Renormalization of the 1π exchange in higher partial wave

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- "Weinberg counting" works
- Quantitative analysis of the cutoff dependence for the NN system ....
- ... and what we propose to do about it
- Comparison to experimental phase shifts
- Application to the 3N bound state
- Implications for the power counting
- Conclusions and Outlook

in collaboration with Bira van Kolck & Rob Timmermans

(see AN,UvK,RGET, nucl-th/0506005)

“Weinberg counting“ works

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“Weinberg counting” works !

The nuclear potential can be expanded according to a power counting and the Schrödinger equation needs to be solved non-perturbatively

1) results are in a good (and with each order improving) agreement with NN, 3N, ... data (Ordóñez et al. PRC 53, 2086
Epelbaum et al. NPA 671, 295; NPA 747,632
Entem et al. PRC 68, 041001)

2) extraction of LEC’s from πN and NN data agree well
(Büttiker et al. NPA 668, 97 & Rentmeester et al. PRC 67, 044001)

Cutoffs are needed to regularize the Schrödinger equation.
Here we want to quantify the dependence of observables (phase shifts) on these cutoffs in LO, namely with 1π exchange.

Are the NN phase shifts cutoff independent?
Is Weinberg power counting consistent in LO in the 3N system?
Do 3NF need to be promoted to higher orders as in pionless EFT?

Where and why are results cutoff dependent?

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In higher partial waves, in LO, the NN interaction is the 1π exchange w/o contact terms.

Cutoff dependence can be expected since this is a singular interaction.

$$V_{1\pi}(\vec{r}) = \frac{m_\pi^3}{12\pi}\left(\frac{g_A}{2f_\pi}\right)^2 \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,[T(r)\;S_{12} + Y(r)\;\vec{\sigma}_1\cdot\vec{\sigma}_2]$$

$$T(r) = \frac{e^{-m_\pi r}}{m_\pi r}\left[1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right]$$

$$Y(r) = \frac{e^{-m_\pi r}}{m_\pi r},$$

⇒ No unique solution in partial waves, for which the $1/r^3$ part (tensor force) is attractive (see e.g. Frank et al. RMP 43,36)

This implies that one necessarily finds dependence on the regulator in attractive triplets

Which triplets are attractive?

- Look at $\tau_1 \cdot \tau_2 \; S_{12}$

$\tau_1 \cdot \tau_2 \; S_{12}$

	$s=1$	$l=j-1$	$l=j$	$l=j+1$
$t=1$	$l'=j-1$	$-2\,\frac{j-1}{2j+1}$	0	$6\frac{\sqrt{j(j+1)}}{2j+1}$
	$l'=j$	0	2	0
	$l'=j+1$	$6\frac{\sqrt{j(j+1)}}{2j+1}$	0	$-2\,\frac{j+2}{2j+1}$
$t=0$	$l'=j-1$	$6\,\frac{j-1}{2j+1}$	0	$-18\frac{\sqrt{j(j+1)}}{2j+1}$
	$l'=j$	0	-6	0
	$l'=j+1$	$-18\frac{\sqrt{j(j+1)}}{2j+1}$	0	$6\,\frac{j+2}{2j+1}$

We can expect a problematic cutoff dependence in ${}^3P_0$, ${}^3P_2$-${}^3F_2$, ${}^3D_2$, ${}^3D_3$-${}^3H_3$ ...

1) Are singlets and repulsive triplets cutoff independent? Is any form of renormalization necessary for those partial waves?

2) In which range of cutoffs are observables (phase shifts) dependent of the cutoffs?
Is there a range of cutoffs, which is "optimal",
as suggested in e.g. J. Gegelia et al., nucl-th/0403052.

Numerical approach

The calculations were performed in momentum space

(the most important results were confirmed in configuration space)

1π exchange

$$V_{1\pi}(\vec{q}) = -\frac{1}{(2\pi)^3}\left(\frac{g_A}{2f_\pi}\right)^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{(\vec{\sigma}_1 \cdot \vec{q})\,(\vec{\sigma}_2 \cdot \vec{q})}{\vec{q}^2 + m_\pi^2}$$

regularized with $f(p', p) = e^{-(p^4 + p'^4)/\Lambda^4}$.

The LS equation

$$T_{ll'}(p,p') = V_{ll'}(p,p') + \sum_{l''} \int dp''\, p''^2\, V_{ll''}(p,p'')\, \frac{m_N}{m_N E + i\varepsilon - p''^2} T_{l''l'}(p'',p')$$

is solved for Λ between 2 fm^{-1} and 20 fm^{-1}.

This range starts for values a little bit smaller than the ones usually used and extends to values well beyond Λ_{QCD}

This study is performed using the physical π mass.

We won't be able to learn anything about the π mass dependence of counter terms.

Singlet phase shifts

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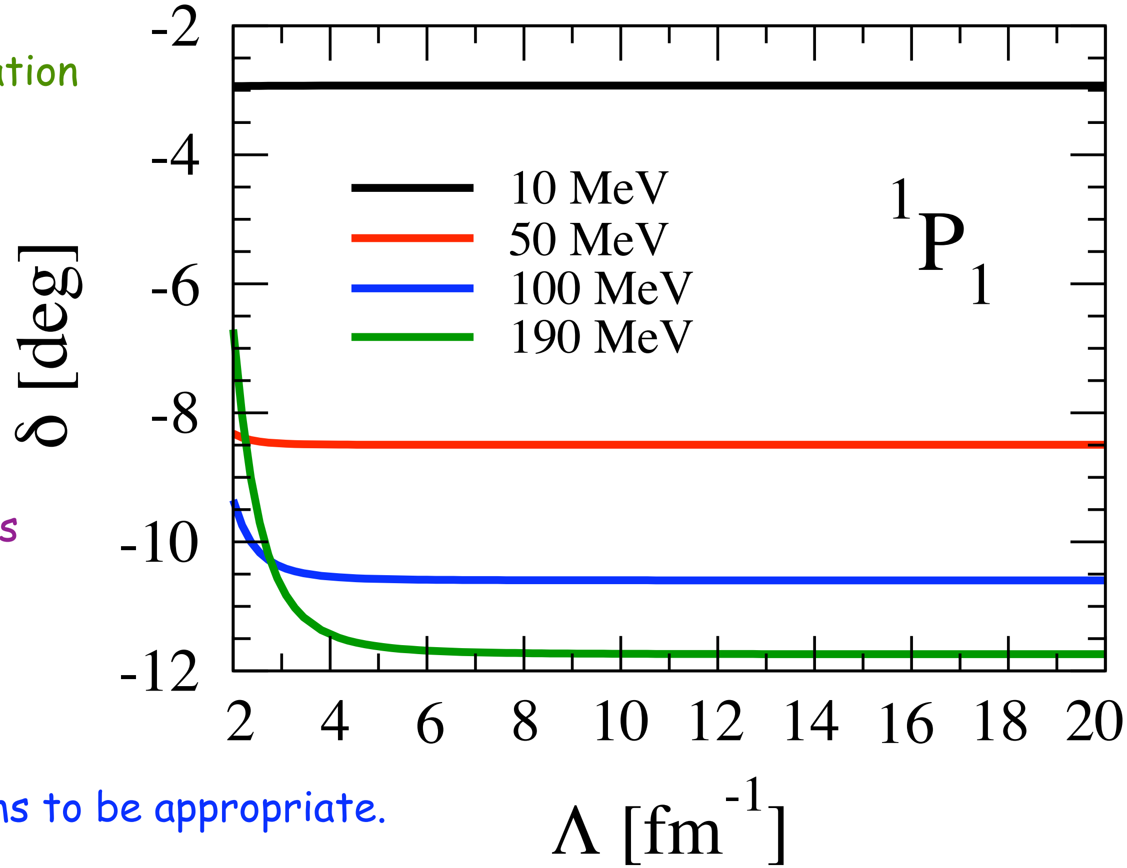
Numerically, one does not observe any cutoff dependence in singlets for large cutoffs.

This indicates that no renormalization is necessary in these channels.



The speed of convergence depends on the energy
(but not on the partial wave)

Up to 100 MeV, Λ = 3-4 fm^{-1} seems to be appropriate.

Also Λ = 2.5 fm^{-1} leads to a reasonably independent result.

For 190 MeV, Λ = 5-6 fm^{-1} insures almost converged results.

Numerically, no inconsistency of the power counting is found in singlets.

Λ dependence of singlet waves

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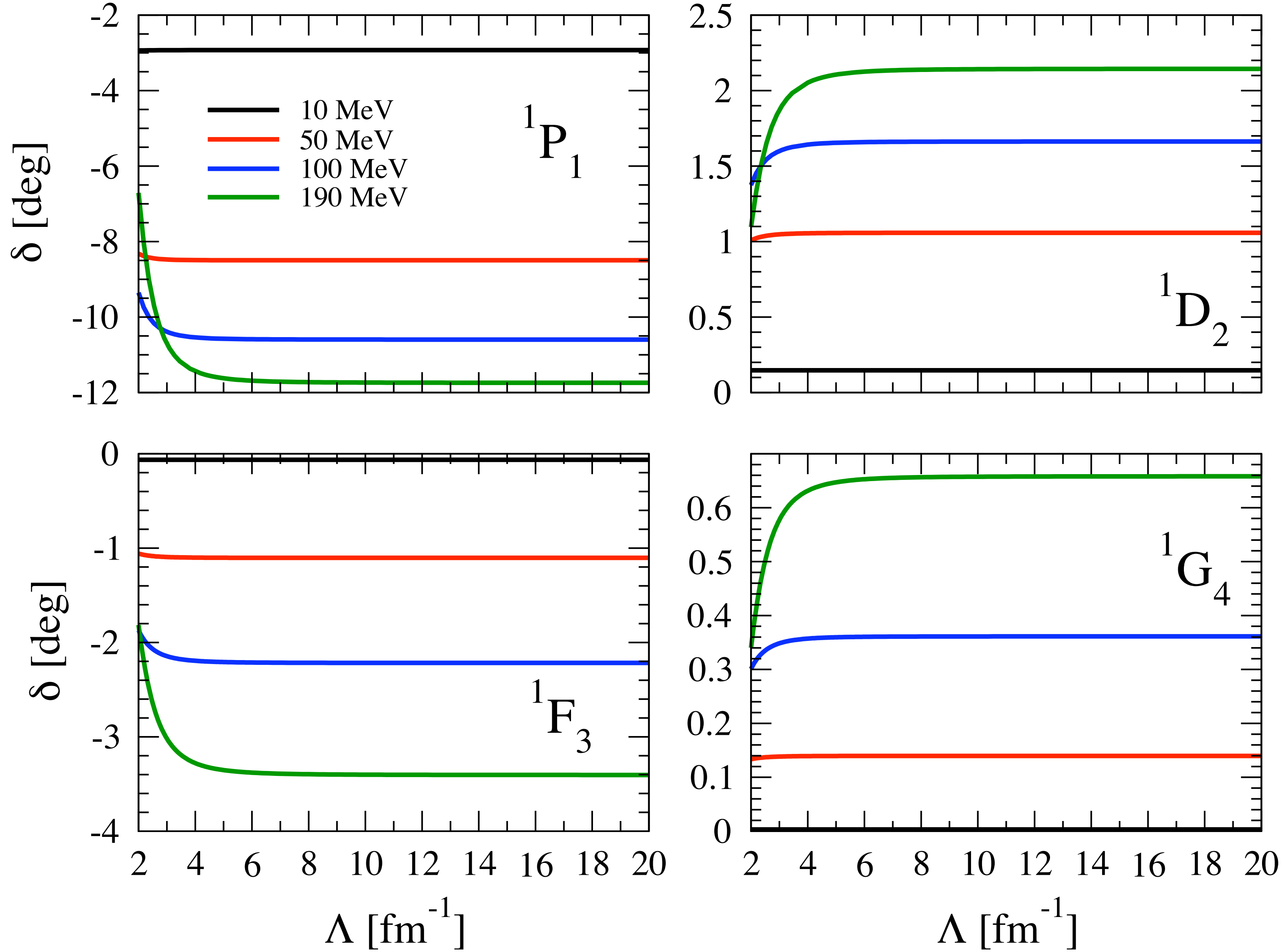

Repulsive triplet phase shifts

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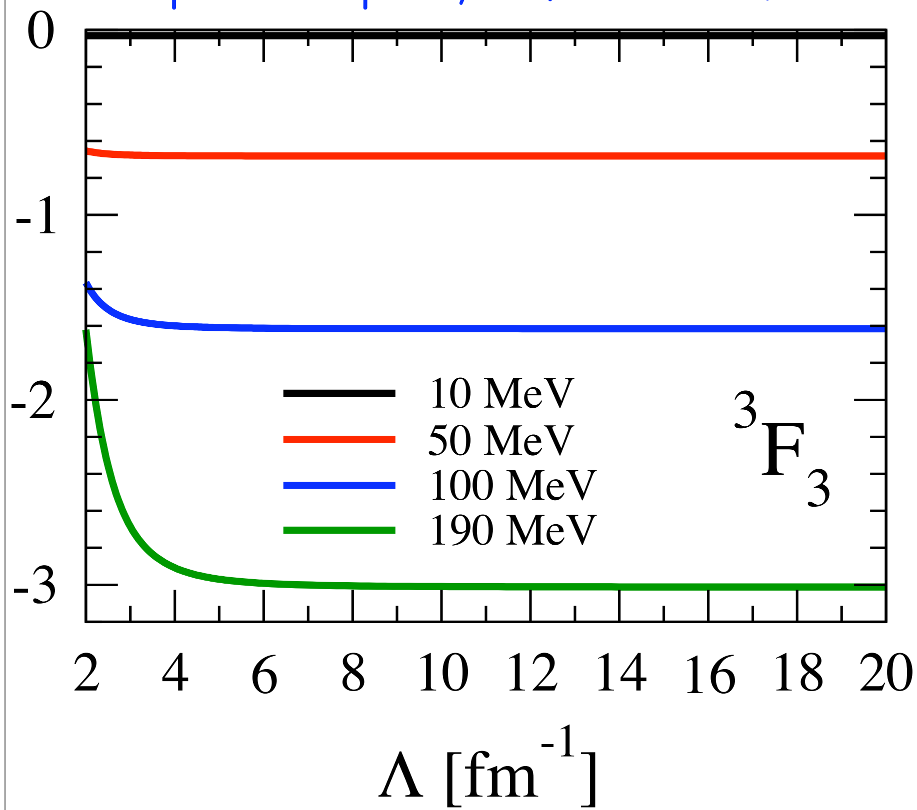
In repulsive triplets, we find the same behavior for the Λ dependence.



Numerically, there is no inconsistency of the power counting seen in repulsive triplets.

No terms appear which scale like Λ^n as was argued e.g. by Kaplan et al. NPB 478,629 and recently by Gegelia et al. nucl-th/0403052 based on a perturbative calculation

The perturbative series does not reflect the renormalization behavior for this non-perturbative problem (see Beane et al. NPA 700,377).

This is here confirmed for higher partial waves and the regularization in momentum space.

But what happens for attractive triplets?

Attractive triplet phase shifts

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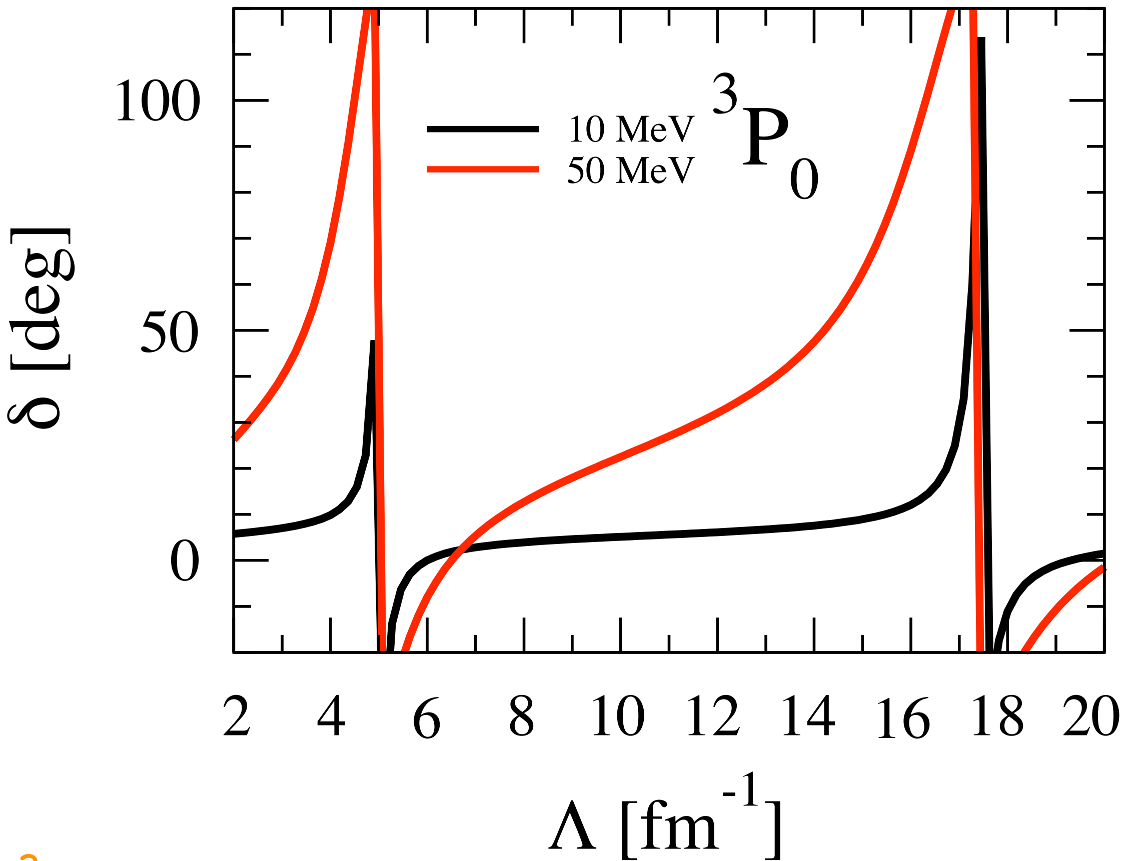
We find the expected cutoff dependence for attractive triplet channels



We confirm the limit cycle behavior for attractive singular interactions

Size and slope within the cycles depends on the partial wave

The slope of the "plateau region" also depends on the energy

$^3P_0$ is the worst case, because the variation is strongest for small cutoffs and the slope is the steepest one.

Is a renormalization possible for these partial waves?

Attractive triplet phase shifts

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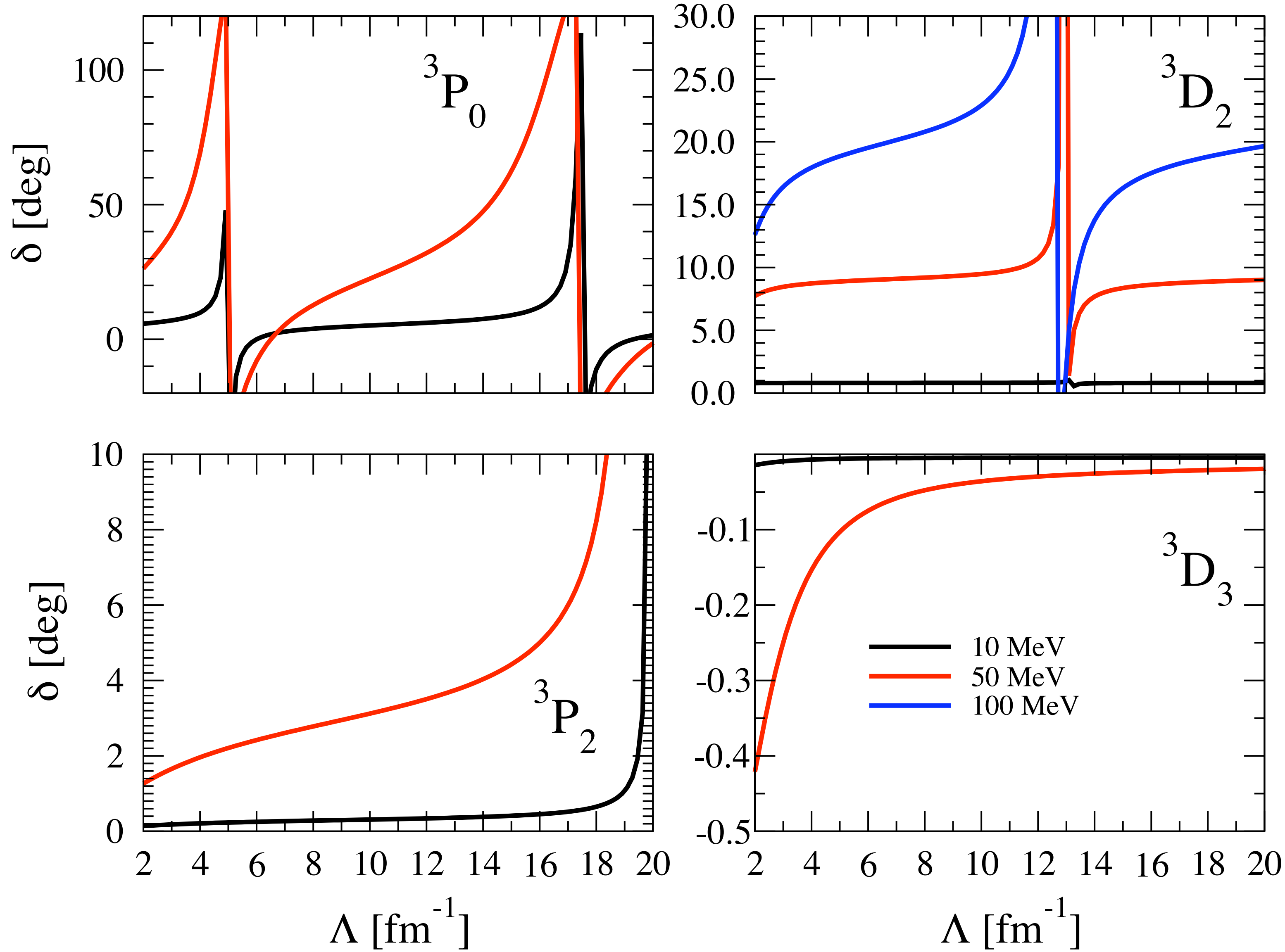
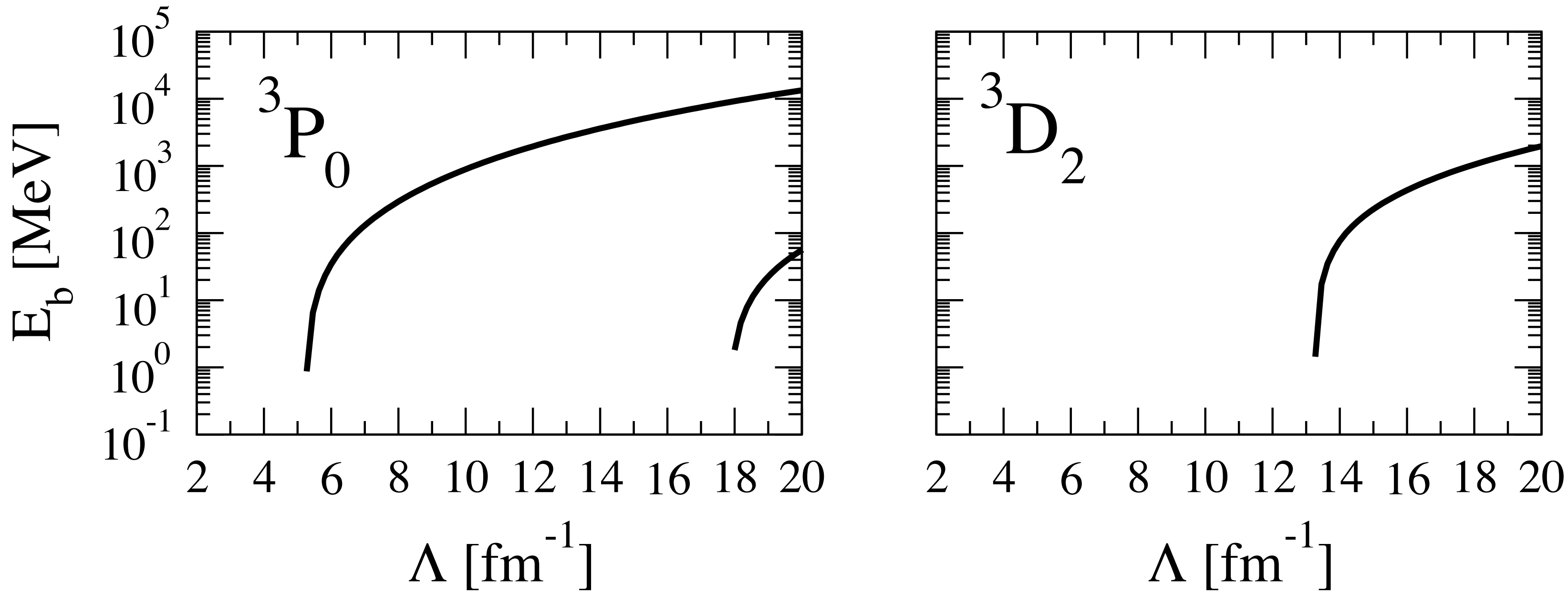


Binding energies of spurious bound states

- This cutoff dependence is of course induced by spurious bound states
- For $\Lambda \leq 20\ \text{fm}^{-1}$, we find bound states in $^3P_0$, $^3D_2$ (and almost in $^3P_2$-$^3F_2$)



- The binding energies increase very rapidly to several hundred MeV

Counter terms in triplet channels

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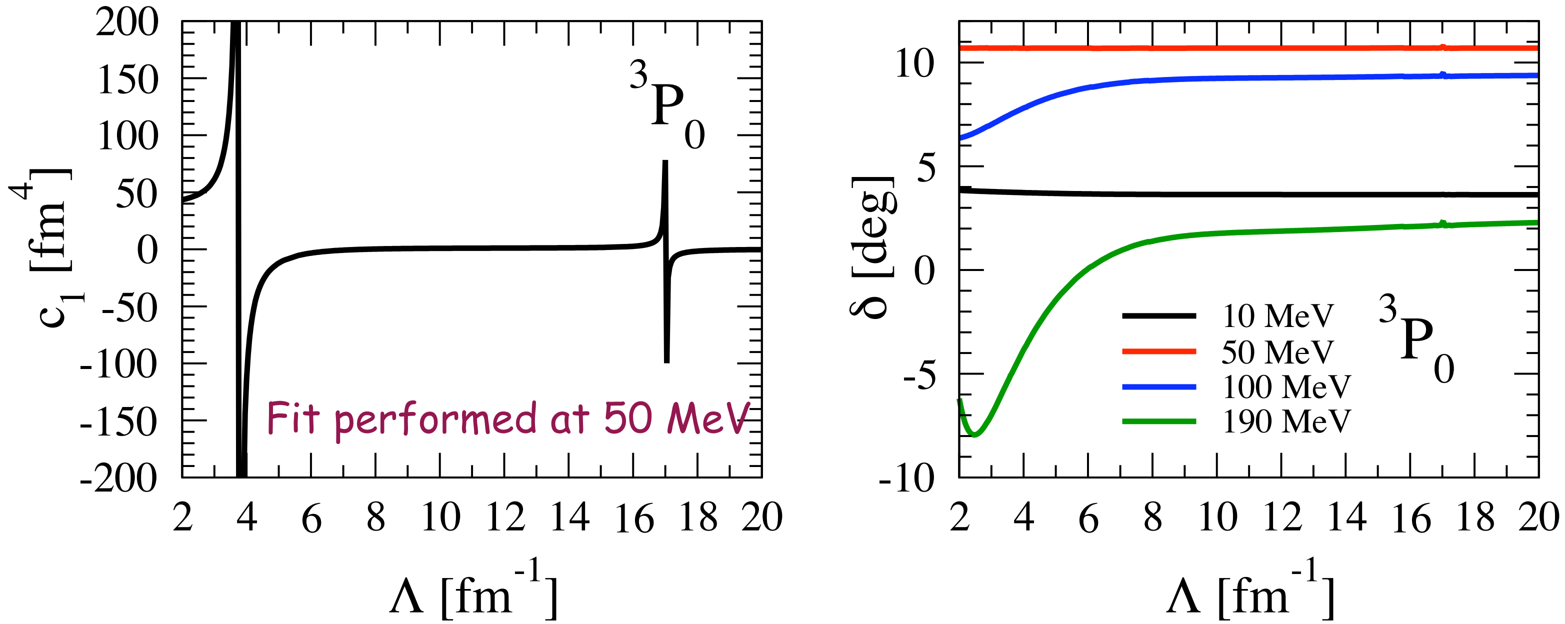
The renormalization of singular interactions is possible with
1 counter term (boundary condition) per partial wave (see e.g. Frank et al.)

In LO, this requires the promotion of counter terms from naïvely higher orders.

We use $V_i = \frac{c_i}{(2\pi)^3}\, p' p$ $\quad V_d = \frac{c_d}{(2\pi)^3}\, p'^2 p^2.$ in P, D-waves, respectively.



As expected, we obtain Λ independence for all energies. The partial wave can be renormalized with one counter term (The same is true in $^3P_2$-$^3F_2$ and $^3D_2$ and $^3S_1$-$^3D_1$)

Counter terms in other triplet channels

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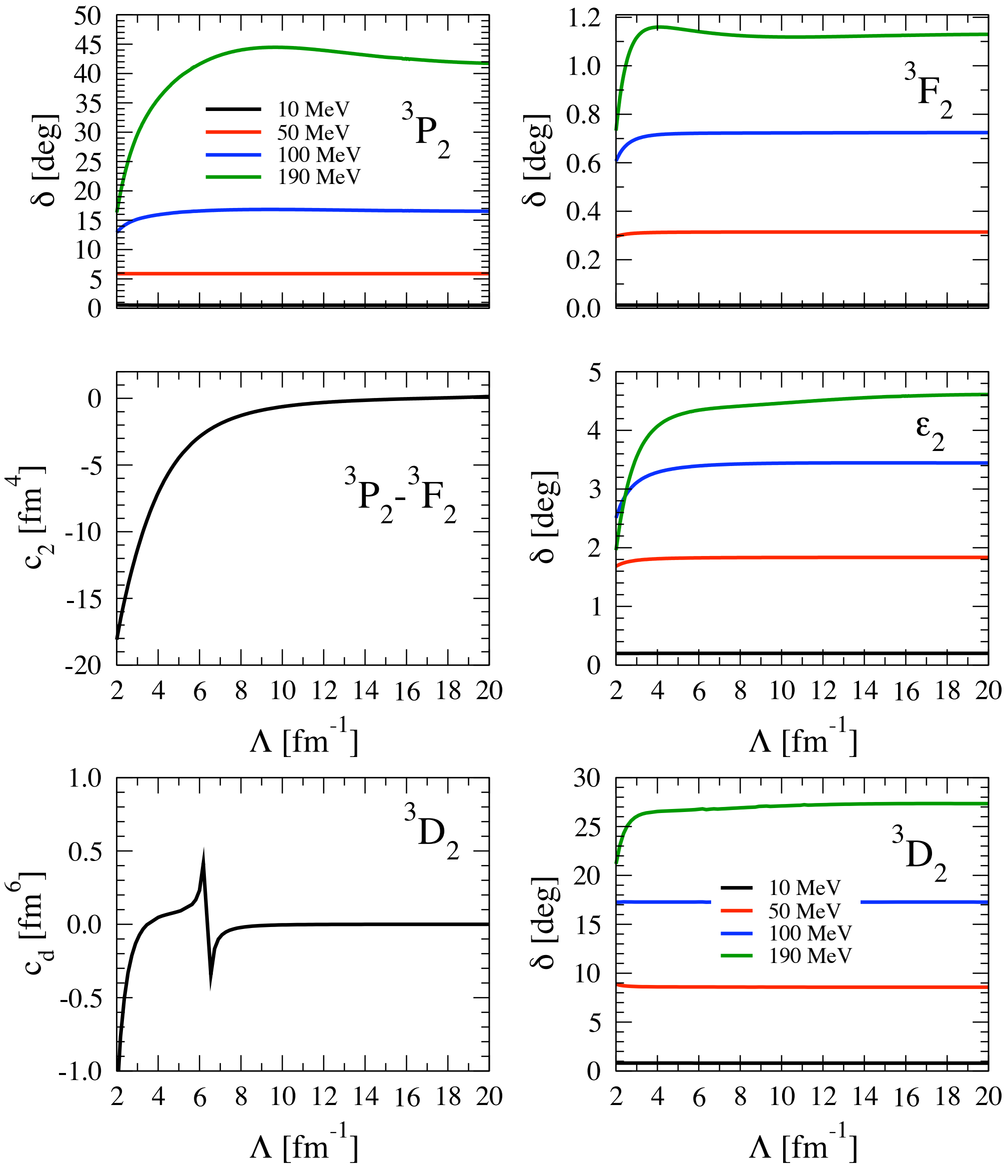

Fit performed at 50 MeV

Fit performed at 50 MeV

Fit performed at 100 MeV

Spurious bound states in the triplet channels

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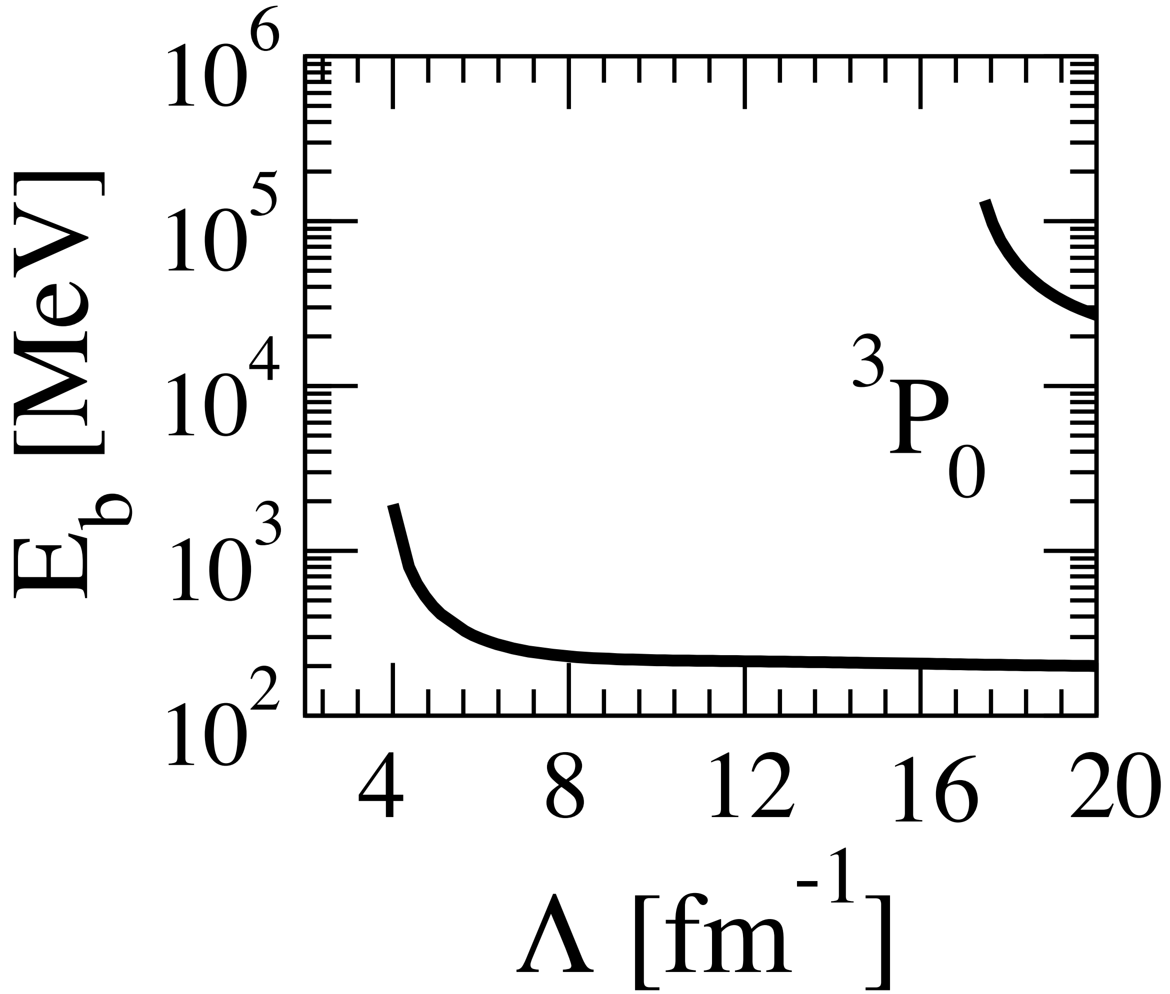
We still find spurious bound states.

They are, however, deeply bound for all cutoffs.

Obviously, they do not influence low energy phase shifts.

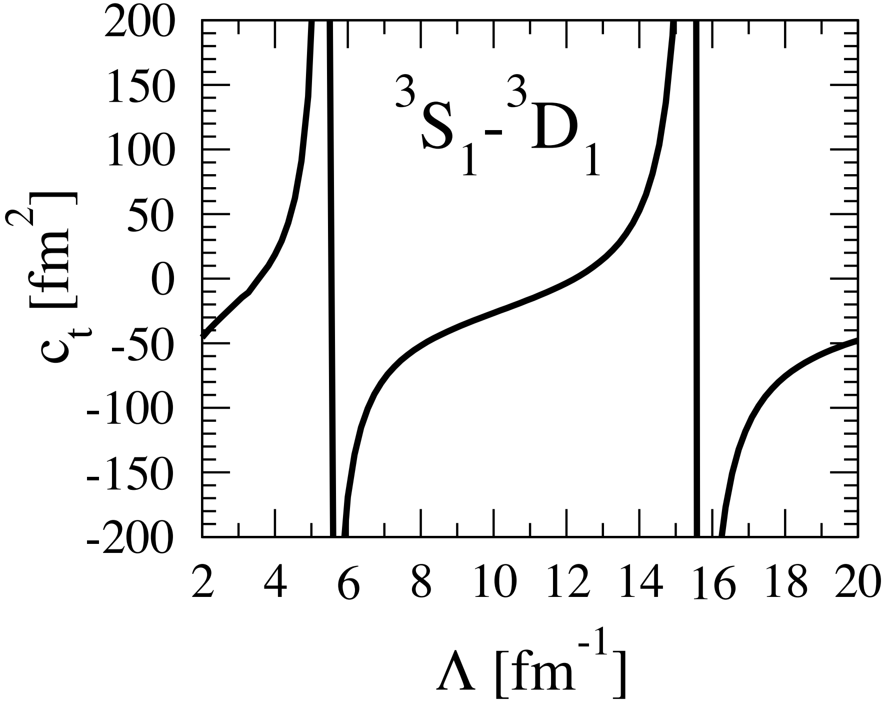


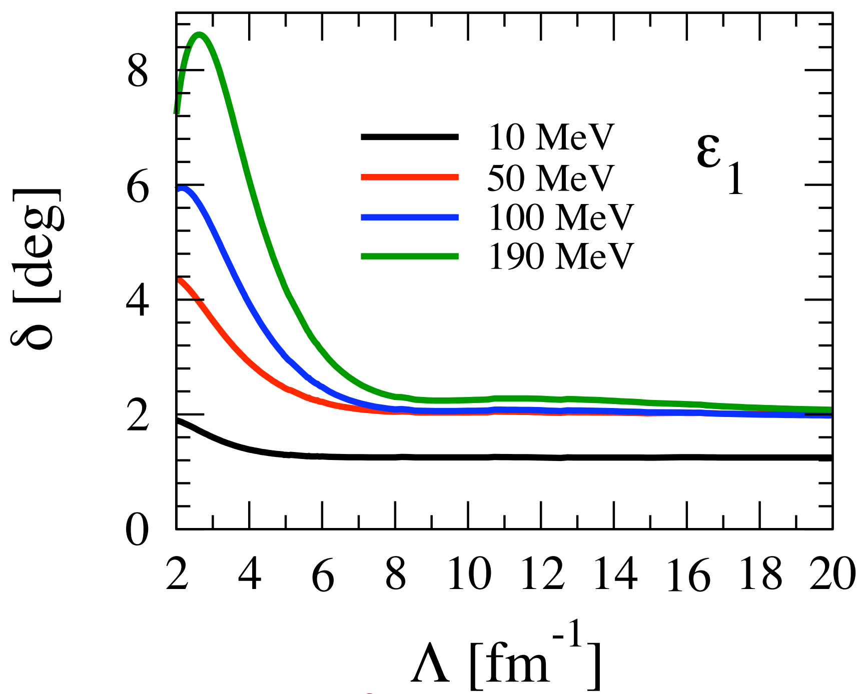
Counter terms in S-wave channels

S-waves were previously investigated by Frederico et al. NPA 653,209;
Beane et al. NPA 700, 377;
Valderrama et al. PRC 70,044006;
Valderrama et al. nucl-th/0504067 ...

We reconfirm that the LO counter terms absorb the cutoff dependence also for our momentum space regulator.

The deuteron binding energy converges to **-1.92 MeV** (or can be fitted to experiment).





The fit was performed for the $^3S_1$ phase shift at 10 MeV.

Phase shifts singlets & repulsive triplets

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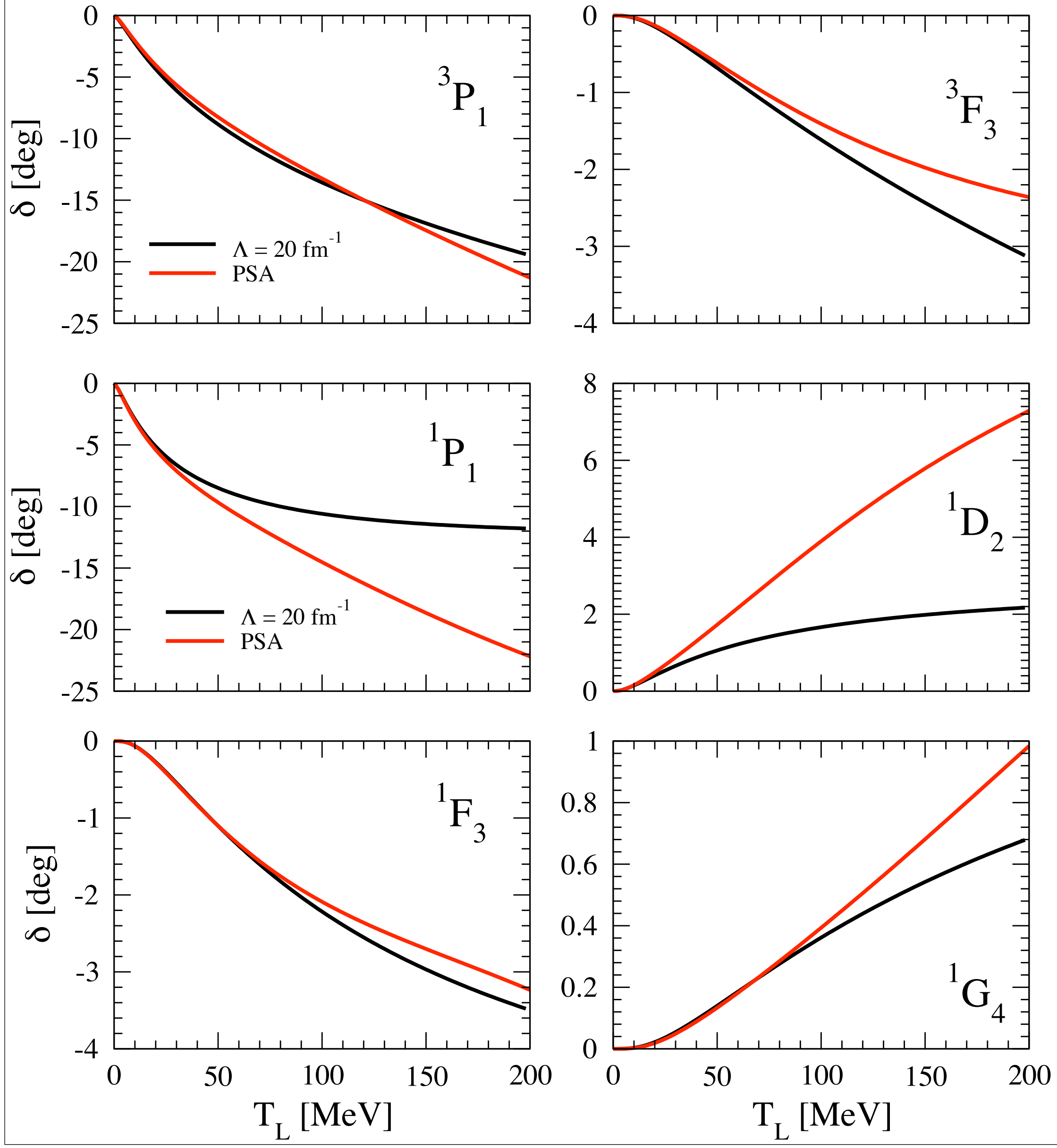


All predictions are qualitatively similar for smaller cutoffs (see e.g. Epelbaum et al. NPA 671,295)

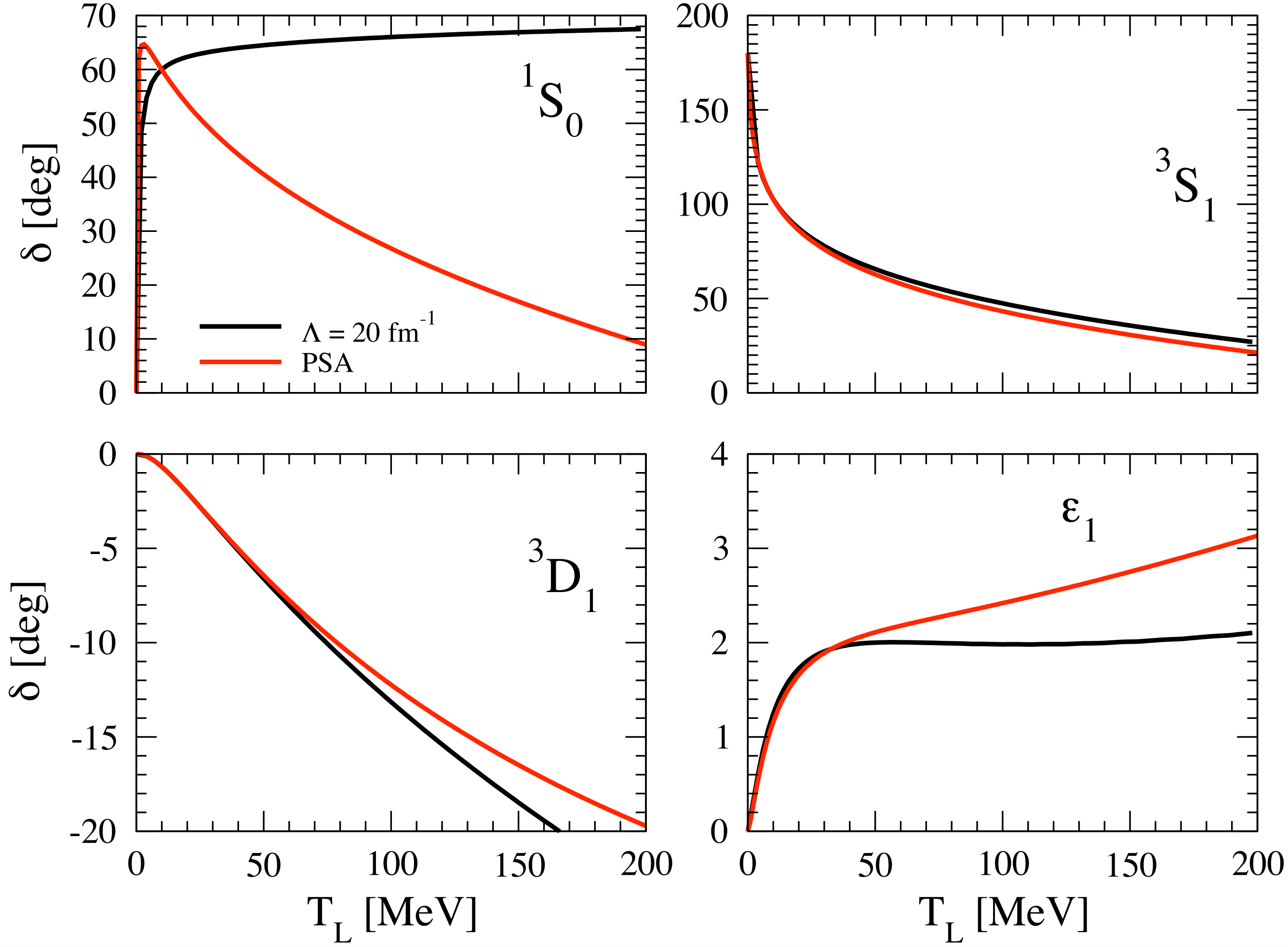
Phase shifts in these channels are complete predictions.

S-wave phase shifts

Again the predictions are similar for smaller cutoffs.

ϵ_1 is now underpredicted, the prediction seems to be improved.

The $^1S_0$ prediction is still poor.



Attractive triplet phase shifts (I)

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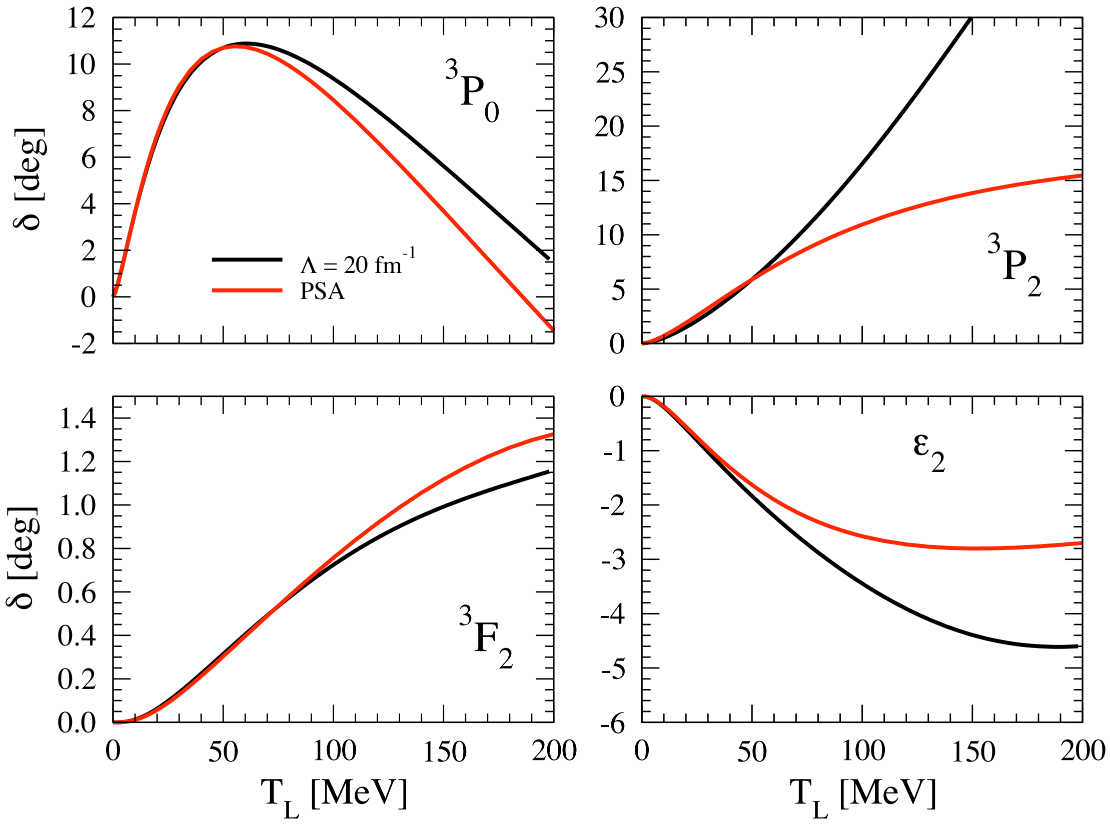

We obtain a very good prediction of the energy dependence for $^3P_0$!

For $^3P_2$-$^3F_2$, the LO predictions are better for smaller cutoffs,

but the NLO predictions for small cutoffs are similar than the ones here.

Attractive triplet phase shifts (II)

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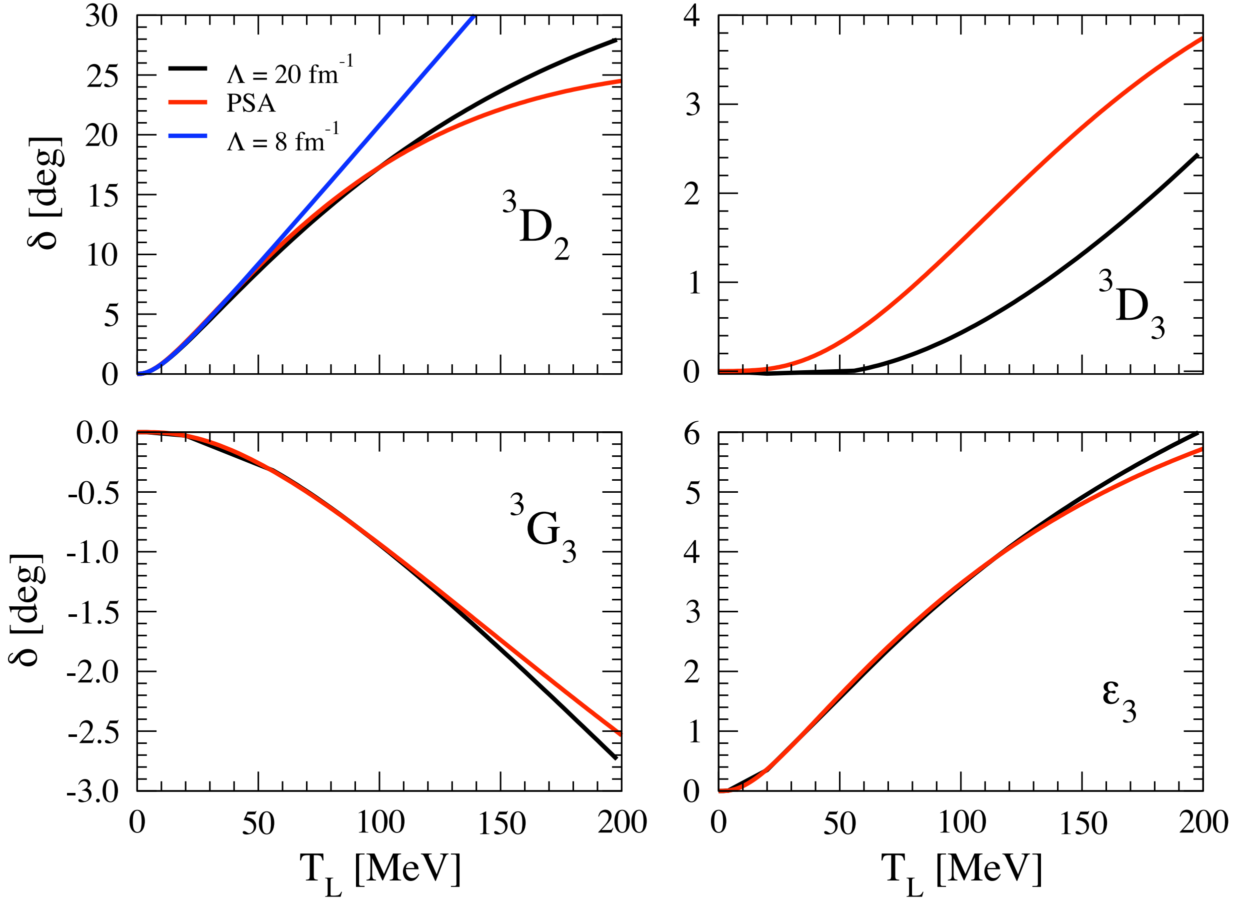

The increase of the cutoffs does not compromise the description of data.

For cutoffs in the plateau region, the data is well described without counter terms.

(see especially $^3D_2$)

Attractive triplet phase shifts (III)

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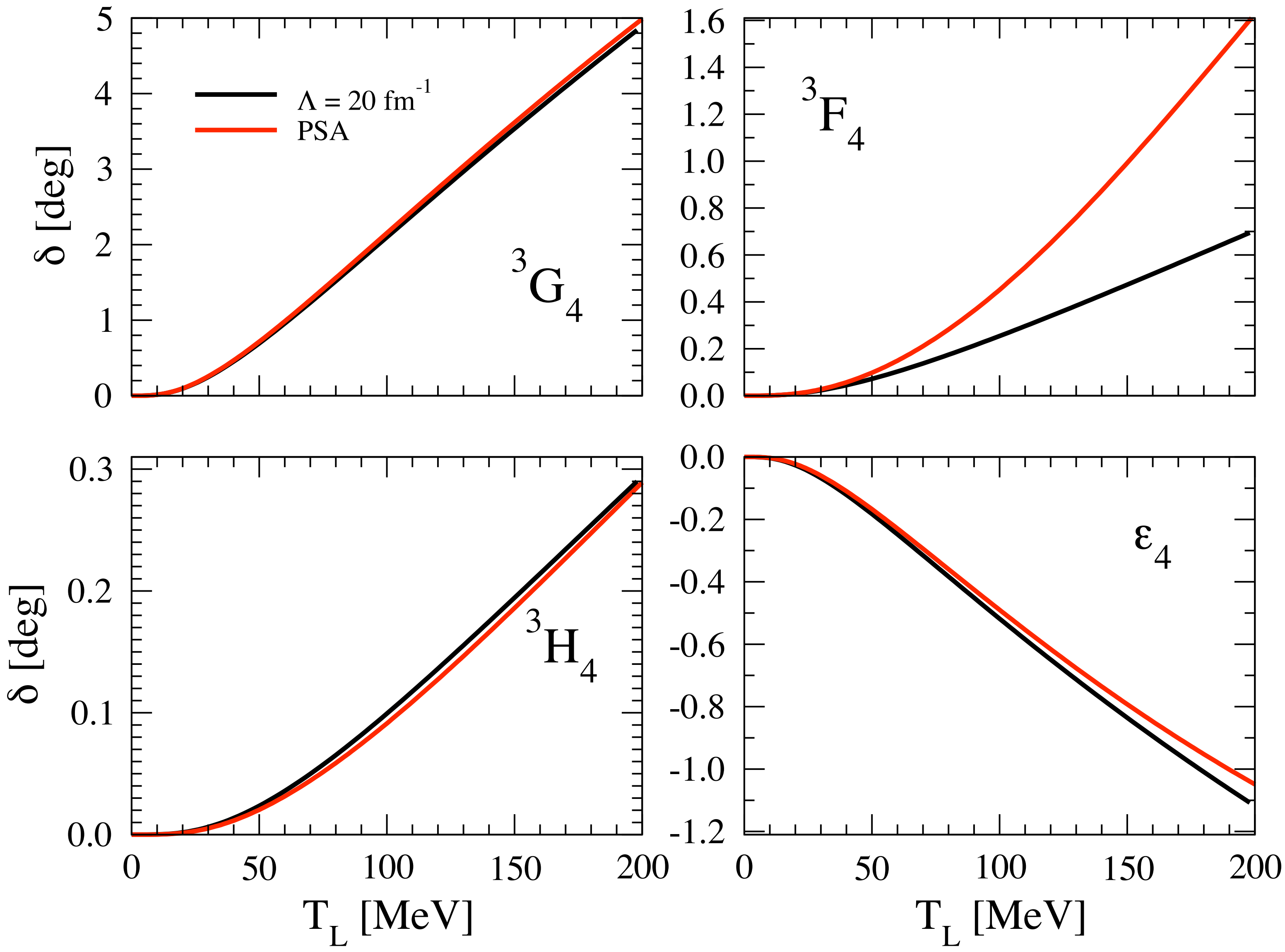

Deuteron w/ E=2.23 MeV

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- We performed a refit that reproduces the deuteron binding energy.
  Phase are still well described.

$\Lambda [fm^{-1}]$	E [MeV]	T [MeV]	P_D[%]	A_S [$fm^{-1/2}$]	η	r [fm]	Q_d [fm^2]	n
2	2.225	28.91	5.24	0.839	0.030	1.889	0.3005	1
3	2.225	38.45	8.09	0.855	0.028	1.913	0.2942	1
4	2.225	45.48	8.23	0.866	0.027	1.933	0.2827	1
5	2.225	53.53	7.49	0.867	0.025	1.935	0.2747	1
6	2.224	62.33	6.94	0.866	0.025	1.932	0.2704	2
7	2.225	70.16	6.73	0.865	0.025	1.928	0.2683	2
8	2.225	75.95	6.76	0.864	0.026	1.926	0.2676	2
10	2.227	81.99	7.00	0.864	0.026	1.925	0.2674	2
12	2.227	85.80	7.14	0.864	0.026	1.925	0.2675	2
14	2.224	91.94	7.14	0.863	0.026	1.926	0.2675	2
∞	2.225	--	7.88	0.8681	0.026	1.9351	0.2762	---
Expt.	2.225	—	—	0.8846	0.026	1.9671	0.2859	1

(∞ from Valderrama et al. nucl-th/0506047)

Towards 3N: Subtracting the bound states

To solve the 3N problem, we need the t-matrix.

The spurious bound states are cumbersome in few-body calculations!

⇨ We subtract the spurious bound states from the NN t-matrix and solve the Faddeev equation with the modified one.

$$\bar{V} = V + |\chi\rangle \; \lambda \; \langle\chi| \; ,$$

⇨ $$\bar{t} = t - |\eta\rangle \; \frac{1}{\langle\chi|G_0|\eta\rangle} \; \langle\eta| \; ,$$

$$|\eta\rangle = |\chi\rangle \; + t \; G_0|\chi\rangle$$

The expectation value based on the non-subtracted potential agrees with the binding energy.

3N binding energy

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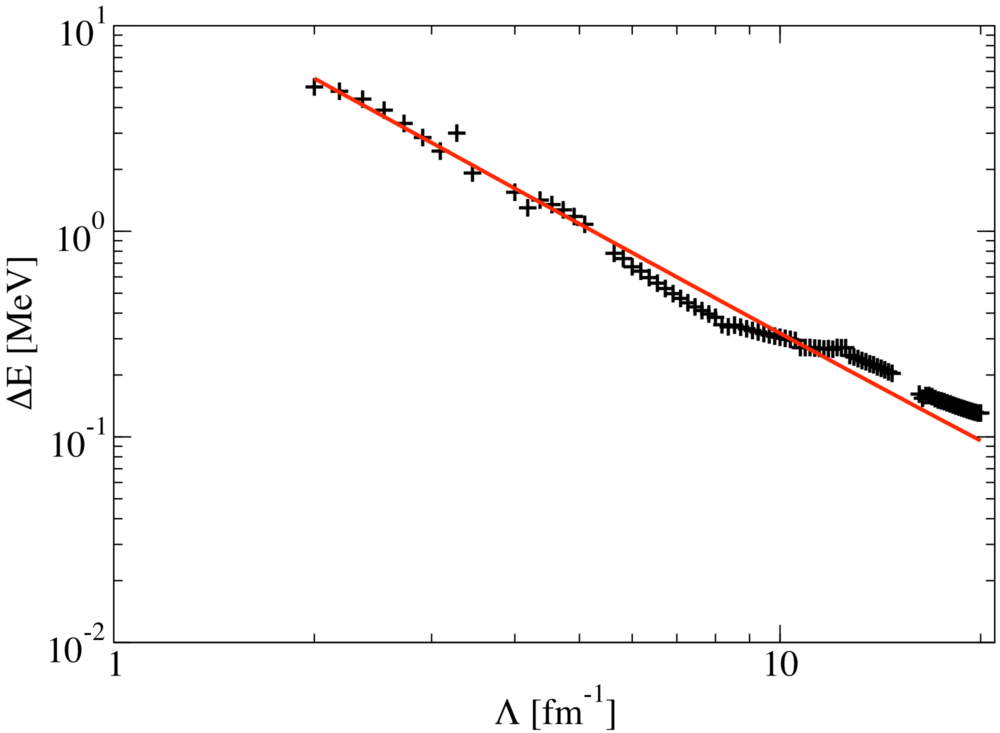

$$E(\Lambda) = E_0 \left[1 + \left(\frac{C}{\Lambda} \right)^x \right]$$

$$E_0 \approx -3.6 \text{ MeV}$$
$$x \approx 1.8$$
$$C \approx 2.54 \text{ fm}^{-1}$$

The calculation confirms that no 3NF's are necessary in LO.

(The additional physics input compared to pionless EFT pays off)

The binding energy prediction is rather small, larger values are obtained for small cutoffs. NLO will be extremely interesting to see rate of convergence - $^1S_0$?.

3N binding energy non-log-scale

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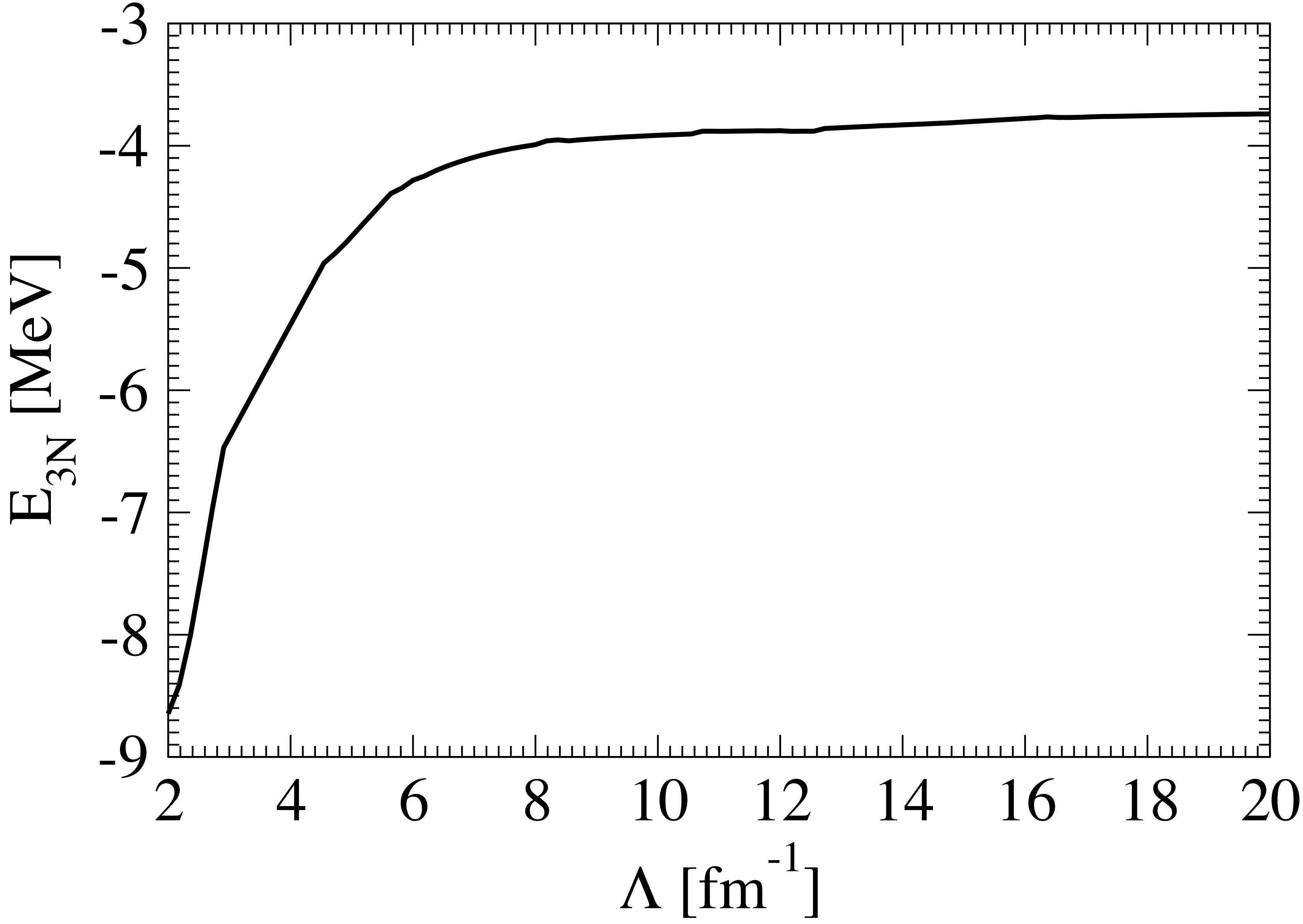

Implications for the power counting

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- In principle, for arbitrary large cutoffs, we require an infinite set of counter terms

  → this upsets the power counting for few-body systems, where all angular momenta contribute

- However, everyone agrees that we don't need all partial waves
  Can we formulate that in terms of power counting?

- What is the suppression of $G_0 V$?

- Weinberg (due to infrared enhancement): $$G_0 V = \frac{m_N Q}{4\pi} \frac{1}{f_\pi^2} \alpha_{ll'}$$

- so pions are non-perturbative for $$\frac{m_N Q}{4\pi} \frac{1}{f_\pi^2} \alpha_{ll'} > 1$$

→ $$Q > 100 \text{ MeV} \quad \text{for } \alpha_{ll'} = 1$$

This estimate agrees with Fleming et al. NPA 677,313.

How is V suppressed for large l?

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- Can we estimate the suppression factor?
  We use Goldberger-Watson estimate for the complete infinite order interaction (e.g. range d = $1/m_\pi$; non-singular). This excludes the chiral limit!

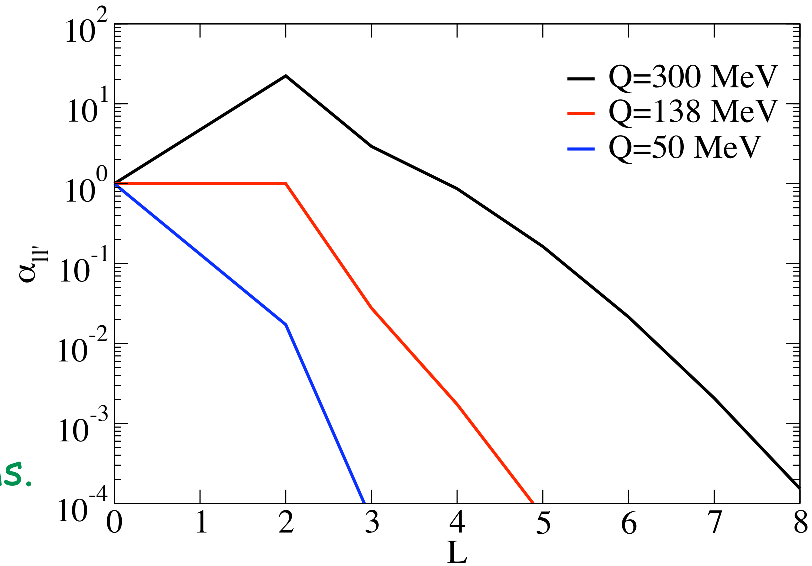
$\alpha_{ll}(Q)$

$$\tan\delta_l = \left(\frac{Qd}{l+1/2}\right)^{2l+1} \to \left(\frac{Q}{m_\pi l}\right)^{2l+1} \propto V_{ll}$$

$$\rightarrow \quad \frac{V_{l+1,l+1}}{V_{ll}} \approx \left(\frac{Q}{l m_\pi}\right)^2 \quad \rightarrow \quad V_{l+1,l+1} \approx \left(\frac{Q}{m_\pi}\right)^{2l} \frac{1}{(l!)^2}$$

So the range of the interaction guarantees that higher partial waves drop very quickly with l.

We can treat higher partial waves perturbatively and renormalization is possible with the available counter terms.



Power counting suggestion

- **In theory**
  - promote a finite number of counter terms, so that a renormalization for lower partial waves is possible, even when it is necessary to sum the interaction to all orders.
  - treat high partial waves perturbatively, then there is no problem with the renormalization.

- **In practice**
  Treat π exchange non-perturbative in all partial waves for a restricted cutoff range
  - promote a finite number of counter terms, so that a renormalization for lower partial waves is possible, even if the interaction is summed to all orders.
  - restrict the cutoff to a number, which you have before determined numerically (look for a plateau region)
  - use your favorite code to sum to all orders, because this will not differ from the perturbative treatment anyway

Conclusions

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- We quantitatively studied the cutoff dependence of the LO chiral interaction
  - despite common believe, we did not find a renormalization problem in our numerical calculation in most partial waves;
    non-renormalizability is related to the singularity of the interaction
    (to the quantum mechanical problem of the LS equation)
  - some (P-wave) attractive triplet channels need additional counter terms for renormalization, these can be promoted from naïvely higher orders
  - alternatively, one finds sensible predictions for cutoffs in "plateau regions"
- We find that cutoff independence is reached for Λ slightly larger than the 2.5 to 3 fm^{-1} usually used (5 to 8 fm^{-1})
- Also the 3N binding energy is Λ independent; there is no 3NF necessary in LO
- We argued that a consistent power counting requires that the interaction is treated perturbatively in higher partial waves and that this is possible.

Outlook

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- NLO,NNLO results are in preparation
  - In which partial waves do we observe Λ dependence?
  - Does the range of "good" Λ 's increase towards smaller values?
  - What happens to the 3N binding energy?
    Is the small binding energy related to the $^1S_0$ phase only?