Meson-Baryon coupling constants from QCD

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*Nuclear Forces and QCD: Never the Twain Shall Meet? @ ECT**

6/30/2005

Contents

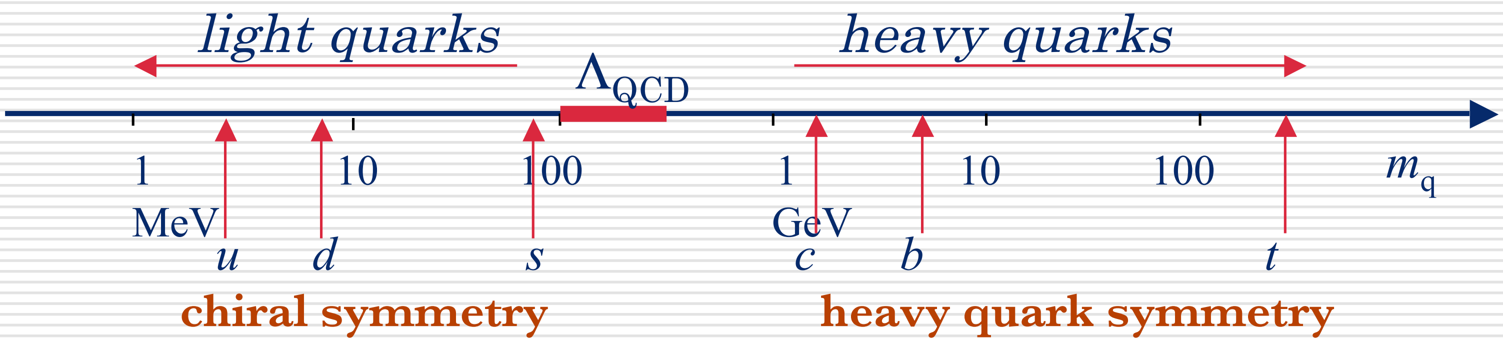
Flavors in QCD

$$\mathcal{L} = \bar{q}(i\not{D} - M)q - \frac{1}{2}\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}]$$

$$M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

$$m_u \simeq m_d \ll m_s \sim \Lambda_{QCD}$$

$$m_s / \Lambda_{QCD} \sim 1$$



Strangeness is most sensitive to QCD

Strangeness in nuclear/hadron physics

- Rich spectra in strange-quark systems
  Probed by K, Λ, Σ, Ξ – nuclear reactions
  - $S = -1$ Λ, Σ, K nucleus
  - $S = -2$ ΛΛ, Ξ nucleus
  - $S = +1$ Θ^+ nucleus ?
  - $S = 0$ η, φ in nuclear medium
- The strangeness may go deep inside nuclear matter.
  It may probe properties of hadrons in dense nuclear medium.
- It may create new forms of nucleus or matter
  ex. compact nucleus with a bound kaon
  Akaishi-Yamazaki-Dote, Iwasaki, Kishimoto

Strangeness in nuclear/hadron physics

- Strangeness appears in Compact Stars:
  Neutron stars (and quark stars) are
  hyper-heavy-nuclei with ***strangeness***
  - Possibility of stable strange hadron matter
  - Strangeness changes equation of states
    Kaon condensation
    Λ, Σ, Ξ (Y) mixtures
  - Understanding the YN, YY interactions is critical.

New narrow hadron resonances

- Θ^+ (1540) by LEPS, SPring-8 ($\Gamma < 10$ MeV)
- X (3872) by Belle, KEK ($\Gamma < 2.3$ MeV)
- D_s*(2317) and D_s*(2463) by BaBar ($\Gamma < 10$ MeV)
- S^0 (3115) (K^-pnn) ($\Gamma < 21$ MeV)

  and ${}^3_{K^-}H$ (K^-ppn) ($\Gamma < 25$ MeV) at 12GeV PS (KEK)

  Are they deep K^- bound states?

- "Common" features

  narrow compared with neighbours

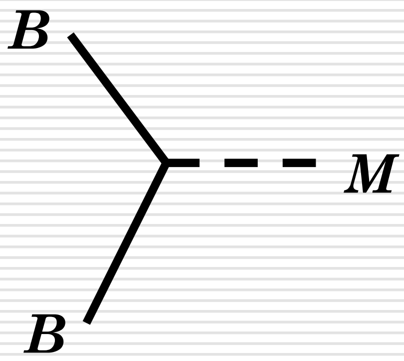
  contain heavy quarks (s, c, ..)

Baryon-baryon interactions

- NN interaction
  - OPE by Yukawa (1934)
  - OBEP supplemented by short range repulsion
  - Exchanged mesons mass < 1 GeV
- YN and YY interactions (Y: hyperons Λ, Σ, Ξ...)
  - Few direct scattering experiments
  - Not enough experimental data for partial wave analysis
  - Spin-dependent forces have not been well determined
  - Theories (models) are based on the flavor SU(3) symmetry.

SU(3) symmetry of MBB vertices

Baryon octet	8	N Λ Σ Ξ
Meson octet	8	π η_8 K
singlet	1	η_1



$8 \times 8 = 1 + 8_D + 8_F + 10 + 10 + 27$

- Two independent couplings for 8 x 8 = 8

  F Tr [[B , B] M] F coupling

  D Tr [{ B , B } M] D coupling

$\alpha_F = \frac{F}{F+D}$ is a free parameter in SU(3)

SU(3) symmetry breaking in OBE

- SU(3) of PS-octet baryon coupling constants

a la *Nijmegen group*

$$g_{\pi NN} = g \qquad \text{independent of } \alpha_F$$

$$g_{\eta NN} = \frac{1}{\sqrt{3}}(4\alpha_F - 1)g \qquad \eta \equiv \eta_8$$

$$g_{\pi\Xi\Xi} = (2\alpha_F - 1)g \qquad g_{\eta\Xi\Xi} = \frac{1}{\sqrt{3}}(2\alpha_F + 1)g$$

$$g_{\pi\Sigma\Sigma} = 2\alpha_F g \qquad g_{\eta\Sigma\Sigma} = \frac{2}{\sqrt{3}}(1 - \alpha_F)g$$

$$g_{\pi\Lambda\Sigma} = \frac{2}{\sqrt{3}}(1 - \alpha_F)g \qquad g_{K\Lambda N} = -\frac{1}{\sqrt{3}}(2\alpha_F + 1)g$$

$$g_{K\Sigma N} = (1 - 2\alpha_F)g$$

SU(6) symmetry: spin-flavor

Symmetry of the NR quark model $(u\uparrow\ u\downarrow\ d\uparrow\ d\downarrow\ s\uparrow\ s\downarrow)$

Baryons 56 = (8, 1/2) + (10, 3/2)

$N\ \Lambda\ \Sigma\ \Xi \qquad \Delta\ \Sigma^*\ \Xi^*\ \Omega$

Mesons 1 = (1,0) η_1

35 = (8,0) + (8,1) + (1,1) and mixings

$\pi\ \eta_8\ K \qquad \rho\ \omega\ K^* \qquad \phi \qquad \eta_1 - \eta_8\ /\ \omega - \phi$

$56 \times \overline{56} = 1 + 35 + 405 + 2695$

(8,0) (8,0)

F/D ratio is fixed by the SU(6) symmetry

$\rightarrow F/D = 2/3$ or $\alpha_F = 2/5$

SU(6) symmetry: spin-flavor

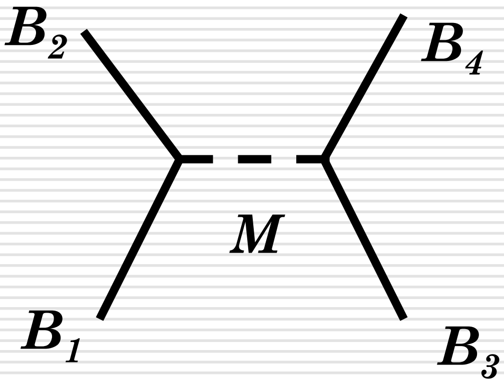
But,

the SU(6) symmetry must be broken significantly
not only by the mass difference of quarks
but also by spin-dep. interactions of quarks

ex N (J=1/2) – Δ (J=3/2) mass difference

SU(3) symmetry breaking in OBE

- SU(3) is broken by $m_s >> m_u, m_d$ in QCD.
- meson mass differences
  - range of the potential
- baryon mass differences
  - recoil correction at vertex



$$g\bar{\psi}_{B'}\gamma_5\psi_B\phi_M \simeq \frac{g}{2\sqrt{M_{B'}M_B}}\chi^\dagger_{B'}\vec{\sigma}\cdot\vec{q}\chi_B$$

- form factors
  - size of the meson and baryons

$$g(q^2) = g(q^2 = m^2) \times \frac{m^2 - \Lambda^2}{q^2 - \Lambda^2}$$

Λ=0.7-1.5 GeV

So...

- Why do we believe the SU(3)/SU(6) relations of the couplings??

  What does QCD predict for the F/D ratio in SU(3)?

  How strong is the SU(3) violation of the couplings?

- Direct computation of the coupling constants from QCD

  Lattice QCD a pioneering attempt by K.F. Liu et al.

  QCD sum rule

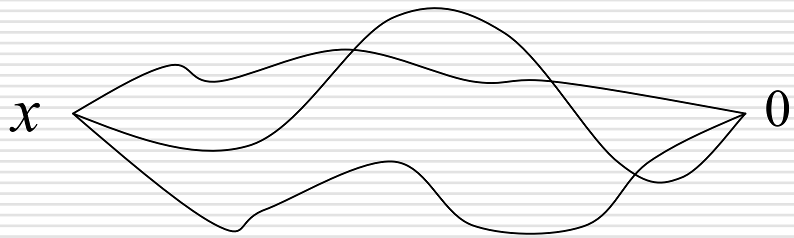
QCD Sum Rules

Shifman, Vainshtein, Zakharov (1979)
Ioffe (1981)
Reinders, Rubinstein, Yazaki (1983)

Correlation function of composite operators

$$\Pi(q) \equiv i \int d^4x e^{iq\cdot x} \langle 0|T(J(x)\bar{J}(0))|0\rangle$$

J(x): local composite operator



QCD Sum Rules

Interpolating field operators

mesons

$$J_\rho(x) = \bar{q}(x)\gamma^\mu \frac{\vec{\tau}}{2} q(x) \qquad J_\pi(x) = \bar{q}(x)\gamma^5 \frac{\vec{\tau}}{2} q(x)$$

baryons

$$J_N(x) = \epsilon_{abc}[(u_a^T C\gamma^\mu u_b)\gamma^5\gamma_\mu d_c]$$

$$J_\Lambda(x) = \sqrt{\frac{2}{3}}\epsilon_{abc}\left([u_a^T C\gamma_\mu s_b]\gamma_5\gamma^\mu d_c - [d_a^T C\gamma_\mu s_b]\gamma_5\gamma^\mu u_c\right)$$

$$J_{\Sigma^0}(x) = \sqrt{2}\epsilon_{abc}\left([u_a^T C\gamma_\mu s_b]\gamma_5\gamma^\mu d_c + [d_a^T C\gamma_\mu s_b]\gamma_5\gamma^\mu u_c\right).$$

QCD Sum Rules

$$\Pi(q) \equiv i \int d^4x e^{iq\cdot x} \langle 0|T(J(x)\bar{J}(0))|0\rangle$$

(1) OPE (Operator Product Expansion) side

$$\Pi(q_E^2) = \sum_n C_n(q_E^2) \langle 0|O_n(0)|0\rangle \qquad q_E^2 \equiv -q^2 \to \infty$$

(2) Phenomenological side

parametrization of the spectral function at $q^2 = m^2$

$$\Pi(q^2) = \frac{1}{\pi} \int ds \frac{\rho(s)}{s - q^2} \qquad \rho(s) = \lambda\delta(s - m^2) + \theta(s - s_0)\rho(s)$$

s_0 : threshold for excited states

QCD Sum Rules

(3) They are "related" by using the analyticity of the correlator: dispersion relation

$$\Pi(q^2) = \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\Pi(s)}{s-q^2}$$

QCD duality threshold s_0

$$\mathrm{Im}\Pi^{\mathrm{OPE}}(s) = \mathrm{Im}\Pi^{\mathrm{PH}}(s) \quad \text{for } s > s_0$$

$$\int_0^{s_0} \frac{\mathrm{Im}\Pi^{\mathrm{OPE}}(s)}{s-q^2} ds = \int_0^{s_0} \frac{\mathrm{Im}\Pi^{\mathrm{PH}}(s)}{s-q^2} ds$$

QCD Sum Rules

(4) To improve:

Borel transformation M^2

$$\Pi(q^2=-q_E^2) \to \mathcal{B}_{M^2}\Pi \equiv \tilde{\Pi}(M^2) = \lim_{q_E^2, n\to\infty, M^2 \equiv q_E^2/n = \text{finite}} \frac{(q_E^2)^{n+1}}{n!}\left(-\frac{d}{dq_E^2}\right)^n \Pi(q_E^2)$$

Borel sum rule for the imaginary part of $\Pi\ (s = q_E^{\ 2})$

$$\mathcal{B}_{M^2}\int_0^{s_0} \frac{\mathrm{Im}\Pi(s)}{s+q_E^2} ds = \int_0^{s_0} e^{-s/M^2}\mathrm{Im}\Pi(s)ds$$

$$\int_0^{s_0} e^{-s/M^2}\mathrm{Im}\Pi^{\mathrm{OPE}}(s)ds = \int_0^{s_0} e^{-s/M^2}\mathrm{Im}\Pi^{\mathrm{PH}}(s)ds$$

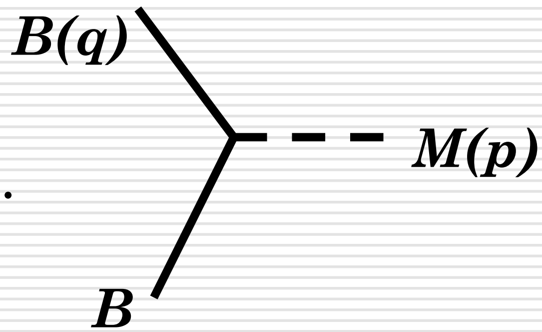
QCD Sum rules for coupling constants

Reinders-Rubinstein -Yazaki (1983)

3 point correlation function

$$\Pi(q,p) \equiv i \int d^4x\, d^4y\, e^{iq\cdot x + ip\cdot y} \langle 0|T(J_{B'}(x) J_M(y) \bar{J}_B(0))|0\rangle$$

OPE gives $\Pi(q,p) \sim \frac{1}{p^2}\langle \bar{q}q\rangle + \ldots$



This pion pole term at $p^2=0$ gives $g_{MBB}(0)$.

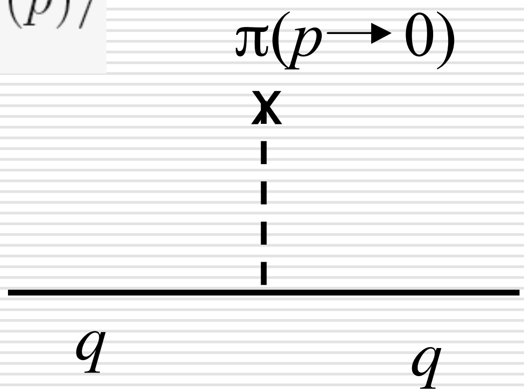
But the OPE is valid only at $p^2 \to -\infty$ and $q^2 \to -\infty$.
The sum rule is contaminated by contribution from excited mesons.

QCD Sum rules for coupling constants

Shiomi-Hatsuda (1995)

2 point correlator with a pion

$$\Pi(q,p) \equiv i\int d^4x\, e^{iq\cdot x}\langle 0|T(J_{B'}(x)\bar{J}_B(0))|M(p)\rangle$$



phenomenological side

$$\frac{1}{q^2-M_N^2}g_{\pi NN}i\gamma^5\frac{1}{q^2-M_N^2}$$

The double pole is the signature of NNπ.

QCD Sum rules for coupling constants

Birse-Krippa (1996) criticized to take the soft-pion limit.

soft-pion relation ($p \to 0$)

$$
\begin{aligned}
T^a(q,p) &\overset{p\to 0}{=} -\frac{i}{f_\pi}\int d^4x e^{iq\cdot x}\langle 0|\left[Q_5^a, T(B(x)\bar{B}(0))\right]|0\rangle \\
&= \frac{i}{2f_\pi}\left\{\gamma^5\tau^a, \int d^4x e^{iq\cdot x}\langle 0|T(B(x)\bar{B}(0))|0\rangle\right\}_+
\end{aligned}
$$

Baryon 2-point correlator

➔ $$\frac{M_N}{f_\pi} = \frac{g_{\pi NN}}{g_A} \qquad \text{with } g_A = 1$$

This is not independent from the nucleon mass SR with the GT relation

QCD Sum rules for coupling constants

Soft Meson Limits in SU(3)

Ioffe Current $$B_{\rm Ioffe} = [(u^T C\gamma^\mu u)\gamma^5\gamma_\mu d]_{C=1}$$

↓ SU(3)

$$[Q_5^a, B_{\rm Ioffe}^b(x)]_{ET} = d_{abc} B_{\rm Ioffe}^c(x)$$

PS Meson – Baryon couplings are pure D couplings!!??

QCD Sum rules for coupling constants

General current

$$B_p = [(u^T C d)\gamma^5 u]_1 + t[(u^T C \gamma^5 d) u]_1$$

$$[Q_5^a, B^b(x)]_{ET} = \left(\frac{1+t}{2} i f_{abc} + \frac{1-t}{2} d_{abc}\right) B^c(x)$$

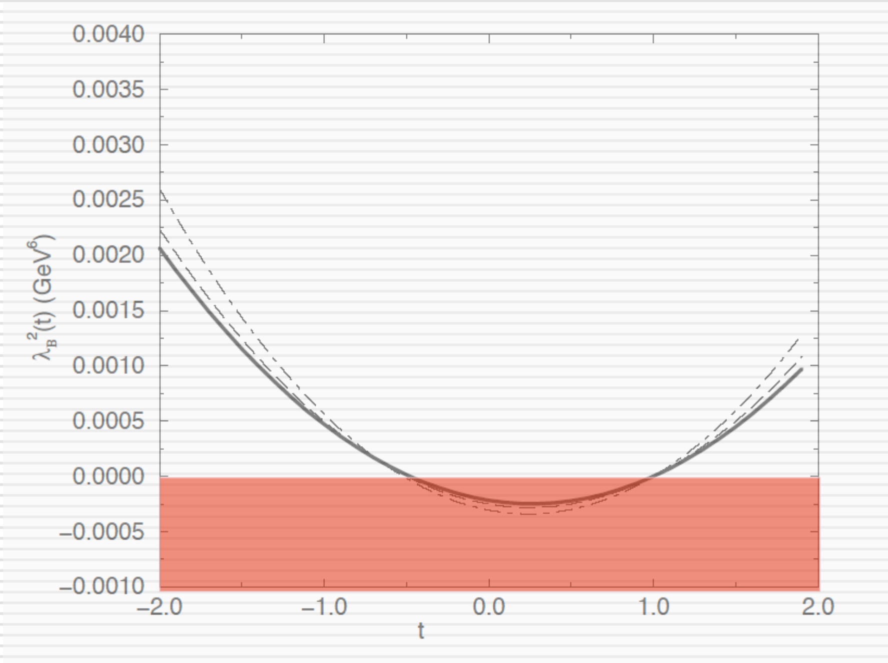
F coupling + D coupling

$t = -1$ (Ioffe) (3 , 3) + (3 , 3) → D coupling

$t = +1$ (8 , 1) + (1 , 8) → F coupling

QCD Sum rules for coupling constants

$$\frac{F}{D} = \frac{1+t}{1-t}$$ = 2/3 (SU(6)) for t = – 1/5 ? **Furnstahl**



However, the baryon sum rule does not work at $t \sim -1/5$.

$|\text{coupling}|^2 < 0$

Conclusion

Soft-pion limit should not be taken for the PS meson-baryon int.

QCD Sum rules for coupling constants

T. Doi, H. Kim, S.H. Lee, Y. Kondo, M. O.

PL B453 (1999) 97, PR D60 (1999) 034007
NP A662 (2000) 371, NP A678 (2000) 295
Phys. Rep. 398 (2004) 253

G. Erkol, R.G.E. Timmermans, Th.A. Rijken

QCD Sum rules for coupling constants

H. Kim, T. Doi, S.H. Lee, Y. Kondo, M.O

- πNN coupling constant

$$\Pi^{\alpha\beta}(q,p) = i\int d^4x\, e^{iq\cdot x}\langle 0|\mathrm{T}\left[J_N^{\alpha}(x)\bar{J}_N^{\beta}(0)\right]|\pi(p)\rangle$$

$$\Pi(q,p) \;=\; i\gamma_5 \not{p}\, \Pi^{\mathrm{PV}} + i\gamma_5\, \Pi^{\mathrm{PS}} + \boxed{\gamma_5\sigma^{\mu\nu}q_\mu p_\nu\, \Pi^{\mathrm{T}}} + i\gamma_5 \not{q}\, \tilde{\Pi}^{\mathrm{PV}}$$

tensor structure

- nucleon interpolation field

$$J_N(x;t) \;=\; 2\epsilon_{abc}[\,(u_a^T(x)Cd_b(x))\gamma_5 u_c(x) + t\,(u_a^T(x)C\gamma_5 d_b(x))u_c(x)\,]$$

two independent terms mixed by $\tan\theta = t$

QCD Sum rules for coupling constants

Meson matrix elements

$$\langle 0|\bar{q}(0)\Gamma q(x)|\pi(p)\rangle \qquad \Gamma = \gamma^5, \gamma^\mu\gamma^5$$

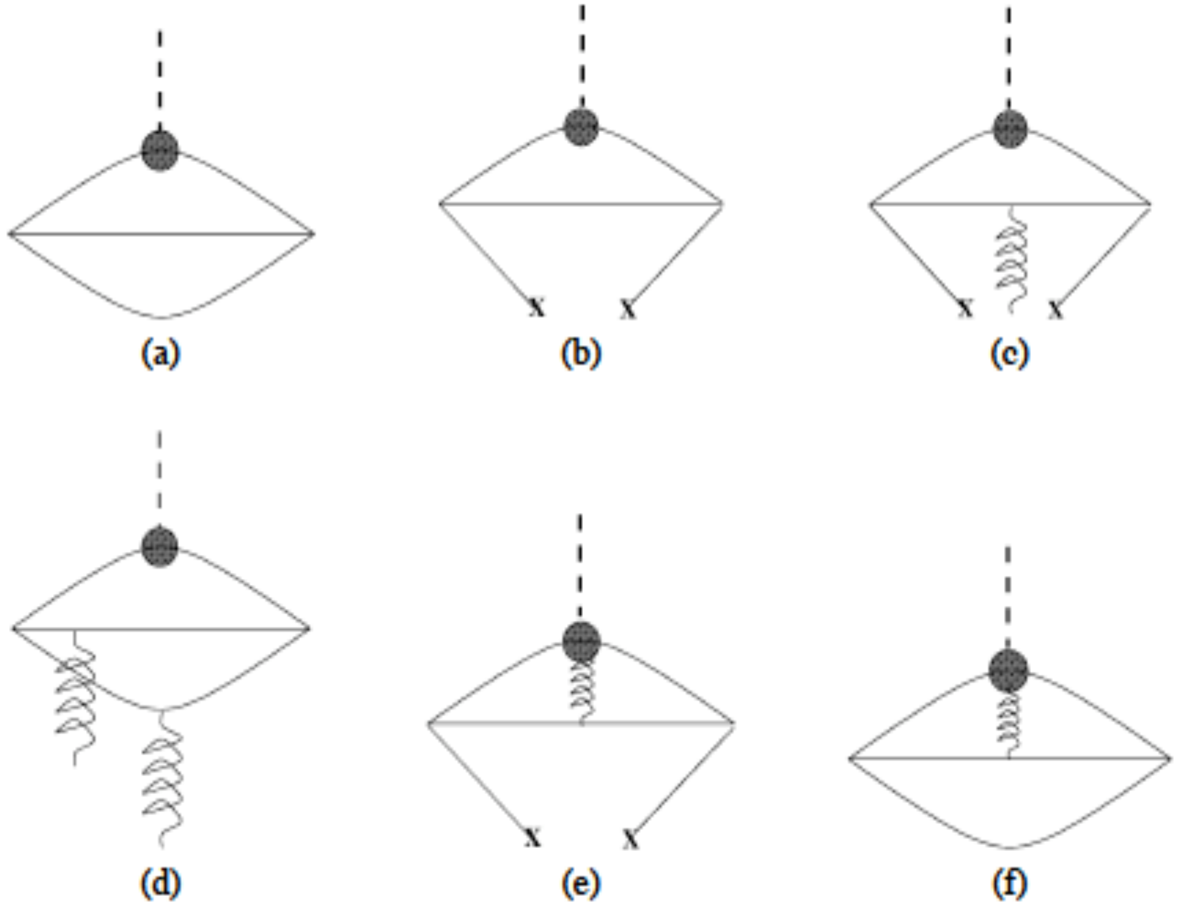
(1) Twist expansion at the light-cone ($x^2 = 0$)

(2) Short distance expansion of the light-cone operators around Euclid $x_E = 0$

$$\langle 0|\bar{u}(0)\gamma_\mu\gamma_5 u(x)|\pi^0(p)\rangle = i f_\pi p_\mu + \frac{1}{2} f_\pi (p\cdot x) p_\mu$$
$$-\frac{1}{18} i f_\pi \delta^2 (p\cdot x) x_\mu + \frac{5}{36} i f_\pi \delta^2 x^2 p_\mu + \frac{5}{72} f_\pi \delta^2 (p\cdot x) x^2 p_\mu - \frac{1}{36} f_\pi \delta^2 (p\cdot x)^2 x_\mu$$

$$\delta^2 = \frac{m_0^2}{4} = 0.2 \ (\mathrm{GeV})^2$$

QCD Sum rules for coupling constants



QCD Sum rules for coupling constants

- for πNN coupling (tensor term)

$$
\begin{aligned}
&g_{\pi N}\lambda_N^2(1+A^{\mathrm{T}}_{\pi N}M^2)e^{-m_N^2/M^2} \\
&= \frac{1}{96\pi^2 f_\pi}(10+4t-14t^2)\langle\bar{q}q\rangle M^4 E_0(x) - \frac{f_\pi}{3}(-1-2t+3t^2)\langle\bar{q}q\rangle M^2 \\
&\quad -\frac{1}{54}f_\pi\delta^2(-1-26t+27t^2)\langle\bar{q}q\rangle + \frac{1}{72\cdot 12 f_\pi}(17+2t-19t^2)\langle\bar{q}q\rangle\left\langle\frac{\alpha_s}{\pi}\mathcal{G}^2\right\rangle \\
&\quad +\frac{f_\pi}{72}(-5-6t+11t^2)m_0^2\langle\bar{q}q\rangle
\end{aligned}
$$

QCD Sum rules for coupling constants

In principle, all the Dirac structures should be equivalent.
But, in practice, not all of them make a legitimate sum rule.

- Three (technical) check points

(1) coupling scheme independence

PS coupling *v.s.* PV coupling

which are equivalent for the on-mass-shell baryons

(2) suppression of the single pole term

induced by excited baryons

require mild dependence on Borel mass M

(3) consistent *t* dependence

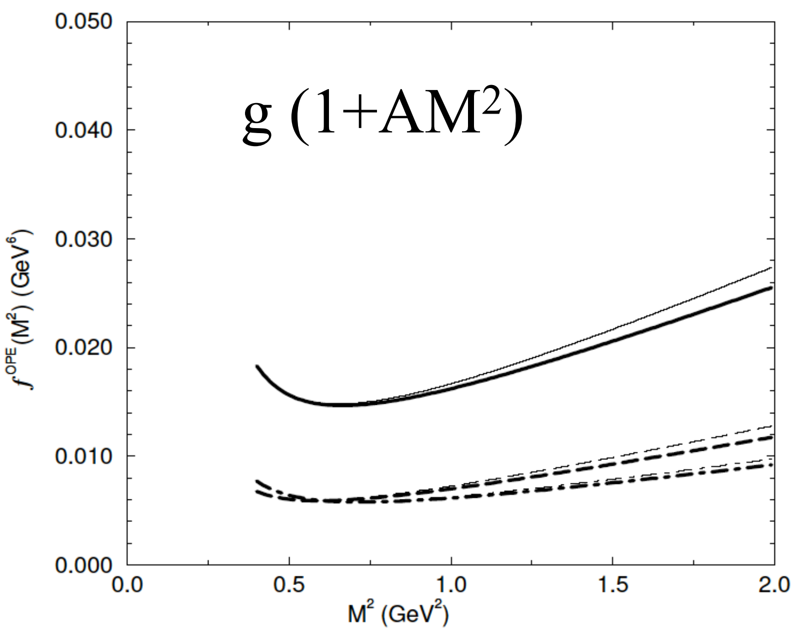
require consistency within the sum rules and also with the baryon mass sum rule

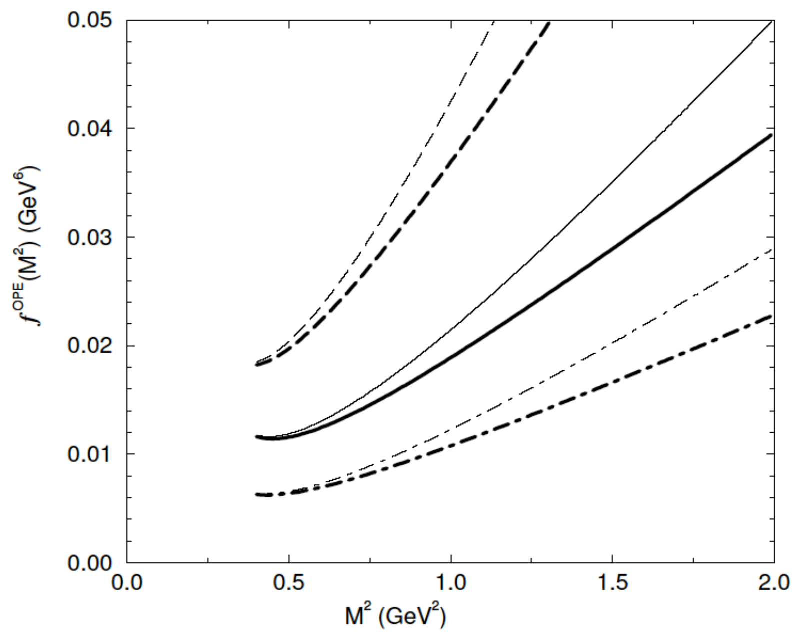
QCD Sum rules for coupling constants

Status	PV $i\gamma^5 \gamma q$	PS $i\gamma^5 q^2$	T $i\gamma^5 \sigma_{\mu\nu} p^\mu q^\nu$
coupling scheme dependence	No	OK	Good
single pole term	No	Good	Good
t dependence	-	No	Good

Single pole contribution

T sum rule *v.s.* PV sum rule

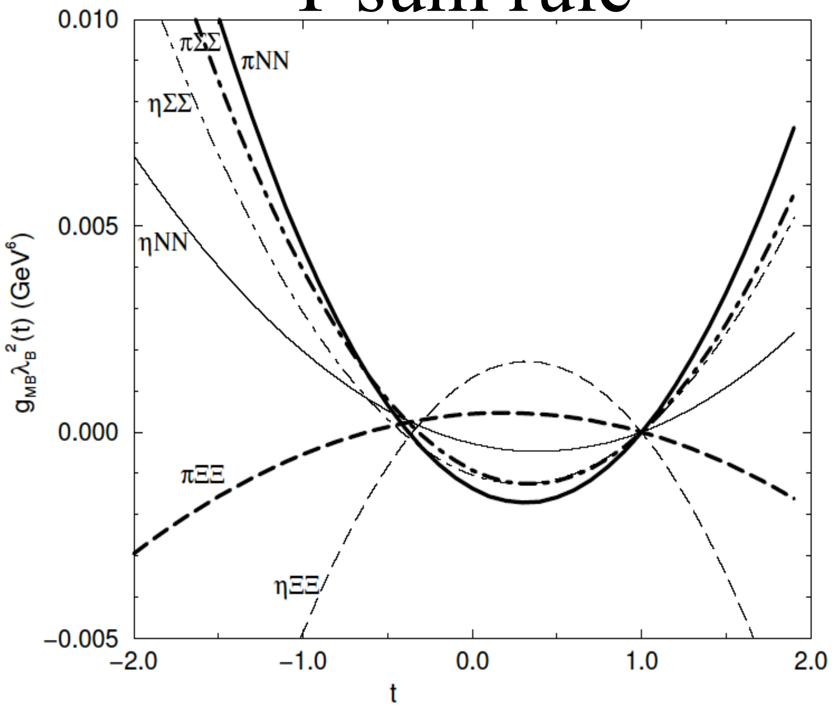


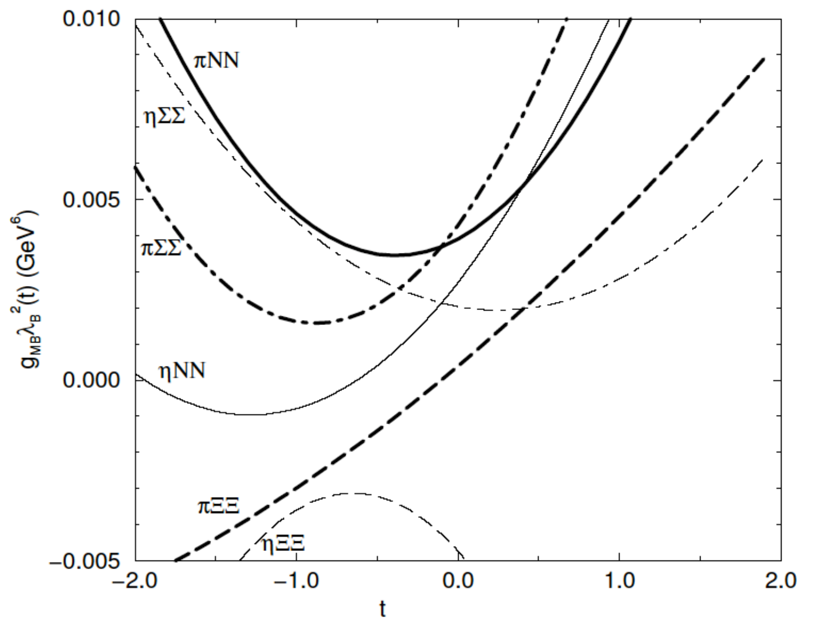


Consistent *t*-dependence

SU(3) limit $\lambda_N(t) = \lambda_\Xi(t) = \lambda_\Sigma(t)$ linear in t

T sum rule *v.s.* PS sum rule





Sum rule parameters

TABLE II: QCD parameters. We always assume that $\langle \bar{d}d \rangle = \langle \bar{u}u \rangle$, and employ the same $m_0^2 \equiv \frac{\langle \bar{q} g_s \sigma \cdot G q \rangle}{\langle \bar{q} q \rangle}$ for the u, d and s quarks. The OPE terms which are proportional to m_u or m_d are neglected, that is equivalent to the choice $m_u = m_d = 0$.

$\langle \bar{u}u \rangle$	$\langle \bar{s}s \rangle / \langle \bar{u}u \rangle$	m_s	m_0^2	δ^2	$\langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle$
$-(0.23\ \mathrm{GeV})^3$	0.8	0.12GeV	$0.8\ \mathrm{GeV}^2$	$0.2\ \mathrm{GeV}^2$	$(0.33\ \mathrm{GeV})^4$

They are taken to be 0 in the SU(3) limit

Results in SU(3) limit

- Ratios of MBB' couplings in the SU(3) limit

  Borel window 0.9 < M^2 <1.5 GeV^2

  $g(\eta_8 NN)/ g(\pi NN) = 0.36$

  $g(\eta_8 \Xi\Xi)/g(\pi\Xi\Xi) = 4.0$, $g(\eta_8 \Sigma\Sigma)/g(\pi\Sigma\Sigma) = 0.97$

F/D =0.68 α_F=0.405

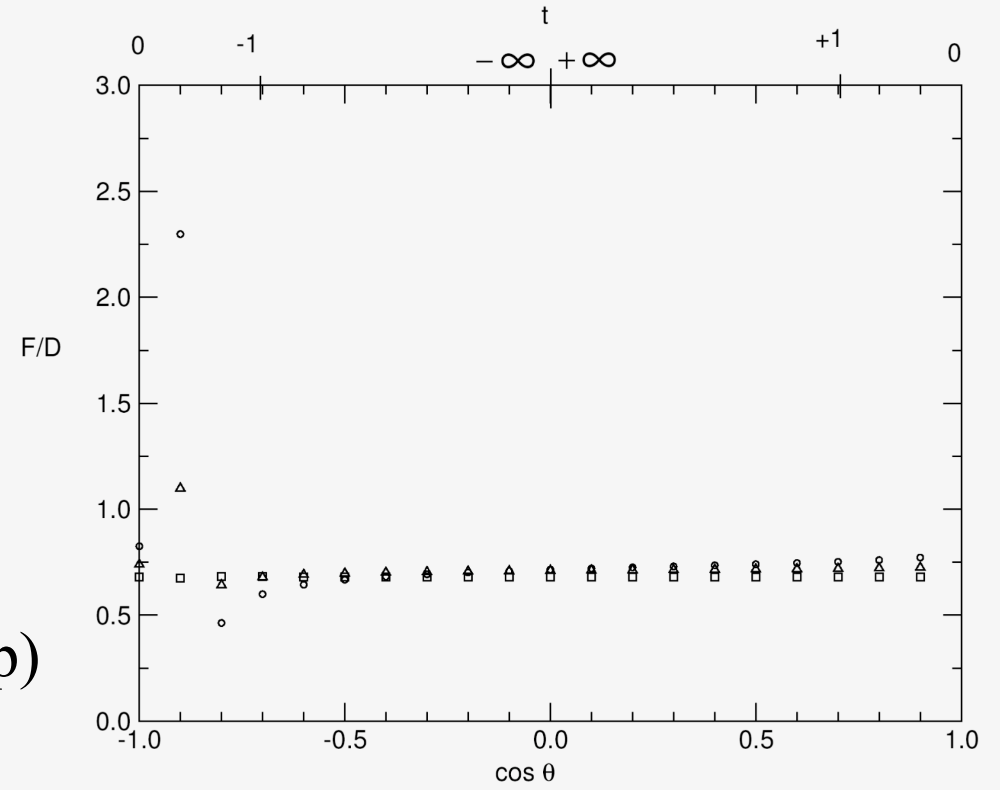
compared with $\alpha_F = 0.4$ for SU(6)

$\alpha_F = 0.464, 0.409$ for Nijmegen D, F

$\alpha_F = 0.36$ from the octet baryon β decays

F/D ratio *v.s.* cosθ for T sum rule

Tensor sum rule   Doi, Kim, Oka, PRC52 (2000)

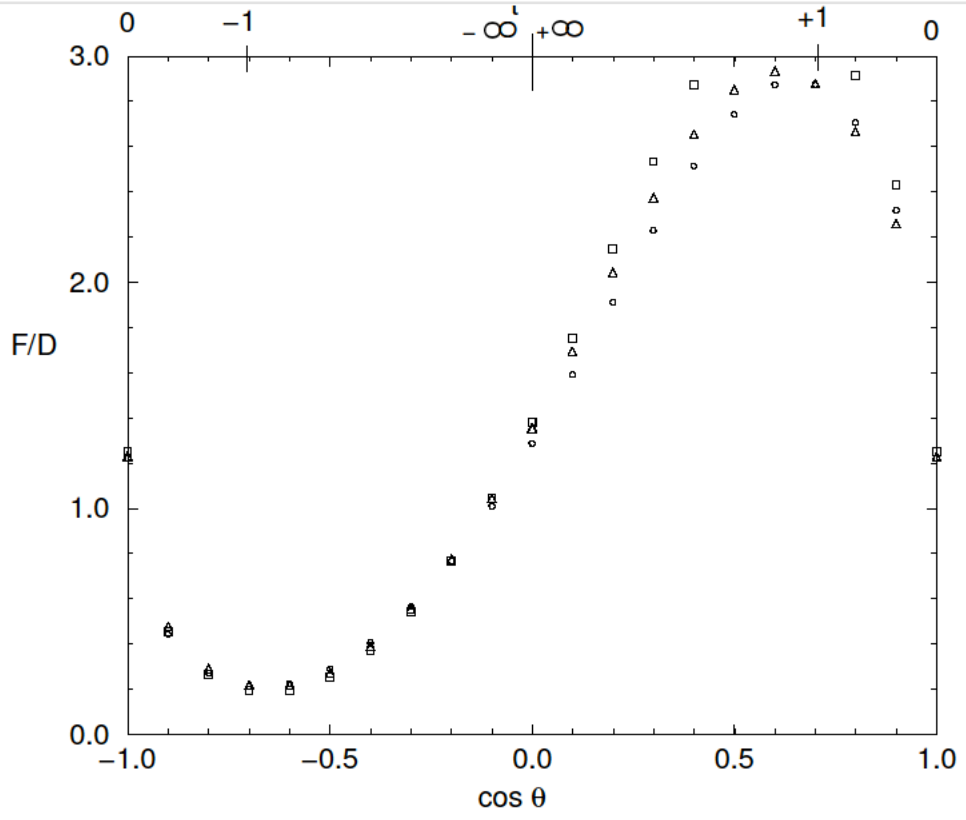


F/D = 0.65 ± 0.10

2/3 for SU(6)

0.57 from g_A (exp)

PS sum rule



strong t dependence

πNN coupling constant

- Absolute value of the coupling constants

Eliminate $|\lambda|^2$ by using the (chiral odd) mass sum rule

$g_{\pi NN} \sim 9.6 \pm 0.6$ (exp. ~13.4)

The mass sum rule seems responsible for this discrepancy.

SU(3) Violation in πBB and ηBB couplings

Sources of SU(3) violation (partial) up to $O(q \sim \sqrt{m_s})$

– quark condensate $\langle \bar{s}s \rangle \sim 0.8 \langle \bar{u}u \rangle$

– decay constants $f_\eta \sim 1.2 f_\pi$

Ratios of coupling constants: g_η / g_π

no ambiguities from the baryon – current coupling

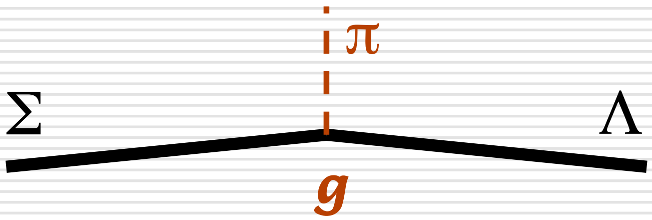
g(ηNN)/ g(πNN)	= 0.39	(0.36) @ SU(3)
g(ηΞΞ)/g(πΞΞ)	= 3.0	(4.0) 30%
g(ηΣΣ)/g(πΣΣ)	= 1.1	(0.97) 15%

$O(q \sim \sqrt{m_s})$ *corrections only*

πΛΣ coupling constant

- Off-diagonal couplings

  no double pole term for $M_\Lambda \neq M_\Sigma$



- Projected correlation function method

  Y. Kondo, O. Morimatsu Nucl. Phys. A717 (2003)

$$\Pi_+(pr, qs, k) = \bar{u}(\boldsymbol{p}r)\gamma_0 \Pi(p, k)\gamma_0 u(\boldsymbol{q}s),$$

$$\boxed{\Gamma_+(pr, qs, k)} = (p_0 - M)(p_0 - E_q - \omega_k)\Pi_+(pr, qs, k),$$

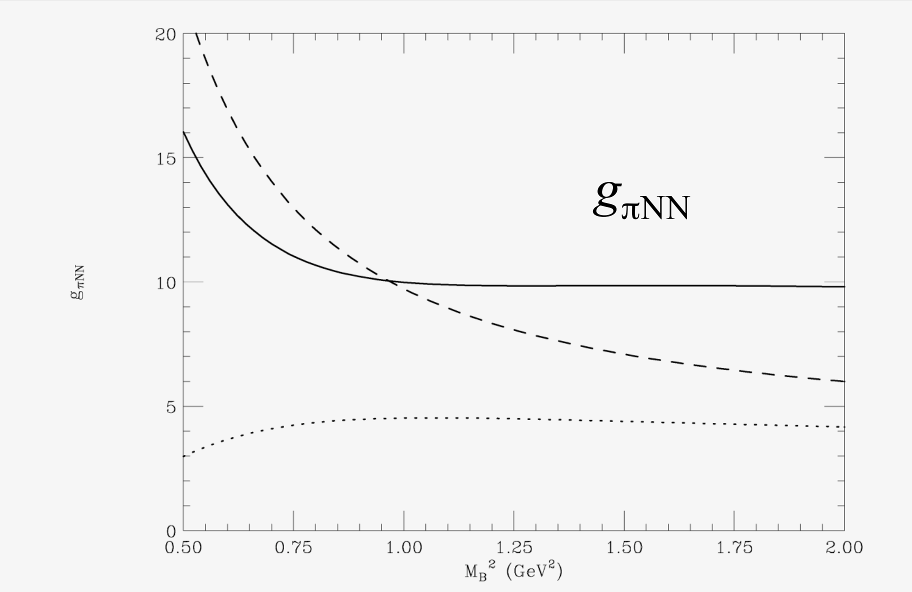
on-masss-shell vertex function prop to the coupling cons.

Projected correlation function

- πNN coupling in the projection method

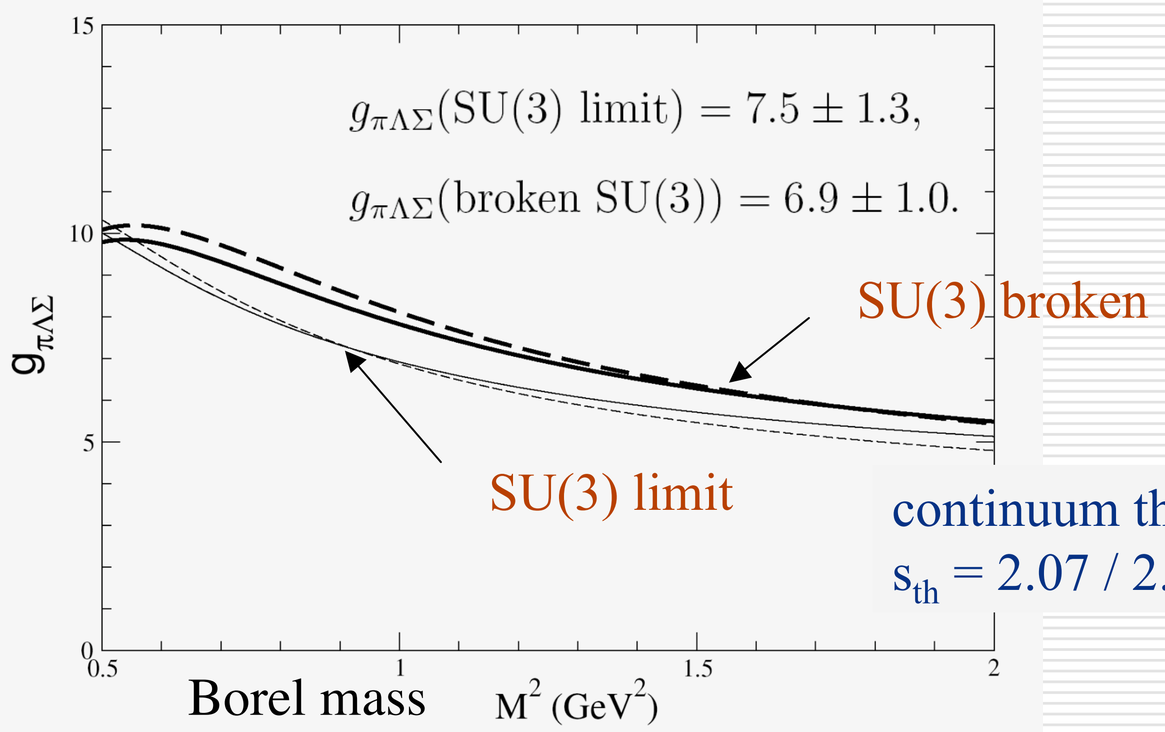
Kondo-Morimatsu, Nucl. Phys. A717 (2003)

They found that it gives better Borel stability.



πΛΣ coupling constant

Doi, Kondo, Oka, Phys. Rep. 398 (2004) 293



continuum threshold
s_{th} = 2.07 / 2.57 GeV^2

$\pi\Lambda\Sigma$ coupling constant

- Deviation from the SU(3) limit is small.

$$g_{\pi\Lambda\Sigma}(\mathrm{SU(3)\ limit}) = 7.5 \pm 1.3,$$

$$g_{\pi\Lambda\Sigma}(\mathrm{broken\ SU(3)}) = 6.9 \pm 1.0.$$

- Comparison
  - Lutz, Kolomeitsev ~ 10.4 by fitting exp data
    But we underestimate the πNN coupling.
  - Keil et al. ~ 0.4 $g_{\pi NN}$ indicates a large SU(3) breaking

SU(3) breaking

$$\text{OPE} \approx \left[-\frac{1}{8\pi^2}\frac{\langle \bar{q}q\rangle}{f_\pi}\omega_p - \frac{1}{6\pi^2} f_\pi \omega_p (E_\Lambda + m_\Lambda) + \frac{1}{8\pi^2}\frac{\langle \bar{q}q\rangle}{f_\pi}(m_q - m_s) + \left(-\frac{1}{24\pi^2}\frac{\langle \bar{q}q\rangle}{f_\pi} + \frac{1}{8\pi^2} f_\pi m_q \right)(E_\Lambda + m_\Lambda) \right]$$

pion matrix element (leading)

strange quark mass

baryon mass

The SU(3) breaking terms are sub-leading.
For the π–B–B' couplings, SU(3) breaking is weak.

Scalar meson

- Critical component for nuclear binding

  intermediate attraction in nuclear force

- QCDSR calculation of the σBB couplings

  σNN coupling G. Erkol, R. Timmermans, Th. Rijken

  σΛΛ, σΣΣ, σΞΞ couplings with M.O.

Scalar meson

- Light scalar mesons
  - $^3P_0$ $q\bar{q}$ state

    octet f_0, a_0, K_0 + singlet f_0'

    if ideally mixed

    $$f_q = (u\bar{u} + d\bar{d})/\sqrt{2} \qquad a_0 = (u\bar{d}, (u\bar{u} - d\bar{d})/\sqrt{2}, d\bar{u})$$

    $$f_s = s\bar{s}$$

  - tetra-quark states $qq\bar{q}\bar{q}$

    $$f_q^4 = ud\bar{u}\bar{d} \qquad a_0^4 = (su\bar{s}\bar{d}, (su\bar{s}\bar{u} - sd\bar{s}\bar{d})/\sqrt{2}, sd\bar{s}\bar{u})$$

    $$f_s^4 = (su\bar{s}\bar{u} + sd\bar{s}\bar{d})/\sqrt{2}$$

σ ← f_q, f_q^4

Background field method

- quark propagator in the σ background

$$
\begin{aligned}
\langle 0|T[q^a(x)\bar{q}^b(0)|0\rangle_\sigma \;=\;& g_q^\sigma \sigma\Big[-\frac{\delta^{ab}}{4\pi^2 x^2} - \frac{1}{32\pi^2}\lambda^n_{ab} g_c G^n_{\mu\nu}\sigma^{\mu\nu}\ln(-x^2) \\
&+ \frac{i\,\delta^{ab}}{48}\langle\bar{q}q\rangle\hat{x} - \frac{\delta^{ab}\chi}{12}\langle\bar{q}q\rangle + \frac{i\,\delta^{ab}x^2}{2^7\times 3^2}\langle g_c\bar{q}\sigma\cdot Gq\rangle\hat{x} \\
&- \frac{\delta^{ab}x^2}{192}\chi_G\langle g_c\bar{q}\sigma\cdot Gq\rangle - \frac{\delta^{ab}\langle g_c^2 G^2\rangle}{2^9\times 3\pi^2}x^2\ln(-x^2)\Big].
\end{aligned}
$$

scalar susceptibility χ is defined by $\chi\langle\bar{q}q\rangle = \frac{1}{2}T(0)$

$$
T(p^2) = i\int d^4x e^{ip\cdot x}\left\langle 0\middle|T[\bar{u}(x)u(x)+\bar{d}(x)d(x),\bar{u}(0)u(0)+\bar{d}(0)d(0)]\middle|0\right\rangle,
$$

$$
\chi = -10\pm 1\ \text{GeV}^{-1}
$$

Background field method

- OPE of the nucleon correlator (chiral odd part)

$$\Pi^q_\sigma(q) = g^\sigma_q \frac{1}{(2\pi)^4}\left[a_q \ln(-q^2) - \chi \frac{4}{3q^2} a_q^2 + \frac{m_0^2}{2q^2} a_q - (\chi + \chi_G) \frac{m_0^2}{6q^4} a_q^2\right]$$

$$a_q \equiv -(2\pi)^2\langle\bar{q}q\rangle,\ b \equiv \langle g_c^2 G^2\rangle$$

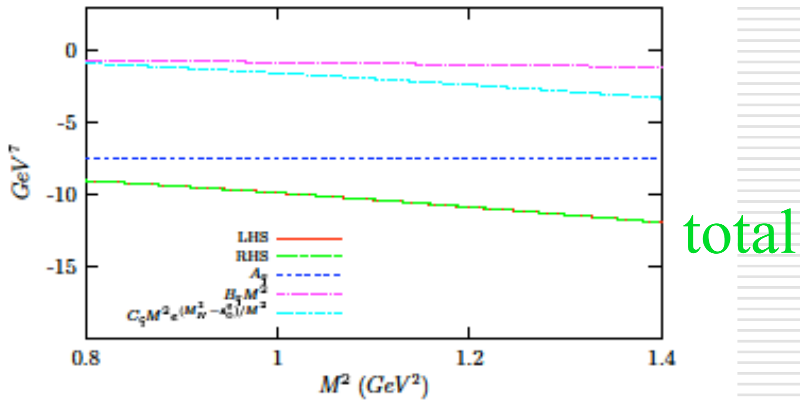
- phenomenological side

$$\Pi^\sigma(q) = \frac{\langle 0|\eta_N|N\rangle}{q^2 - M_N^2}\ \langle N|\sigma N\rangle\ \frac{\langle N|\bar{\eta}_N|0\rangle}{q^2 - M_N^2} = -|\lambda_N|^2 \frac{\hat{q} + M_N}{q^2 - M_N^2} g_{NN\sigma} \frac{\hat{q} + M_N}{q^2 - M_N^2}$$

σNN coupling constant

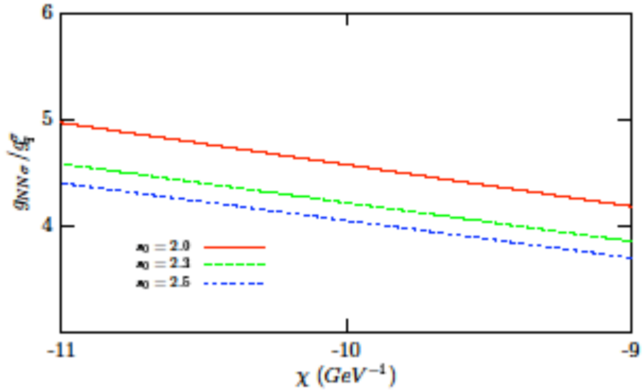
- the Borel mass dependence of the LHS and the RHS

  for 0.8 GeV$^2 \leq M^2 \leq$ 1.4 GeV^2



- the dependence of $g_{NN\sigma}/g_q^\sigma$ on χ

$$g_{NN\sigma}/g_q^\sigma = 3.9 \pm 1.0$$



Meson Baryon Couplings in QCD Sum

σNN coupling constant

- Assuming the value of $g^{\sigma}_{u,d} = 3.7$ (taken from Riska-Brown)

  $g_{\sigma NN} = 14.4 \pm 3.7$ is obtained,

  which is compared with

  $(g_{\sigma NN})_{NSC89} = 16.9$

Other σBB coupling constants: SU(3)

- For other σBB couplings, we obtain in the SU(3) limit

$$g_{NN\sigma}/g_q^\sigma = 4.0, \qquad g_{\Lambda\Lambda\sigma}/g_q^\sigma = 1.7\,, \qquad g_{\Xi\Xi\sigma}/g_q^\sigma = 0.3\,, \qquad g_{\Sigma\Sigma\sigma}/g_q^\sigma = 3.6\,.$$

weak σΛΛ coupling favored by ΛΛ-nuclear data.

for scalar = $q\bar{q}$ with ideal mixing

$$\alpha_s = F/(F+D) = 0.55$$
$$g/g_q^\sigma = g_{NNa_0}/g_q^\sigma = 3.3$$
$$g_1/g_q^\sigma = 3.2$$

for scalar = $qq\overline{qq}$ with ideal mixing

$$\alpha_s = F/(F+D) = 0.55$$
$$g/g_q^\sigma = g_{NNa_0}/g_q^\sigma = -2.3$$
$$g_1/g_q^\sigma = 4.6$$

Conclusion

- Formulate the QCD sum rule for the meson-baryon coupling constants
  - pion-vacuum matrix elements of the two point baryon correlation function.
  - need to go beyond the soft-pion limit.
  - The tensor Dirac structure, $\gamma^5\sigma_{\mu\nu}$, gives the most robust and reliable sum rule.
  - Off diagonal couplings require projection method.

    ex. $\pi\Lambda\Sigma$ coupling

Conclusion

- Results
  - SU(3) limit F/D $= 0.65 \pm 0.10$

    consistent with SU(6), larger than β decay exp.
  - The SR underestimates πNN coupling constant

    $g_{\pi NN} \sim 9.6 \pm 0.6$ (exp. ~13.4)
  - In general, SU(3) breaking ~ 15–30% order is expected.

    $\langle \bar{s}s \rangle < \langle \bar{u}u \rangle = \langle \bar{d}d \rangle$ and $f_\eta < f_\pi < f_K$
  - But, it is weak for πΛΣ coupling and is expected to be weak for other πBB' couplings. (Nijmegen potential)

Conclusion

- The background field method is applied to scalar σ baryon couplings.
  - A strong σNN coupling consistent with OBE models is obtained.
  - A relatively weak σΛΛ coupling is obtained, which is favored by recent experimenal data of ΛΛ hypernuclei.
- Further studies
  - Other mesons: KNΛ, KNΣ, ηΛΛ, Ξ ...

    The p expansion is questionable as $p^2 = m^2$ is large.

    The background field method may work.
  - Weak hadronic couplings: πNΛ, πNΣ, KNN, ...