CHIRAL DYNAMICS OF NUCLEAR MATTER

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ECT* Workshop: 28.6.05

- Introduction
- Chiral Expansion of nuclear EQoS
- $O(k_f^4)$: 1π- and iterated 1π-exchange
  - Results and problems
- Inclusion of $\pi N \Delta$-dynamics
  - Complex single-particle potential
  - Energy density functional
  - Finite temperatures
  - Asymmetry energy, Neutron matter
  - Spin stability, Isovector potential
  - Fermi liquid parameters
- Λ-hyperons in nuclear medium
- Summary and Outlook

Introduction

- Problem of **nuclear binding** central in nuclear physics
- 1. step: Infinite nuclear matter: $N/Z = 1$, $e \to 0$
- Empirical saturation point

$$\boxed{\begin{aligned} \rho_0 &\simeq 0.16\,\text{fm}^{-3} \\ \bar{E}_0 &\approx -16\,\text{MeV} \\ K &= (260 \pm 30)\,\text{MeV} \end{aligned}}$$

$d_{NN} \simeq 1.8\,\text{fm} = 1.3\,m_\pi^{-1}$!

$\rho = \frac{2k_f^3}{3\pi^2}$, $k_{f0} \simeq 270\,\text{MeV}$! $\simeq 2m_\pi$

$K = k_{f0}^2 \frac{\partial^2 \bar{E}}{\partial k_f^2}\Big|_{k_{f0}}$

"compressibility"

- σ-ω meanfield approach of Walecka et al.
- "Realistic" NN-potentials + sophisticated many-body techniques
- ....
- Role of pion-dynamics: 2π-exchange ?
- Effective chiral field theory

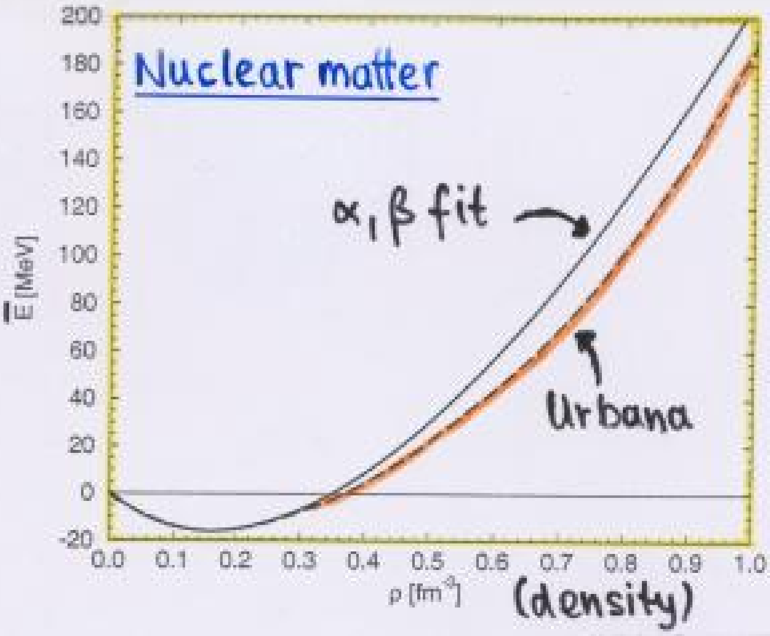
Simple but realistic parametrization

$$\boxed{\bar{E}(k_f) = \frac{3k_f^2}{10M} - \alpha \frac{k_f^3}{M^2} + \beta \frac{k_f^4}{M^3}}$$

- fit $\alpha = 5.3$, $\beta = 12.2$ to saturation point

$\to$ predict $K = 236\,\text{MeV}$ ✓

Energy per particle



Chiral Expansion of nuclear matter EQoS

ChPT Loop-Expansion	$\leftrightarrow$	Expansion of energy per particle $\bar{E}(k_f)$ in powers of k_f, modulo functions $f_n(k_f/m_\pi)$

$k_{f0} \simeq 2m_\pi$ (small scales)

Basic ingredient for finite density calculations

- Particle-hole or in-medium nucleon propagator

$$\boxed{(\not{p}+M)\left[\underbrace{\frac{i}{p^2-M^2+i\epsilon}}_{\text{in vacuo}} - \underbrace{2\pi\,\delta(p^2-M^2)\,\theta(p_0)\,\theta(k_f-|\vec{p}|)}_{\text{medium-insertion (filled Fermi se}}\right]}$$

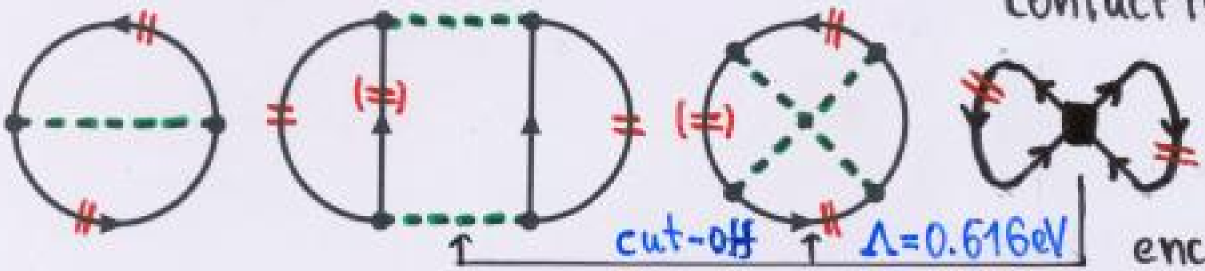
- Organize in number of medium-insertions

  1: Renormalization of nucleon mass in vacuum
  2: Many-body contr. from 2-body interaction
  3: " 3-body "
  $\leftrightarrow$ Pauli-blocking on 2-body term
  ⋮

UP TO FOURTH ORDER

- Kinetic energy $\mathcal{O}(k_f^2)$
- 1π-exchange $\mathcal{O}(k_f^3)$
- iterated 1π-exchange $\mathcal{O}(k_f^4)$

adjustable contact term

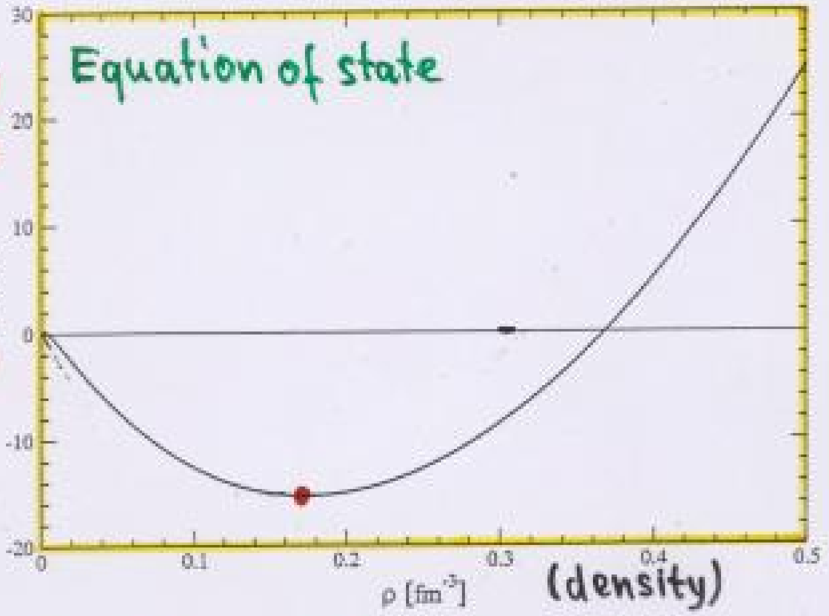


encodes unresolved short-dist. dynamics

$\bar{E}_0 = -15.3\,\mathrm{MeV}$

$\rho_0 = 0.17\,\mathrm{fm}^{-3}$

$K = 252\,\mathrm{MeV}$



$m_\pi = 135\,\mathrm{MeV}$

$g_{\pi N} = 13.2$

Chiral Saturation Mechanism

- take chiral limit $m_\pi = 0$

$$\bar{E}(k_f) = \frac{3k_f^2}{10M} - \alpha\frac{k_f^3}{M^2} + \beta\frac{k_f^4}{M^3}$$

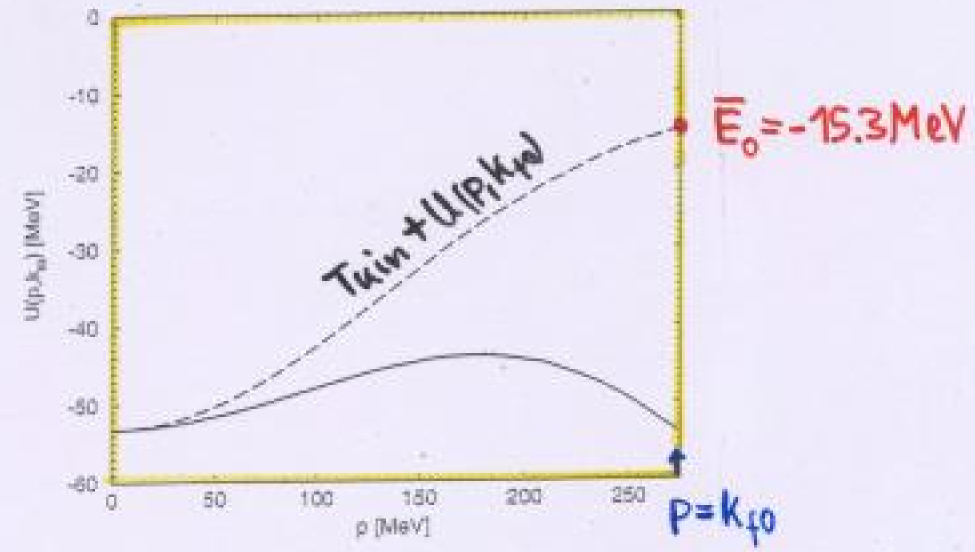
realistic parametrization

$$\boxed{\beta = \frac{3}{70}\left(\frac{g_{\pi N}}{4\pi}\right)^4\left(4\pi^2 + 237 - 24\ln 2\right) - \frac{3}{56} = 13.5\,!}$$

mainly Pauli-blocking in iterated 1π-exchange

Further nuclear matter properties

- Single-particle potential $U(p, k_f)$



- momentum dependence too strong: $M^*(k_{f0}) \simeq 2.9M$!
- density of states at Fermi surface not well described
- Critical temperature of liquid-gas phase transition too high: $T_c \simeq 25.5\,\mathrm{MeV}$

Isospin properties

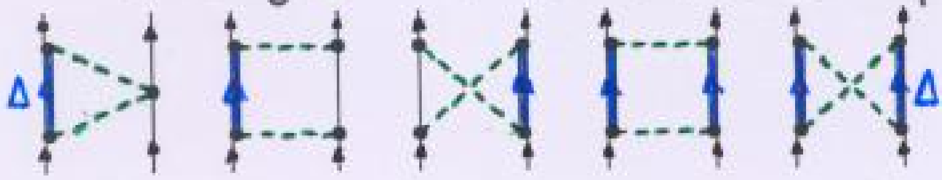
- Asymmetry energy $A(k_f)$: $A_0 \simeq 38\,\mathrm{MeV}$ ok ✓
  but downward bending at $\rho \gtrsim 0.2\,\mathrm{fm}^{-3}$
- Neutron matter equation of state $\bar{E}_n(k_n)$: (unbound!)
  only rough agreement with realistic calc. at low densities,
  "unrealistic" downward bending at $\rho_n \gtrsim 0.2\,\mathrm{fm}^{-3}$

$\boxed{\text{Missing long-range dynamics}}$

- Scheme of Lutz et al. PLB474(2000)7:
  Contact interaction iterated with 1π-exchange (2nd order)
  Problems even more severe !

INCLUSION OF $\pi N \Delta$-DYNAMICS

- $\Delta(1232)$-resonance: prominent in low energy πN-scatt.
- 2π-exchange with Δ-excit.: isoscalar central NN-attract.
- mass-splitting $\Delta = 293$ MeV: small scale comparable to k_{f0}



- Two-body terms

  subtracted dispersion relation to isolate long-range comp.

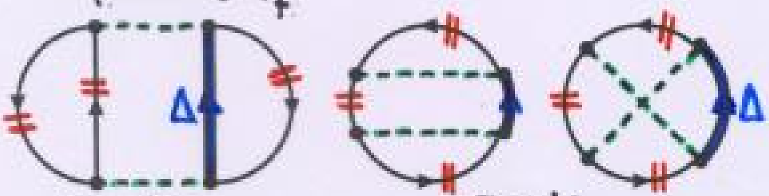
$$\bar{E}(k_f)^{(2F)} = \frac{1}{8\pi^3}\int_{2m_\pi}^{\infty} d\mu\, \mathrm{Im}(V_C + 3W_C + 2\mu^2 V_T + 6\mu^2 W_T)\Big\{3\mu k_f - \frac{4k_f^3}{3\mu} + \frac{8k_f^5}{5\mu^3} - \frac{\mu^3}{2k_f} - 4\mu^2 \arctan\frac{2k_f}{\mu} + \frac{\mu^3}{8k_f^3}(12k_f^2+\mu^2)\ln\Big(1+\frac{4k_f^2}{\mu^2}\Big)\Big\}$$

$$\bar{E}(k_f)^{(ct)} = B_3\frac{k_f^3}{M^2} + B_5\frac{k_f^5}{M^4}$$

B_3 encodes all unresolved short-distance dynamics, $B_5 \approx 0$

- Three-body terms

repulsive k_f^6



Hartree Fock

attractive 3N-contact interaction

$\zeta = 1/4$

$$\bar{E}(k_f)^{(3H)} = \frac{g_A^4 m_\pi^6}{\Delta(2\pi f_\pi)^4}\Big[\frac{2}{3}u^6\zeta + u^2 - 3u^4 + 5u^3\arctan 2u - \frac{1}{4}(1+9u^2)\ln(1+4u^2)\Big]$$

$$\bar{E}(k_f)^{(3F)} = -\frac{3g_A^4 m_\pi^6 u^{-3}}{4\Delta(4\pi f_\pi)^4}\int_0^u dx\Big[2G_S^2(x,u) + G_T^2(x,u)\Big]$$

$u = k_f/m_\pi$

- plus irreducible 2π-exchange with no deltas (relatively small)

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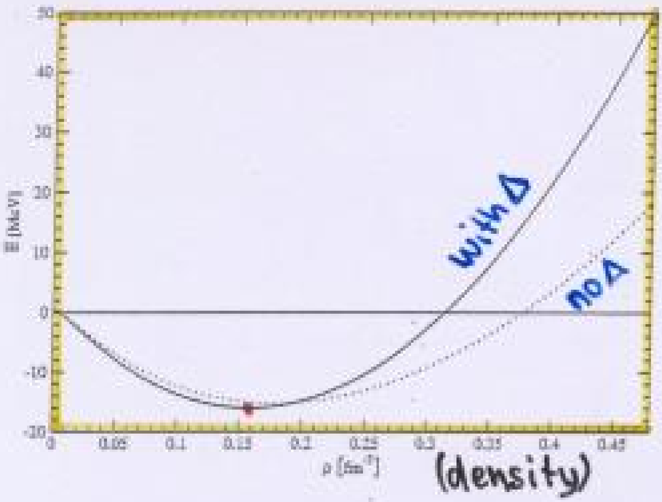
Nuclear matter saturation curve



adjusting $B_3 = -8.0$

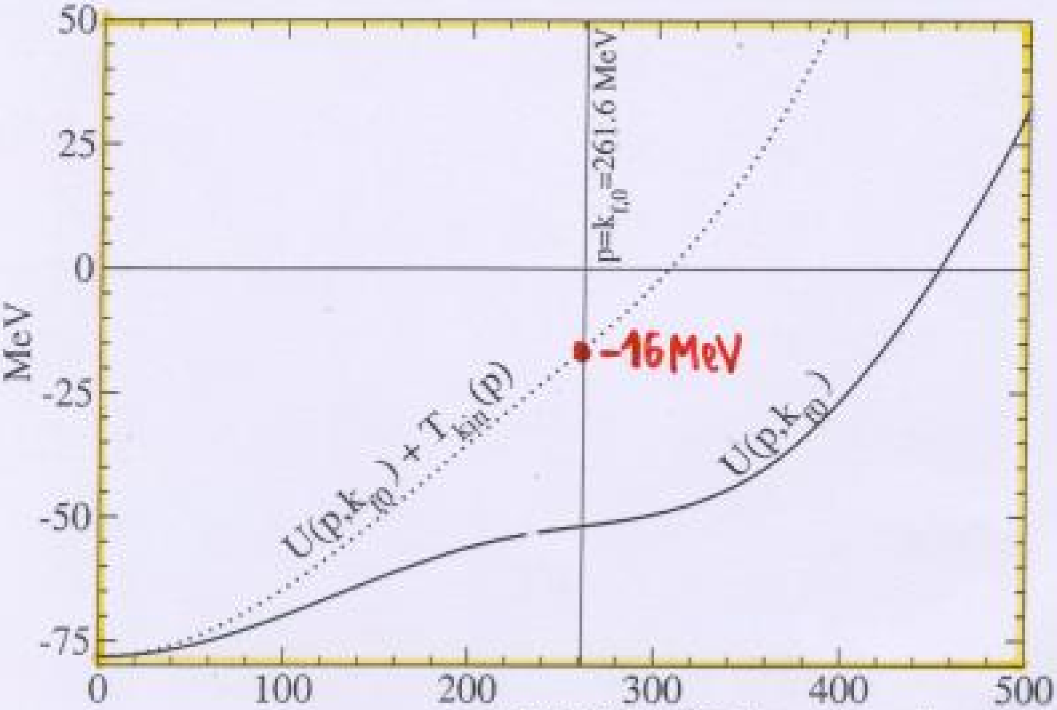
$\bar{E}_0 = -16.0\,\mathrm{MeV}$

$\rho_0 = 0.157\,\mathrm{fm}^{-3}$

$K = 300\,\mathrm{MeV}$

compressibility somewhat high

Real single-particle potential $U(p, k_f)$



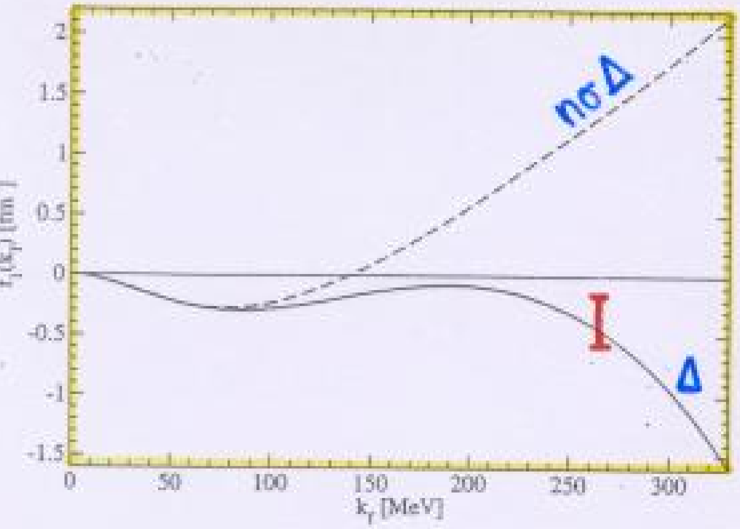
- p-dependence substantially <u>improved</u> by $\pi N \Delta$-dyn.
- <u>realistic</u> effective mass at Fermi surface $p = k_{f0}$

  $M^*(k_{f0}) = 0.88M$
- Hugenholtz-van-Hove theorem <u>exactly</u> fulfilled

$$T_{kin}(k_f) + U(k_f, k_f) = \bar{E}(k_f) + \frac{k_f}{3}\frac{\partial \bar{E}(k_f)}{\partial k_f}$$

Landau parameter: $f_1(k_f) = -\frac{3\pi^2}{2k_f^2} \frac{\partial U(p,k_f)}{\partial p}\Big|_{p=k_f}$

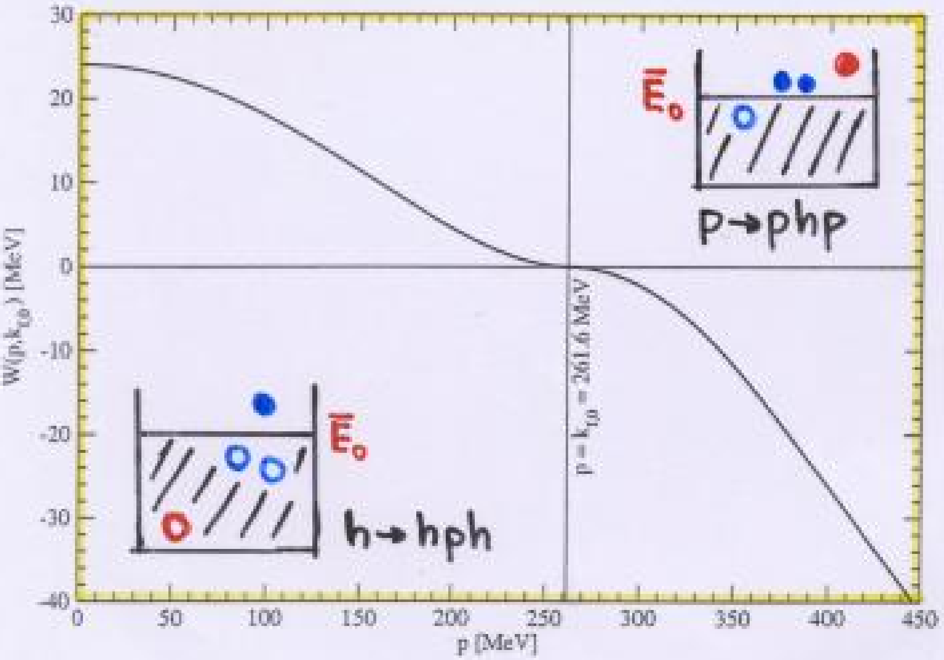
- determines effective N-mass (density of states)



- adjusted short-range parameters drop out
- pure long-range pion dynamics
- semi empirical value $-0.75\,\mathrm{fm}^2 < f_1(k_{f0}) < 0$

Imaginary single-particle potential: $W(p,k_f)$

- width of hole or particle states



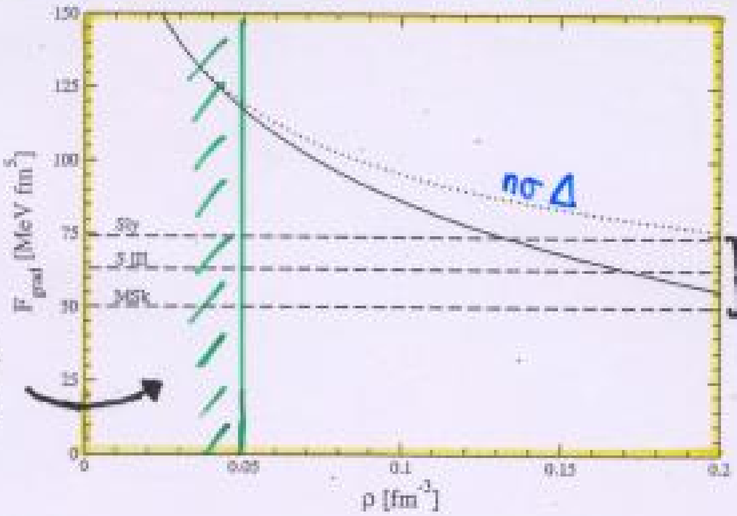
$\pm(p-k_f)^2$ near Fermi surface

- completely parameter free: only iterated 1π-exchange
- $W(0,k_{f0}) = 24$ MeV realistic value
- Luttingers theorem exactly fulfilled

Nuclear energy density functional

- nuclear structure calculations

$$\boxed{\mathcal{E}[\rho,\tau] = \rho\bar{E}(k_f) + \left(\tau - \frac{3}{5}\rho k_f^2\right)\frac{1}{2\tilde{M}^*(\rho)} + (\vec{\nabla}\rho)^2 F_\nabla(\rho) + \dots}$$



DME not applicable

- "Skyrme"-mass $\tilde{M}^*(\rho_0) = 0.64M$

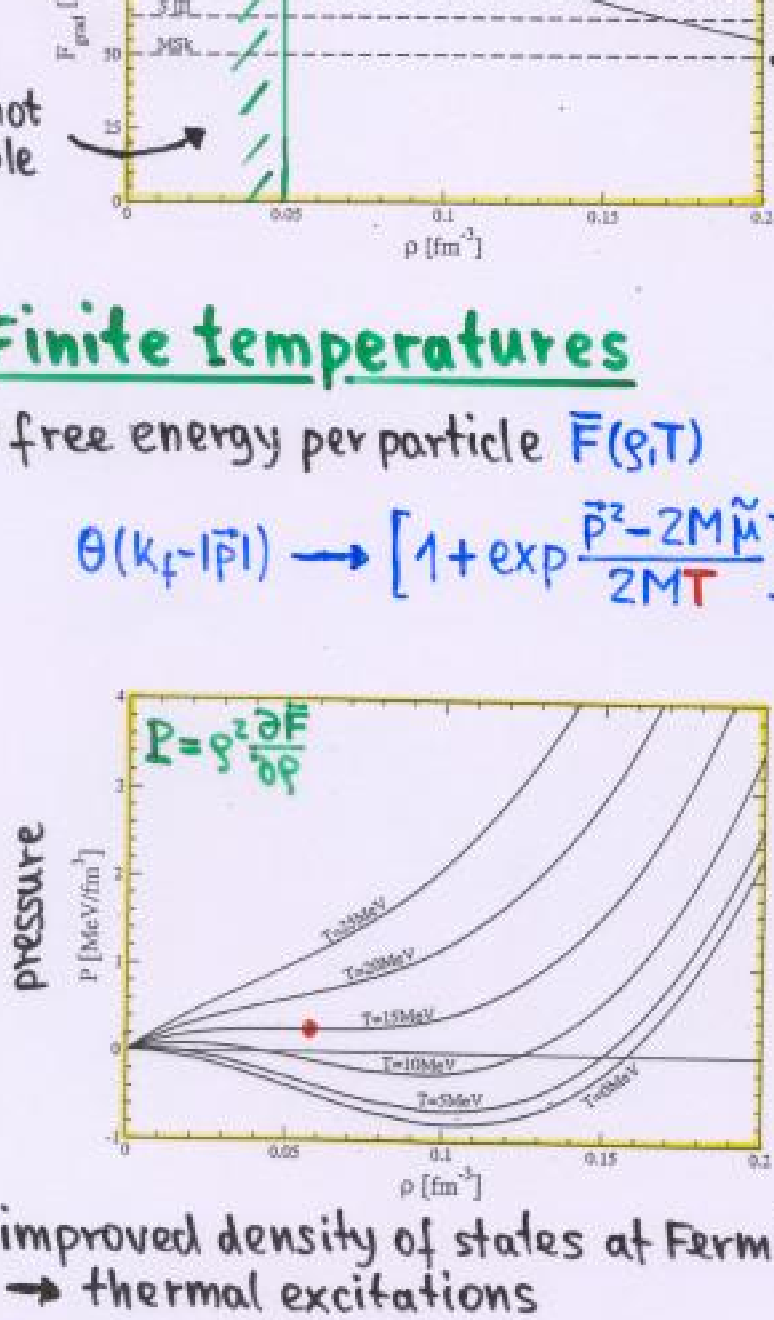
} phenomenological Skyrme forces

Finite temperatures

- free energy per particle $\bar{F}(\rho,T)$

$$\theta(k_f - |\vec{p}|) \longrightarrow \left[1 + \exp\frac{\vec{p}^2 - 2M\tilde{\mu}}{2MT}\right]^{-1}, \quad \tilde{\mu} = \tilde{\mu}(\rho,T)$$

related to density



- liquid-gas phase transition
- critical point
  $T_c = 15\,\text{MeV}$
  $\rho_c = \rho_0/3 = 0.053\,\text{fm}^{-3}$

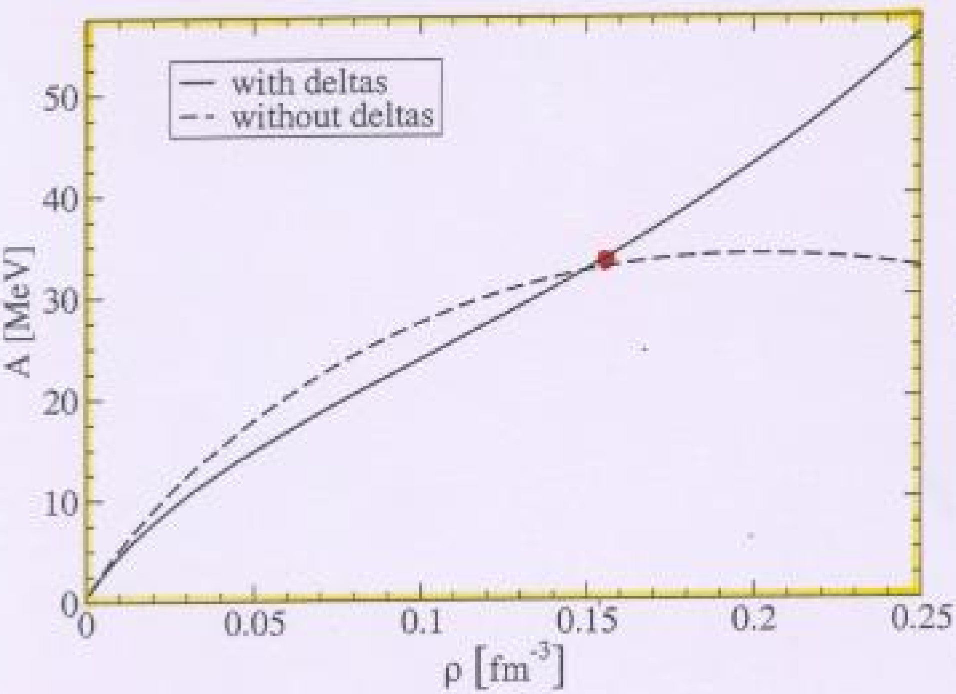
- improved density of states at Fermi surface
  → thermal excitations
- "empirical" $T_c = (16.6 \pm 0.9)\,\text{MeV}$ from heavy ion collisions

1a

ASYMMETRY ENERGY

- isospin-asymmetric nuclear matter

$$\boxed{\bar{E}(k_p, k_n) = \bar{E}(k_f) + \delta^2 A(k_f) + \ldots, \; k_{p,n} = k_f(1 \pm \delta)^{1/3}}$$

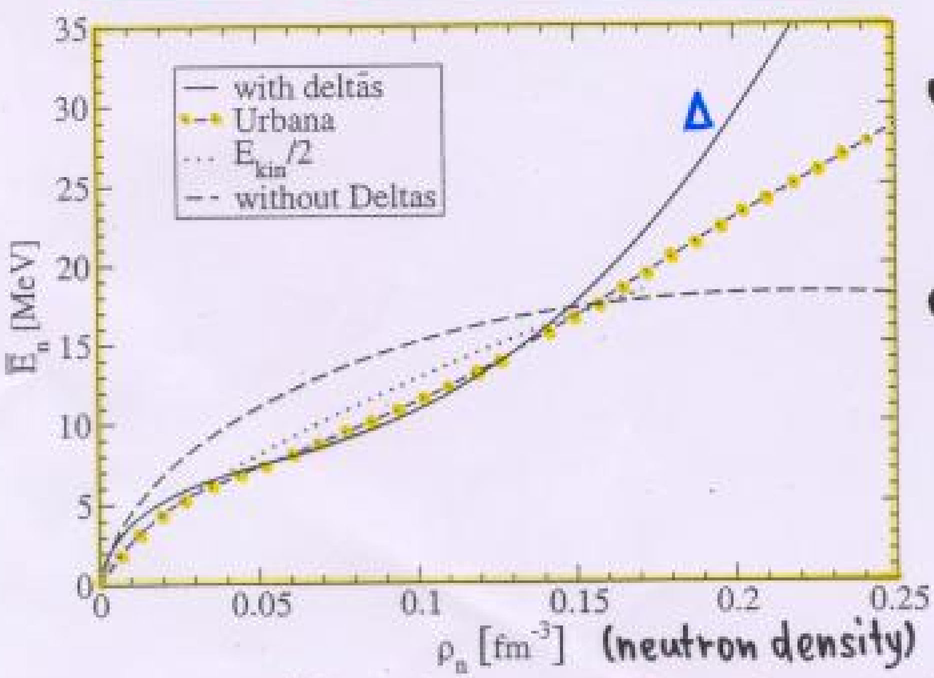


- at saturation density

$A_0 = 34\,\text{MeV}$

- notorious downward bending eliminated

PURE NEUTRON MATTER



(neutron density)

- good agreement with Urbana at low $\rho_n < 0.15\,\text{fm}^{-3}$
- downward bending eliminated

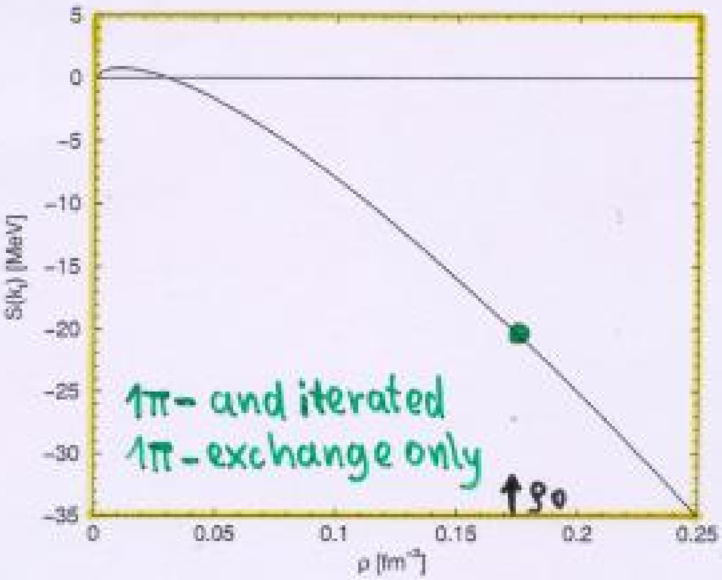
- Improved isospin properties from inclusion of chiral $\pi N \Delta$-dynamics!

Spin stability

- Energy per particle of spin-polarized nuclear matter

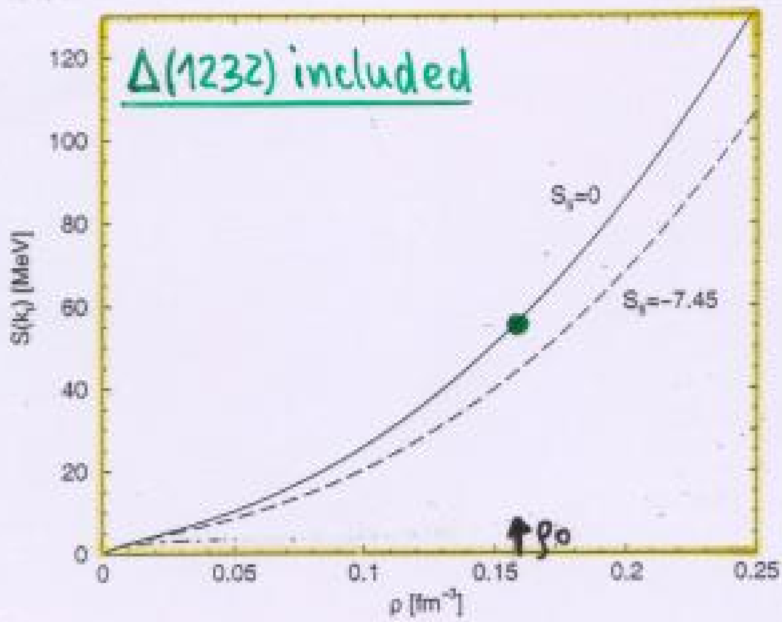
$$\boxed{\bar{E}_{pol}(k_{f\uparrow},k_{f\downarrow}) = \bar{E}(k_f) + \eta^2 S(k_f),\ k_{f\uparrow,\downarrow} = k_f(1\pm\eta)^{1/3}}$$

- Spin-asymmetry energy $S(k_f)>0$ must be positive



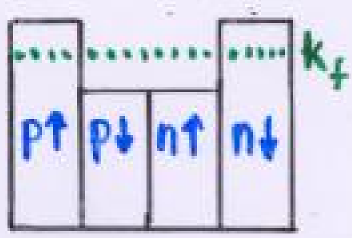
to this approx.: spin-unstable

independent of regularization $\bar{E}(k_{f0})$, $A(k_{f0})$



Inclusion of 2π-exchange with Δ-excitat. guarantees spin-stability

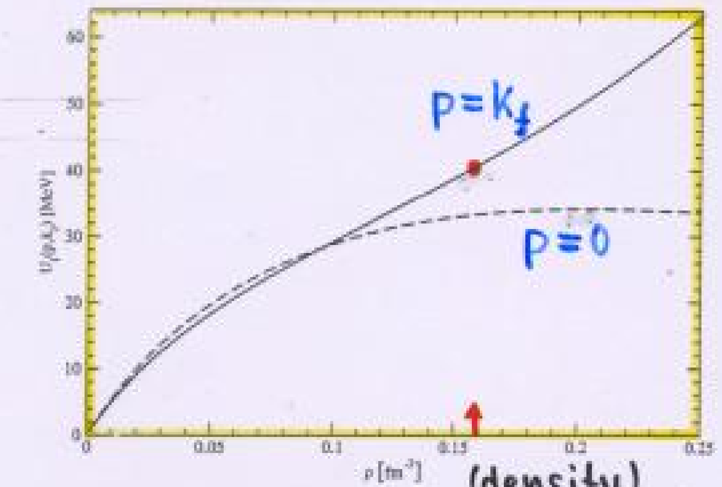
- Spin-isospin stability: generic feature of π-exchange dynamics



Isovector single-particle potential

- different mean-fields for protons and neutrons in isospin-asymmetric nuclear matter

$$\boxed{U(p,k_f) - U_I(p,k_f)\,\tau_3\,\frac{\rho_n-\rho_p}{\rho_n+\rho_p}+\dots}$$



prediction:

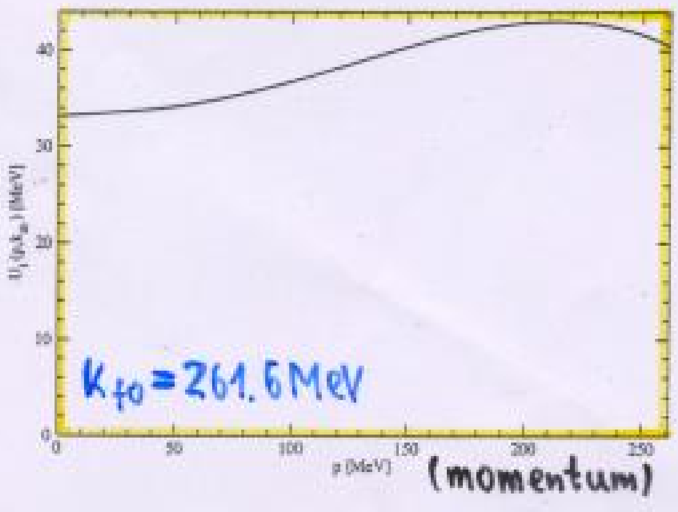
$U_I(k_{f0},k_{f0}) = 40.4\,\mathrm{MeV}$

in good agreement with optical potential

- Generalized Hugenholtz-van-Hove theorem

$$\boxed{U_I(k_f,k_f) = 2A(k_f) - \frac{k_f^2}{3M} + \frac{k_f^4}{6M^3} - \frac{k_f}{3}\frac{\partial U(p,k_f)}{\partial p}\bigg|_{p=k_f}}$$

exactly fulfilled!



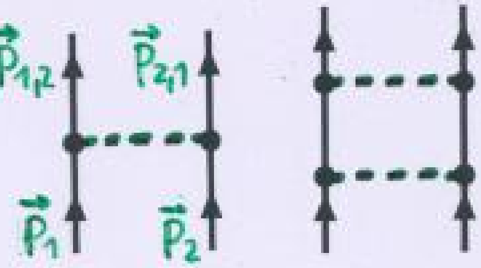
moderate p-depend. of $U_I(p,k_{f0})$

Landau parameters

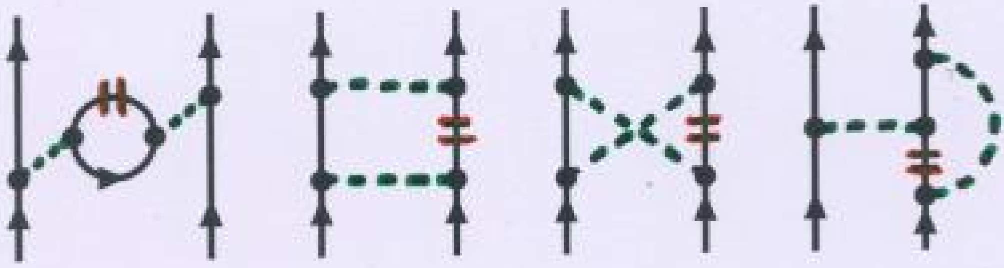
- Interaction of quasi-nucleons at Fermi surface

$$\boxed{\begin{aligned}&(4\pi)^{-2}\int d\Omega_1 d\Omega_2 \langle \vec{p}_1,\vec{p}_2 | V_{eff} | \vec{p}_1,\vec{p}_2 \rangle \\ &= f_0(k_f) + g_0(k_f)\,\vec{\sigma}_1\cdot\vec{\sigma}_2 + f_0'(k_f)\,\vec{\tau}_1\cdot\vec{\tau}_2 + g_0'(k_f)\,\vec{\sigma}_1\cdot\vec{\sigma}_2\,\vec{\tau}_1\cdot\vec{\tau}_2\end{aligned}}$$

$|\vec{p}_1| = |\vec{p}_2| = k_f$

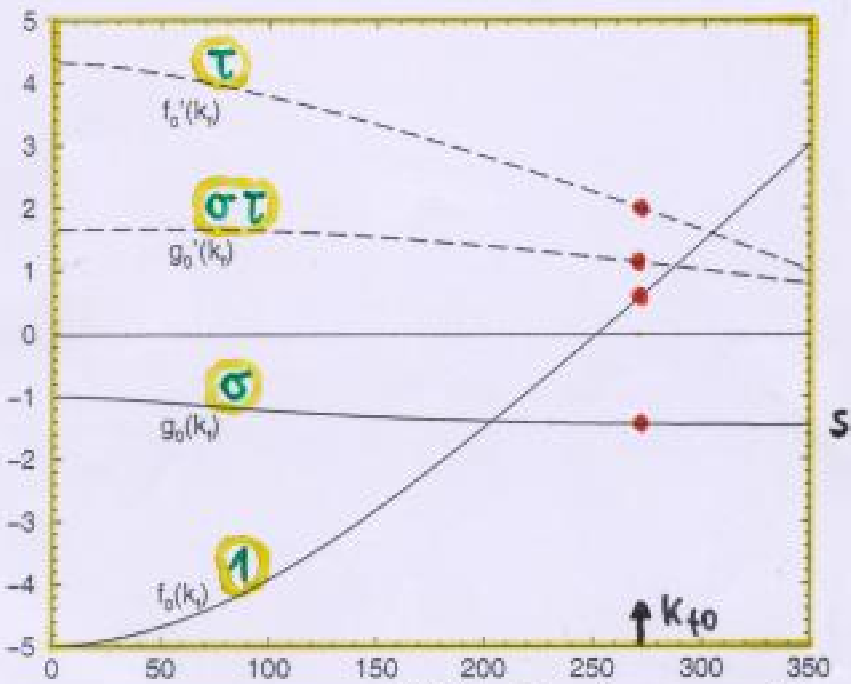


+ crossed terms



"medium modificat."

- 1π- and iterated 1π-exch.

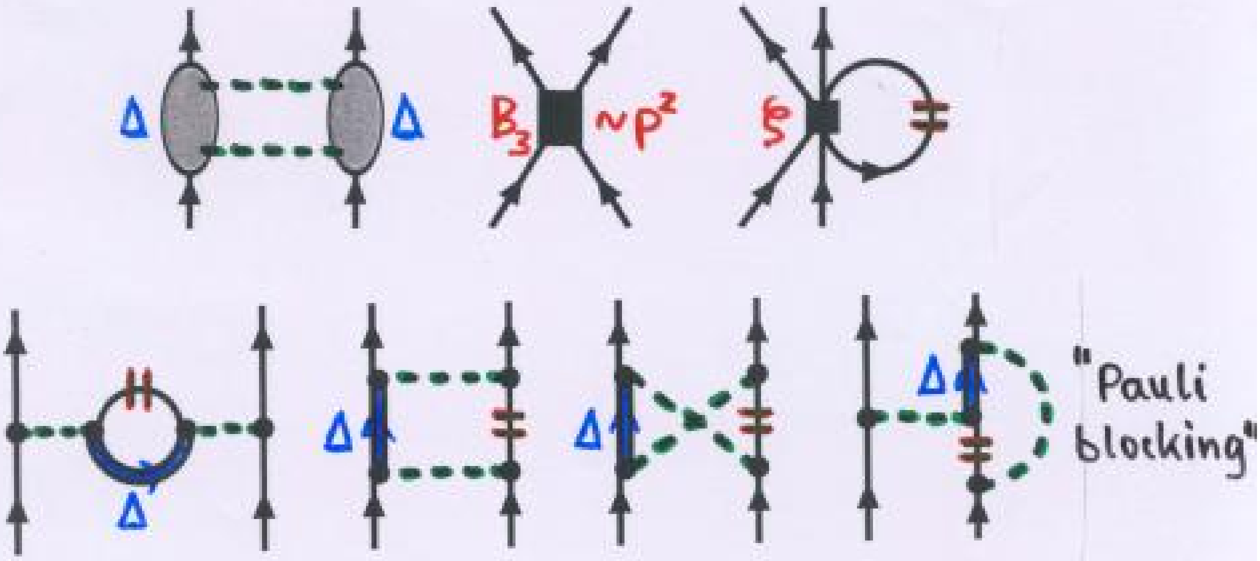


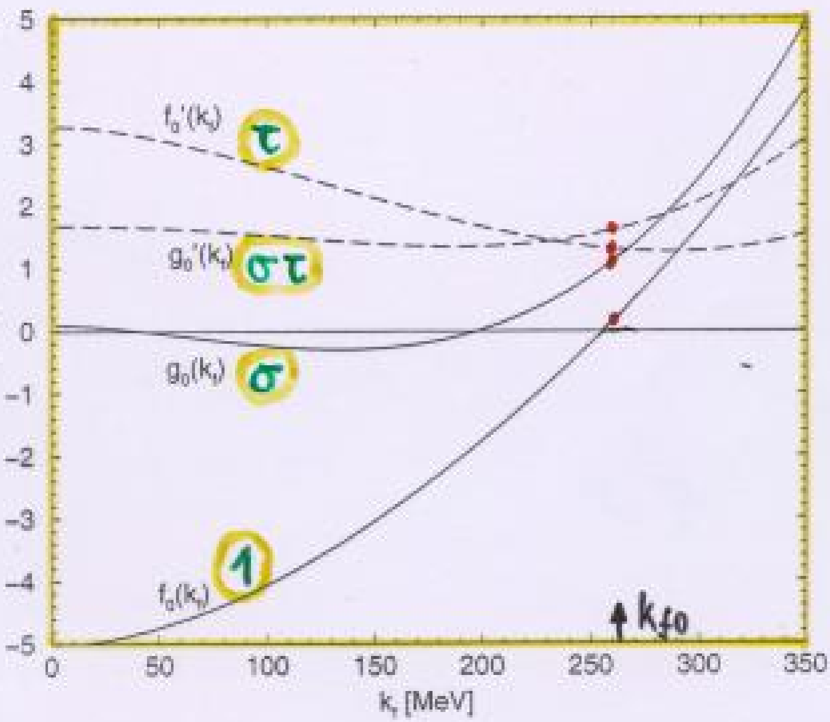
spin-instability

Fermi momentum

- $g_0'(2m_\pi) = (\underset{1\pi}{0.13} + \underset{2\pi}{0.99} + \underset{\text{Pauli}}{0.02})\,\text{fm}^2 = 1.14\,\text{fm}^2$ (emp.: $1.6\,\text{fm}^2$)
- Landau relations to compressibility K and asymmetry energy $A(k_{f0})$ exactly fulfilled

Landau parameters:
Chiral $\pi N\Delta$-dynamics included





$N_0 = \frac{2}{\pi^2} k_{f0} M^* \simeq 1\,\mathrm{fm}^{-2}$ (density of states)

Tensor interaction:

$h_0 = 5.0\,\mathrm{fm}^2$ twice 1π-exch.

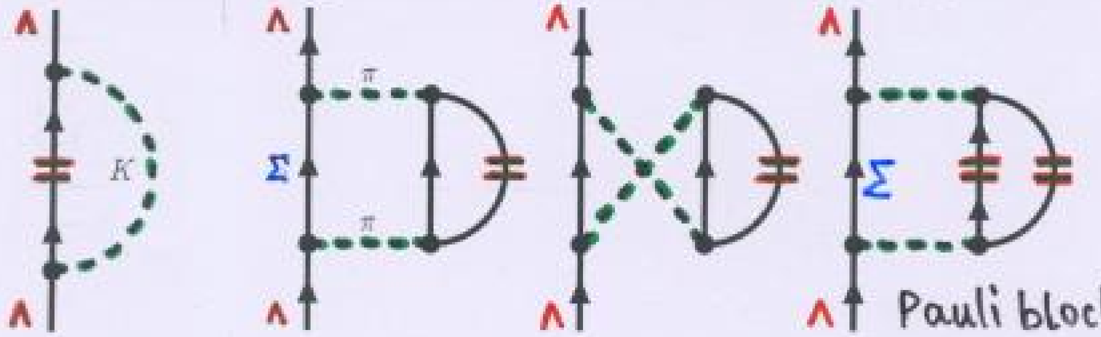
$h_0' \simeq -0.8\,\mathrm{fm}^2$

$$\left.\begin{aligned} f_0(k_{f0}) &= 0.20\,\mathrm{fm}^2 \\ g_0(k_{f0}) &= 1.15\,\mathrm{fm}^2 \\ f_0'(k_{f0}) &= 1.30\,\mathrm{fm}^2 \\ g_0'(k_{f0}) &= 1.62\,\mathrm{fm}^2 \end{aligned}\right\} \text{close to empirical values}$$

- additional short-range p^2-terms estimated from realistic NN-potentials (σ, $\sigma\tau$-channel)

Λ-hyperons in nuclear medium

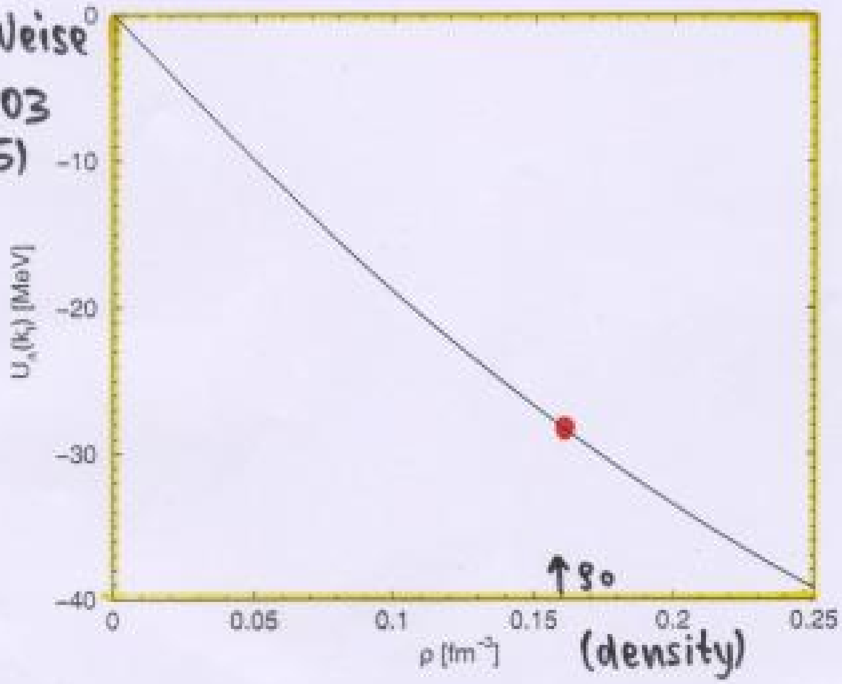
- Spectroscopy of Λ-hypernuclei:
  Λ-nucleus potential ≈ half as deep as N-nucleus pot.
  Λ-spinorbit interaction extremely small
- Leading long-range ΛN-interaction from K-exchange and 2π-exchange with intermediate Σ-hyperons



Pauli blocking

Λ-nuclear mean-field ($\vec{p}_\Lambda = 0$)

N. Kaiser, W. Weise
PRC 71, 015203 (2005)



(density)

cutoff scale $\bar{\Lambda} = 0.7$ GeV represents all short-distance dynamics: $U_\Lambda(k_f)^{sh} \sim \rho$

$$U_\Lambda(k_{f0}) = (\underset{1K}{4.2} - \underset{2\pi\Sigma}{39.8} + \underset{\text{Pauli}}{7.5})\,\text{MeV} = -28.1\,\text{MeV}$$

- small mass splitting $M_\Sigma - M_\Lambda = 77$ MeV figures prominently

Λ-nuclear spin-orbit coupling

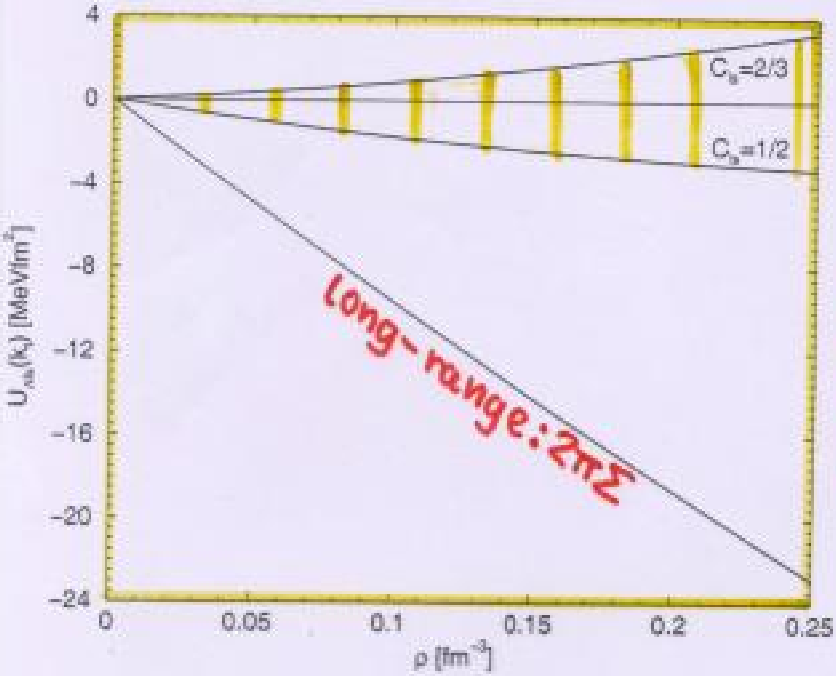
- Spin-dependent part of Λ-selfenergy in weakly inhomogeneous nuclear matter

$$\boxed{\Sigma_{spin} = \frac{i}{2}\vec{\sigma}\cdot(\vec{q}\times\vec{p})\, U_{\Lambda ls}(k_f)}$$

- spin-orbit Hamiltonian $H_{ls} = U_{\Lambda ls}(k_{f0})\frac{1}{2r}\frac{df(r)}{dr}\vec{\sigma}\cdot\vec{L}$
- Iterated 1π-exchange with intermediate Σ-hyperons generates "wrong-sign" spin-orbit interaction
  $\vec{\sigma}\cdot(\vec{l}+\vec{q}/2)\,\vec{\sigma}\cdot(\vec{l}-\vec{q}/2) = \underline{i\vec{\sigma}\times\vec{q}}\cdot\vec{l}+\ldots$; $\underline{\vec{p}}$ from energy denom.
- decomposition: long + short range pieces

$$\boxed{U_{\Lambda ls}(k_f) = U_{\Lambda ls}(k_f)^{(2\pi\Sigma)} + C_{ls}\frac{M_N^2}{M_\Lambda^2}\,U_{Nls}(k_f)}$$

shell model
$35\,\mathrm{MeV fm}^2\,\rho/\rho_0$



Cancellation between (model-independent) long-range terms and short-range contributions

$$U_{\Lambda ls}(k_{f0}) = (\underset{\text{short}}{24.8\,C_{ls}} \underset{2\pi\Sigma}{-16.7} \underset{\text{Pauli}}{+1.6})\,\mathrm{MeV fm}^2$$

- Long-range spin-orbit coupling from iterated 1π-exch. not a relativistic effect: linear in M_B!

SUMMARY + OUTLOOK

- Chiral Expansion of nuclear matter EQoS

$$\bar{E}(k_f) = \sum_{n=2}^{5} k_f^n \, f_n(k_f/m_\pi, \Delta/m_\pi)$$

- Saturation from Pauli-blocking on iterated 1π-exchange

  Problem: single-particle + isospin properties

- Substantial improvement by including $\pi N\Delta$-dynam.

  $M^*(k_{f0}) = 0.88M$, $T_c = 15$ MeV, $A(k_f)$, $E_n(k_n)$, ...

- $\pi N\Delta$-dynamics guarantees spin-stability
- Landau parameters: $f_0, g_0, f_0', g_0', h_0, h_0'$

  quasi-particle interaction at Fermi surface

- Λ-hyperons in nuclear medium:

  potential depth U_Λ and spin-orbit coupl. $U_{\Lambda ls}$

- Open questions:
  - "Convergence"/higher orders

    $\bar{E}_0 = -16$ MeV small number, needs finetuning

  - Relation of contact couplings $B_3, B_{n3}, \ldots$ to realistic NN-potentials (or QCD)

Lesson: Good saturation alone is insufficient, all (semi) empirical nuclear matter properties