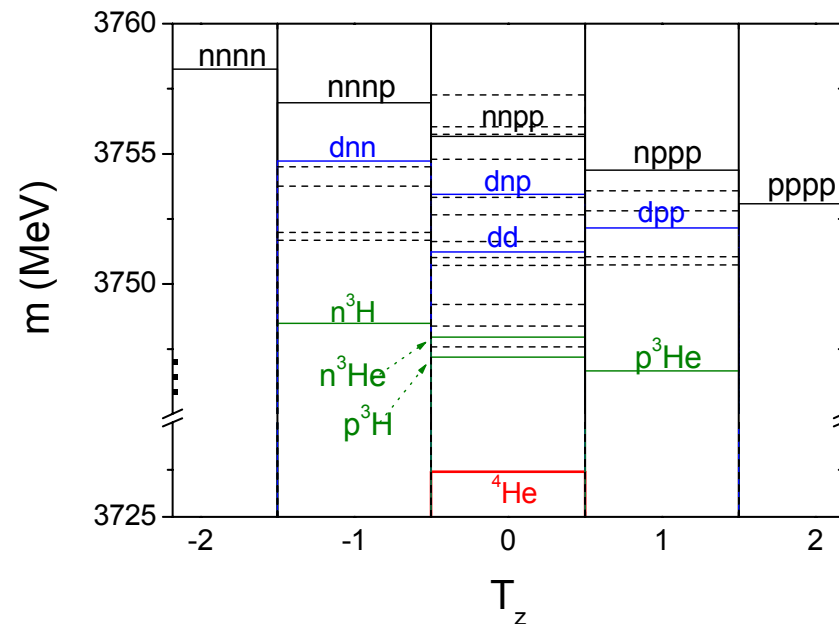


Hadronic resonances in four nucleon systems

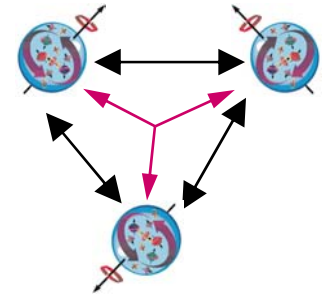
In collaboration with J. Carbonell, *LPSC Grenoble, France*



Hadronic resonances in four nucleon systems

Why four-nucleon systems?

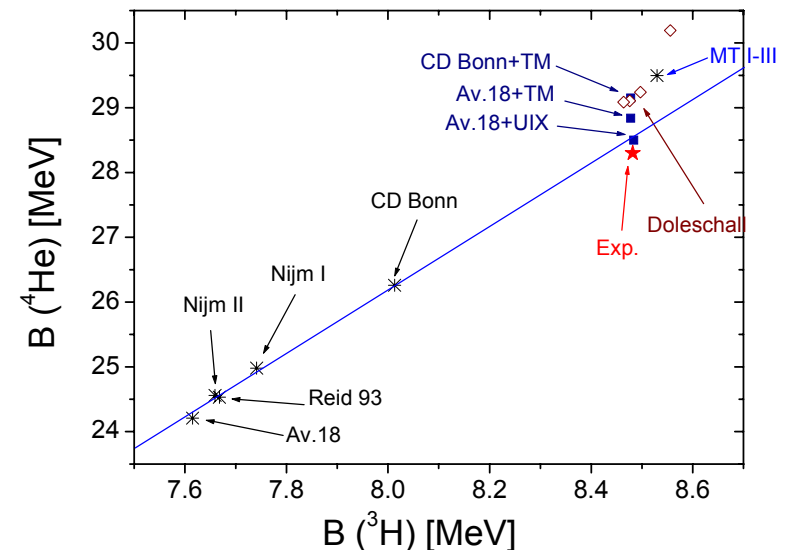
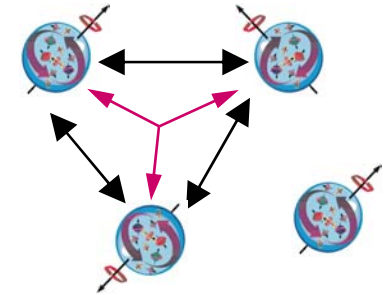
- $2N$ – is made to work (by establishing pot. param.)...
- $3N$ – problem starts, requires one more fit ($3NF$ force)...
however $3NF$ does not manifest much in N -d scattering at low en.



Hadronic resonances in four nucleon systems

Why four-nucleon systems?

- $2N$ – is made to work (by establishing pot. param.)...
- $3N$ – problem starts, requires one more fit ($3NF$ force)...
however $3NF$ does not manifest much in N - d scattering at low en.
- Binding energies of heavier nuclei?
loss of individuality
few observables to control
binding energies are correlated!

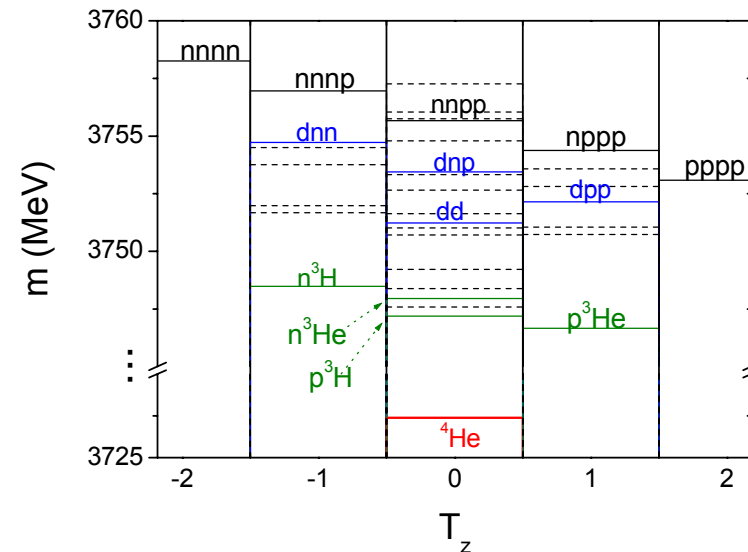
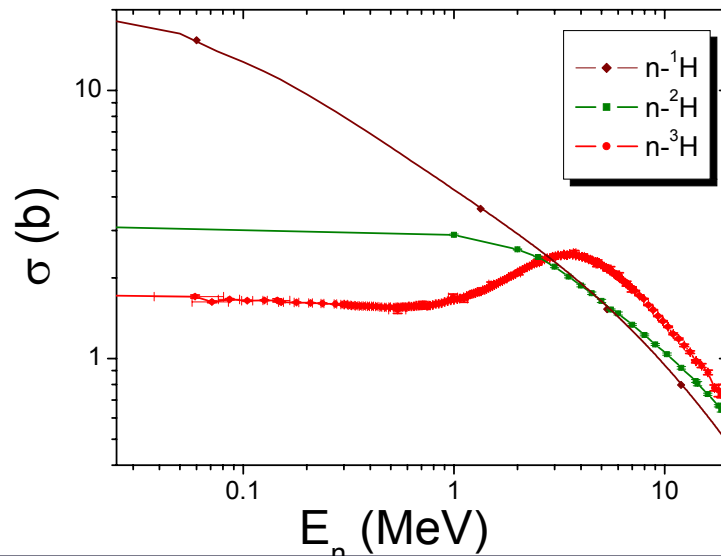
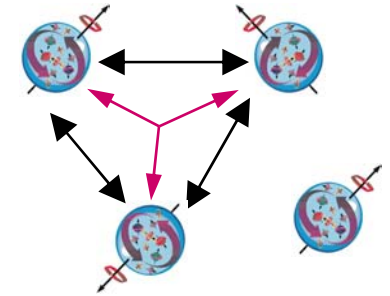


A. Nogga et al: Phys.Rev. C65 (2002) 054003

Hadronic resonances in four nucleon systems

Why four-nucleon systems?

- $2N$ – is made to work (by establishing pot. param.)...
- $3N$ – problem starts, requires one more fit (3NF force)...
however 3NF does not manifest much for N-d scattering at low en.
- Rich structure of $4N$ continuum. **Negative parity resonances!!**

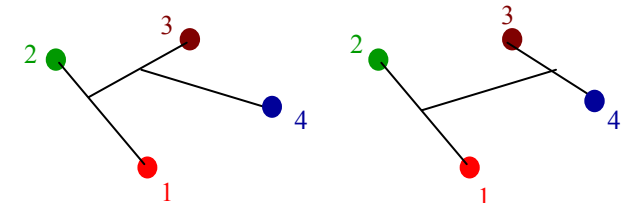


Hadronic resonances in four nucleon systems

- Faddeev-Yakubovski equations in configuration space

$$\boxed{12} \rightarrow (E - \hat{H}_0 - V_{12})K_{12,3}^4 = V_{12} [K_{23,1}^4 + K_{23,4}^1 + K_{13,2}^4 + K_{13,4}^2 + H_{13,24} + H_{23,14}]$$

$$\boxed{6} \rightarrow (E - \hat{H}_0 - V_{12})H_{12,34} = V_{12} [K_{34,1}^2 + K_{34,2}^1 + H_{34,12}]$$



$$\Psi(\vec{x}, \vec{y}, \vec{z}) = \sum_{i < j, k, l=1}^4 K_{ij,k}^l(\vec{x}_{ij,k}^l, \vec{y}_{ij,k}^l, \vec{z}_{ij,k}^l) + \sum_{i < j, k < l=1}^4 H_{ij}^{kl}(\vec{x}_{ij}^{kl}, \vec{y}_{ij}^{kl}, \vec{z}_{ij}^{kl})$$

- Solution is searched by decomposing Faddeev-Yakubovski components in the partial wave basis

$$K_{\alpha}(\vec{x}_{\alpha}, \vec{y}_{\alpha}, \vec{z}_{\alpha}) = \sum_{LST} \frac{F_{\alpha LST}(x_{\alpha}, y_{\alpha}, z_{\alpha})}{x_{\alpha} y_{\alpha} z_{\alpha}} [L_{\alpha}(\hat{x}_{\alpha}, \hat{y}_{\alpha}, \hat{z}_{\alpha}) \otimes S_{\alpha} \otimes T_{\alpha}]$$

$$H_{\beta}(\vec{x}_{\beta}, \vec{y}_{\beta}, \vec{z}_{\beta}) = \sum_{LST} \frac{F_{\beta LST}(x_{\beta}, y_{\beta}, z_{\beta})}{x_{\beta} y_{\beta} z_{\beta}} [L_{\beta}(\hat{x}_{\beta}, \hat{y}_{\beta}, \hat{z}_{\beta}) \otimes S_{\beta} \otimes T_{\beta}]$$

Basis is infinite

- Radial parts of the amplitudes $F_{\alpha LST}(x_{\alpha}, y_{\alpha}, z_{\alpha})$ are developed in the spline basis, converting differential equations into linear algebra problem.

Hadronic resonances in four nucleon systems

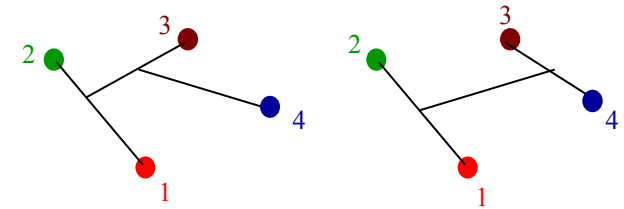
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$$\boxed{12} \rightarrow (E - \hat{H}_0 - V_{12})K_{12,3}^4 = V_{12} [K_{23,1}^4 + K_{23,4}^1 + K_{13,2}^4 + K_{13,4}^2 + H_{13,24} + H_{23,14}]$$

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$$V_{12}^{Short} + \sum_{i < j=1}^4 V_{ij}^{Long}$$

$$V_{12}^{Short}$$



- Coulomb? **Mercuriev-like decomposition**

$$V_{ij}(x) = V_{ij}^{Short}(x, y, z) + V_{ij}^{Long}(x, y, z)$$

Hadronic resonances in four nucleon systems

- Faddeev-Yakubovski equations in configuration space

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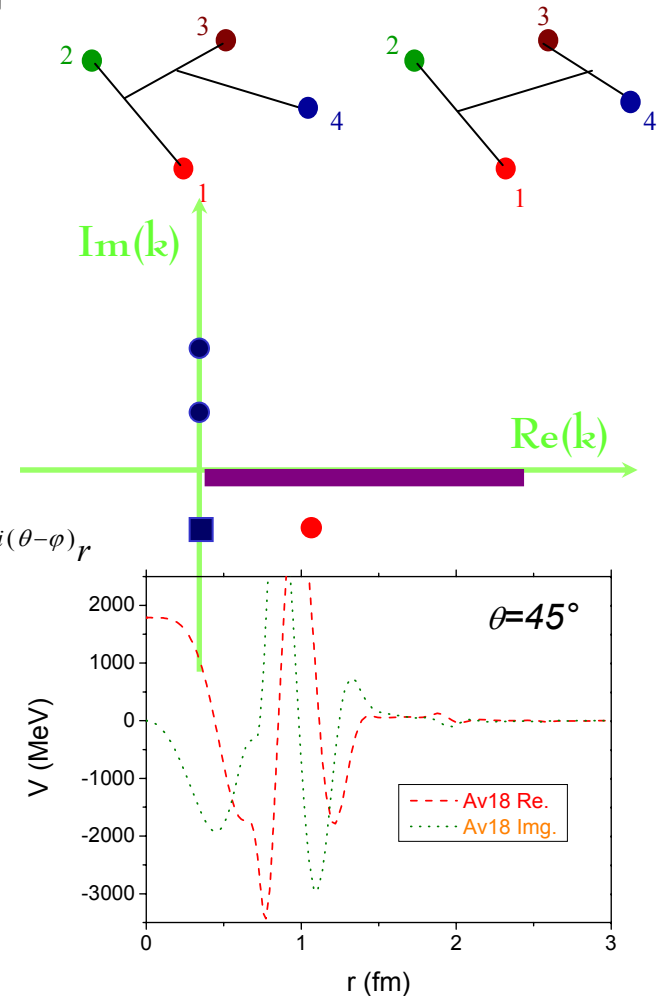
$$\boxed{6} \rightarrow (E - \hat{H}_0 - V_{12})H_{12,34} = V_{12} [K_{34,1}^2 + K_{34,2}^1 + H_{34,12}]$$

- Coulomb? Merkuriev-like decomposition

- Direct solution for resonant states!

- ✓ Complex scaling method

$$r \rightarrow re^{i\theta} \quad \Phi(r) \sim e^{ikr} = e^{i|k|e^{-i\varphi}r} \rightarrow e^{i|k|e^{i(\theta-\varphi)}r}$$



Hadronic resonances in four nucleon systems

- Faddeev-Yakubovski equations in configuration space

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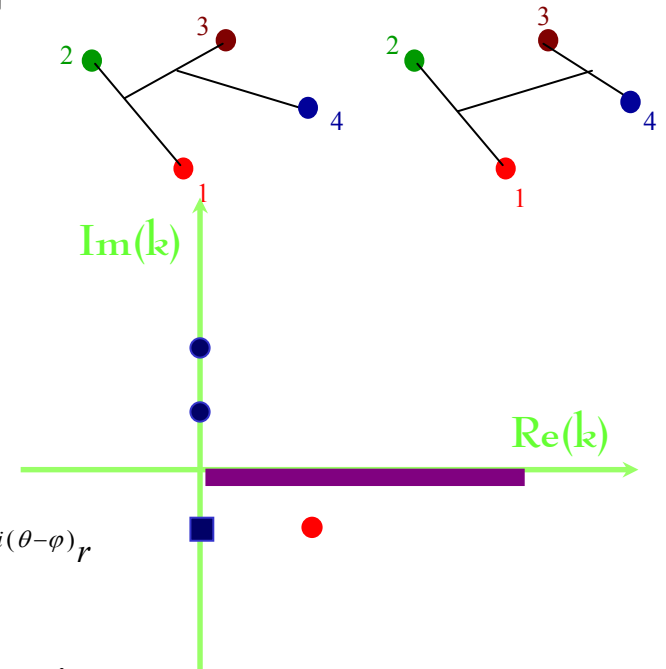
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- ✓ Analytic Continuation in the Coupling Constant

$$\hat{H}' = \hat{H} + \gamma V' \quad k(\gamma) = i f(\gamma - \gamma_0); \quad k(\gamma_0) = 0$$

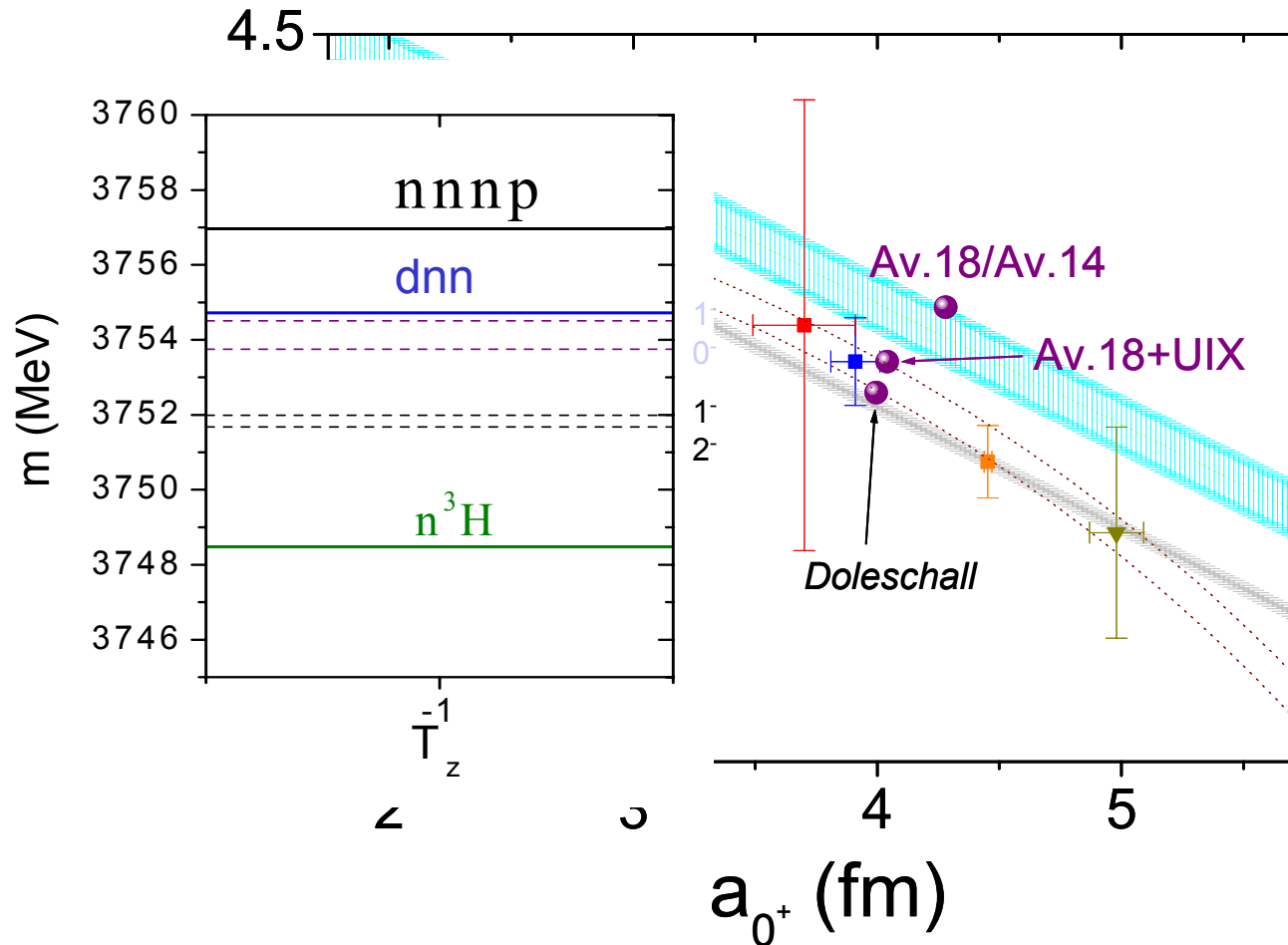


Hadronic resonances in four nucleon systems

The simplest case: n-³H scattering

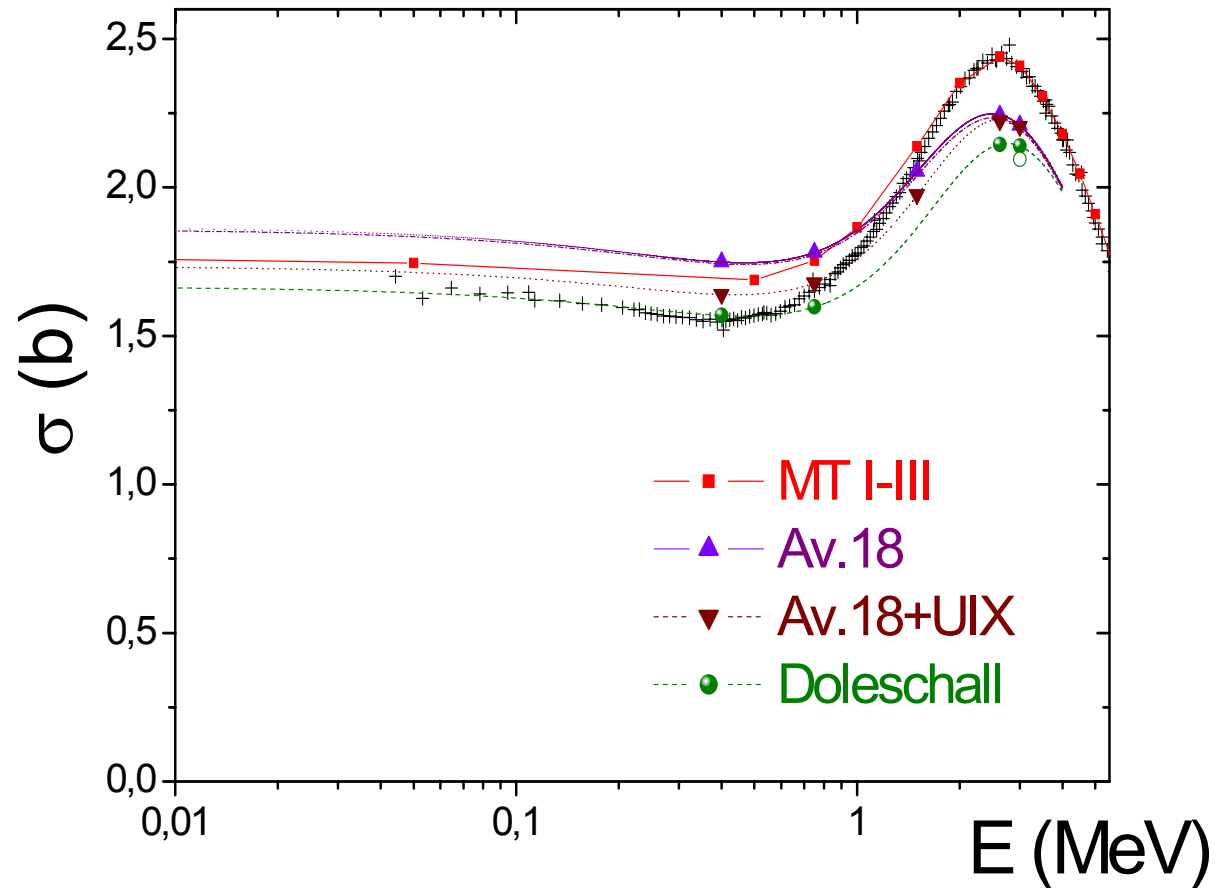
$$\sigma_{E=0} = (a_{0^+}^2 + 3a_{1^+}^2) \pi$$

$$a_c = \frac{a_{0^+} + 3a_{1^+}}{4}$$



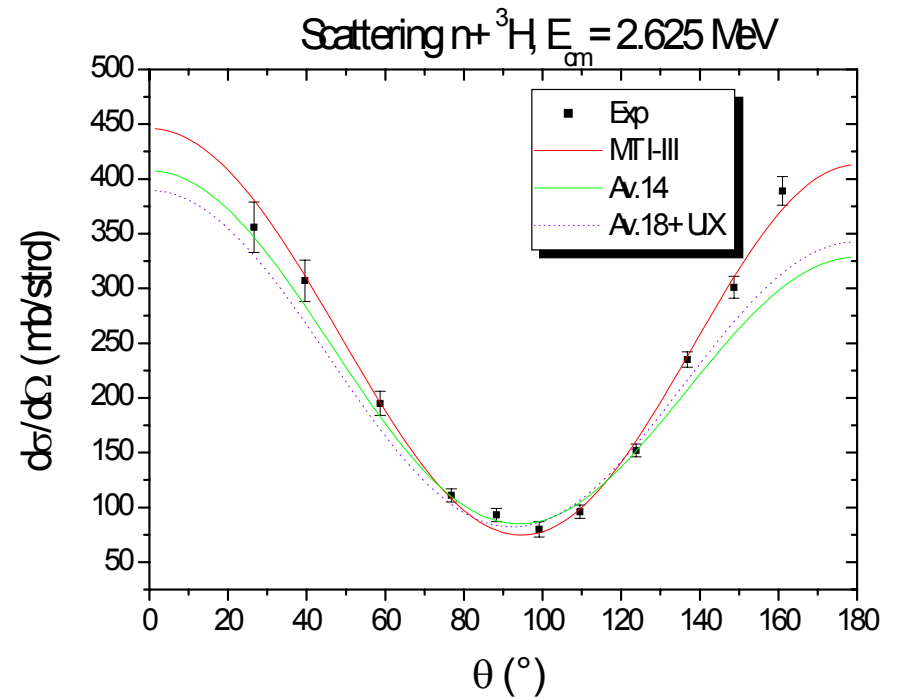
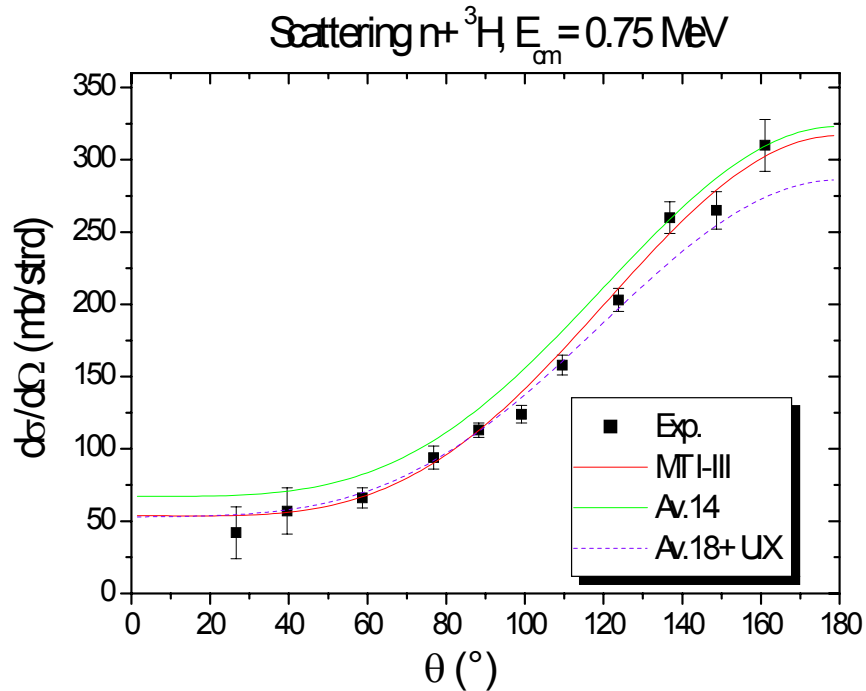
Hadronic resonances in four nucleon systems

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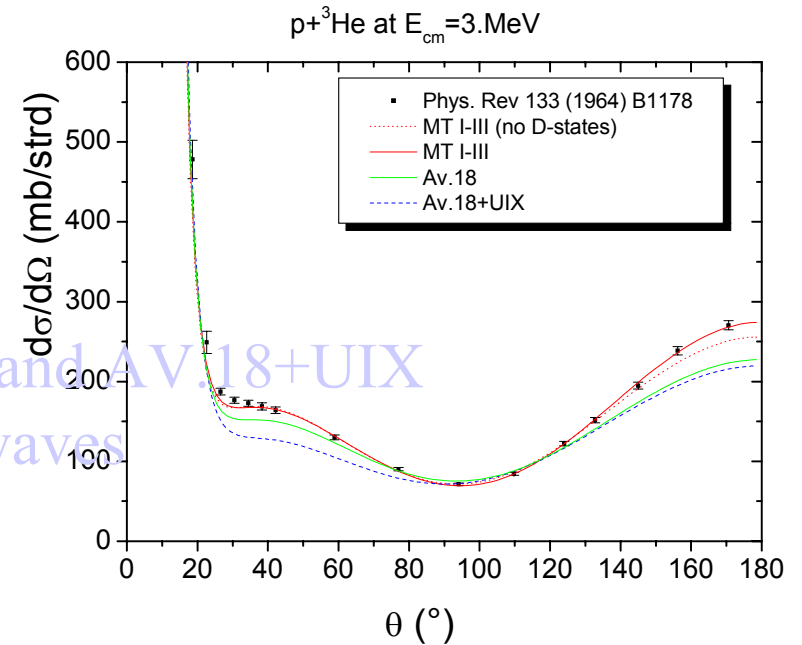
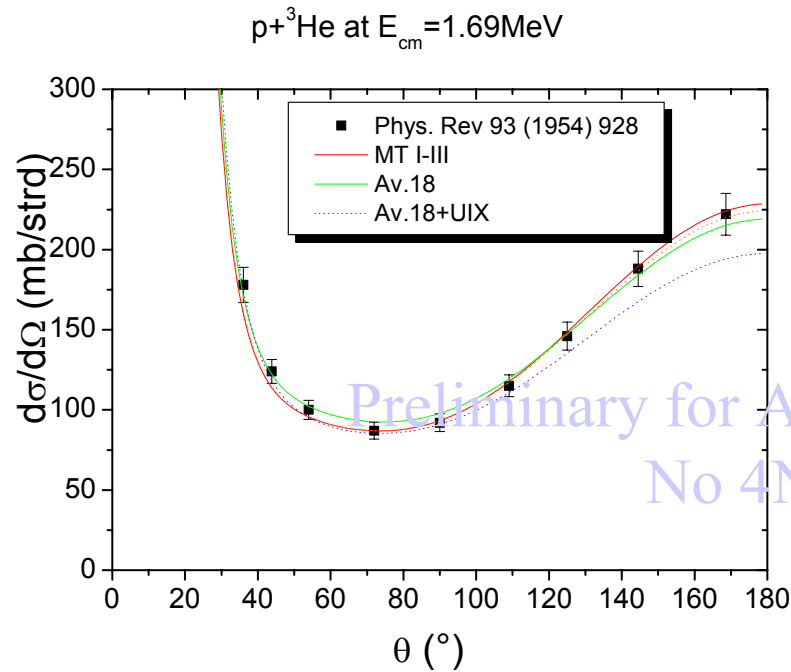


Hadronic resonances in four nucleon systems

The simplest case: n-³H scattering



Hadronic resonances in four nucleon systems

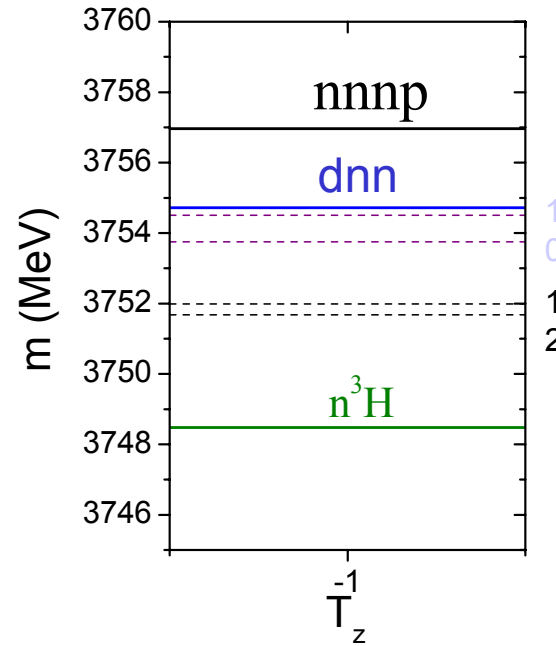
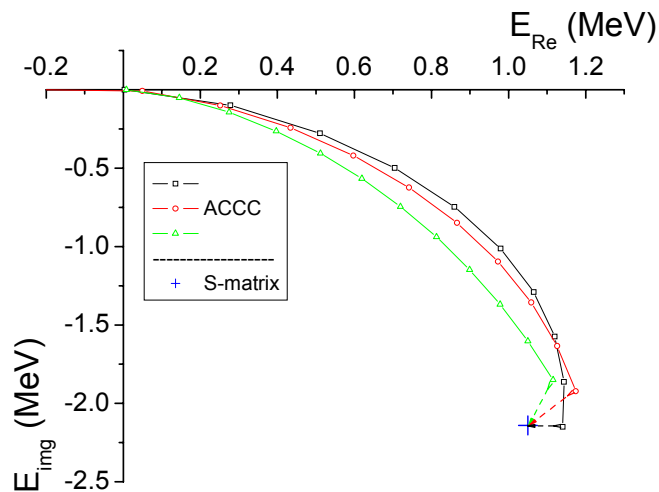
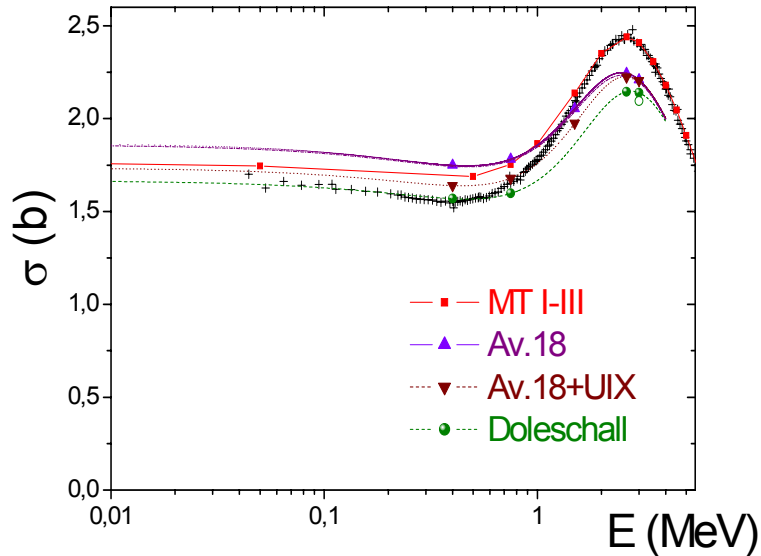


Preliminary for Av.18 and Av.18+UIX
No 4N D-waves

J^{π}	MT I-III	Av.18+UIX	Av.18+UIX (Viviani et al.*)	Exp.
0^+	11.5	10.9	11.5	10.8 ± 2.6
1^+	9.20	8.70	9.13	8.1 ± 0.5 ; 10.2 ± 1.5

*Phys.Rev.Lett. 81 (1998) 1580-1583

Hadronic resonances in four nucleon systems



Resonance position?

R-matrix analysis*

J^π	E_{res} (MeV)
1^-	6.02-6.50i
0^-	5.27-4.46i
1^-	3.50-3.37i
2^-	3.19-2.71i

*D.R. Tilley et al: Nucl. Phys. A 541 (1992) 1.

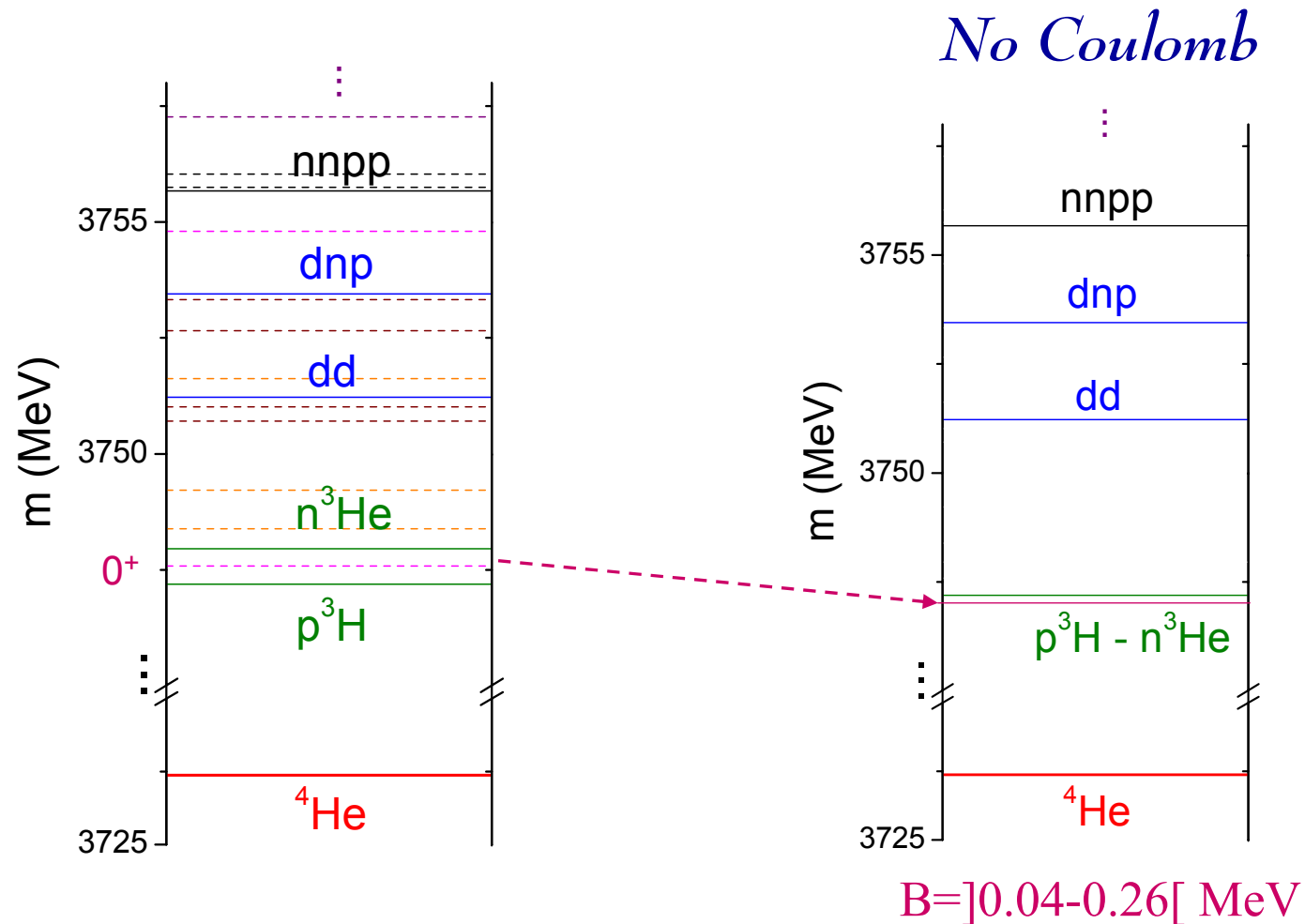
S-matrix pole (MT I-III)

$S_z(L=1)$	E_{res} (MeV)
0	0.35 -2.3i
1	1.03 -2.14i

S-matrix pole (Av.18)

J^π	E_{res} (MeV) \approx
1^-	-0.08 -2.3i
0^-	0.4 -2.8i
1^-	0.80 -2.12i
2^-	1.1 -2.2i

Hadronic resonances in four nucleon systems



Hadronic resonances in four nucleon systems

Exp.: $a_{0^+} \approx [-16, -20] \text{ fm}$

Theory without Coulomb:

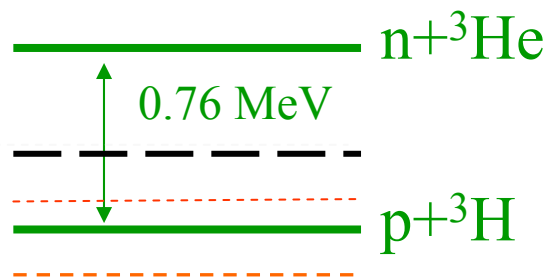
MT I-III: $a_{0^+} = 9.4 \text{ fm}$

With Coulomb:

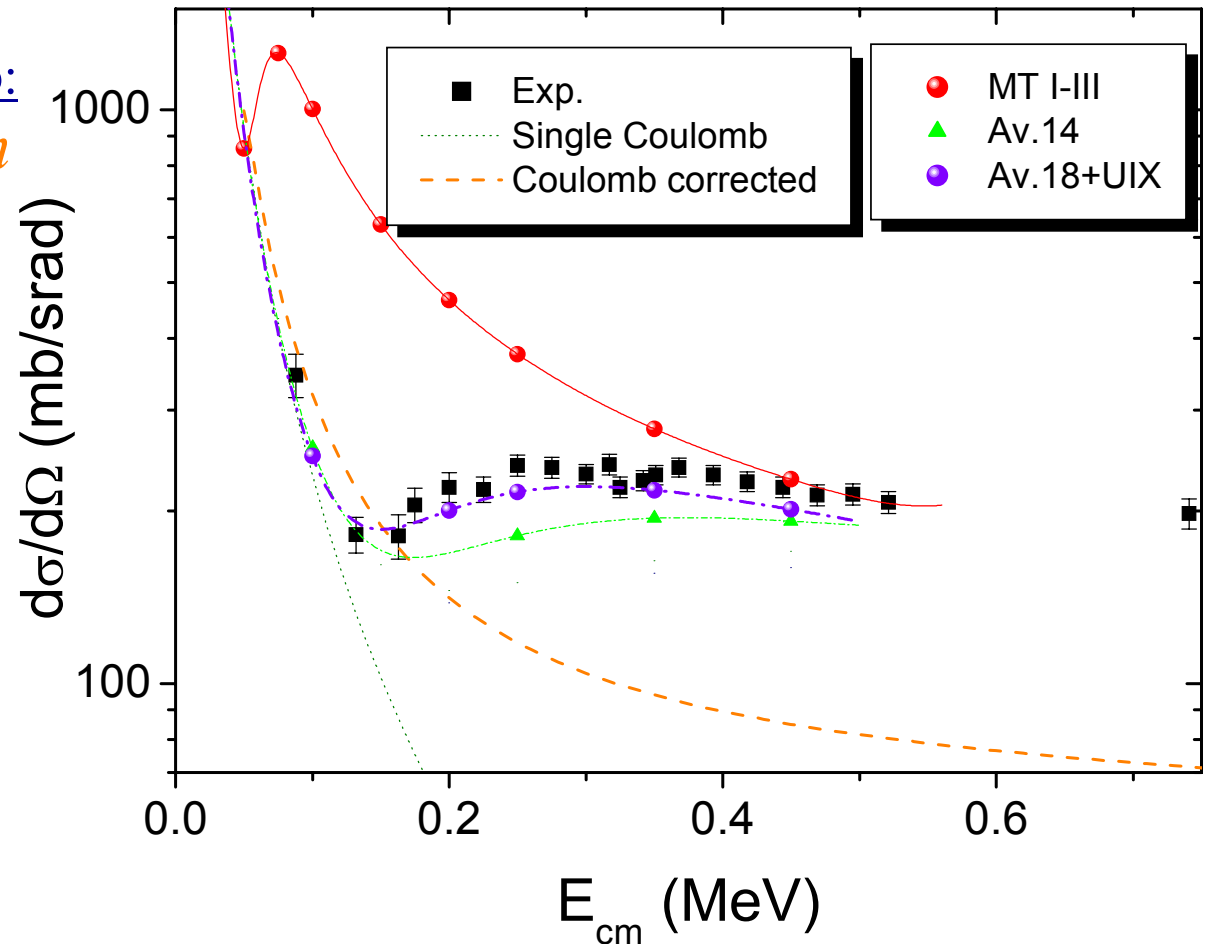
MT I-III: $a_{0^+} = -63 \text{ fm}$

AV14: $a_{0^+} = -13.9 \text{ fm}$

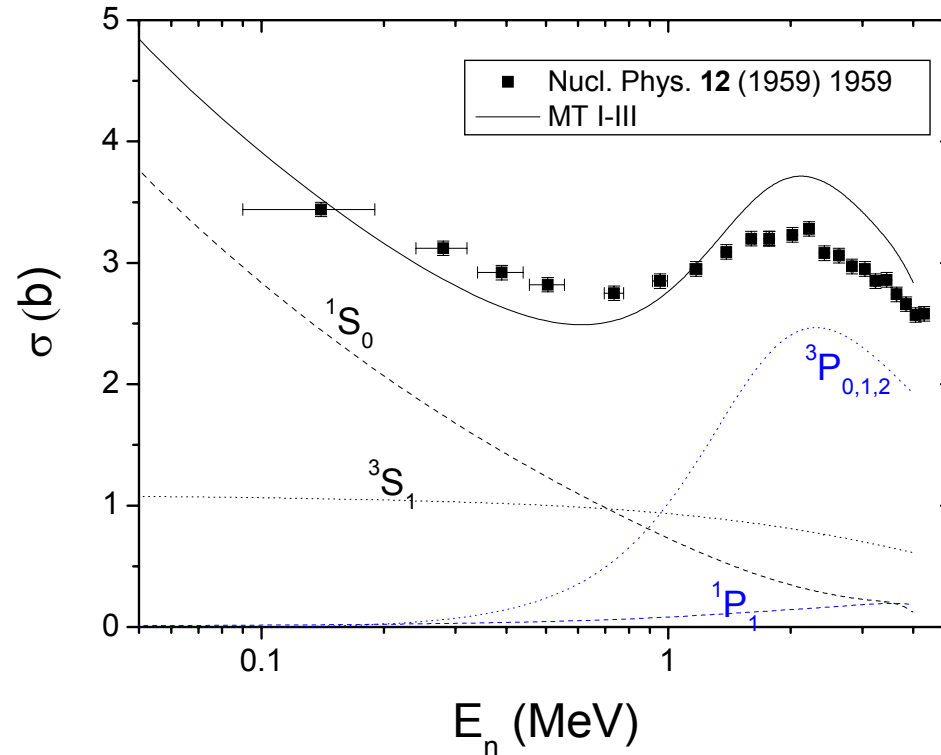
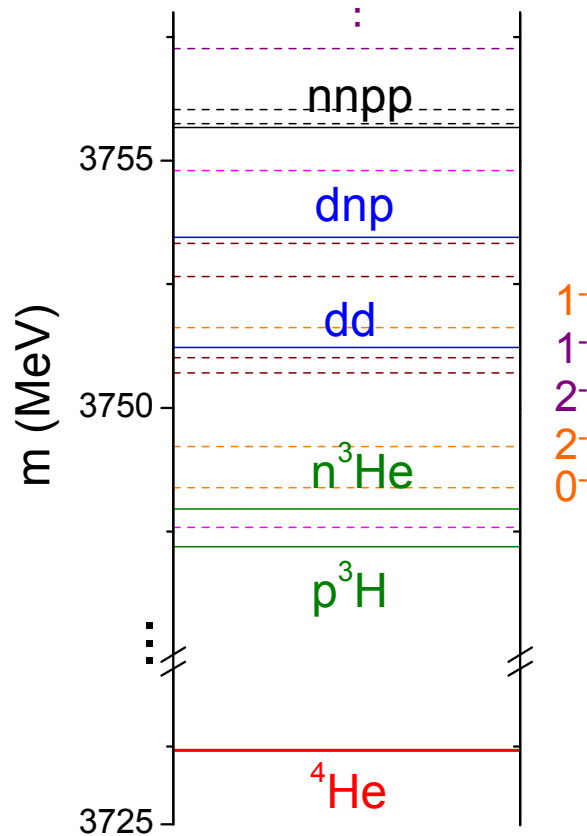
AV18+UIX: $a_{0^+} = -16.5 \text{ fm}$



Scattering $p+{}^3\text{H}$, $\alpha=120^\circ$



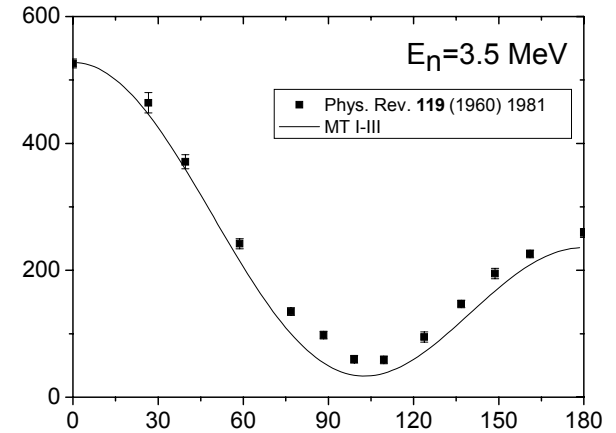
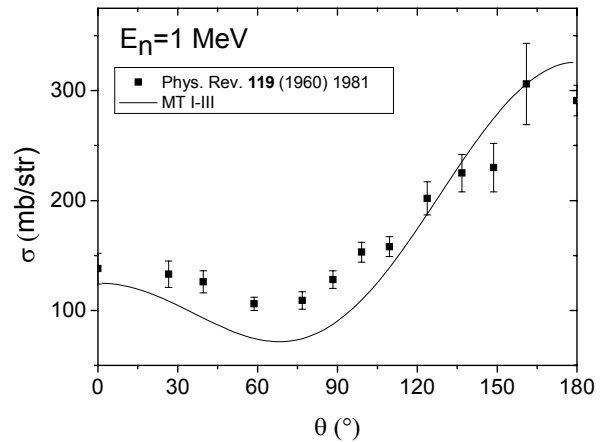
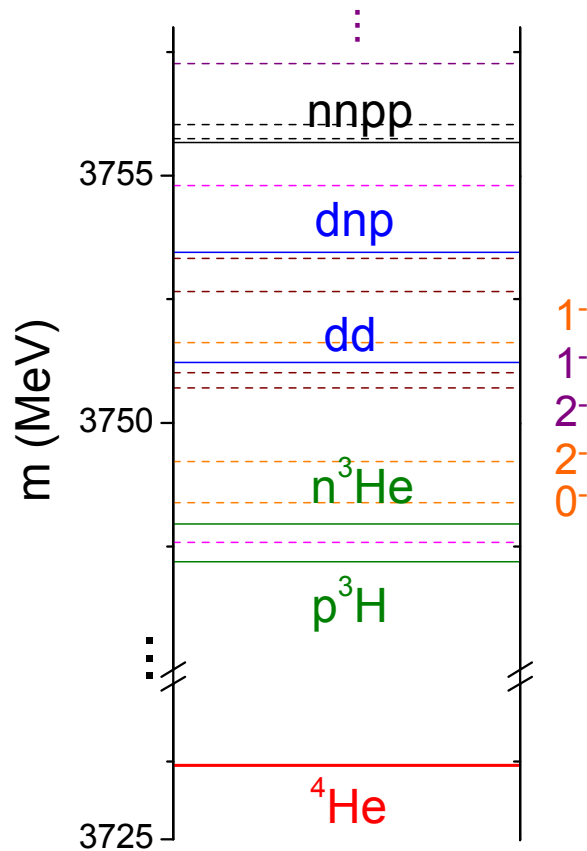
Hadronic resonances in four nucleon systems



J^π	MT I-III	Av.18+UIX (Hofmann et al.*)	Exp.
0^+	8.00 -2.90i	7.62 -4.07i	7.37 -4.448i
1^+	3.34 -0.0083i	3.333 -0.0052i	3.278 -0.001i

*Phys.Rev. C68 (2003) 021002

Hadronic resonances in four nucleon systems



J^π	MT I-III	Av.18+UIX (Hofmann et al*)	Exp.
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Hadronic resonances in four nucleon systems

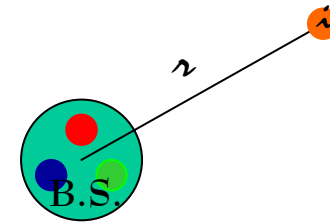
- To what the phaseshifts are sensible?

In bound state we calculate mean values: $\frac{\langle \Psi | V | \Psi \rangle}{\langle \Psi | \Psi \rangle}$.

But scattering wave functions diverge!

- Integral representation of phaseshifts:

$$\sin \delta = -\frac{m}{\hbar^2} \int \Phi_{\alpha}^{B.S.}(\vec{R}_{B.S.}) \hat{j}_l(kr) \left(\sum_{k \neq i} V_{ik} \right) \Psi(\vec{R}_{B.S.}, \vec{r}) dV$$



J^{π}	E(MeV)	1S_0	3SD_1	3P_0	3P_1	3P_2	1P_1
0^+	0.0	76.0	21.4	-1.30	1.94	-0.81	3.60
0^+	3.0	79.8	17.9	-1.05	1.91	-0.58	3.41
0^-	3.0	64.2	28.2	32.8	-29.7	2.31	-0.38
2^-	3.0	39.8	51.7	1.06	-6.90	14.3	-0.16

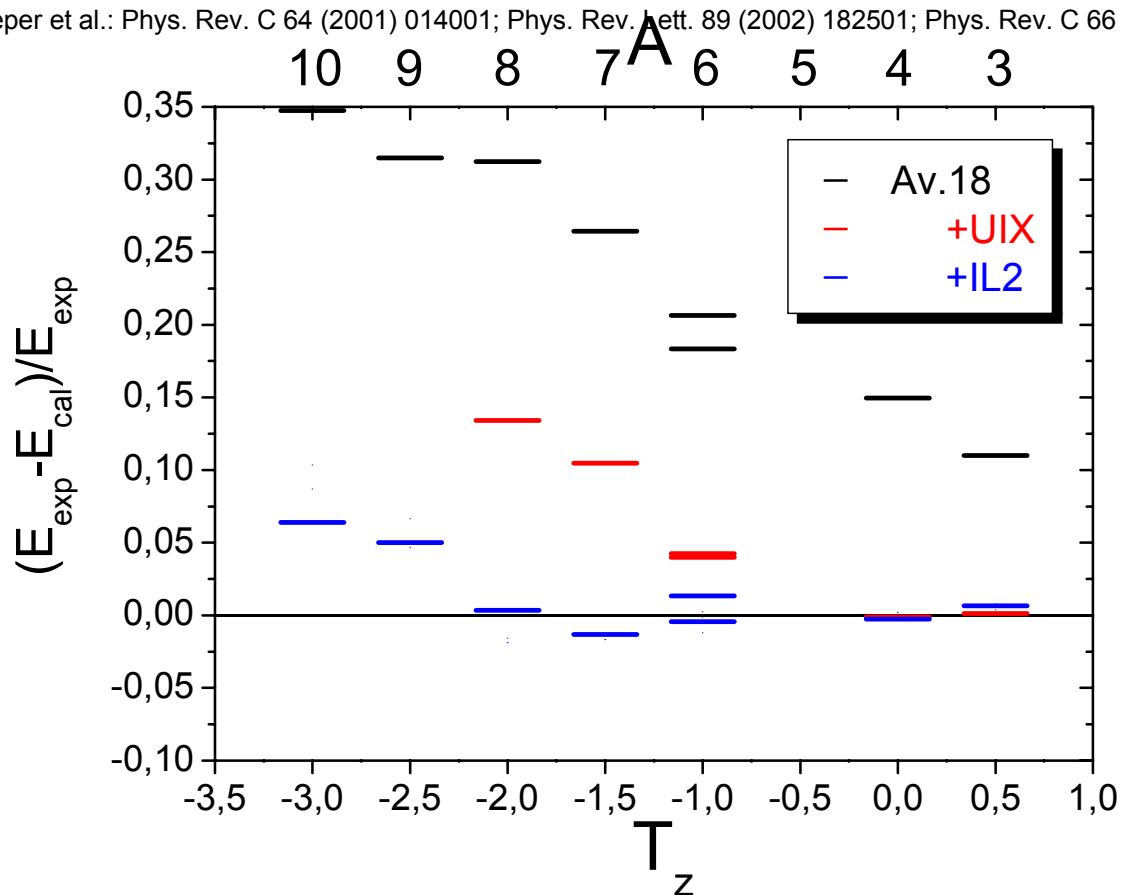
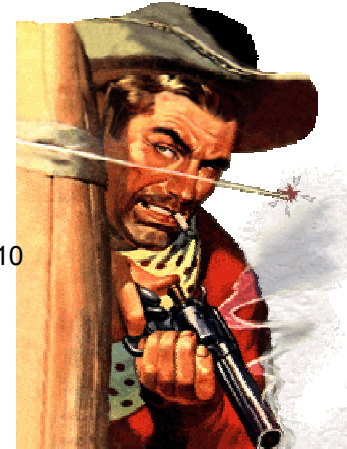
Hadronic resonances in four nucleon systems

✓ A_y and tensor analyzing power...

A. Kievsky et al., Phys. Rev. C 57 (1998) 555; H. Witała et al. Nucl. Phys.A 528 (1991) 48

✓ Binding energies...

C. Pieper et al.: Phys. Rev. C 64 (2001) 014001; Phys. Rev. Lett. 89 (2002) 182501; Phys. Rev. C 66 (2002) 044310



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• **Can we improve n - t cross sections?**

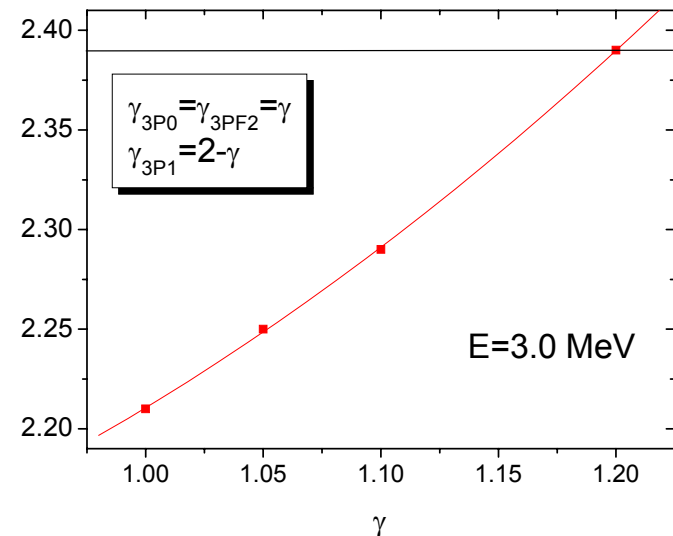
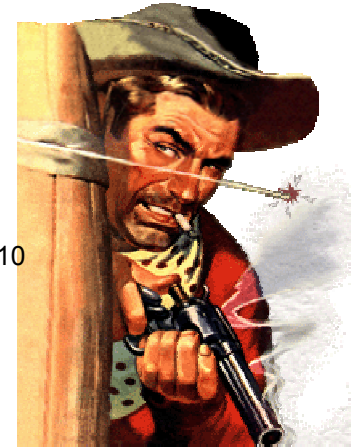
✓ If NN P -waves should be modified: AV18
 nn P -waves should be changed by $\sim 20\%$

✓ Compare to ~ 6 - 8% necessary to improve

A_y in n - d (A. Kievsky et al., Phys. Rev. C 57 (1998) 555; H. Witała et al. Nucl. Phys.A 528 (1991) 48)

✓ One needs still strong, isospin breaking

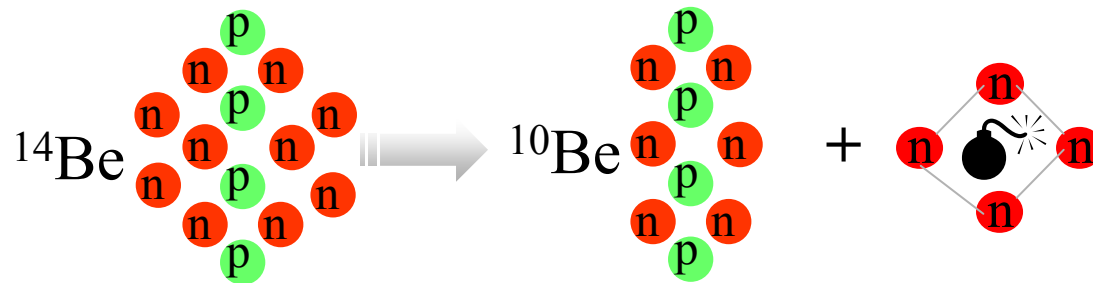
$3NF$... n - 3H puzzle???



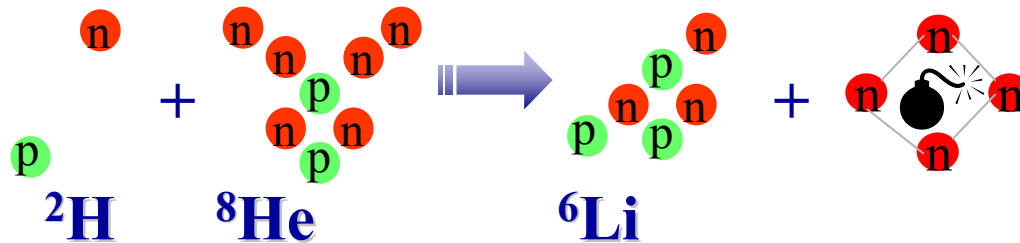
Hadronic resonances in four nucleon systems

Recent experiments in GANIL

F.M. Marqués et al: Phys. Rev. C 65 (2002) 044006 et arxiv:nucl-ex/0504009

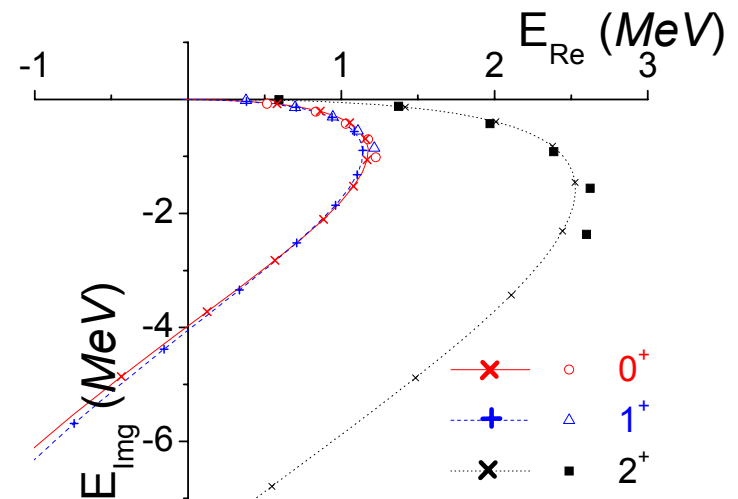
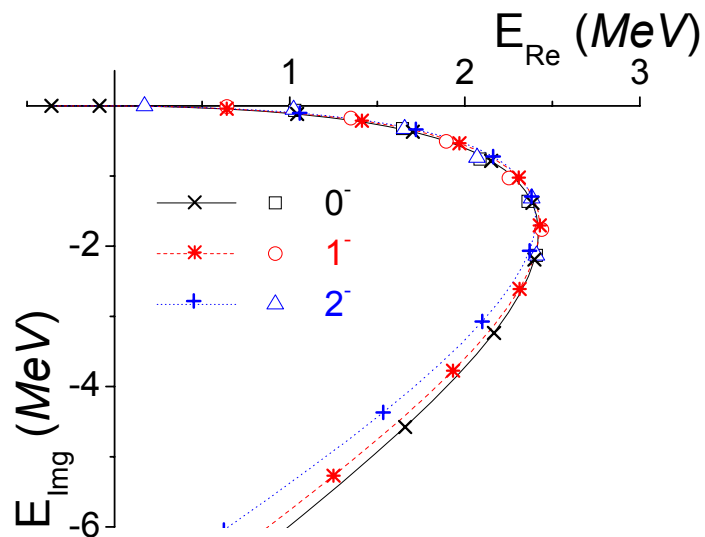


E. Rich et al: proceedings Exxon conference 2004



Hadronic resonances in four nucleon systems

✓ 3n and 4n resonances are explored using realistic NN interaction Reid93 in conjunction with additional attractive force:



All the resonance trajectories end up in III-rd energy quadrant, with $|E_{\text{img}}| > 6$ MeV

Hadronic resonances in four nucleon systems

4N is a mine of surprises and disappointments: A=4 continuum test other dynamical aspects of NN potentials than A=2,3 and bring new, yet not solved, problems. *Realistic potentials 'still' do well at the difficult (p+t) and fail at 'rather simple' (n+t)*

- Problem with negative parity states prior to A_y or break-up: **modified P-waves and/or 3NF with strong CIB needed?**

n-t scattering puzzle?

3n and 4n are non-resonant!!