Electromagnetic structure of few-nucleon systems

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Theoretical framework:

- Realistic Hamiltonians (AV18/UIX)
- Bound and scattering states nuclear wave functions (HH method)
- Realistic model for the nuclear electromagnetic current operator

A=3 EM structure up to 2000

GOOD:

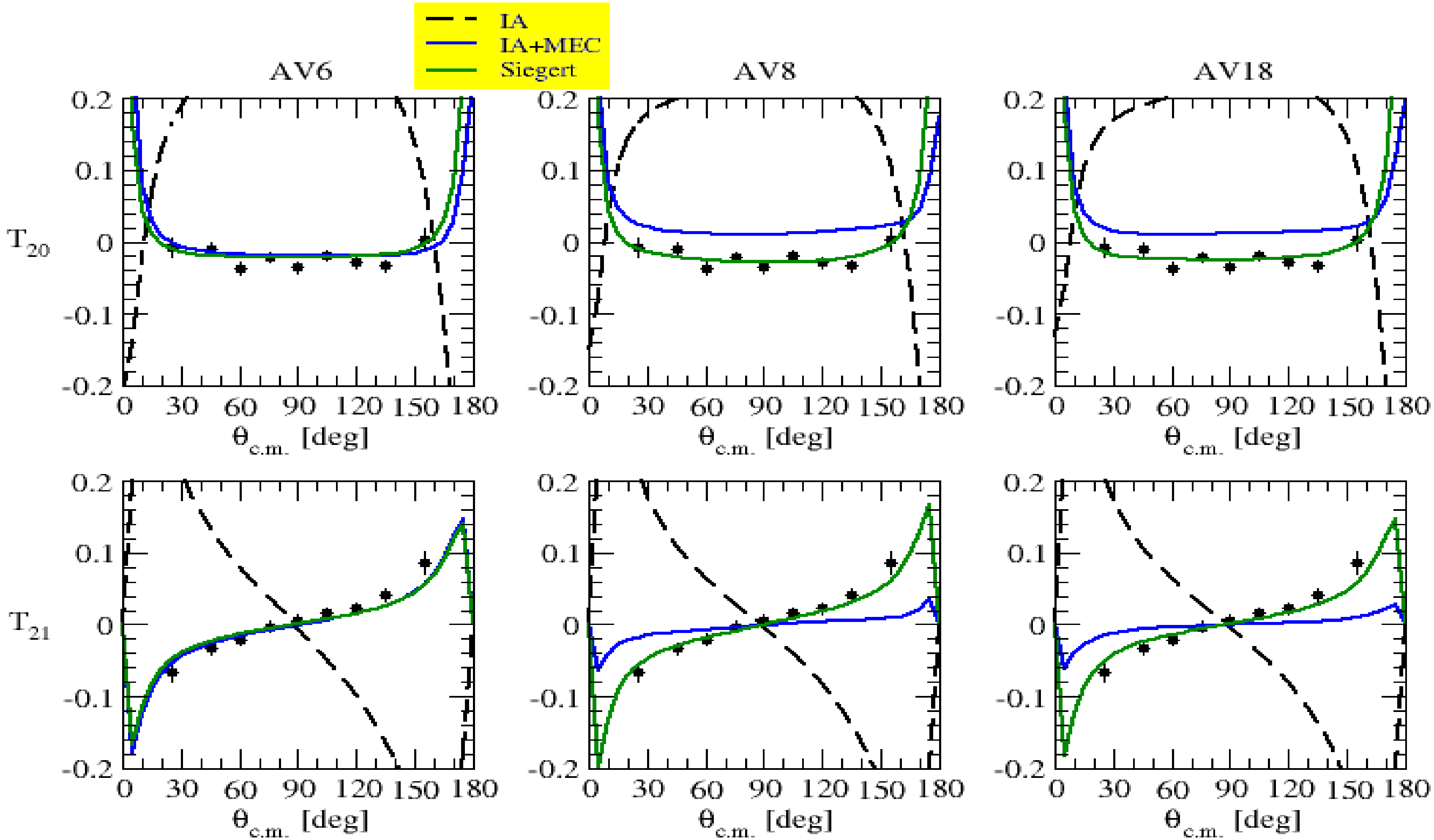
- Charge F.F.
- *pd* radiative capture below DBT (σ, A_y, iT_{11})

Viviani *et al.*, PRC **54**, 534 (1996)
Marcucci *et al.*, PRC **58**, 3069 (1998)
Viviani *et al.*, PRC **61**, 064001 (2000)

BAD:

- Magnetic F.F. in the 1st diffraction region
- *pd* radiative capture below DBT (T_{20}, T_{21})
- *nd* total σ at thermal energy and *pd* S(E=0) (1996 TUNL data)

$p+d \rightarrow {}^3\text{He} + \gamma$ @ 2 MeV (data from Smith & Knutson, 1999)



Therefore:

- Or the HH wave functions are not exact eigenfunction of the Hamiltonian

  → Ruled out by benchmarks calculations and improved accuracy of the calculation

- Or the nuclear current operator is not properly constructed (not conserved)

Wave functions: HH method (see Kievsky's talk of last week)

- Use of hyperspherical coordinates (ρ, φ) instead of the moduli of the Jacobi coordinates
- Expansion of the w.f. on a basis of hyperspherical-harmonics
- Use of variational principles to calculate the hyperradial functions

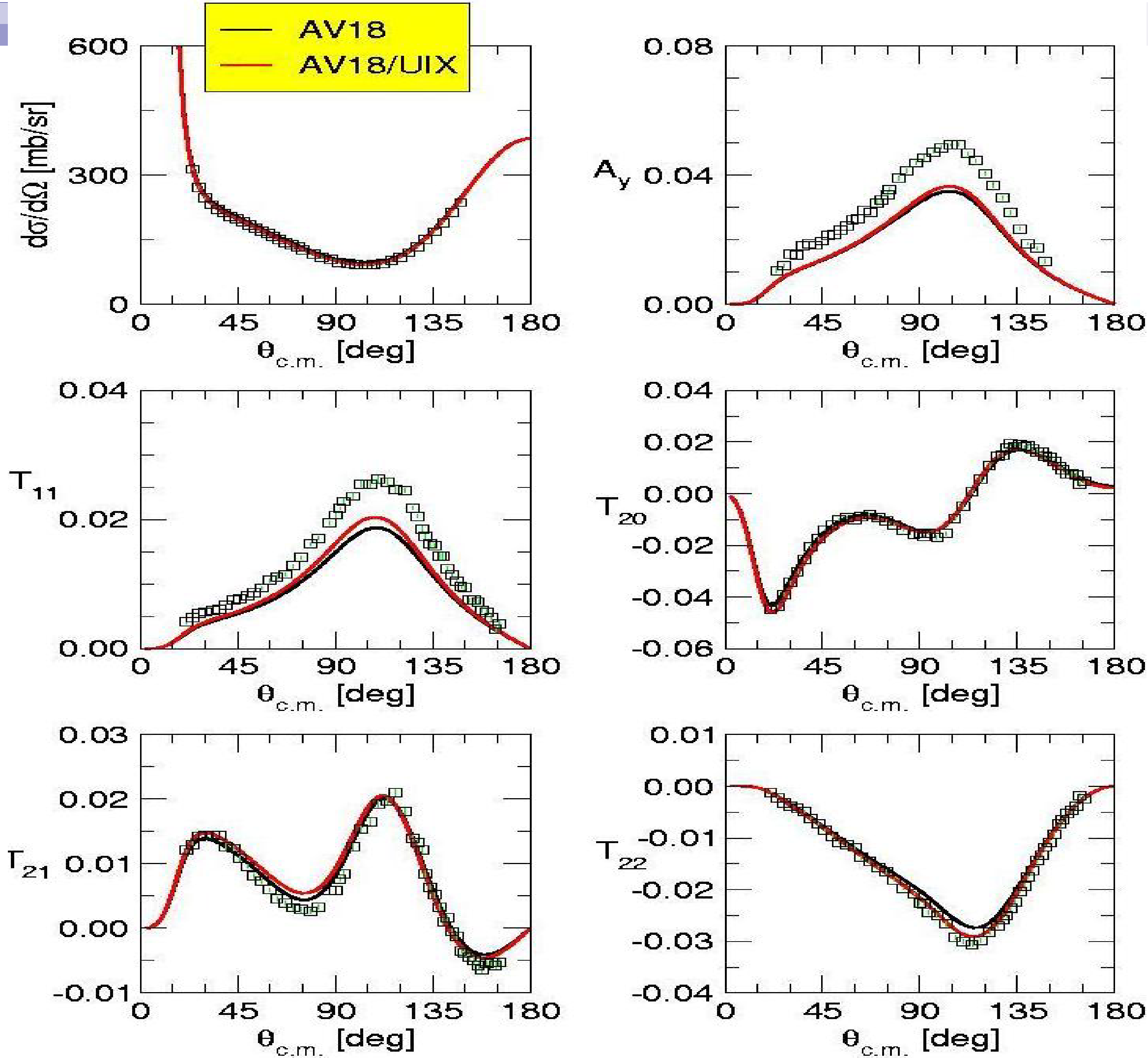
Accurate w.f.s for:

- *A*=3 and 4 bound states
- *pd* and *nd* scattering states, below and above DBT
- *A*=4 scattering states (*p*-^{3}He , *n*-^{3}H)

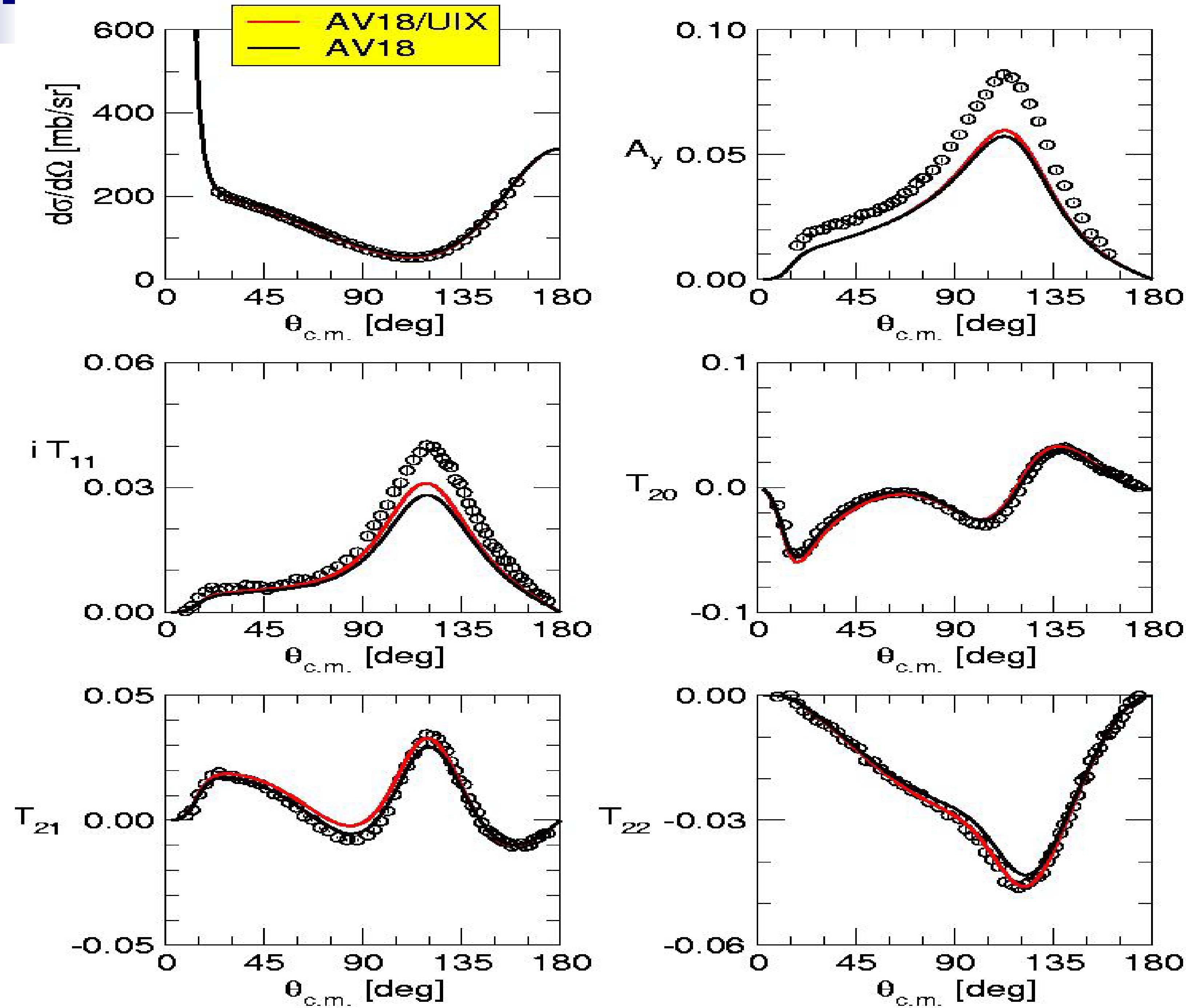
No approximation in the inclusion of the Coulomb interaction

A=3 and 4 binding energies (MeV) (AV18/UIX)

Nucleus	HH	Expt.
^{3}He	**7.750**	**7.718**
^{3}H	**8.479**	**8.482**
^{4}He	**28.47**	**28.30**



pd elastic scatt. at 2.00 MeV



pd elastic scatt. at 3.33 MeV

Present talk:

"*New*" model for the nuclear EM current, tested studying a wide range of A=2 and 3 observables. In particular:

- A=2 and 3 radiative captures
- ^{3}He and ^{3}H electron scattering observables

Marcucci *et al*., nucl-th/0502048, PRC in press

Electromagnetic current operator

Current conservation relation (CCR)

$$\mathbf{q}\cdot\mathbf{j}(\mathbf{q}) = [H, \textstyle\sum_i \rho_i(\mathbf{q})]$$

$$H = T + \sum_{ij} v_{ij} + \sum_{i<j<k} V_{ijk}$$

$$\rho_i \approx e^{i\mathbf{q}\cdot\mathbf{r}_i} [G_E^S(q_\mu) + G_E^V(q_\mu)\boldsymbol{\tau}_{i,z}]/2$$

e_i

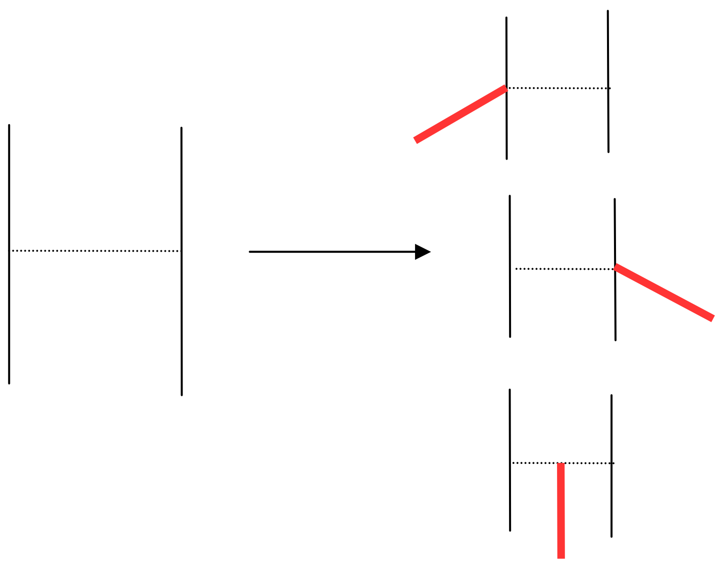
$$\mathbf{j}(\mathbf{q}) = \sum_i \mathbf{j}_i(\mathbf{q}) + \sum_{ij} \mathbf{j}_{ij}(\mathbf{q}) + \sum_{i<j<k} \mathbf{j}_{ijk}(\mathbf{q})$$

- $\mathbf{j}_i(\mathbf{q})$: non-relativistic reduction of single-nucleon covariant current op.
- $\mathbf{j}_{ij}(\mathbf{q})$: two-body current, which has to satisfy CCR with *NN* potential
- $\mathbf{j}_{ijk}(\mathbf{q})$: three-body current, which has to satisfy CCR with TNI

“Old” model for $\mathbf{j}_{ij}(\mathbf{q})$
Riska and Buchmann *et al.*, 1985

Meson-exchange scheme:

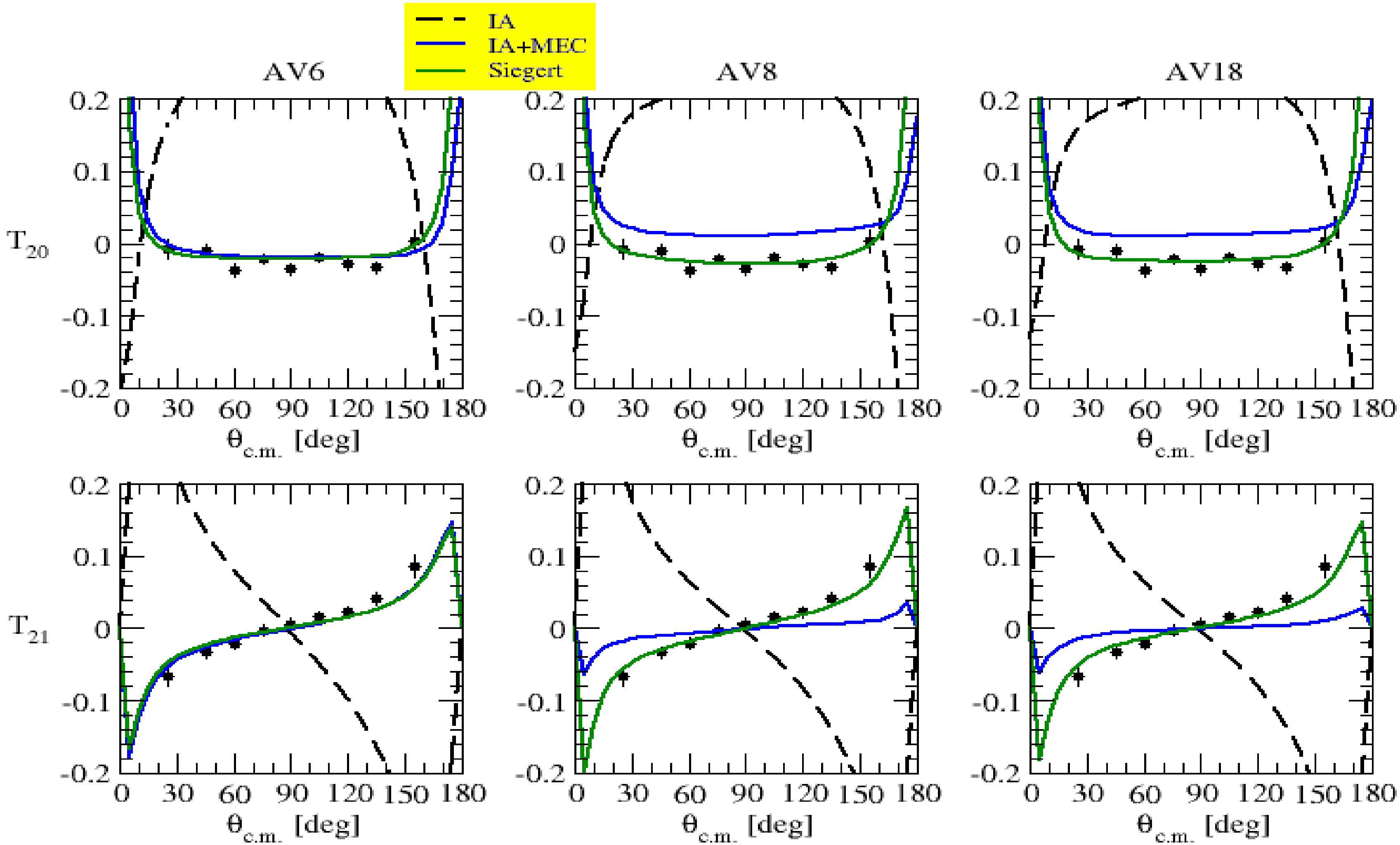
v_{ij} due to exchanges of families of π and ρ mesons



Observations:

- the operators from the *p*-independent part of the *NN* potential (AV6) are “*exactly*” conserved
- the operators from the *p*-dependent part of the *NN* potential (AV8-AV18) are only *approximately* conserved

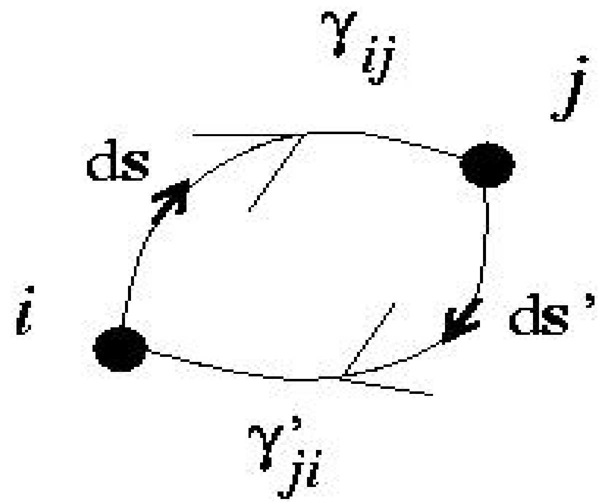
$p+d \rightarrow {}^3\text{He} + \gamma$ @ 2 MeV (data from Smith & Knutson, 1999)



"New" approach for $\mathbf{j}_{ij}(\mathbf{q})$ (I)

[Sachs, PR74, 433 (1948)]

Minimal-substitution scheme:



$$v_{ij} = v_1(ij) + v_2(ij)\ \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

$$\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j = -1 - (1 + \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)\, P_{ij}$$

$$P_{ij} \equiv \mathrm{e}^{\mathbf{r}_{ji}\cdot\partial_i + \mathbf{r}_{ij}\cdot\partial_j}$$

$$\partial_i \rightarrow \partial_i - i\, e_i \boldsymbol{A}(\mathbf{r}_i)$$

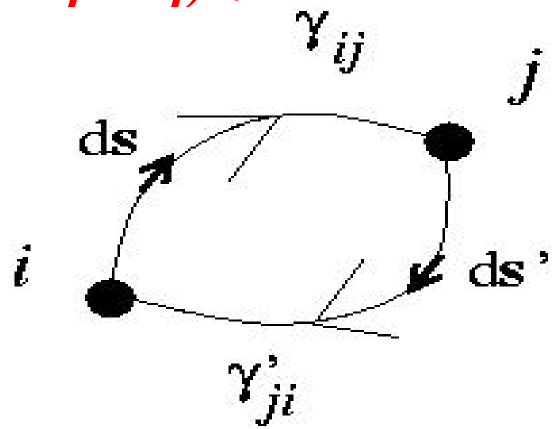
$$P_{ij}^{\boldsymbol{A}} = \mathrm{e}^{-\mathrm{i}\epsilon_i \int_{\boldsymbol{r}_i}^{\boldsymbol{r}_j} d\boldsymbol{s}\cdot\boldsymbol{A}(\boldsymbol{s}) - \mathrm{i}\epsilon_j \int_{\boldsymbol{r}_j}^{\boldsymbol{r}_i} d\boldsymbol{s}'\cdot\boldsymbol{A}(\boldsymbol{s}')}\, P_{ij}$$

"New" approach for $\mathbf{j}_{ij}(\mathbf{q})$ (II) :
Minimal-substitution scheme

$$v_{ij} \rightarrow v_1(ij) + v_2(ij)\,[-1 - (1 + \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)\, P_{ij}^{A}\,]$$

$$= \quad v_{ij} - \int \mathbf{j}_{ij}(\mathbf{x}) \cdot \boldsymbol{A}(\mathbf{x})$$

$$\mathbf{j}_{ij}(\mathbf{q}) = i\, v_2(ij)\; \boldsymbol{e}_i\,(1 + \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \int ds\; \mathrm{e}^{i\mathbf{q}\cdot\mathbf{s}} + \boldsymbol{i} \leftrightarrow \boldsymbol{j}$$

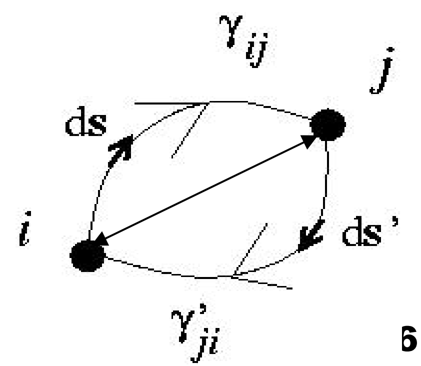


Observations:

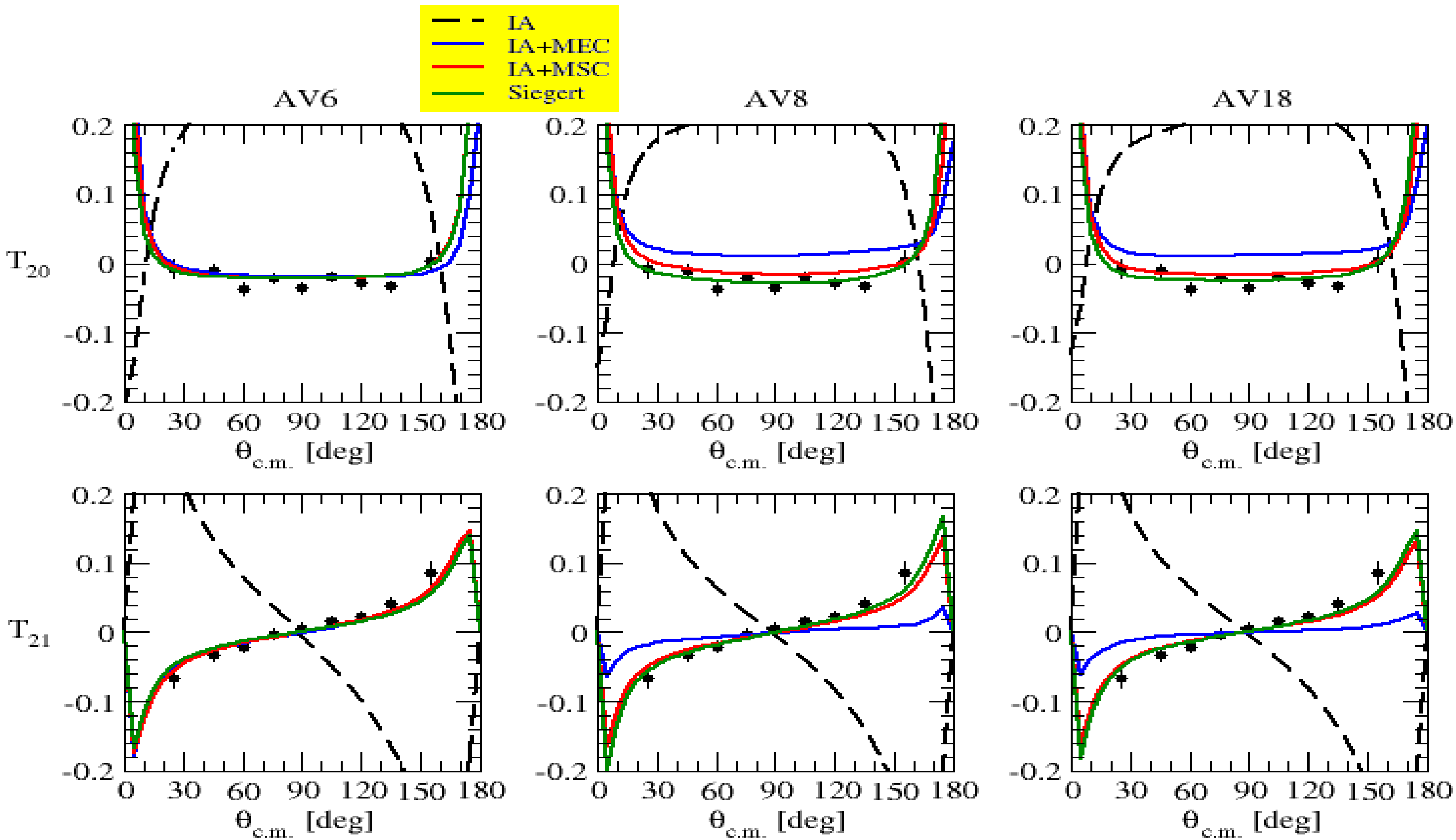
- With the choice of a particular path (ME-path) it is possible to reobtain the MEC currents from OPE potential (AV6)
- for $\mathbf{q}\rightarrow 0$ all $\mathbf{j}_{ij}(\mathbf{q})$ are the same:

$$\mathbf{j}_{ij}(\mathbf{q}) = -\,i\, v_2(ij)\, e_i\,(1+\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)\,\mathbf{r}_{ij} + i \leftrightarrow j$$

- the operators from the *p*-dependent part of the *NN* potential have been obtained using a linear path (LP), and satisfy by construction CCR



$p+d \rightarrow {}^3\mathrm{He} + \gamma$ @ 2 MeV (data from Smith & Knutson, 1999)



"New" model for $\mathbf{j}_{ij}(\mathbf{q})$

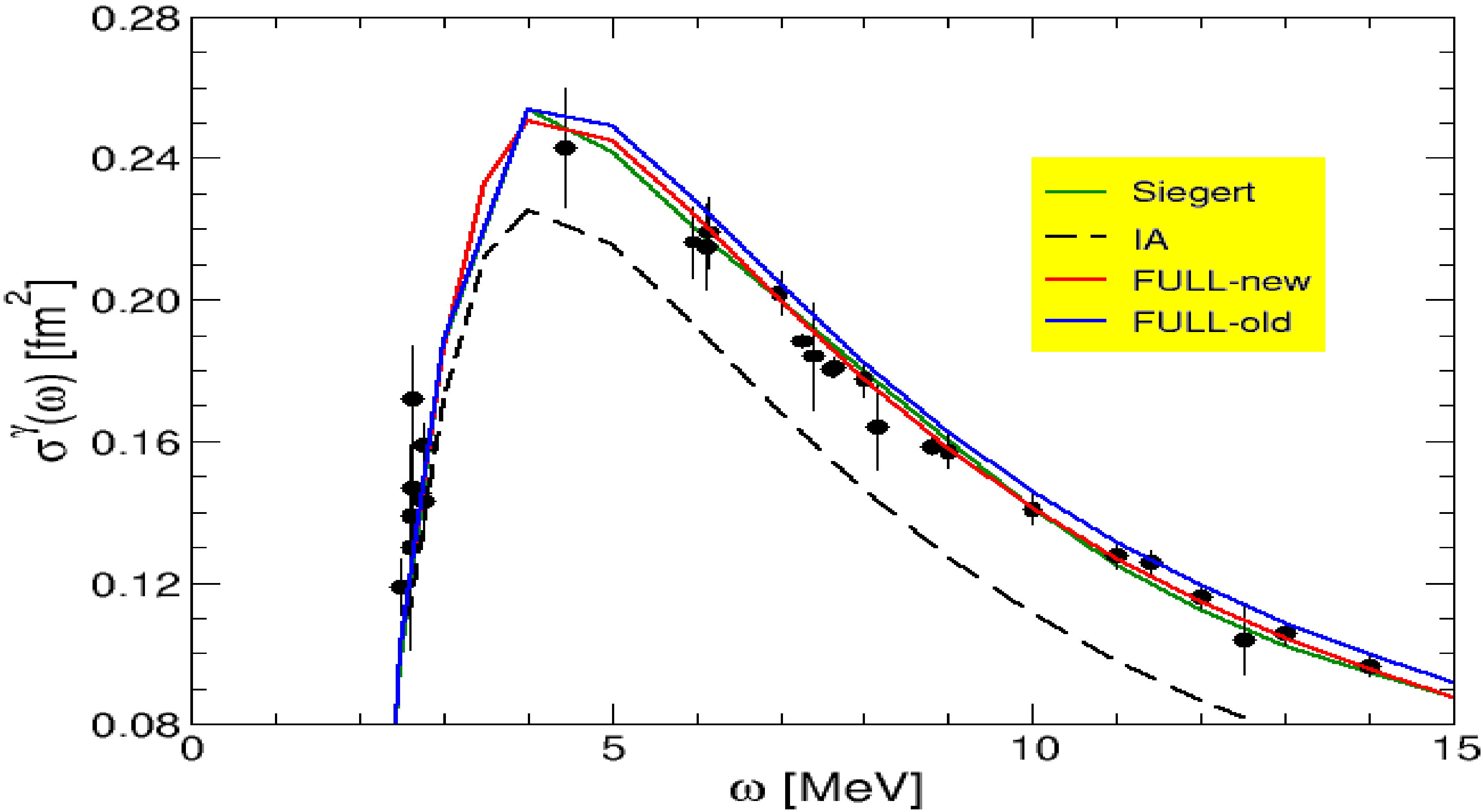
- ME scheme – "old" approach for the currents from the *p*-independent part of the *NN* interaction (AV6)
- MS scheme – "new" approach for the currents from the *p*-dependent part of the *NN* interaction (AV8-AV18) (LP)
- MD currents ($\omega\pi\gamma$, $\rho\pi\gamma$, ...)

$n+p \longrightarrow d+\gamma$ radiative capture at thermal energies

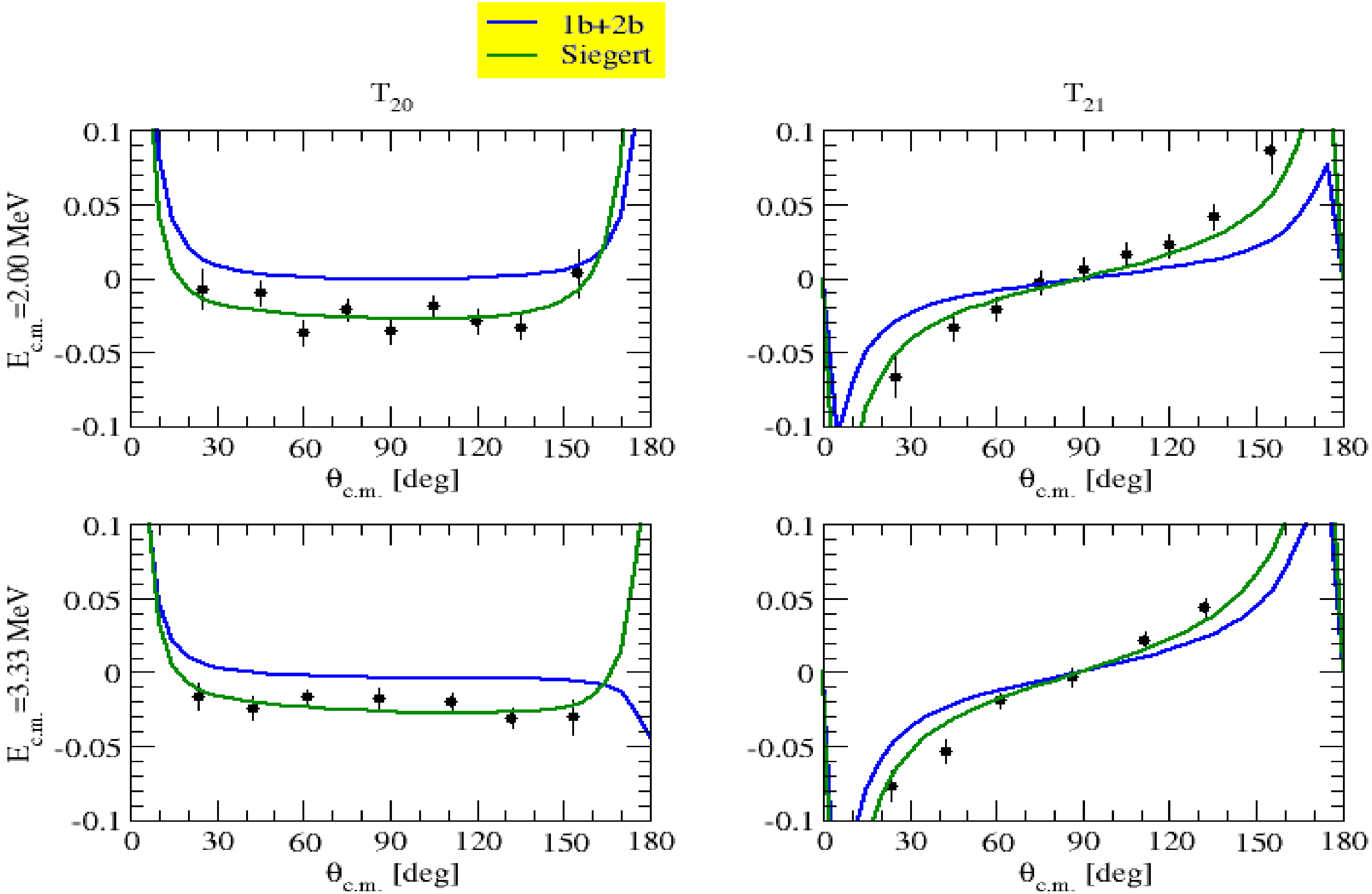
Total cross section [mb] for *np* radiative capture (AV18):

One-body	**304.6**
Full-old	**334.2**
Full-new	**332.7**
Expt.	**332.6(7)**

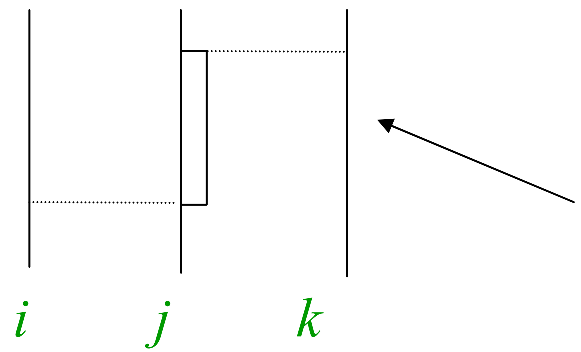
$\gamma + d \longrightarrow n + p$ (AV18)



$p+d \rightarrow {}^3\text{He} + \gamma$ @ 2 and 3.33 MeV (AV18/UIX)



“New” model for $\mathbf{j}_{ijk}(\mathbf{q})$ (I)



Isospin dependence of the Urbana-type potential

$$V_{ijk} \sim v_{jk}^{\dagger}(\Delta N \rightarrow NN)\, v_{ij}(NN \rightarrow \Delta N)$$

Meson-exchange scheme:

$v_{ij}(NN \rightarrow N\Delta)$ due to exchanges of families of π and ρ mesons

"New" model for $\mathbf{j}_{ijk}(\mathbf{q})$ (II)

Minimal-substitution scheme:

$$V_{ijk} \sim F_S(ijk)\ \{\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\, ,\, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k\} + F_A(ijk)\ [\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\, ,\, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k]$$

$\sim \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k \sim P_{ik}$

same as 2b current

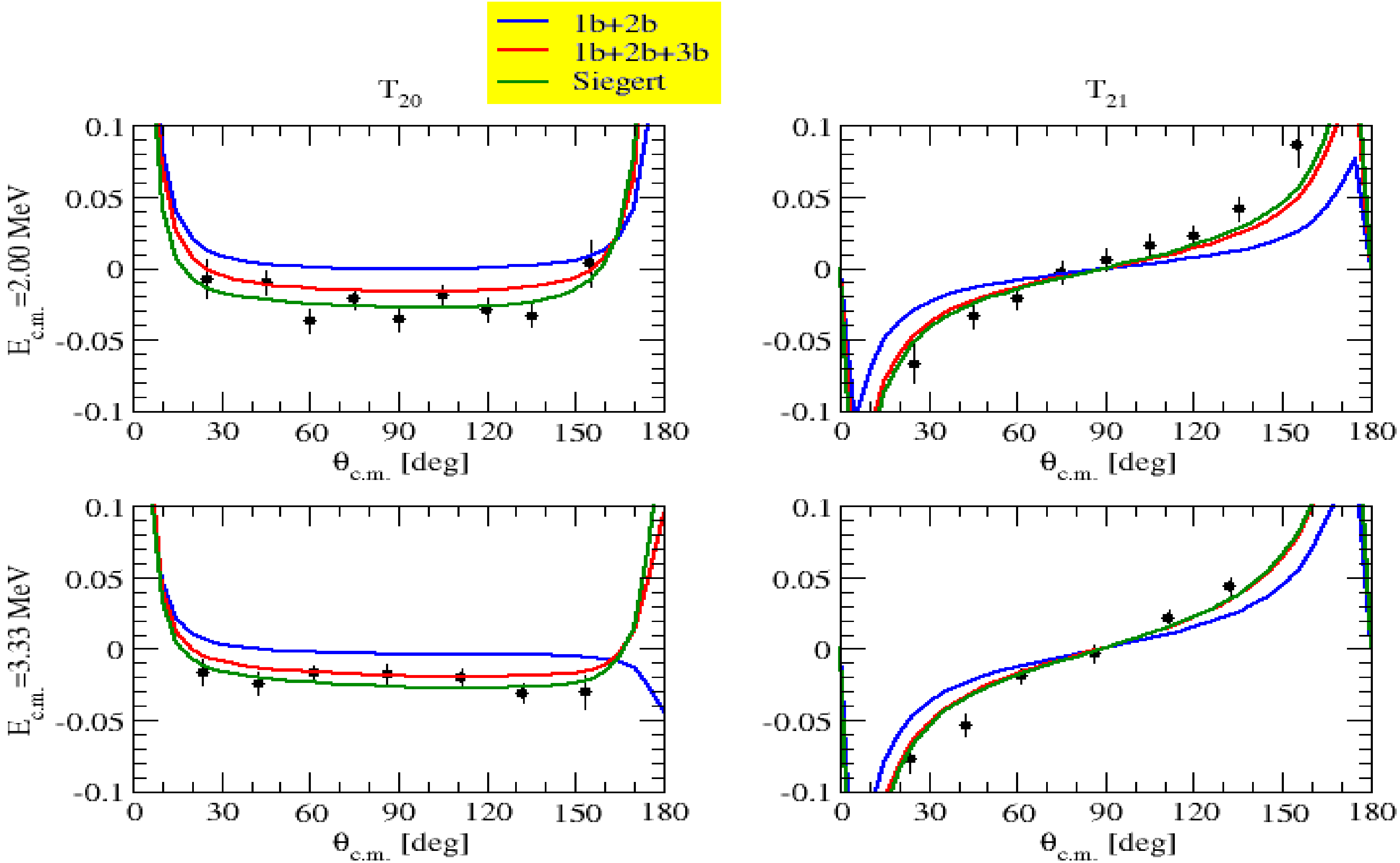
$\sim P_{ij} \cdot P_{jk} - P_{jk} \cdot P_{ij}$

same procedure as 2b current

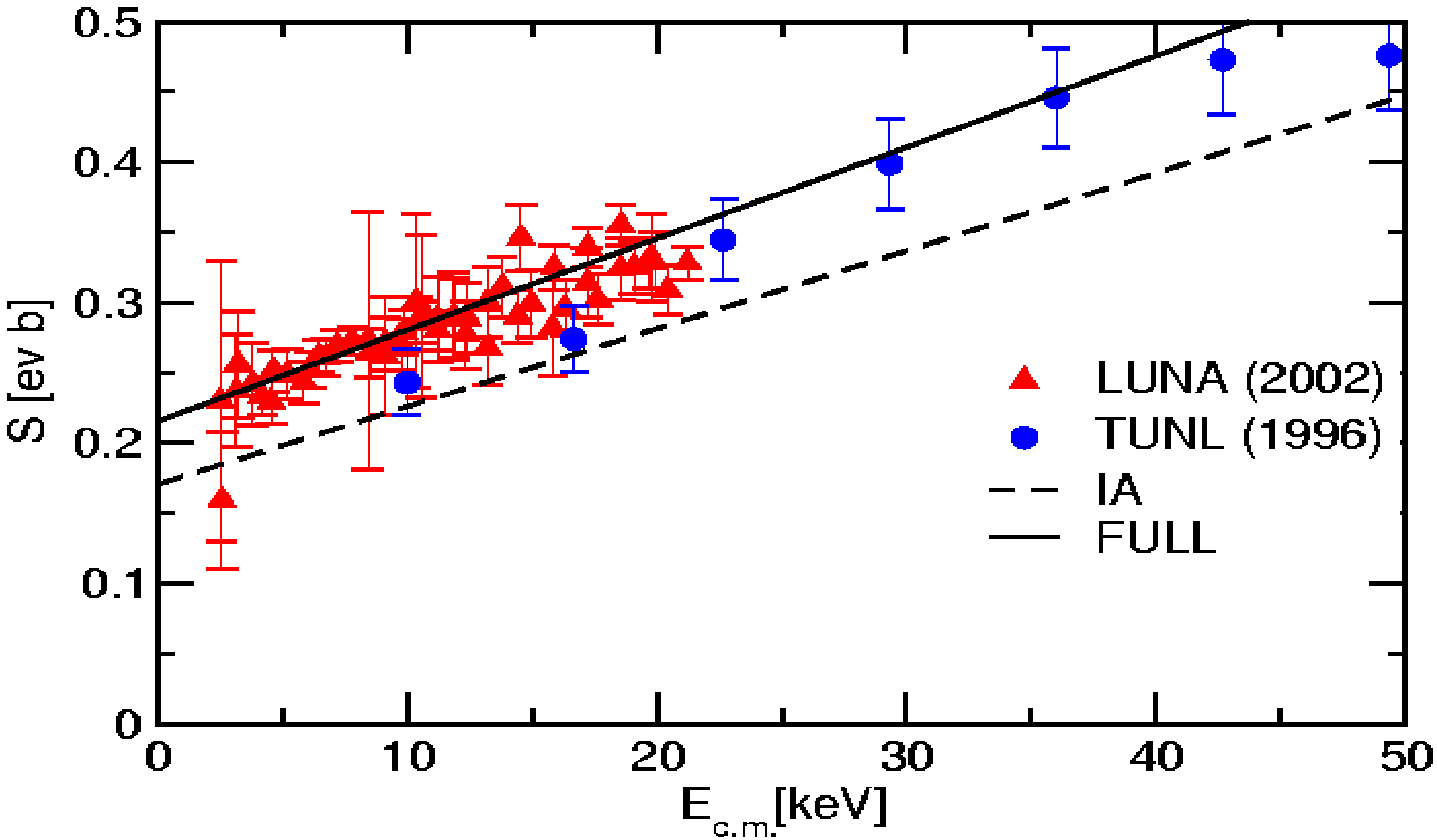
Observations:

- ME and MS schemes give the same results
- The MS scheme can be used also with TM-type potentials
- The full EM current operator satisfies by construction the CCR with the AV18/UIX Hamiltonian model

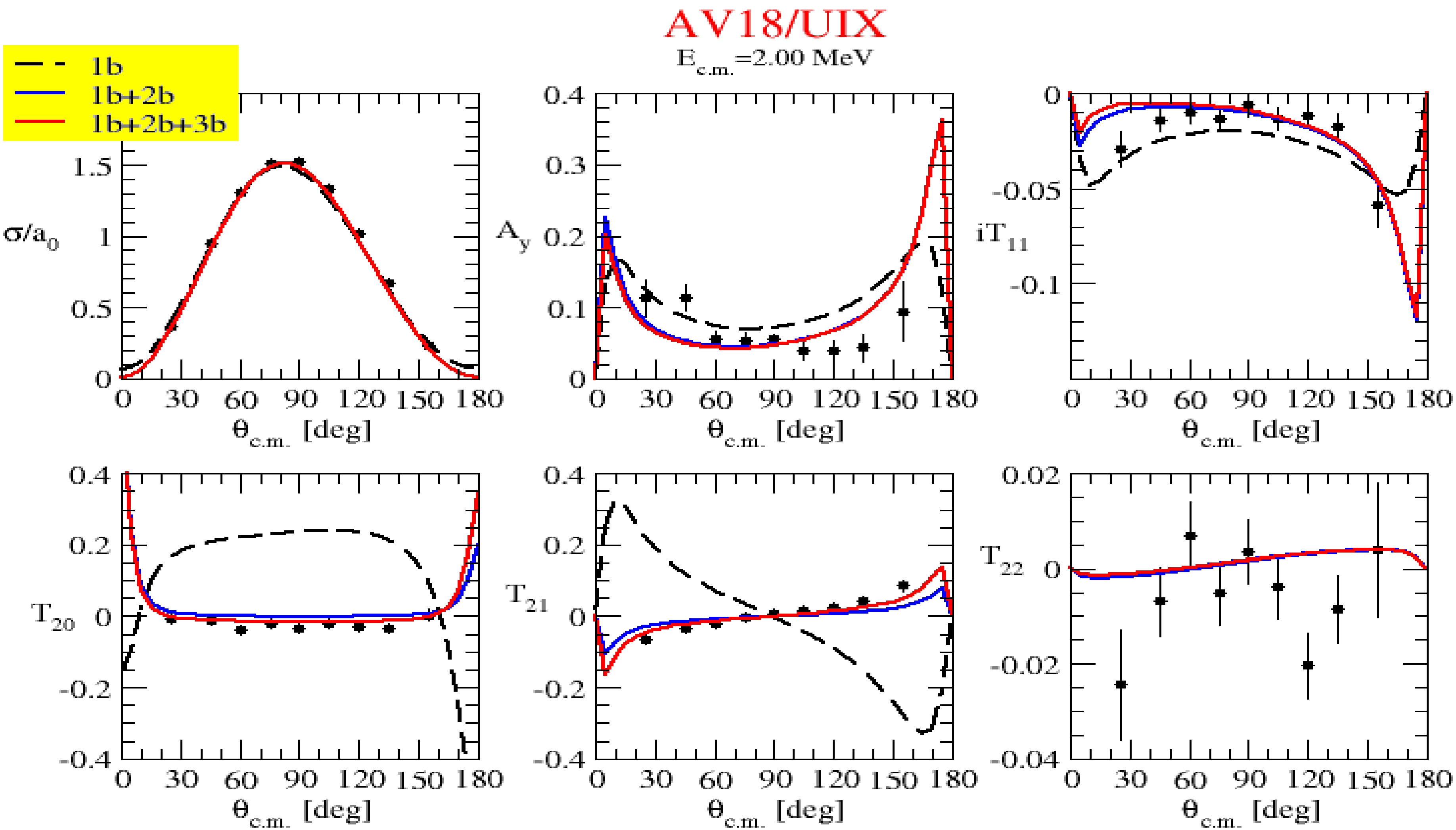
$p+d \rightarrow {}^3\mathrm{He} + \gamma$ @ 2 and 3.33 MeV (AV18/UIX)



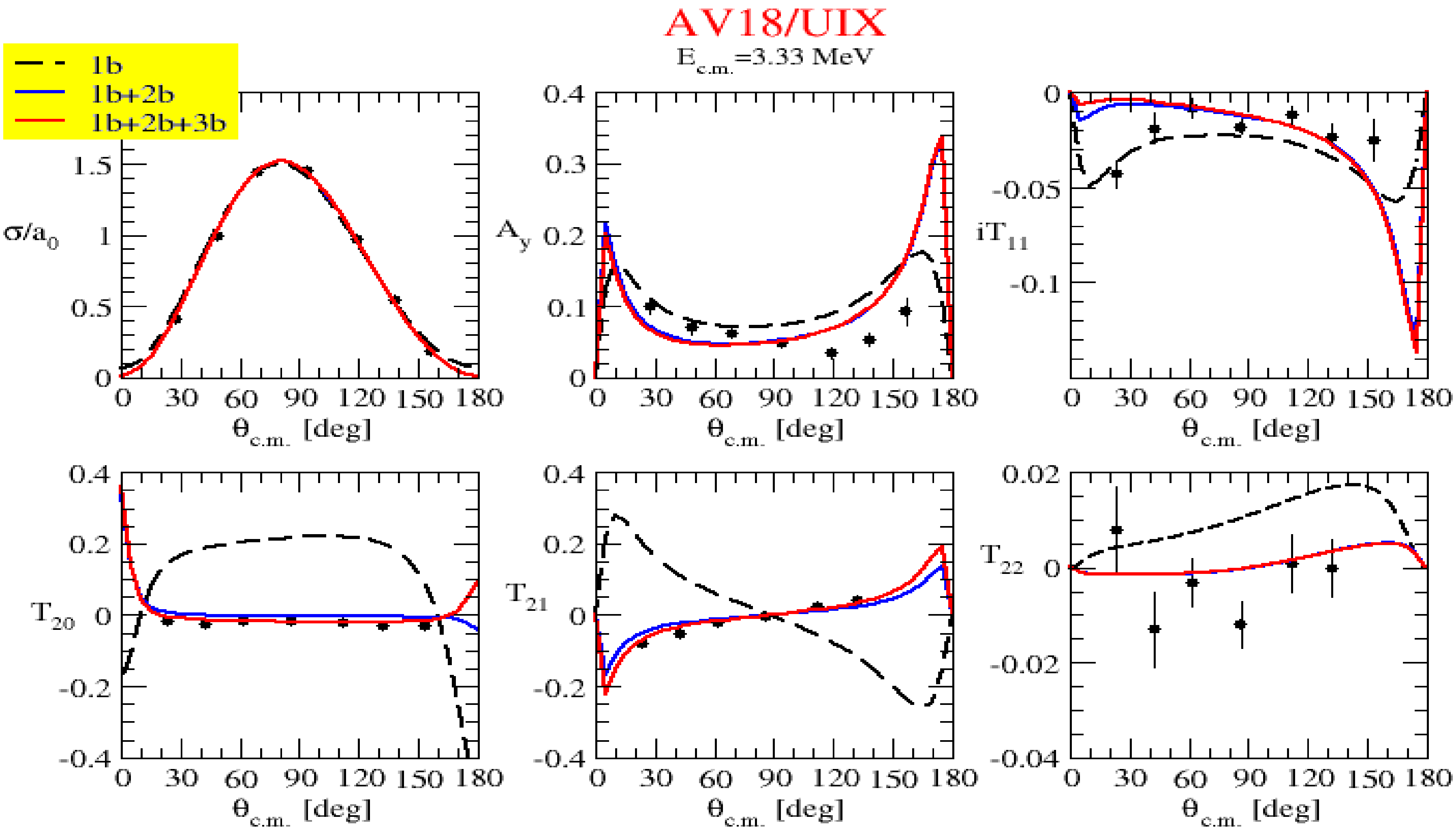
$p+d \rightarrow {}^3\mathrm{He} + \gamma$: S-factor at low energies



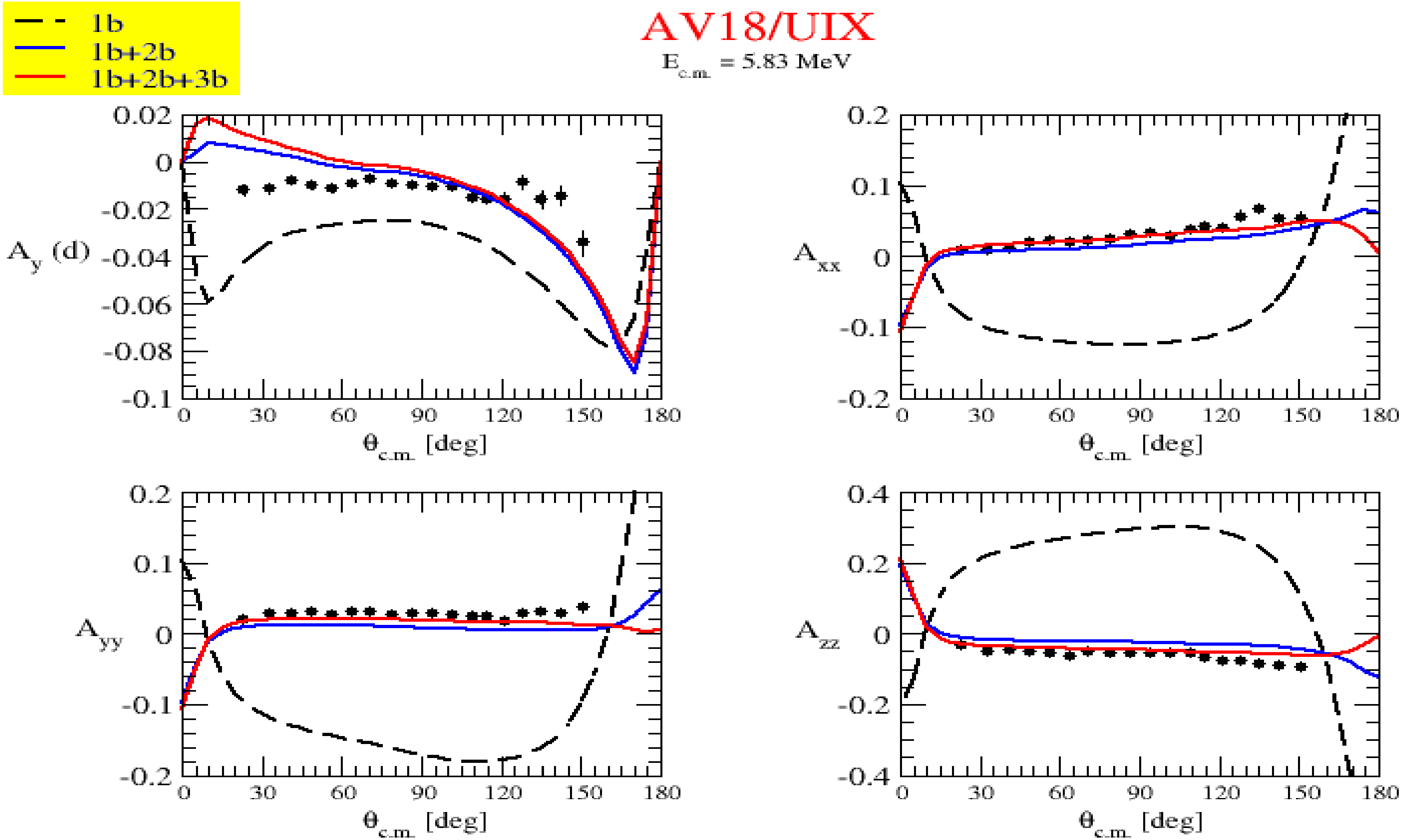
$p+d \rightarrow {}^3\text{He} + \gamma$ (data from Smith & Knutson, 1999)



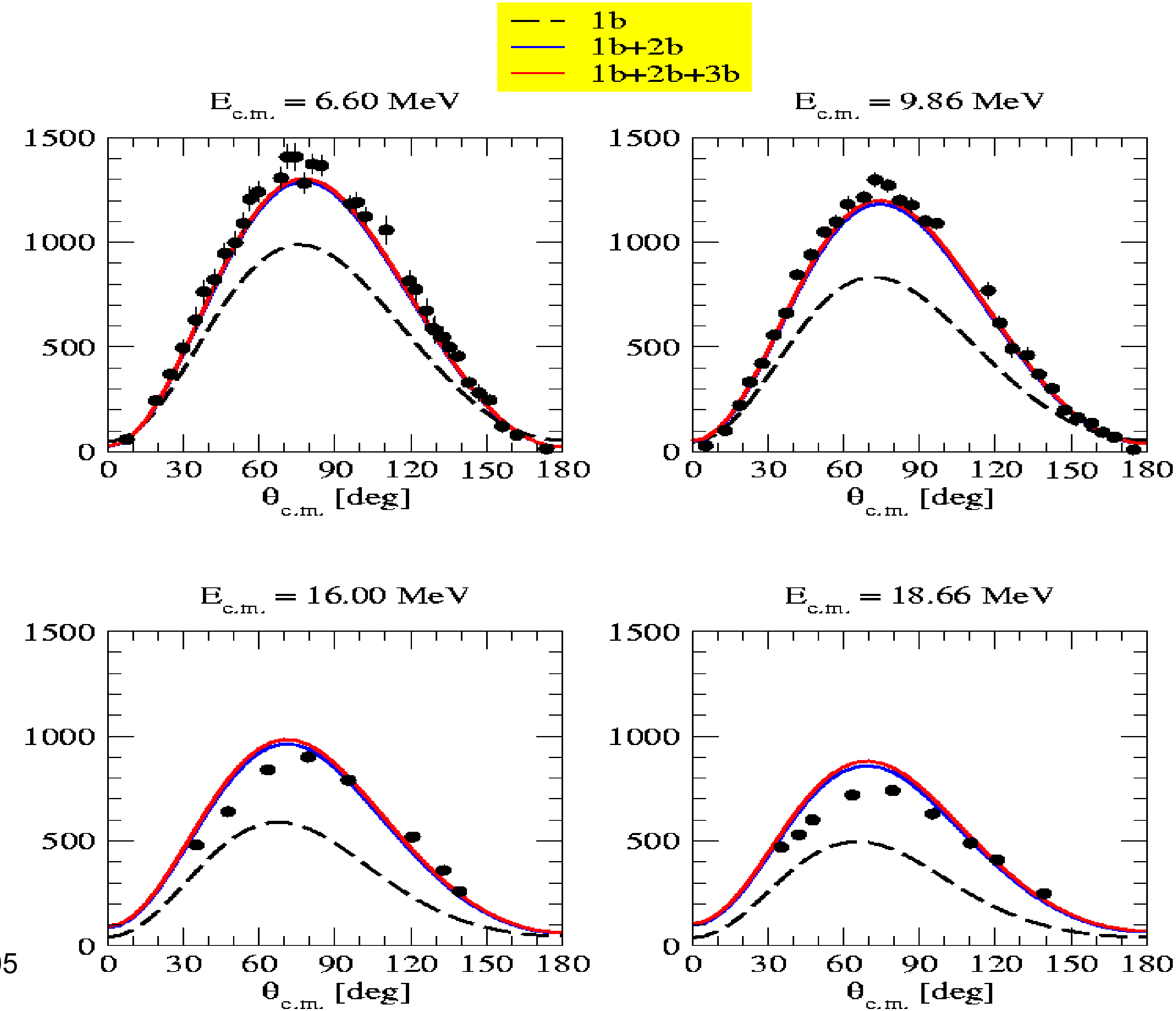
$p+d \rightarrow {}^3\text{He} + \gamma$ (data from Goeckner *et al.*, 1992)



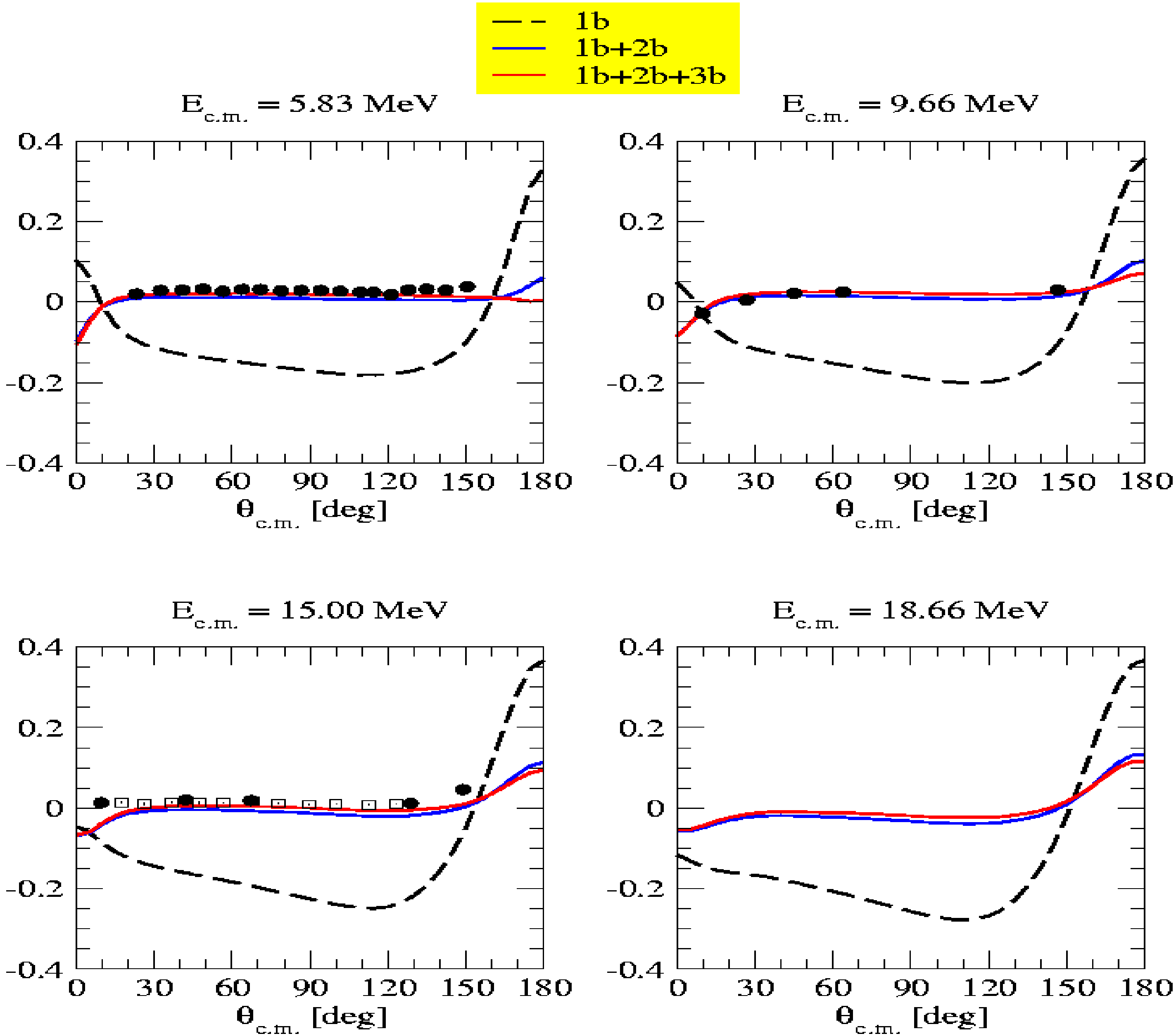
$p+d \rightarrow {}^3\text{He} + \gamma$ (data from Akiyoshi *et al.*, 2001)



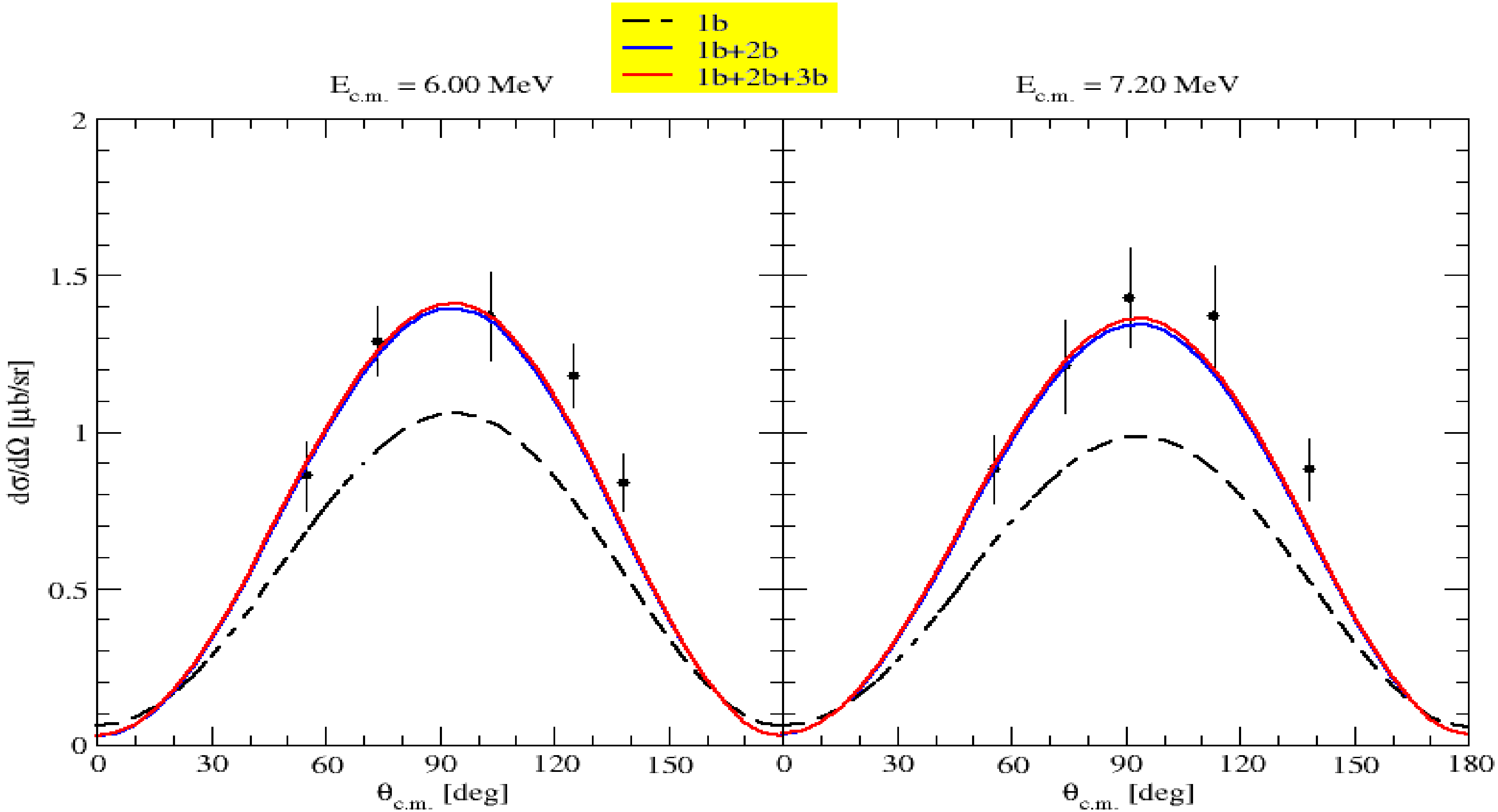
$p+d \rightarrow {}^3He + \gamma$: differential cross section



$p+d \rightarrow {}^3\text{He} + \gamma$: $A_{yy}(d)$

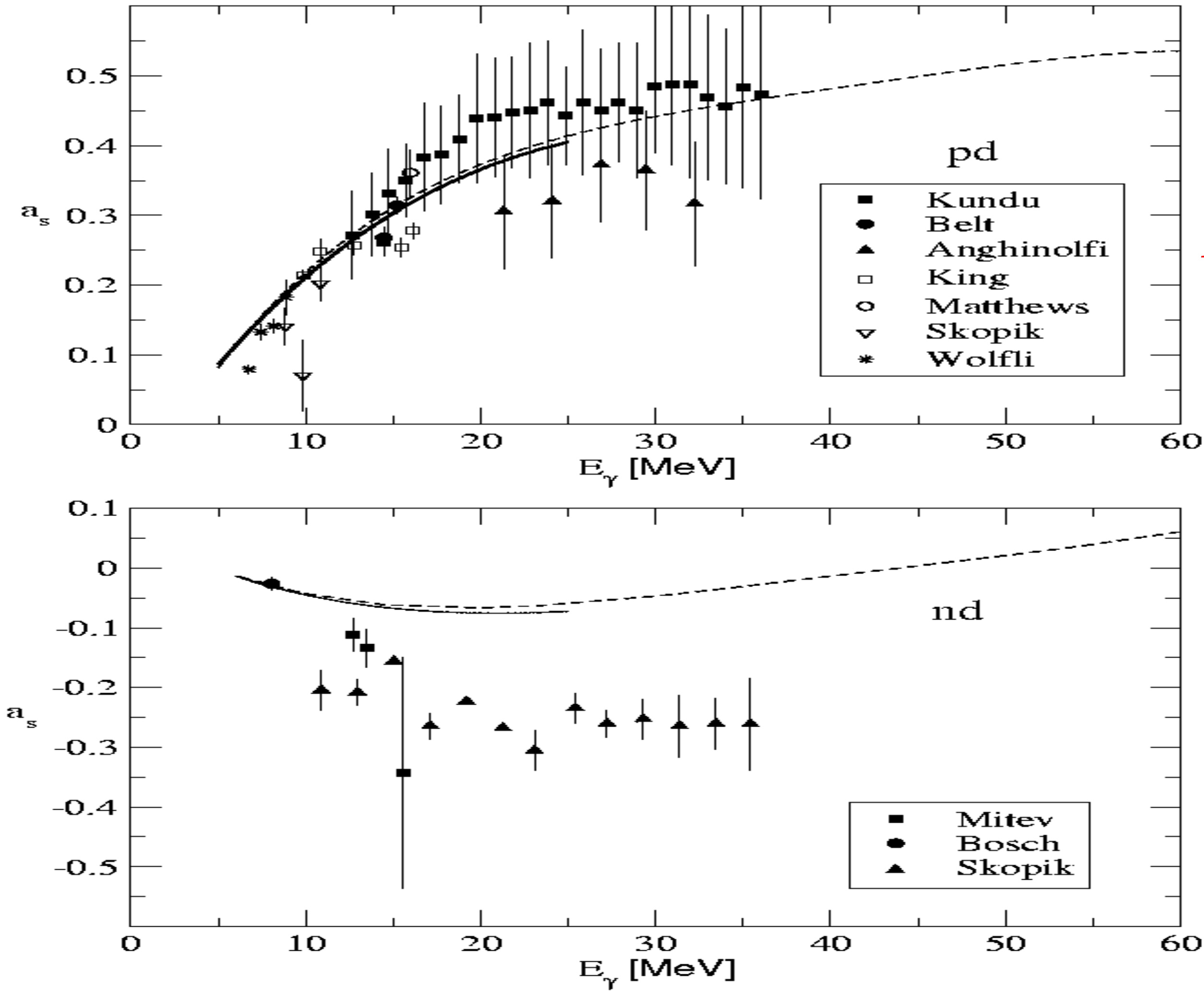


$n+d \rightarrow {}^3H + \gamma$ (data from Mitev *et al.*, 1986)



Foreaft asymmetry

$a_S = [\sigma(55°) - \sigma(125°)] / [\sigma(55°) + \sigma(125°)]$

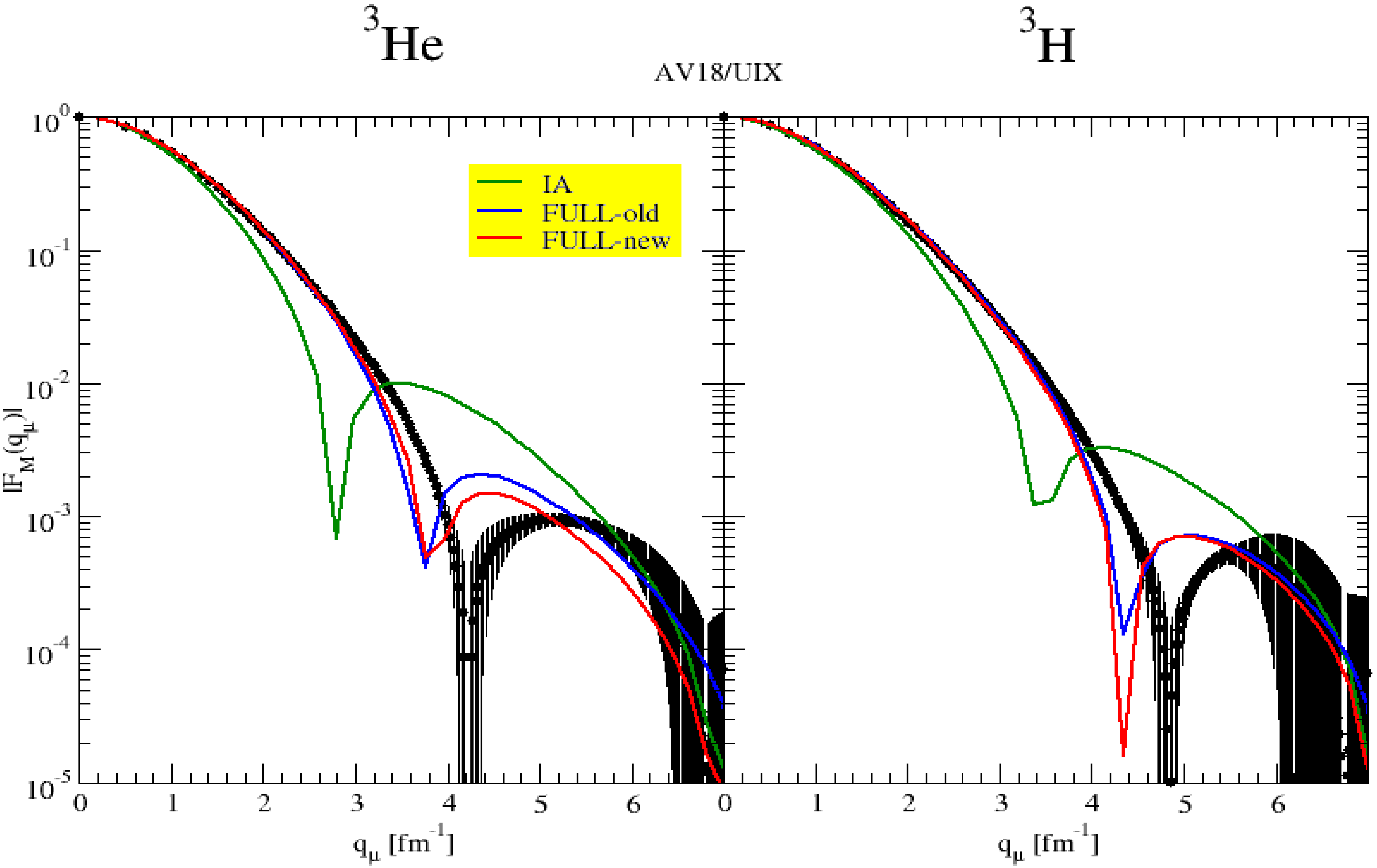


$p + d \rightarrow {}^3He + \gamma$

$n + d \rightarrow {}^3H + \gamma$

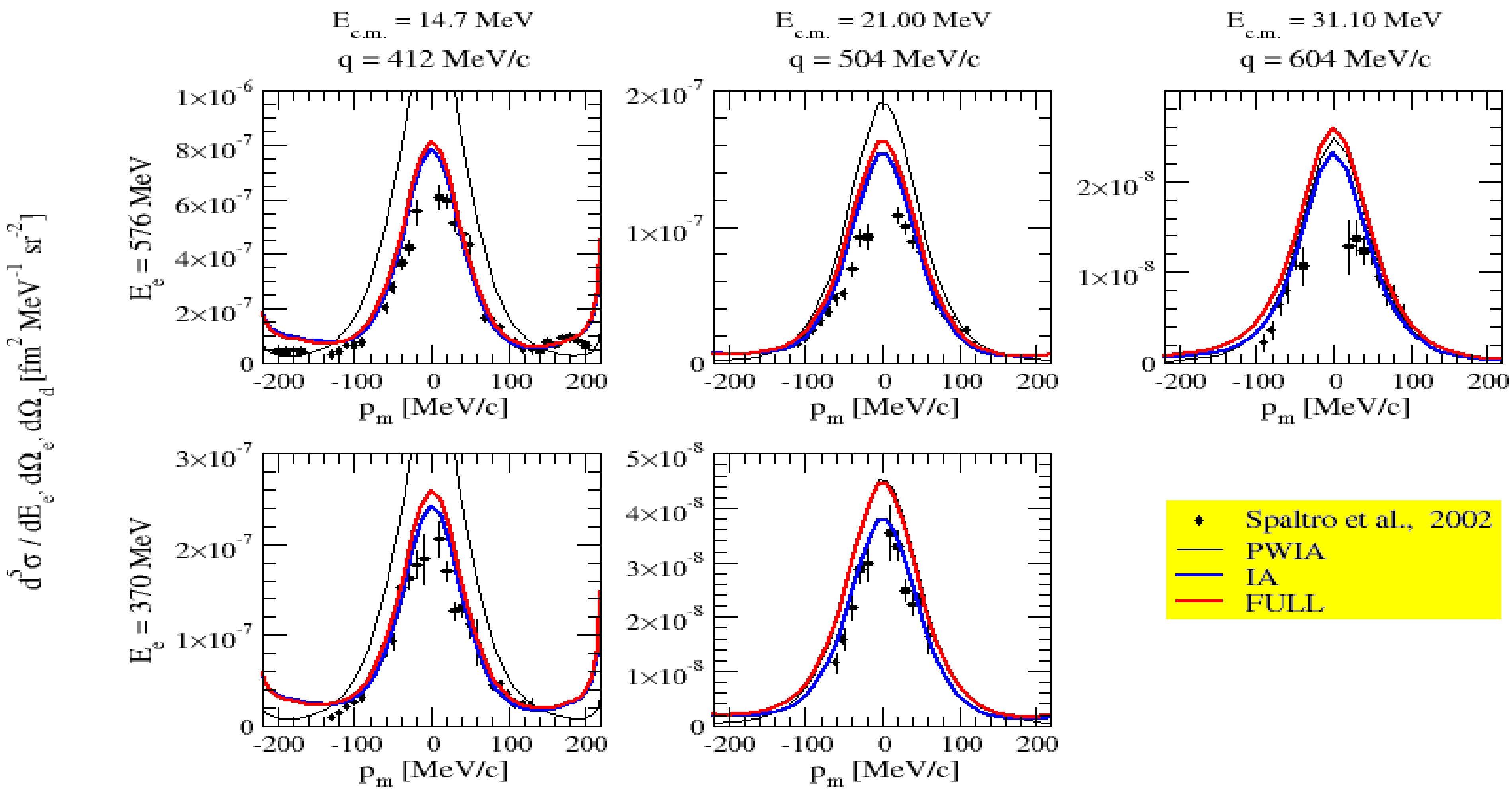
$n+d \rightarrow {}^3\mathrm{H} + \gamma$ radiative capture at thermal energies

Current component	σ_T [mb]
1b	0.227
1b+2b-MI-old	0.462
1b+2b-MI-new	0.418
1b+2b-MI+MD	0.523
Viviani *et al.*, PRC **54**, 1996	0.578
1b+2b+3b	0.556
Expt.	0.508(15)



^{3}He
^{3}H
AV18/UIX
IA
FULL-old
FULL-new
$|F_M(q_\mu)|$
q_μ [fm^{-1}]
q_μ [fm^{-1}]

^{3}He(e,e' d)p in (q,ω)-constant kinematics



Summary

- Model for the nuclear EM current operator which satisfies by construction the CCR with the AV18/UIX
- Overall nice description for $A=2$ and 3 observables, except for vector polarization observables of $p+d \rightarrow {}^3\mathrm{He}+\gamma$, trinucleon MFF, and $n+d \rightarrow {}^3\mathrm{H}+\gamma$ σ_T at thermal energies and a_S
- Small three-body currents effects in tensor observables of $p+d \rightarrow {}^3\mathrm{He}+\gamma$

Outlook

- Further investigation in the A=3 sector: more observables, especially for ^{3}He$(e,e'd)p$ in a "better" energy range
- Implementation for A=4
- Implementation of the theoretical framework to work in p-space

H.H. expansion in *p*-space

$$\Psi(1....A)=\sum_{\{G\}} u_{\{G\}}(r)\; Y_{\{G\}}(\Omega)$$

Fourier Transform

$$\Psi(1....A)=\sum_{\{G\}} w_{\{G\}}(k)\; Y_{\{G\}}(\Omega_k)$$

$$w_{\{G\}}(k)= \frac{(-i)^G}{(2\pi)^3} \int dr \, \frac{r^{D-1}}{(kr)^{D/2-1}} J_{L+1/2}(kr)\, u_{\{G\}}(r)$$

$D=2(A-1)$ $\quad$ $L=G+(D-3)/2$

Expansion of $u_{\{G\}}(r)$:

$$u_{\{G\}}(r) = \sum_{i=1,N} C_i \; e^{-\alpha(i)\, r}$$

Preliminary results: tests with AV18

AV18, p-space, j_{max}=4, vs. r-space results obtained solving the diff. equations

	N=8	N=12	N=16	D.E.
3 ch	-6.944	-6.971	-6.973	-6.980
8 ch	-7.551	-7.581	-7.583	-7.592
12 ch	-7.567	-7.597	-7.599	-7.609
18 ch	-7.574	-7.604	-7.607	-7.617

AV18, r-space, 3 channels

D.E.	j_{max}=4	j_{max}=6	j_{max}=8
-6.980	-6.973	-6.978	-6.979

Preliminary results: CD Bonn 2000

VERY PRELIMINARY !!!

CDBonn 2000, p-space, j_{max}=6

	N=8	N=12	N=16
3 ch	-7.686	-7.695	-7.695
8 ch	-7.926	-7.935	-7.936
12 ch	-7.936	-7.945	-7.946
18 ch	-7.942	-7.952	
23 ch	-7.964	-7.974	

T=1/2 (3 ch – 18 ch)

T=3/2 (23 ch)

Faddeev:		H.H.:	
-8.005			(with $m_p \neq m_n$)
-7.997	vs.	-7.975	(with $\overline{m}$, T=3/2)
-7.972		-7.953	(with m, T=1/2)

A=4 PRELIMINARY RESULTS for B (MeV) M.Viviani

CD-Bonn 2000	H.H. F.Y.	26.23 26.26
N^3LO (Entem & Machleidt)	H.H. F.Y. NCSM	25.09 25.41 25.36
$V_{low\text{-}k}(\Lambda=2.1\ fm^{-1})$ (Napoli group)	H.H.	27.95