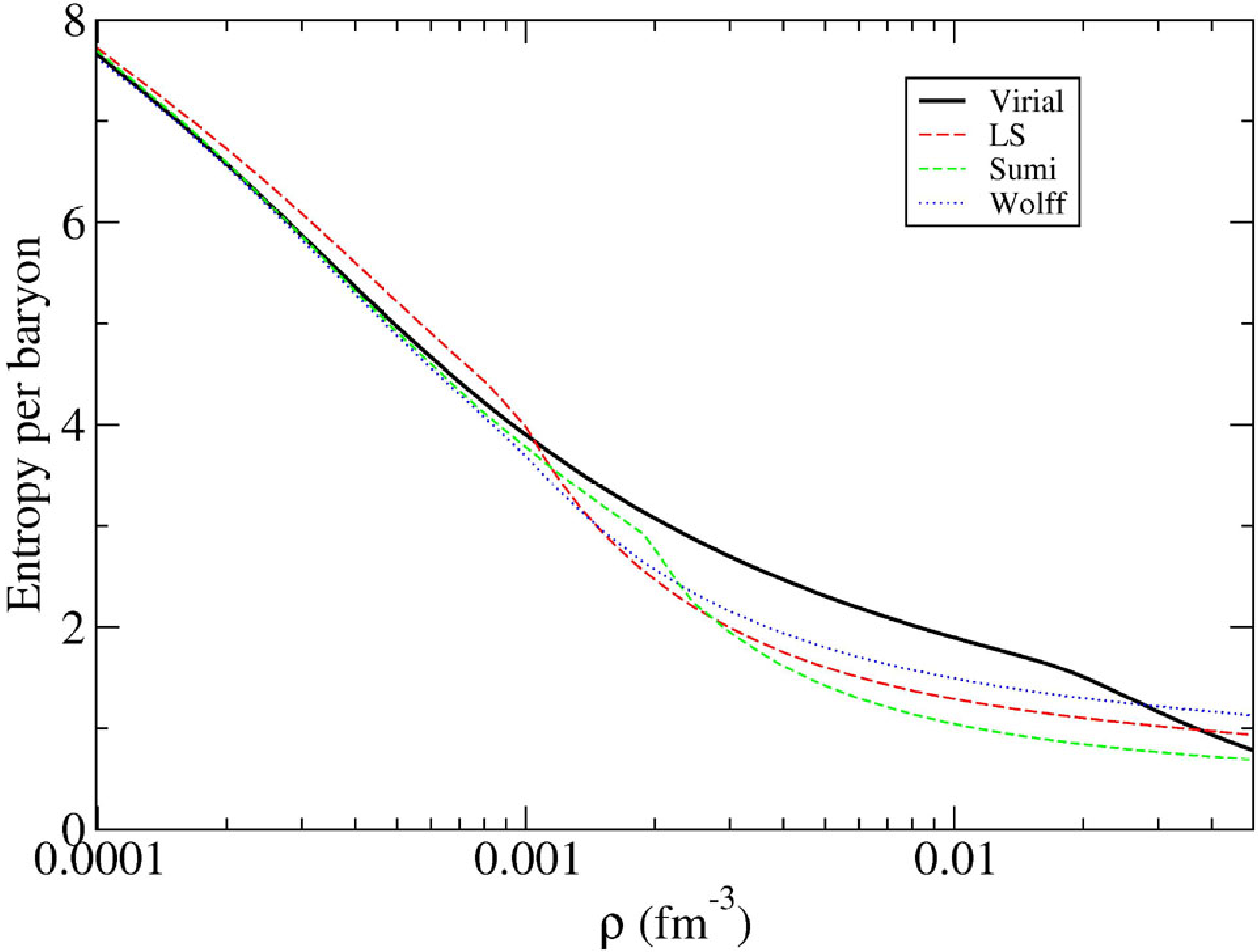
$$+\frac{2T}{\lambda^3}[(z_p^2 + z_n^2)b_n' 2z_p z_n(b_{nuc}' - b_n')]$$

$$+\frac{T}{\lambda_\alpha^3}[z_\alpha^2 b_\alpha' + (z_p + z_n)z_\alpha b_{\alpha n}']$$

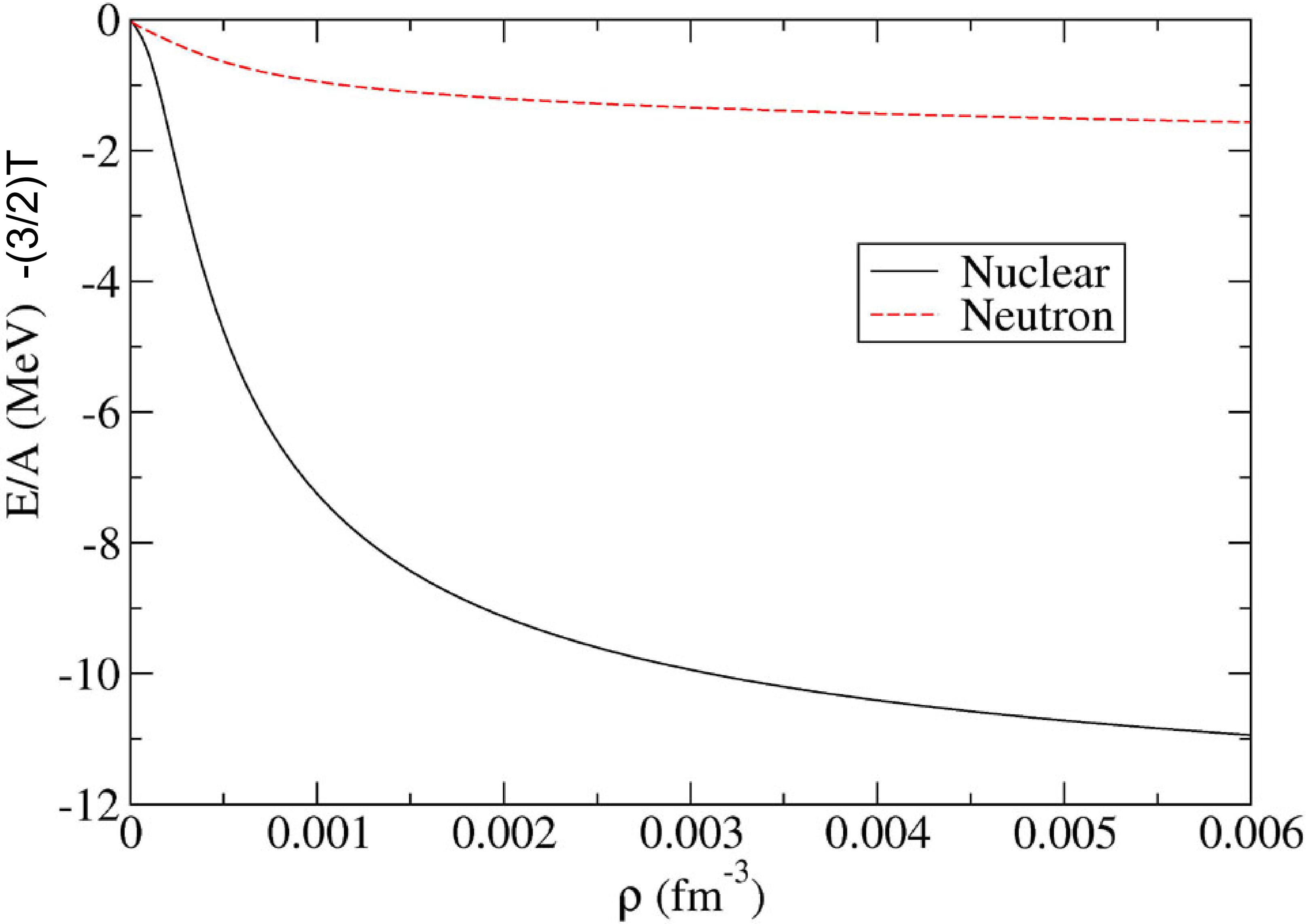
- Energy density is $\epsilon = Ts + \sum_i n_i\mu_i - P$

- Symmetry energy is $S(n,T) = \frac{1}{8}(\frac{\partial^2 E/A}{\partial Y_p^2})_{Y_p=1/2}$

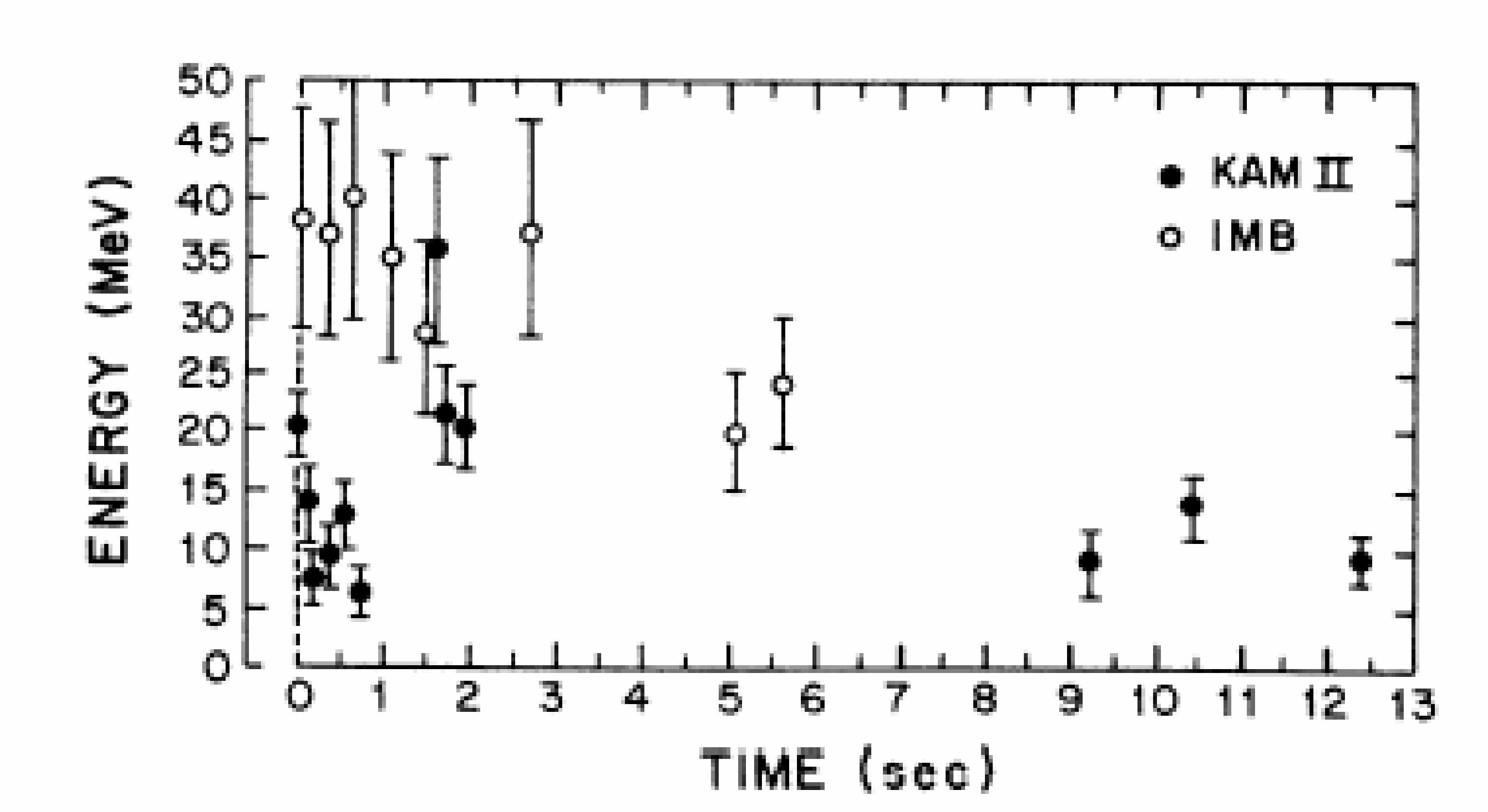
Entropy vs density for symmetric nuclear matter at T=4 MeV



Energy per Particle at T=4 MeV



Neutrino Response



Historic SN1987A data

Neutrino Response

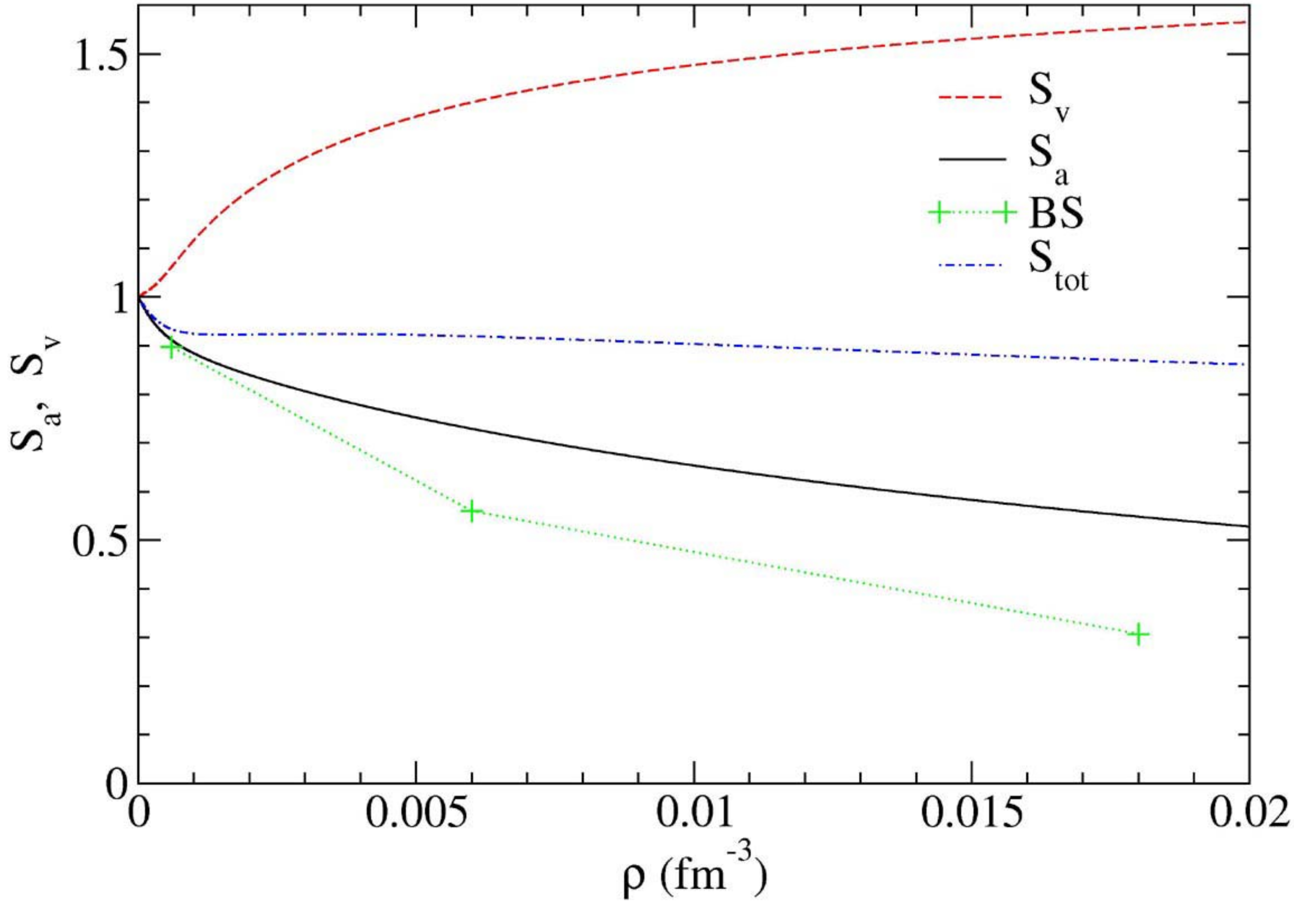
- Static structure factor S_q in $q \rightarrow 0$ limit

  $S_v = S_{q=0} = T/(dP/dn)$
- Axial or spin response from spin polarized neutron matter. z_+ is fugacity of spin up, and z_- spin down, neutrons. $z_a = (z_+/z_-)^{1/2}$

  $S_a = (1/n)\, d/dz_a\, (n_+ - n_-)\, |_{n_+ = n_-}$
- Neutrino-neutron elastic cross section

  $$d\sigma/d\Omega = (G^2 E_\nu{}^2/16\pi^2)\, [(1+\cos\theta) S_v + g_a{}^2 (3-\cos\theta) S_a]$$
- Add contributions from ν-p and ν-α scattering
- Response is only model independent in $q \rightarrow 0$ limit.

ν response T=4 MeV, Y_p=0.3



Total response is given by S_{tot} and this is much larger then traditional RPA calculation of Burrows and Sawyer (BS) because of α contributions.

Future Work

- Calculate nucleon 3rd virial b_3 for neutron and nuclear matter. Example Paulo Bedaque + G. Rupak cond-mat/0206527
- Include heavy nuclei in addition to n, p, and α
  – As a single heavy nucleus with ave. <Z> and <A>.
  – As a distribution of many heavy nuclei (perhaps with simplified N-nucleus scattering).
- Include coulomb interactions.
- Study role of inelastic scattering.
- …

Conclusions

- Virial expansion provides model independent equation of state, composition, entropy, energy, and long wave length responses for nuclear matter at low densities.
- In neutron matter 2nd virial is nearly independent of T but 40% smaller then in unitary limit ($a \to \infty$, $r_s \to 0$).
- Neutron matter EOS scales: $P=T^{5/2}f(n/T^{3/2})$
- Low density nuc matter has clusters and does not scale.
- We describe nuclear matter in n, p, and α coordinates with virial coefficients from NN, Nα, and $\alpha\alpha$ scattering.
- Incorporate d and α bound states and scattering resonances including ^{2}He, N-α p-waves, and ^{8}Be.
- Model independent α mass fraction larger then in widely used Lattimer Swesty model.

- C. J. Horowitz and Achim Schwenk, Indiana University
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