NN Renormalization, the Deuteron, and the Role of the Chiral Couplings

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Based on...

Based on the following works by MPV and E. Ruiz Arriola

- Phys. Lett. B **580**, 149 (2004) [arXiv:nucl-th/0306069].
- Phys. Rev. C **70**, 044006 (2004) [arXiv:nucl-th/0405057].
- arXiv:nucl-th/0407113. (Low energy np parameters with $j \le 5$)
- arXiv:nucl-th/0410020. (OPE+TPE Singlet)
- arXiv:nucl-th/0504067. (OPE deuteron)
- arXiv:nucl-th/0506047 (TPE deuteron)

Contents

The Deuteron

What does the NN Chiral Potentials predict for the deuteron?

This is not an easy question. There are several problems...

The Deuteron

What does the NN Chiral Potentials predict for the deuteron?

- The NN Chiral Potentials are known as a long range expansion

$$\rightarrow \quad U(r) = U_{LO}(r) + U_{NLO}(r) + U_{NNLO}(r) + \ldots$$

  while the short range NN interaction is unknown.

- The NN Chiral Potentials represent a long range trend, so their extrapolation to the origin becomes problematic. They are singular

$$U_{LO}(r) \sim \pm 1/r^3 \quad U_{NLO}(r) \sim \pm 1/r^5 \quad U_{NNLO}(r) \sim \pm 1/r^6$$

- At $NNLO$ there is also the problem of the LEC's (c_1, c_3, c_4). Do their values and uncertainties pose a problem?

The Deuteron

What does the NN Chiral Potentials predict for the deuteron?

So let's look first at an easier example.

Toy Model: Cs-Cs Scattering

In atom-atom scattering there is a scale separation

- At long distances the atom-atom potential is given by the van der Waals force

$$U_{vdW} = -\frac{C_6}{r^6}$$

- At short distances there are various parametrizations

$$\begin{aligned} U_{LJ} &= \frac{C_{12}}{r^{12}} - \frac{C_6}{r^6} \\ U_{^3\Sigma_u} &= \frac{1}{2} B r^{\lambda} e^{-\eta r} - \left(\frac{C_6}{r^6} + \frac{C_8}{r^8} + \frac{C_{10}}{r^{10}} \right) f_c(r) \end{aligned}$$

Cs-Cs Scattering: Scale Separation

The scales for the Cs-Cs case are

- $U_{vdW} = -C_6/r^6 \longrightarrow$ Long range scale

$$R_L = \sqrt[4]{C_6} = 203\,\mathrm{a.u.}$$

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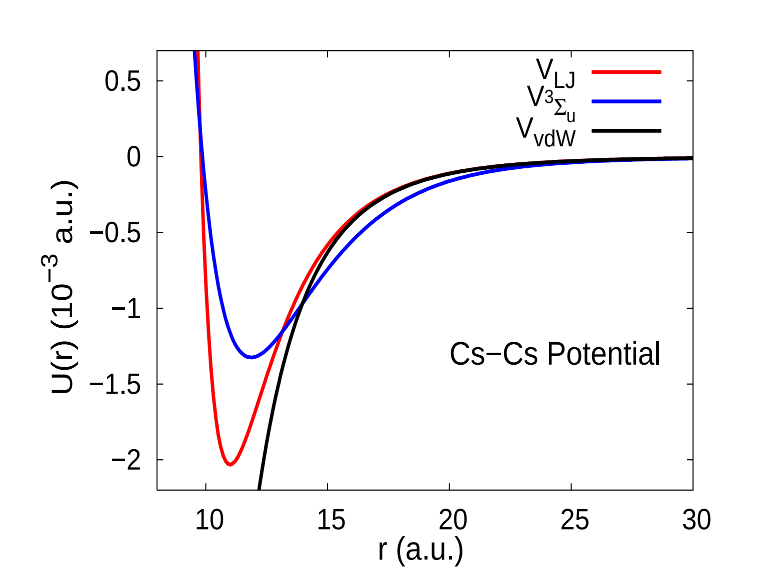

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$\boxed{\lambda \gg R_C}$ $\longrightarrow$ The short distance details should be irrelevant

Cs-Cs Scattering: Scale Separation

Since the short distance details should be irrelevant, the short distance physics must not appear in an explicit way.

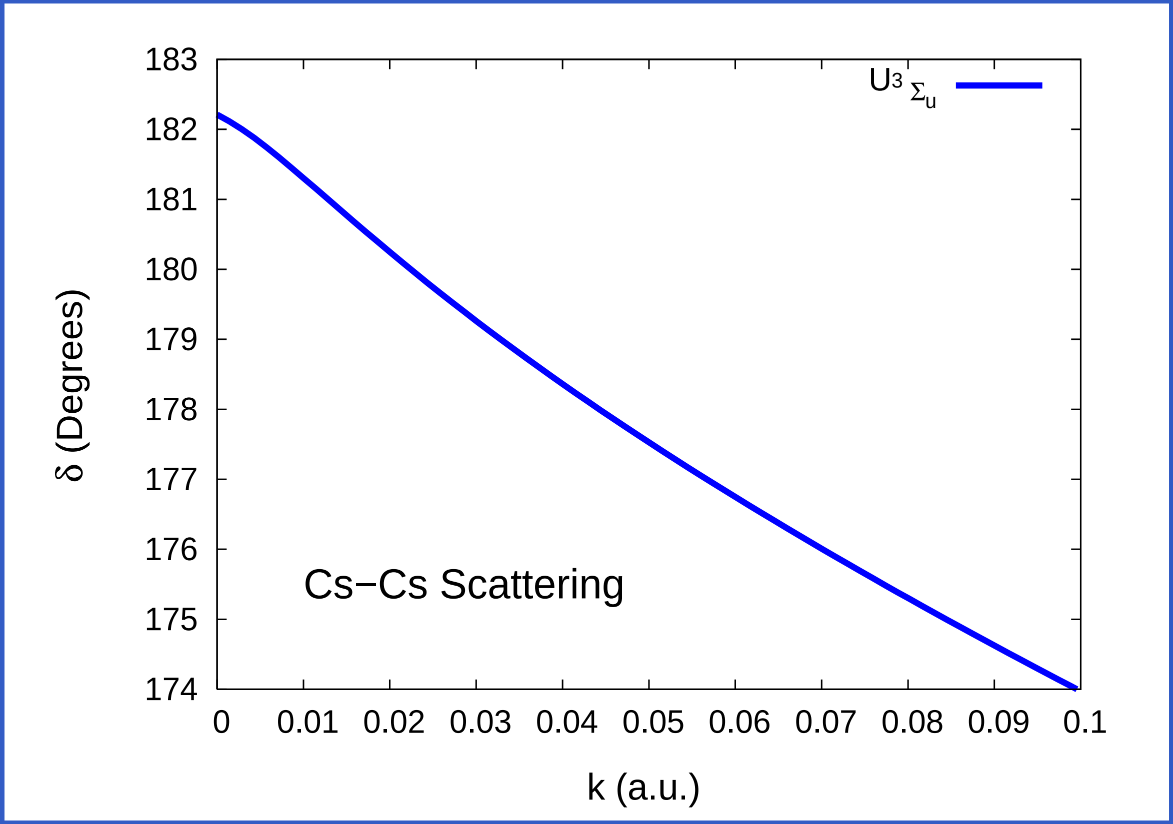
- Schrödinger Eq. + long range potential (U_{vdW})
- The regularity condition at the origin is replaced by the condition of fixing some parameter to its physical value.

A good candidate is the scattering length α, which describes the behaviour of the system at very low energies ($k \to 0$)

$$k \to 0 \quad \Rightarrow \quad \sigma \simeq 4\pi\alpha^2 \quad \text{y} \quad \delta\,(k) \simeq -k\,\alpha$$

Cs-Cs Scattering: Does it work?

Fixing the Scattering Length $\longrightarrow \alpha_{\mathrm{Cs-Cs}} = 68.216\,\mathrm{a.u.}$



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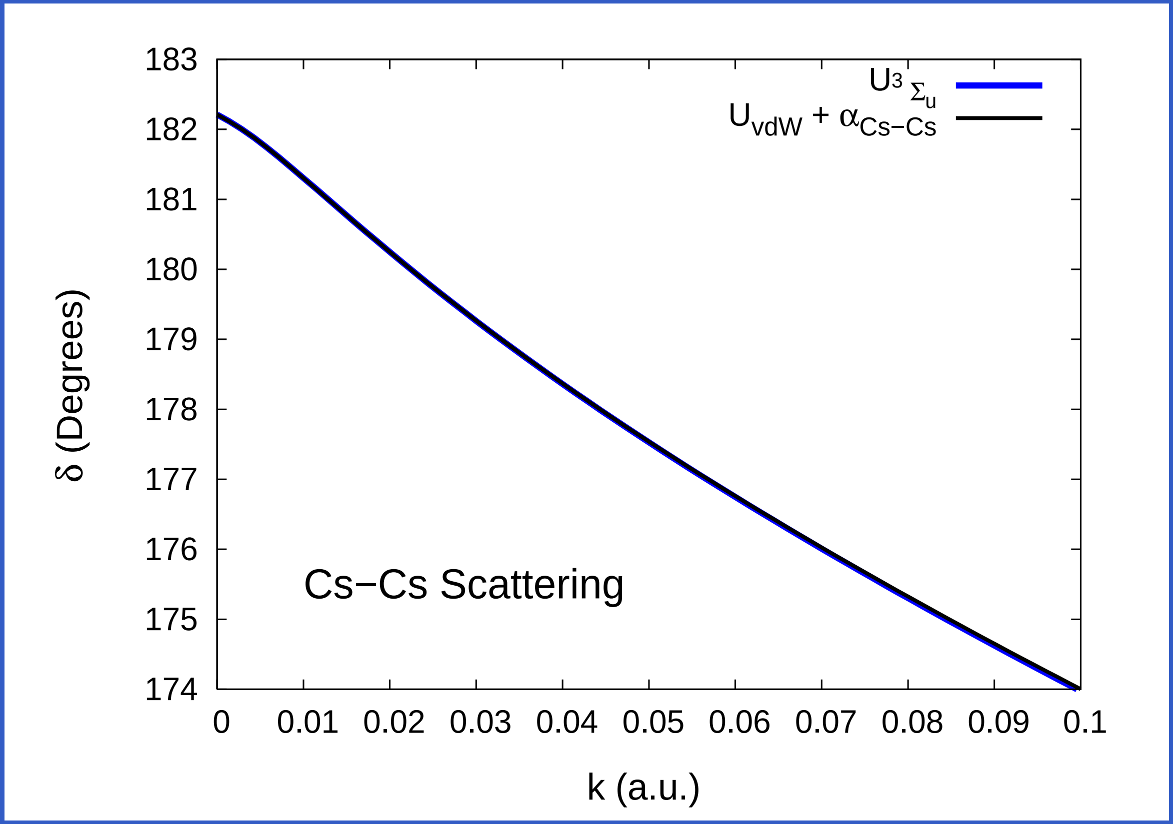

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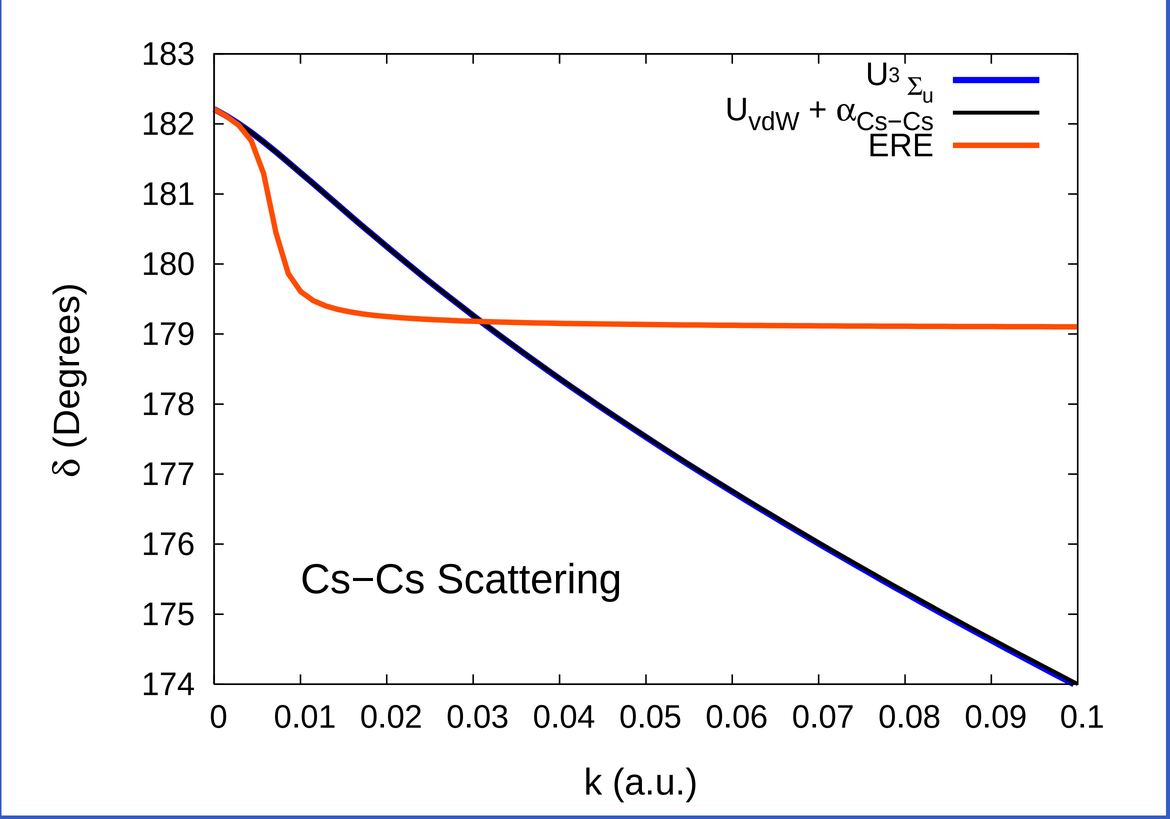


The Idea

We are going to pose the same set of question that in the Cs-Cs case

How do the phase shifts look like if we fix the scattering length to its physical value and assume some U_L as the long range potential?

But in some case, like the deuteron, the relations are going to be a bit more involved.

- Deuteron + OPE $\longrightarrow$ 1 Parameter
- Deuteron + TPE $\longrightarrow$ 3 Parametes!

The One Channel Case

The textbook Quantum Mechanical Problem...

Let’s consider a two body QM system
(described by the reduced Schödinger equation)

$$-u'' + U(r)\, u = k^2\, u \quad \text{(s-wave)}$$

For regular enough potentials (i.e. $\lim_{r\to 0} r^2\, U(r) = 0$)

$$u(r) \longrightarrow a + b\, r$$

Two linearly independent solutions
$\longrightarrow$ We choose the regular one (i.e. $u(0) = 0$)

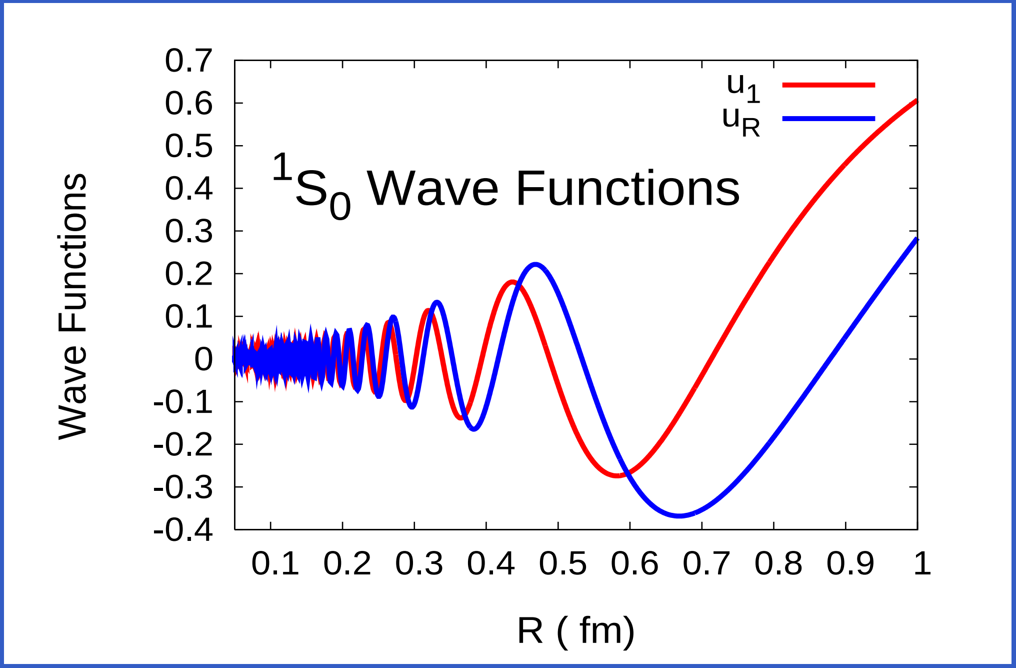
The One Channel Case

But usually our knowledge of the potential is not complete

- The long range piece of the potential is known in the form of some kind of long range expansion. Examples are
  - NN Potential
    $\rightarrow U(r) = U_{LO}(r) + U_{NLO}(r) + U_{NNLO}(r) + \ldots$
  - Atom-Atom Potential
    $\rightarrow U(r) = -\frac{C_6}{r^6} - \frac{C_8}{r^8} - \frac{C_{10}}{r^{10}} - \ldots$
- The long range piece gets very singular near the origin.
- The short range piece is unknown.

(Attractive) Singular Potentials (I)

$u(0) = 0$ is always fulfilled for both independent solutions



The regularity condition does not choose the solution!

(Attractive) Singular Potentials (I)

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For an attractive singular potential that behaves as

$$U(R) \rightarrow -{R_L}^{N-2}/R^N$$

the solutions of the wave function near the origin behave as

$$u(r) \rightarrow C\,(R)^{N/4} \sin\left[\frac{2}{N-2}\left(\frac{R_L}{R}\right)^{\frac{N}{2}-1} + \varphi\right]$$

$\longrightarrow$ We fix the solution by fixing a physical observable!

(Previous treatment in Beane, Bedaque, Childress, Kryjesvky, McGuire and van Kolck)

Singular Potentials (II)

But there is a numerical problem: the oscillations.

$\longrightarrow$ We put a cutoff in everything that constains the wave function, and check for cutoff independence

$$\int_0^\infty dR \to \int_{R_S}^\infty dR \,, \quad u(0) \to u(R_S) \quad \text{and so on}$$

$\longrightarrow$ We can even make an estimation of the cutoff uncertainties

$$I = \int_0^\infty dR\, u^2(R) = \int_{R_S}^\infty dR\, u^2(R) + \Delta I$$

For $U(R) \sim -\frac{1}{R^N} \Rightarrow \Delta I = \int_0^{R_S} dR\, u^2(R) \leq C\, {R_S}^{N/2+1}$

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$$\Delta I \leq C \, {R_S}^{N/2+1} \,, \quad |u\,(R_S)| \leq C' \, {R_S}^{N/4}$$

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$$\Delta\, I \leq C\, {R_S}^{N/2+1} \,, \quad |u\,(R_S)| \leq C'\, {R_S}^{N/4}$$

So attractive singular potentials are not a bug, but a feature.

Long Range Physics Correlations (I)

For example, the effective range is given by

$$r_0 = 2\int_0^\infty dr \left[\left(1 - \frac{r}{\alpha_0}\right)^2 - u_0(r)^2 \right]$$

If α_0 is an input, and u_0 is normalized as to reproduce the scattering length ($u_0\,(R) \to 1 - R/\alpha_0$), then

$$r_0 = A + \frac{B}{\alpha_0} + \frac{C}{\alpha_0^2}$$

where A, B and C depends only on the long range potential.

Long Range Physics Correlations (II)

In Atom-Atom scattering, the van der Waals potential induces the next correlation between α_0 and r_0

$$r_0 = \frac{16\, R\, \Gamma(\frac{5}{4})^2}{3\,\pi} - \frac{4\, R^2}{3\, \alpha_0} + \frac{4\, R^3\, \Gamma(\frac{3}{4})^2}{3\, \alpha_0^2\, \pi}$$

(Gribakin & Flambaum)

Atom	α_0	r_0	$r_0(vdW)$
Li-Li	36.9	66.5 $({}^1\Sigma_g)$	66.296
Na-Na	34.936	187.5 $({}^1\Sigma_g)$	187.316
Cs-Cs	68.216	624.55 $({}^3\Sigma_u)$	624.013

Long Range Physics Correlations (III)

The Chiral NN Potentials also induces the next correlations between α_0 and r_0

$$r_0 = 1.308 - \frac{4.548}{\alpha_0} + \frac{5.193}{\alpha_0^2} \qquad \text{(OPE)}$$

$$r_0 = 2.122 - \frac{4.889}{\alpha_0} + \frac{5.499}{\alpha_0^2} \qquad \text{(TPE NLO)}$$

$$r_0 = 2.617 - \frac{5.641}{\alpha_0} + \frac{5.953}{\alpha_0^2} \qquad \text{(TPE NNLO)}$$

For TPE NNLO we take $c_1 = -0.81 \mathrm{GeV}^{-1}$, $c_3 = -3.20 \mathrm{GeV}^{-1}$, $c_4 = 5.40 \mathrm{GeV}^{-1}$ (Entem & Machleidt)

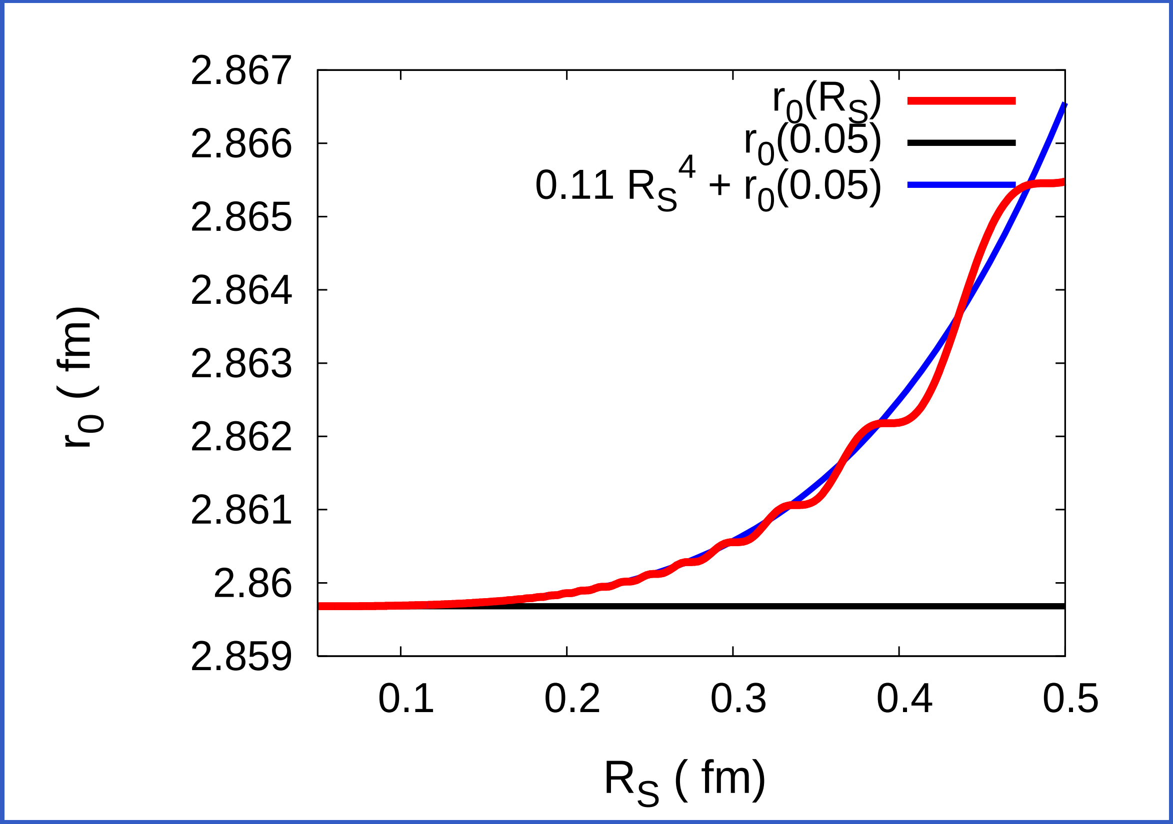
Long Range Physics Correlation (IV)

$^1S_0$	OPE	TPE NLO	TPE NNLO	Exp	Nijm II
$\alpha(\mathrm{fm})$	Input	Input	Input	-23.74(2)	-23.73
$r_0(\mathrm{fm})$	1.46	2.29	2.86	2.77(5)	2.67
$v_2(\mathrm{fm}^3)$	-2.04	-1.02	-0.36	-	-0.48
$v_3(\mathrm{fm}^5)$	9.29	6.09	4.86	-	3.96
$v_4(\mathrm{fm}^7)$	-50.60	-35.16	-27.64	-	-19.88

There is convergence ($LO \to NLO \to NNLO \to \ldots$).

Long Range Physics Correlation (V)

Is the effective range really converging on the cutoff?



Scattering States (I)

We compute the Scattering States from orthogonality to the zero energy state (or to any other state that we choose)

$$\int_0^\infty dR\, u_0\,(R) u_k\,(R) = 0 \to {u_0}'(0)\, u_k(0) - u_0(0)\, {u_k}'(0) = 0$$

That is equivalent to the following condition

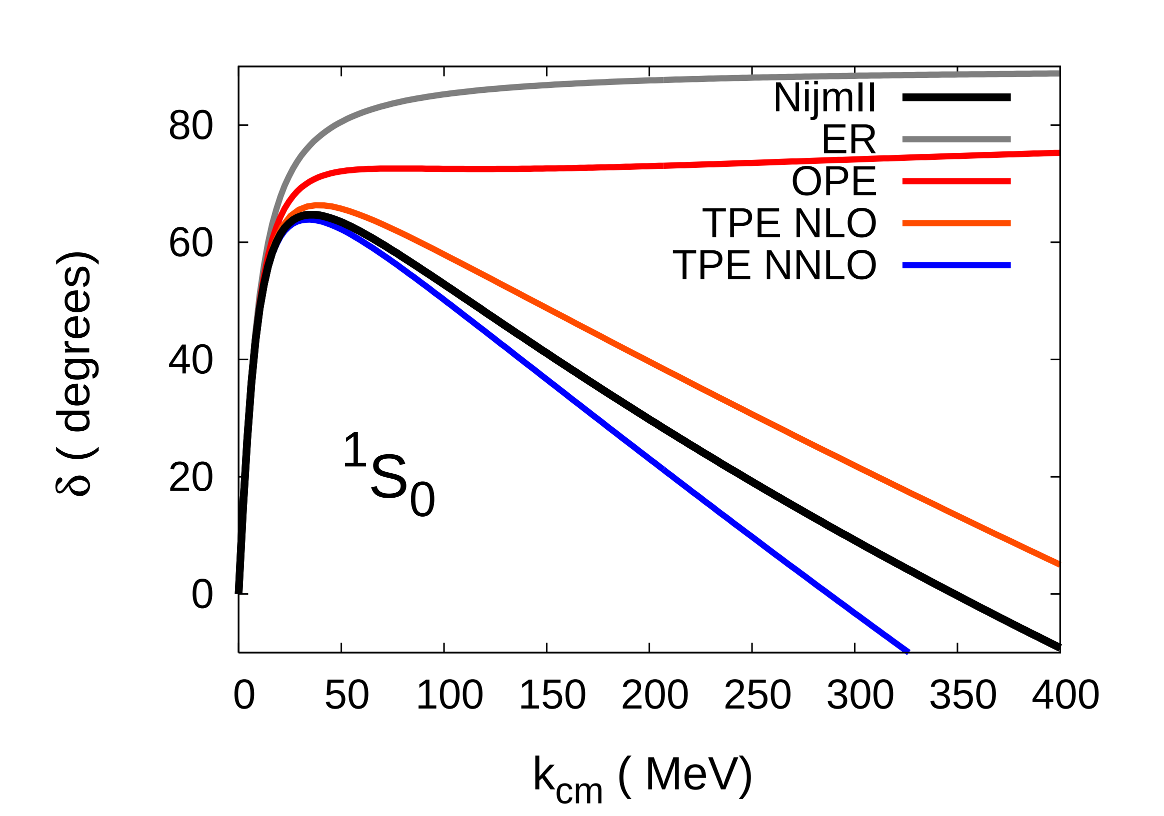
$$\frac{{u_0}'(0)}{u_0(0)} = \frac{{u_k}'(0)}{u_k(0)}$$

Phase shifts can be extracted from the asymptotic behaviour

$$u_k\,(R) \to \sin\left(k\, R + \delta_0\,(k)\right)$$

Scattering States (II)

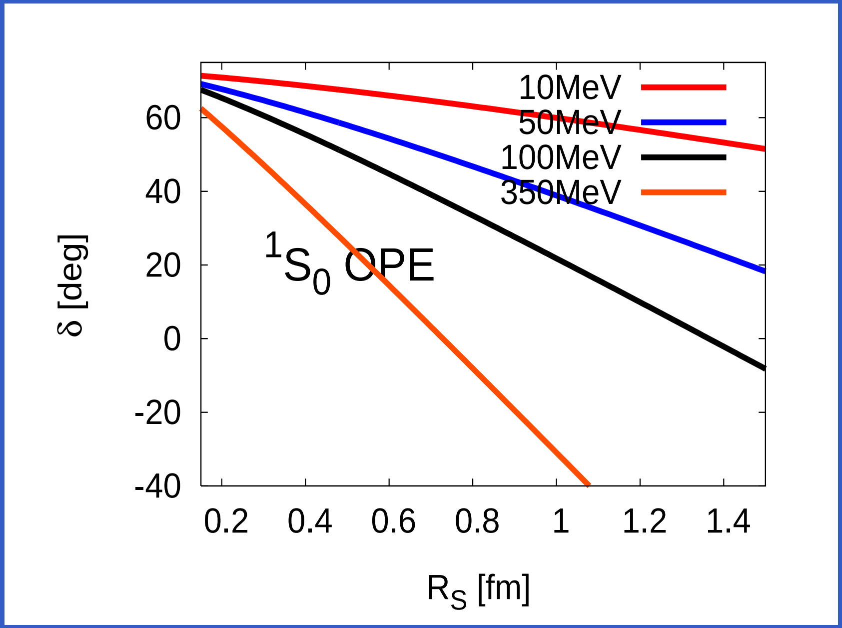
Results for the $^1S_0$ channel (fixing α_0)

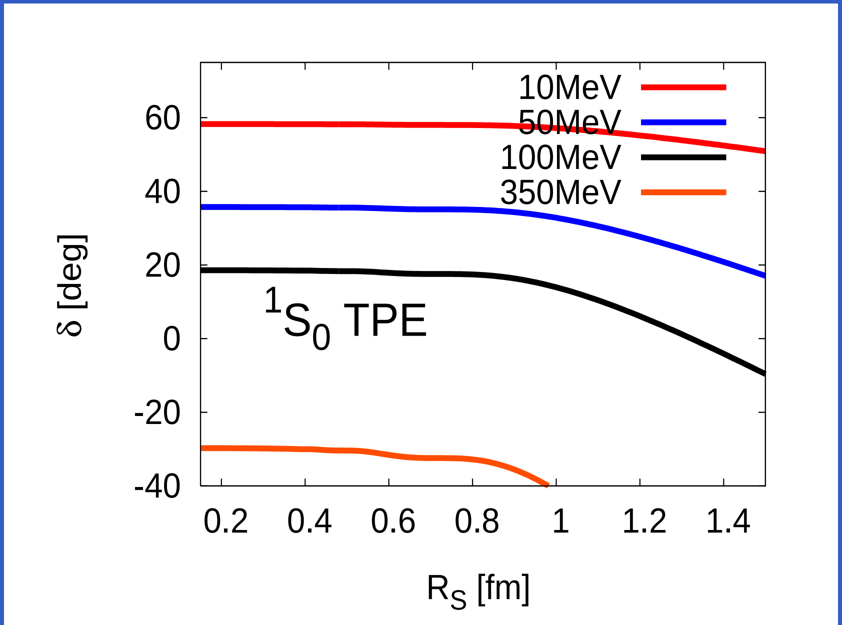


Scattering States (III)

Are the phase shifts really converging on the cutoff?

(NTvK (Nogga, Timmermans, van Kolck) test)





$\longrightarrow$ TPE converges better than OPE!

Cutoff Thoughts

Epelbaum et al. find cut-off dependence at LO, NLO, $NNLO$ but not that much at N^3LO . Why ?

The more singular the potential the better, the singularity smooths the cut-off dependence.

Thus, the only explanation is that the N^3LO is more singular and attractive. Facing singularities is essential

In conclusion,
The cut-off must be removed, but because of the attractive singularity the cut-off dependence is tamed and one can at high orders take smaller cut-offs.

A happy end?

The Deuteron Case

The Deuteron Case is as the Singlet Case, but a bit more involved, since there are two channels

- Singularity Structure related to the potential eigenvalues.

$$D\,U\,D^{-1} = U_D = \begin{pmatrix} U_+ & 0 \\ 0 & U_- \end{pmatrix}$$

- At short enough distances the equations decouple for singular potentials

$$-u_{(+,-)}''(R) + U_{(+,-)}\,u_{(+,-)}\,(R) = M\,E\,u_{(+,-)}\,(R)$$

The Deuteron Case (II)

There are three cases, depending on the sign of the eigenvalues of the potential

- One eigenvalue is attractive and other is repulsive
  $\longrightarrow$ We will need one constraint for the attractive channel (e.g. The deuteron binding energy).
- Both eigenchannels are attractive
  $\longrightarrow$ We will need three contraints to determine the solution. (e.g. The deuteron binding and mixing for the bound state + the $^3S_1$ scattering length for the scattering states).
- Both eigenchannels are repulsive $\longrightarrow$ We cannot fix anything!

The Deuteron Case (III)

So, to which case does the deuteron belong?

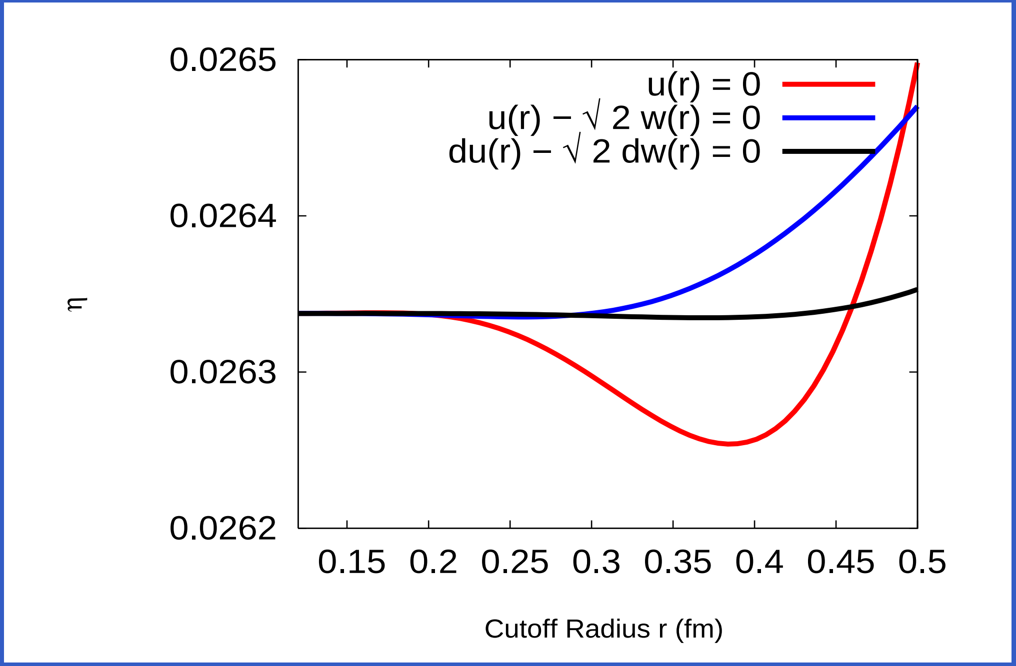
- Deuteron + OPE $\longrightarrow$ 1 Parameter (e.g. $\gamma = \sqrt{M\,E}$)
- Deuteron + TPE NLO $\longrightarrow$ 0 Parameters
- Deuteron + TPE NNLO $\longrightarrow$ 3 Parameters (e.g. γ, η, α_0)

(For Deuteron + OPE see BBSvK)

Deuteron + OPE

The eigenvalues of the potential are attractive-repulsive.

$\rightarrow$ We take $\gamma = \sqrt{M\,E}$ as the input parameter



$\rightarrow$ We predict the mixing ($\eta_{OPE} = 0.02633$)

Deuteron + OPE

Other OPE predictions when γ is given as input are

	η	$A_S(\mathrm{fm}^{-1/2})$	$r_m(\mathrm{fm})$	$Q_d(\mathrm{fm}^2)$	P_D
OPE	0.02633	0.8681	1.9351	0.2762	7.88 %
Exp.	0.0256	0.8846	1.971	0.2859	-

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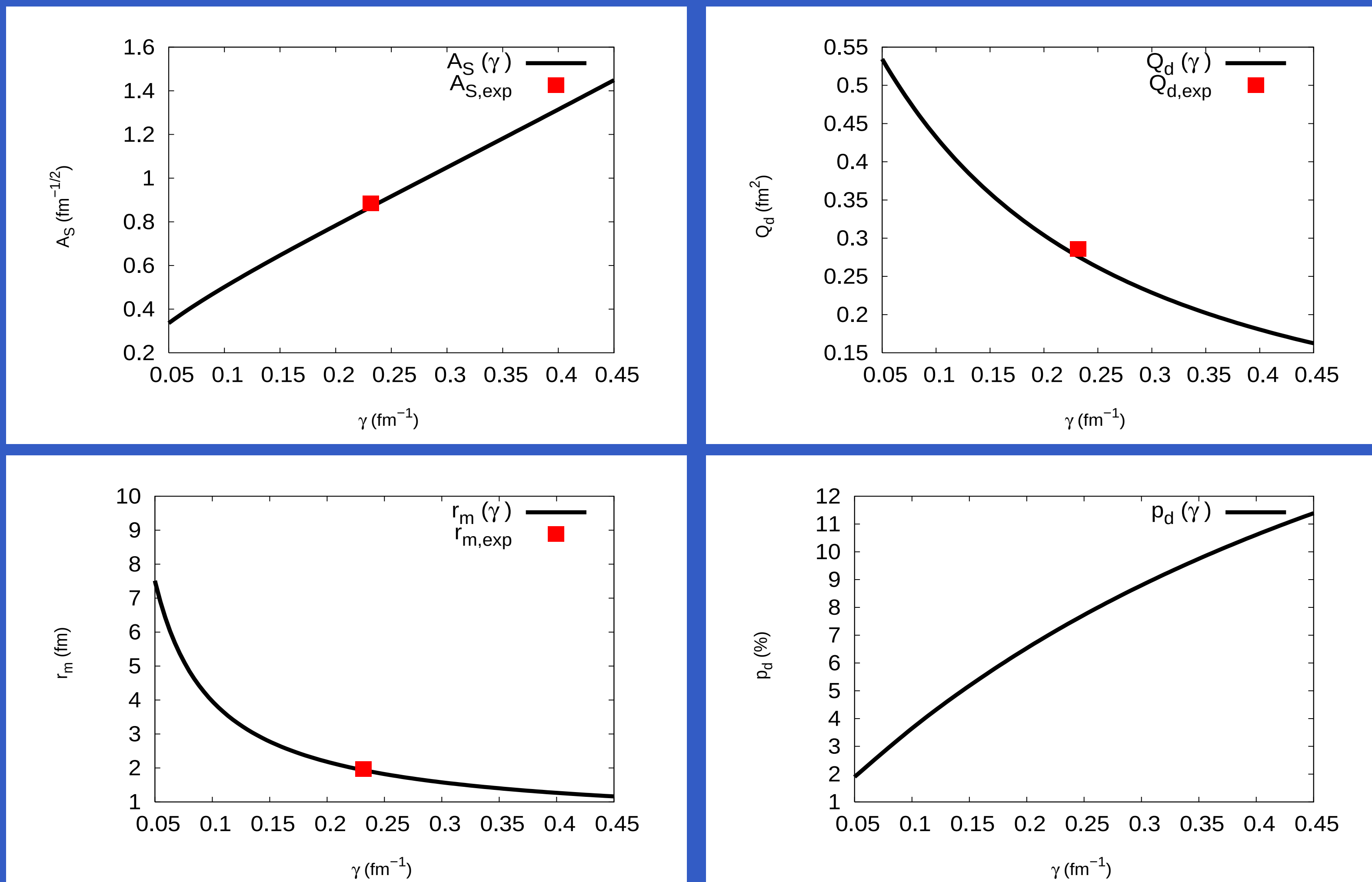
We can check too for what OPE predicts when γ is changed

For example, let's check what happens with A_S, r_m, Q_d and P_D.

Deuteron + OPE



Deuteron + TPE (NNLO)

The eigenvalues of the potential are attractive-attractive.

- We take γ, η and α_0 as input
- We can check for TPE correlations by changing γ, η and α_0
- TPE NNLO $\longrightarrow$ Chiral LEC's (c_1, c_3, c_4)
  $\longrightarrow$ There are different determinations of the LEC's
  $\longrightarrow$ Let's use some of them, and see what happens!

Deuteron + TPE (NNLO)

Set	Source	$c_1 (\mathrm{GeV}^{-1})$	$c_3 (\mathrm{GeV}^{-1})$	$c_4 (\mathrm{GeV}^{-1})$
Set I	(BM) πN	-0.81$\pm$ 0.15	-4.69$\pm$ 1.34	3.40 $\pm$ 0.04
Set II	(RTdS) NN	-0.76	-5.08	4.70
Set III	(EMa) NN	-0.81	-3.40	3.40
Set IV	(EMb) NN	-0.81	-3.20	5.40

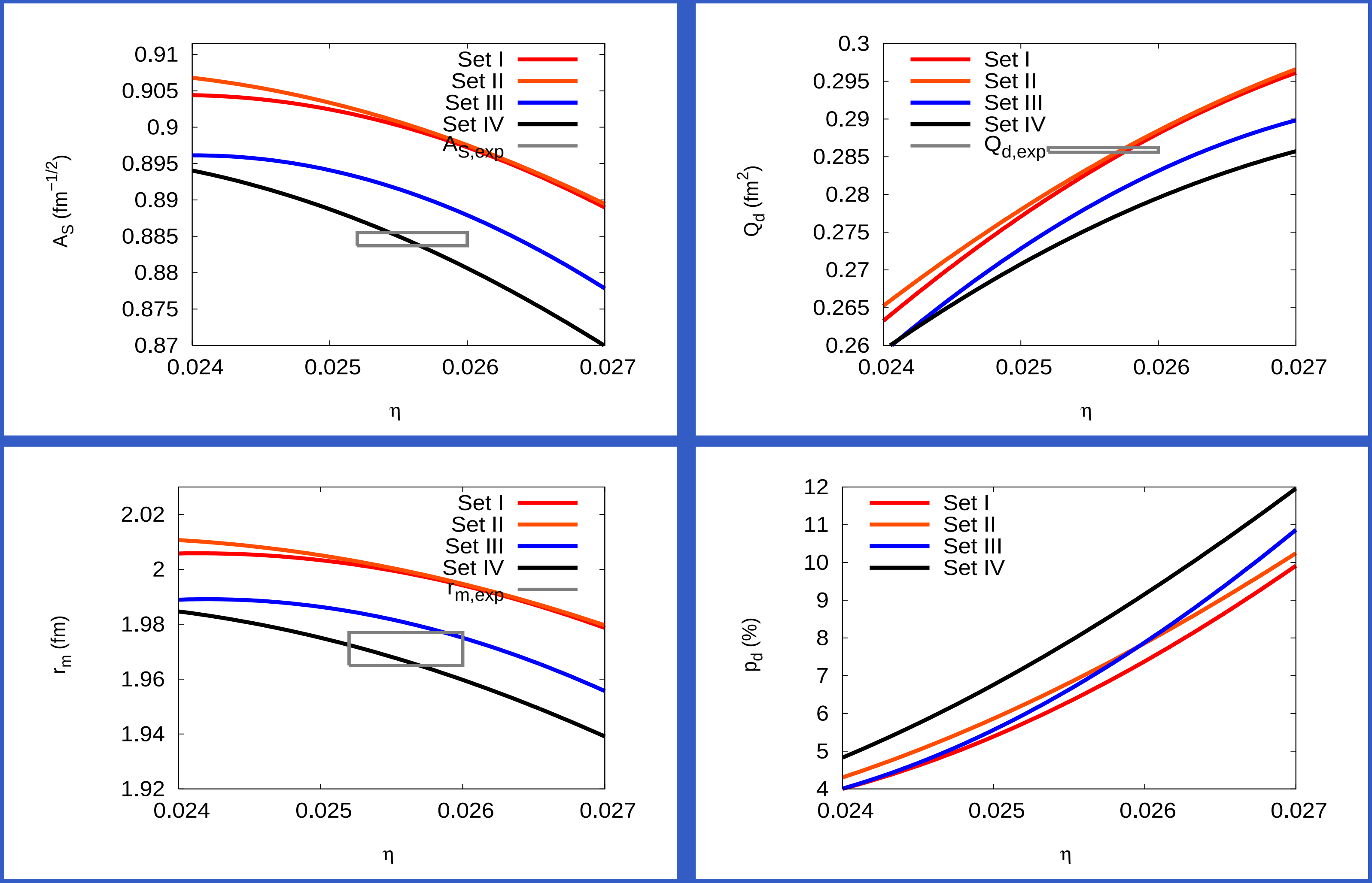
Then we can compute for each set and see for differences.

BM $\longrightarrow$ Büttiker and Meißner

RTdS $\longrightarrow$ Rentmeester, Timmermans, de Swart

EMa, EMb $\longrightarrow$ Entem and Machleidt

Deuteron + TPE (NNLO)



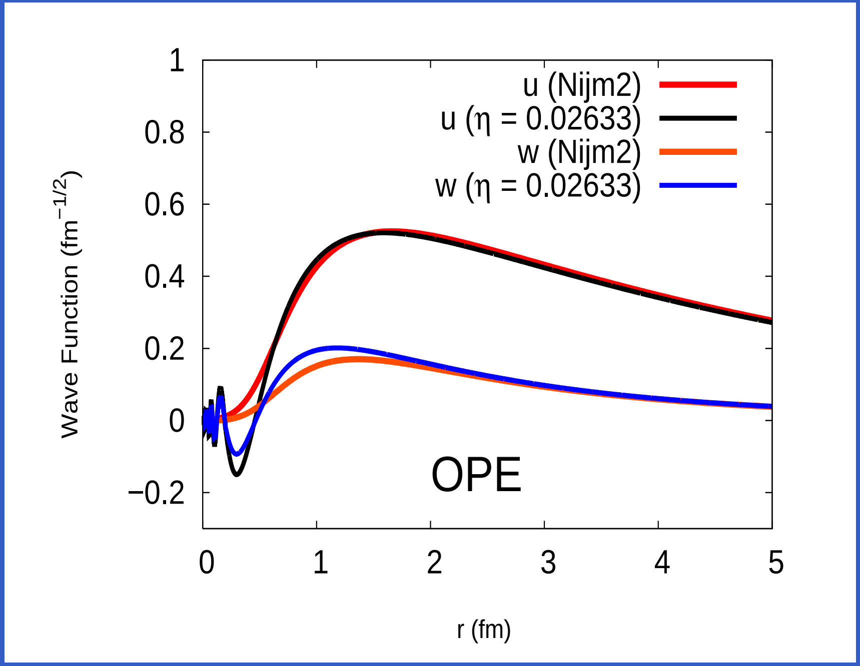
Deuteron + TPE (NNLO)

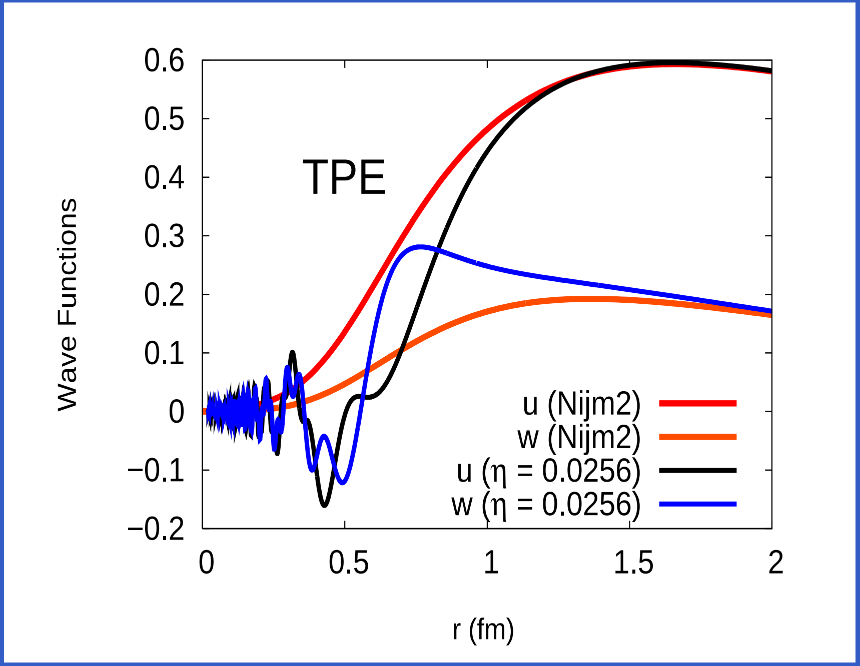
Set	η	$A_S(\mathrm{fm}^{-1/2})$	$r_d(\mathrm{fm})$	$Q_d(\mathrm{fm}^2)$	P_D
OPE	0.02634	0.8681(1)	1.9351(5)	0.2762(1)	7.88(1)%
Set I	Input	0.900(2)	1.999(4)	0.284(4)	6(1)%
Set II	Input	0.900(2)	1.999(4)	0.285(4)	7(1)%
Set III	Input	0.891(3)	1.981(5)	0.279(4)	7(1)%
Set IV	Input	0.884(4)	1.967(6)	0.276(3)	8(1)%
N^3LO	0.0256	0.8843	1.968	0.275	4.51 %
NijmII	0.02521	0.8845(8)	1.9675	0.2707	5.635%
Reid93	0.02514	0.8845(8)	1.9686	0.2703	5.699%
Exp.	0.0256(4)	0.8846(9)	1.971(6)	0.2859(3)	5.67(4)%

N^3LO computation by Entem and Machleidt.

Deuteron + TPE (NNLO)

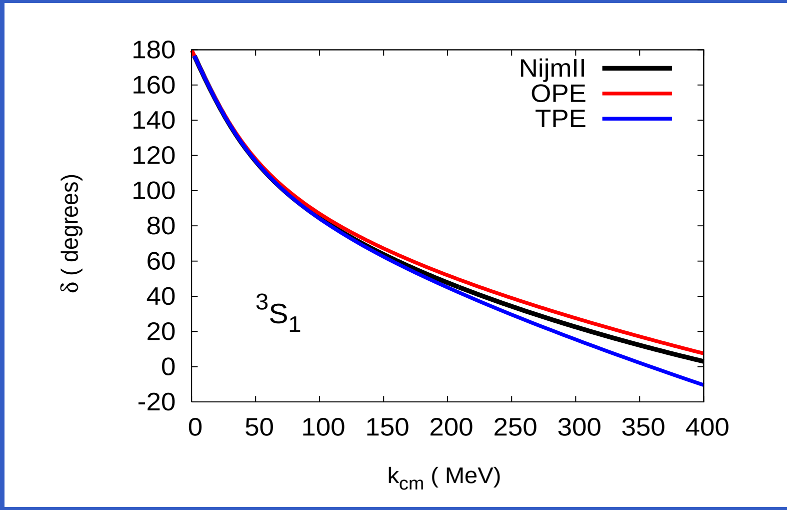
The wave function for OPE and TPE

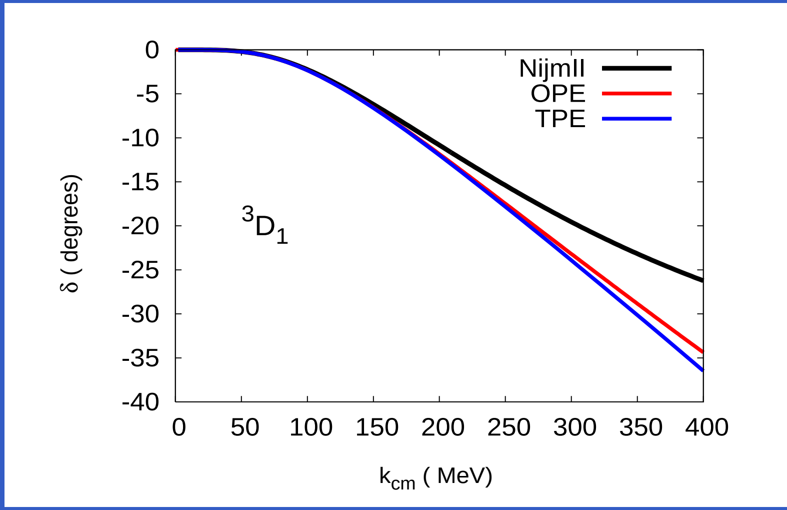


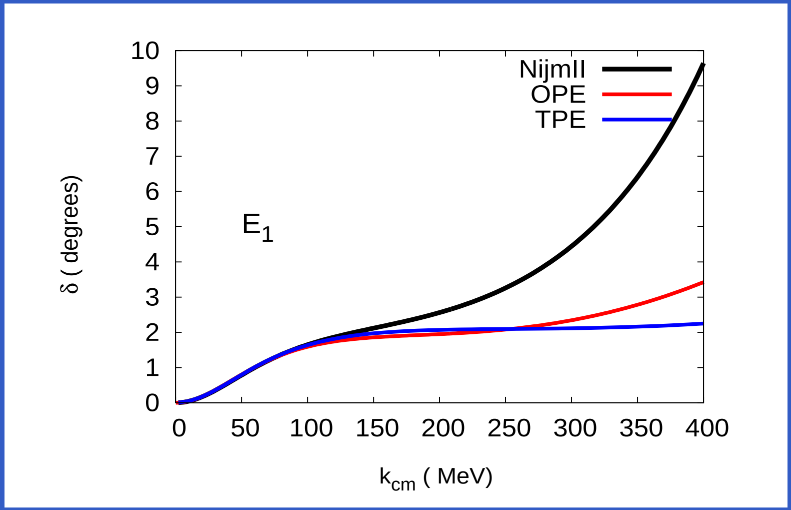


$\longrightarrow$ Deeply Bound States!

Deuteron Phase Shifts

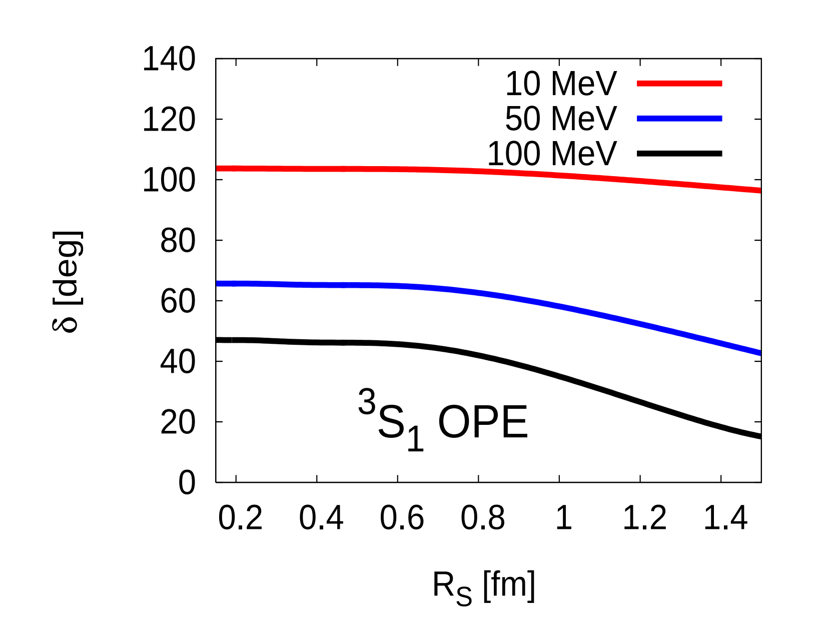


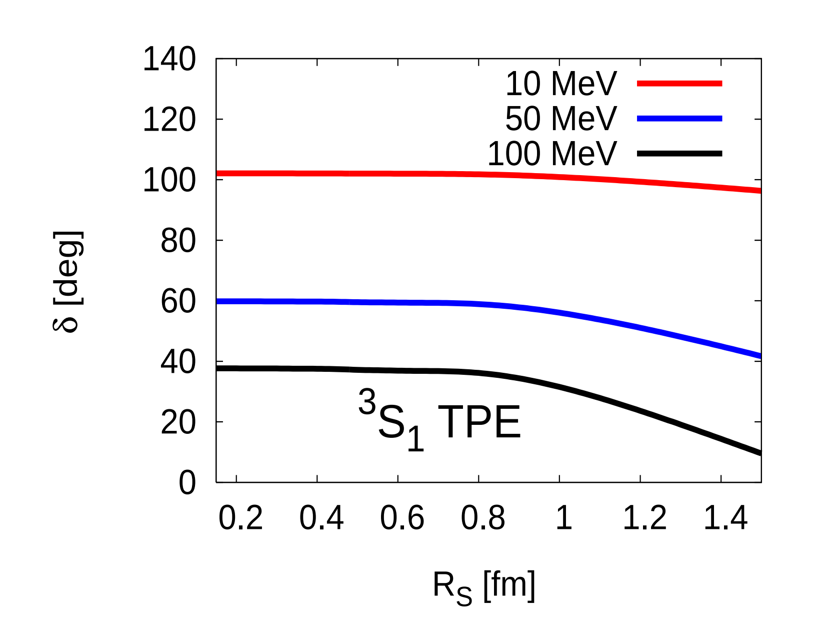




Deuteron Phase Shifts (II)

We perform again a NTvK convergence check





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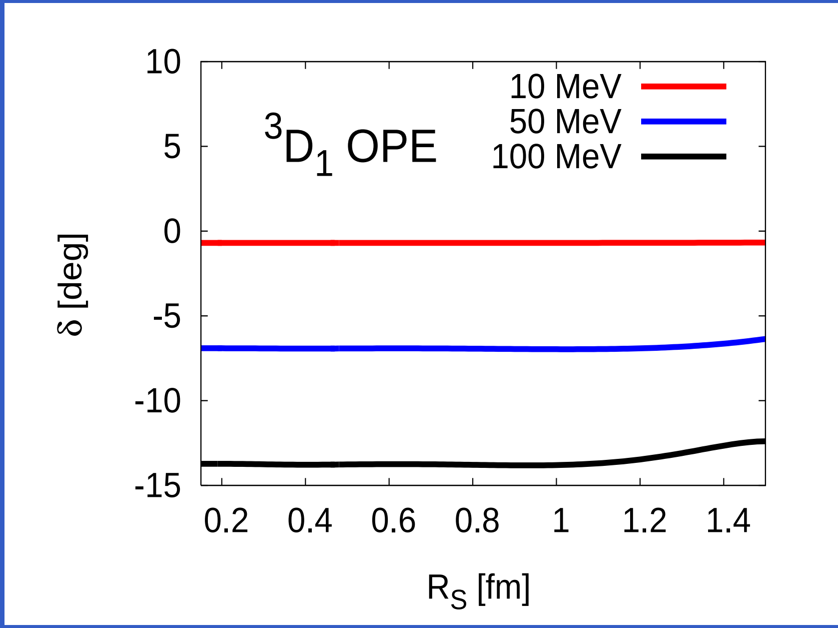

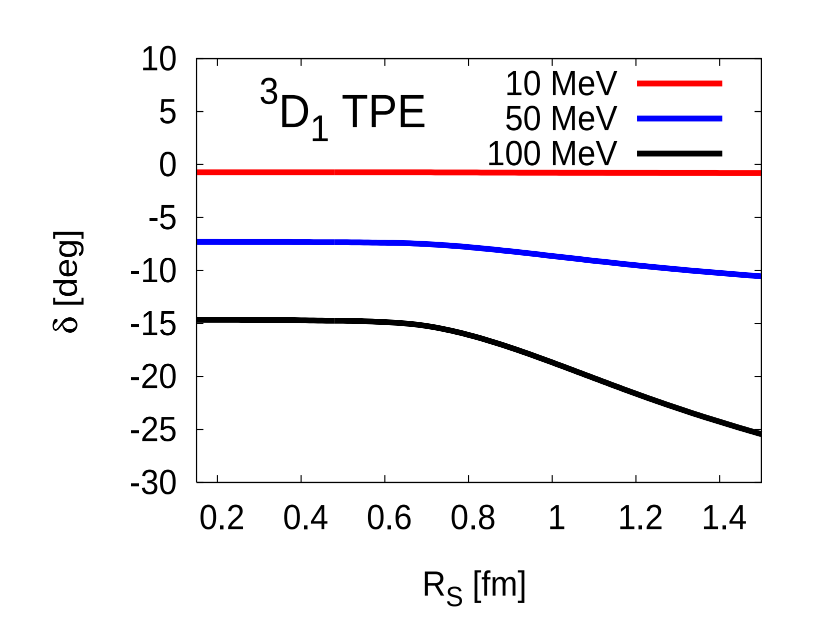

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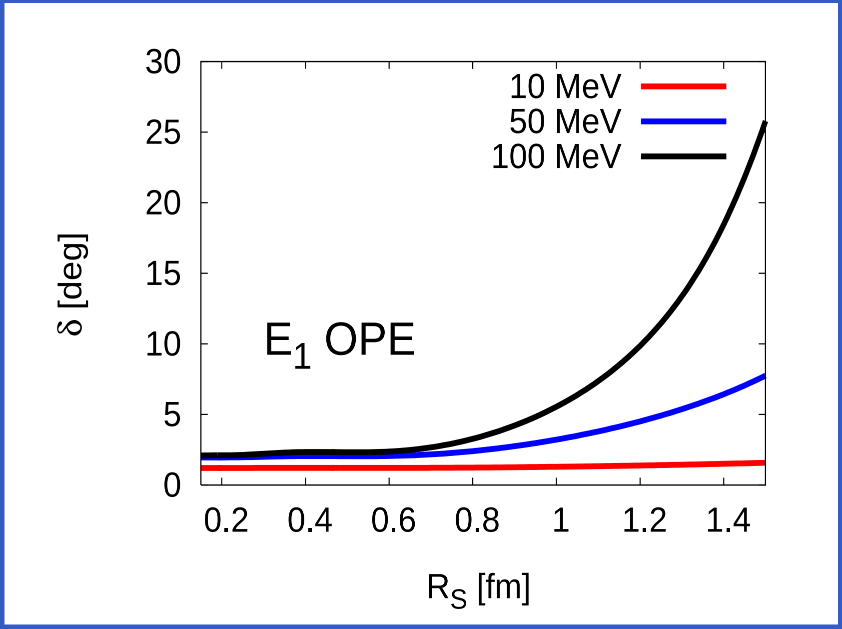
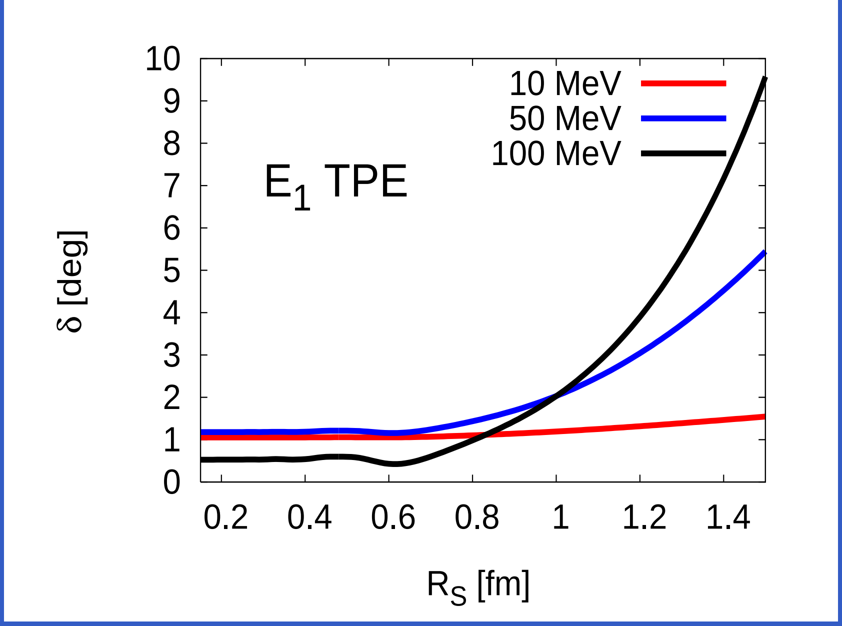
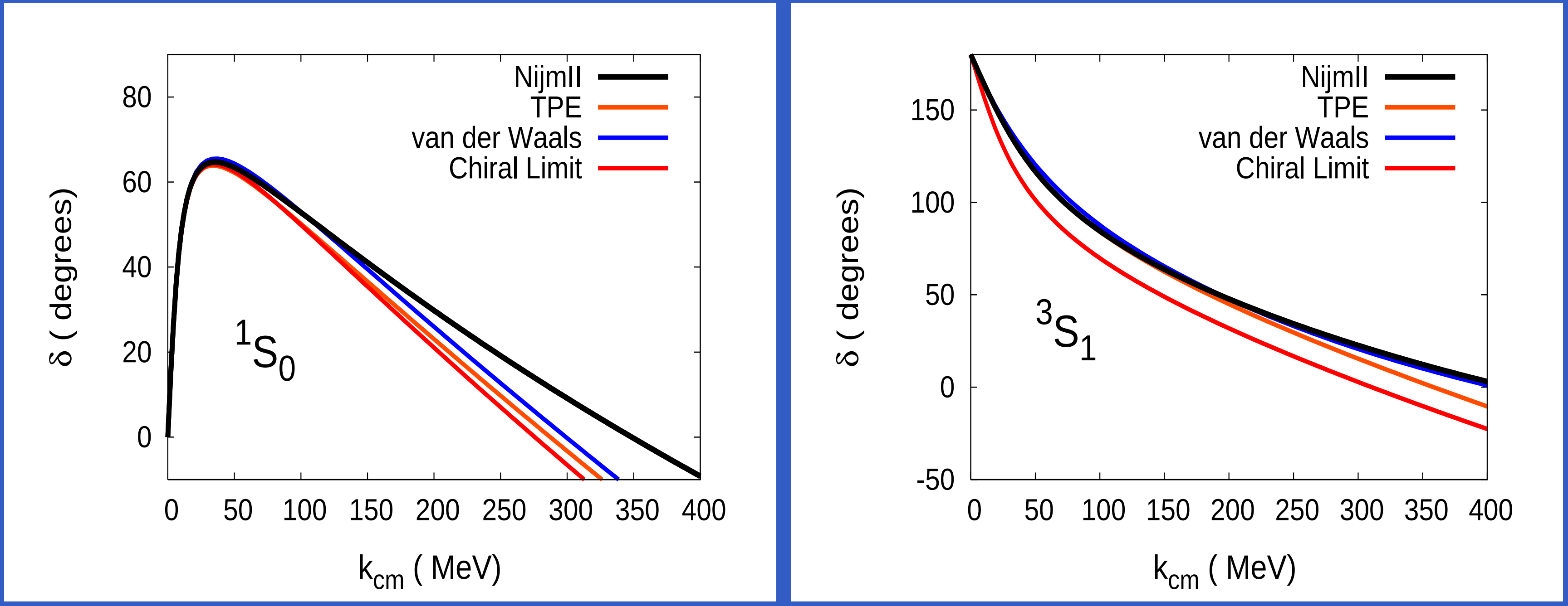




Deuteron Phase Shifts (III)

There is also a van der Waals dominance on the s-waves



$\longrightarrow$ Connection with the Liquid Drop Model?

Chiral Couplings

In Chiral Perturbation Theory both πN and NN low energy data should described by the same set of constants.

For this to be true

Uncertainties of the LEC's (c_1, c_3 and c_4) must be smaller than the cutoff uncertainties.

$\Rightarrow$ LEC's uncertainties must be estimated somehow!

Error Estimations (I)

We perform a Monte Carlo error propagation of the uncertainties of the LEC's and the input parameters to assess the errors

We take $g_{\pi NN} = 13.1 \pm 0.1$, $\alpha_{0,s} = -23.77 \pm 0.05$, $\alpha_{0,s} = 5.419 \pm 0.007$, $\eta = 0.0256 \pm 0.0004$.

And for the chiral couplings

Set I $\rightarrow c_1 = -0.81 \pm 0.15$, $c_3 = -4.69 \pm 1.34$, $c_4 = 3.40 \pm 0.04$
Set II $\rightarrow c_1 = -0.76 \pm 0.07$, $c_3 = -5.08 \pm 0.24$, $c_4 = 4.78 \pm 0.10$
Set IV $\rightarrow c_1 = -0.81$, $c_3 = -3.20 \pm 0.16$, $c_4 = 5.40 \pm 1.65$

Error Estimations (II)

	Set I	Set II	Set IV	Exp.
$r_{0,s}$	$2.92^{+0.08}_{-0.04}$	$2.97^{+0.03}_{-0.02}$	$2.86^{+0.04}_{-0.03}$	2.77 ± 0.05
$r_{0,t}$	$1.36^{+0.33}_{-0.75}$	$1.48^{+0.14}_{-0.25}$	$1.76^{+0.03}_{-0.06}$	1.753 ± 0.008
A_s	$0.899^{+0.008}_{-0.009}$	$0.900^{+0.003}_{-0.004}$	$0.884^{+0.005}_{-0.008}$	0.8849 ± 0.0009
Q_d	$0.284^{+0.005}_{-0.007}$	$0.284^{+0.005}_{-0.004}$	$0.276^{+0.004}_{-0.004}$	0.2859 ± 0.0003
r_m	$1.998^{+0.015}_{-0.019}$	$1.998^{+0.007}_{-0.007}$	$1.965^{+0.011}_{-0.014}$	1.971 ± 0.006
P_d	$6.6^{+1.0}_{-0.9}$	$7.1^{+0.9}_{-0.9}$	$8.3^{+1.4}_{-1.5}$	–
α_{02}	$2.26^{+0.51}_{-0.39}$	$2.20^{+0.23}_{-0.16}$	$1.67^{+0.13}_{-0.13}$	–
α_2	$2.6^{+2.8}_{-6.6}$	$3.1^{+1.4}_{-2.8}$	$6.17^{+0.39}_{-0.75}$	–

Error Estimations (III)

Let's make a determination of the LEC's by minimizing the next χ^2

$$\chi^2 = \left(\frac{r_{0,s} - r_{0,s}^{\exp}}{\Delta r_{0,s}}\right)^2 + \left(\frac{r_{0,t} - r_{0,t}^{\exp}}{\Delta r_{0,t}}\right)^2 + \left(\frac{A_S - A_S^{\exp}}{\Delta A_S}\right)^2$$

And let's also do a Monte Carlo in all the input parameters.

$$\begin{aligned} c_1 &= -1.13^{+0.02}_{-0.04}\,\mathrm{GeV}^{-1} \\ c_3 &= -2.60^{+0.18}_{-0.23}\,\mathrm{GeV}^{-1} \\ c_4 &= +3.40^{+0.25}_{-0.40}\,\mathrm{GeV}^{-1} \end{aligned}$$

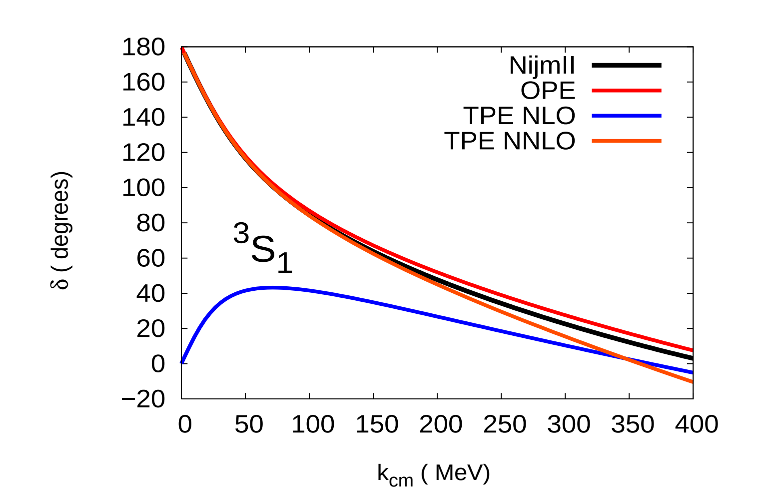
The NLO Problem

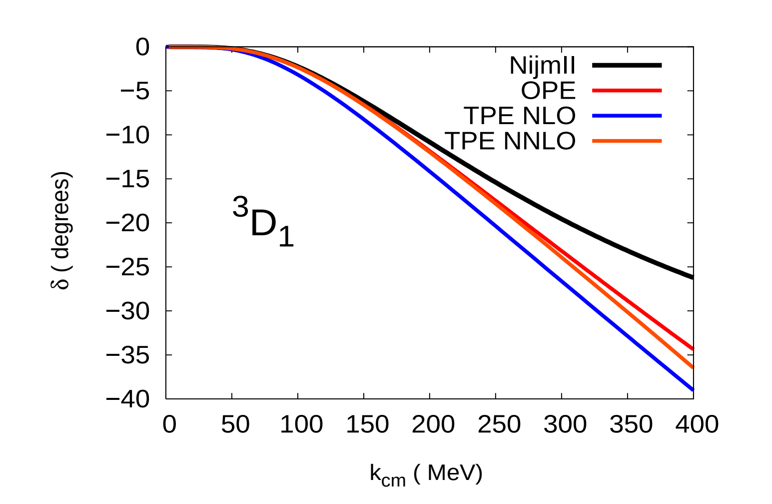
The TPE NLO is repulsive-repulsive in the Deuteron Channel.

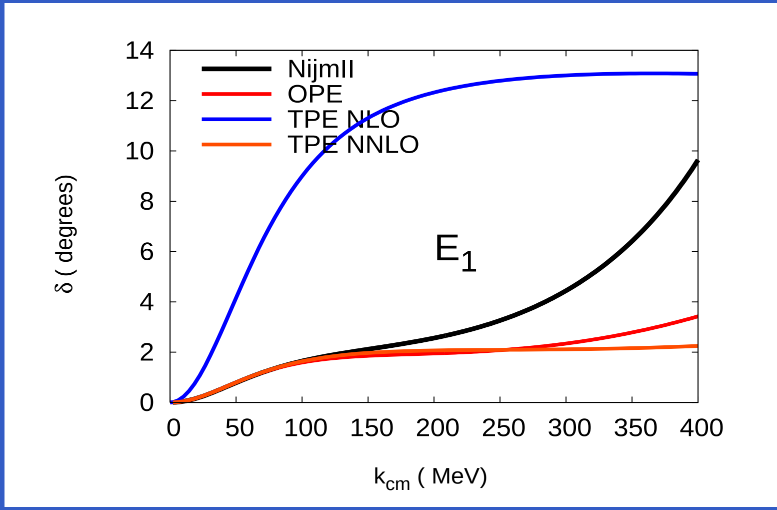
$\rightarrow$ We cannot fix anything!

If we take a look to the phase shifts...

The NLO Problem

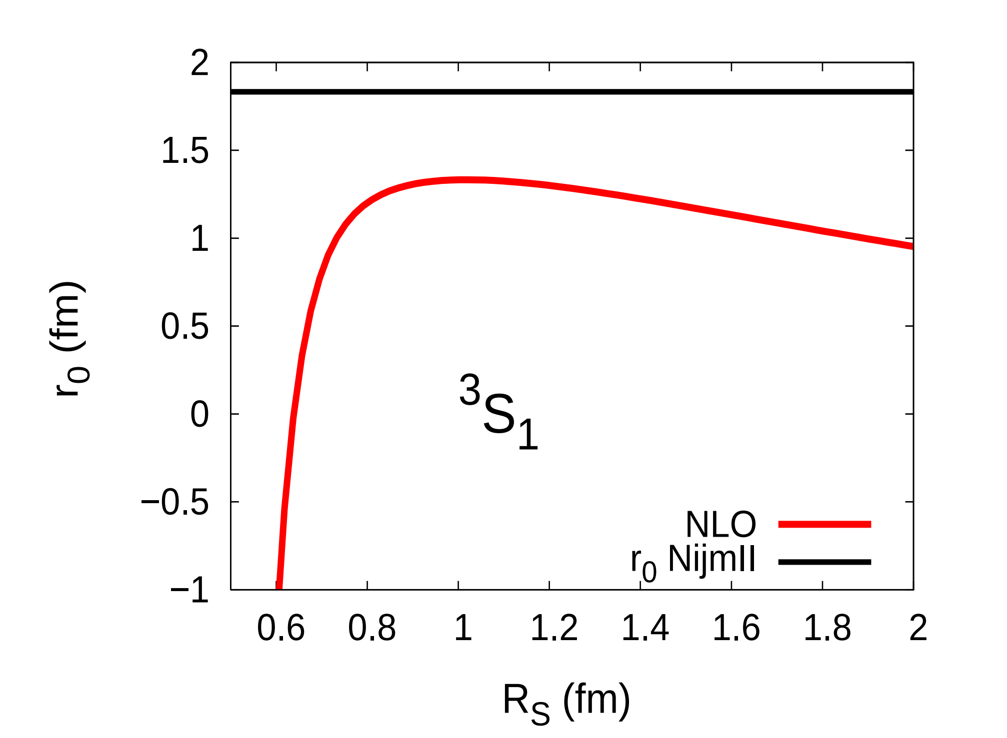






The NLO Problem

What happens if we try to fix the scattering length?



$\longrightarrow$ The effective range diverges!

The NLO + Δ

(prompted by a recent discussion with D. Phillips)

The contribution of the Δ to the NN potential is attractive

$\Delta \simeq 293\mathrm{MeV} \sim 2\,m_\pi \rightarrow$ Since the Δ splitting is small, why not include its contribution at NLO?

Recipe: drop all terms at NNLO which do not contain c_3 or c_4 and set $c_3 = -\frac{{g_A}^2}{2\,\Delta}$ and $c_4 = \frac{{g_A}^2}{4\,\Delta}$

$$(c_3 \simeq -2.71\mathrm{GeV}^{-1},\, c_4 \simeq 1.35\mathrm{GeV}^{-1})$$

The NLO + Δ

The potential is again attractive-attractive $\rightarrow$ 3 Parameters

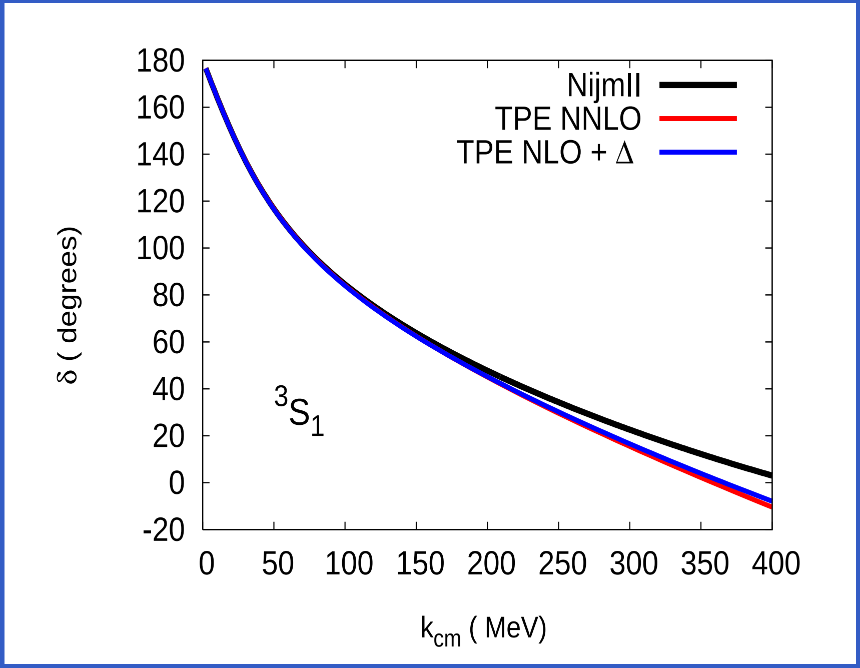
	η	$A_S(\mathrm{fm}^{-1/2})$	$r_m(\mathrm{fm})$	$Q_d(\mathrm{fm}^2)$	P_D
OPE	0.02633	0.8681	1.9351	0.2762	7.88 %
NLO + Δ	Input	0.8868	1.972	0.276	5.66 %
NNLO (EMb)	Input	0.884	1.967	0.276	8 %
Exp.	0.0256	0.8846	1.971	0.2859	-

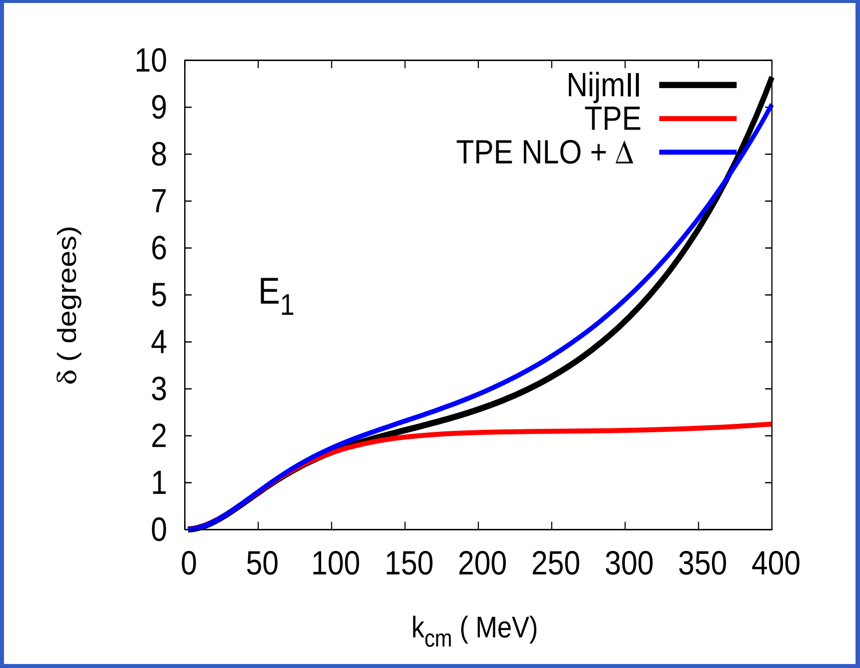
The convergence $LO \rightarrow NLO + \Delta \rightarrow NNLO \rightarrow \ldots$ is restored!

We need the Δ contribution for NLO convergence!

The NLO + Δ

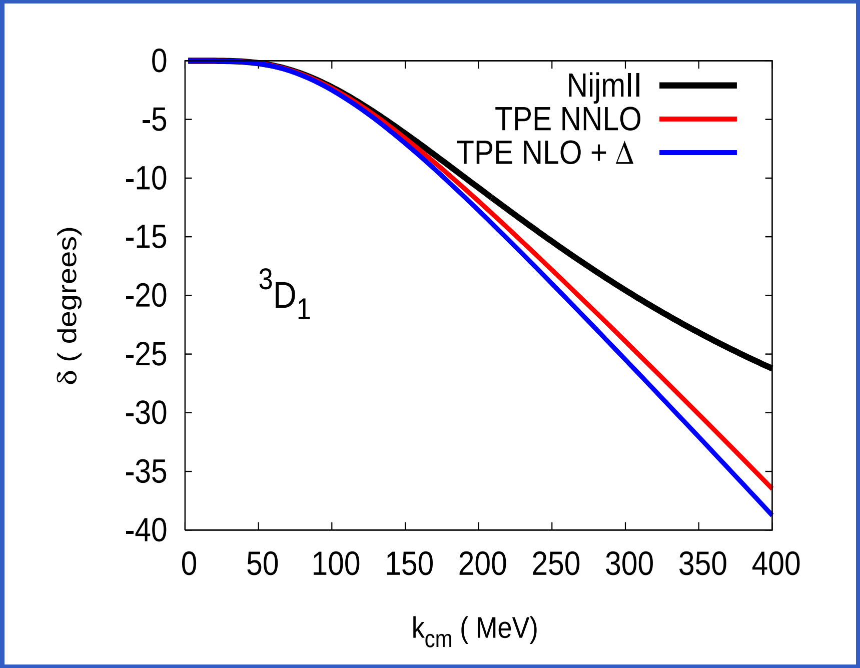
These phase shifts look good, but...

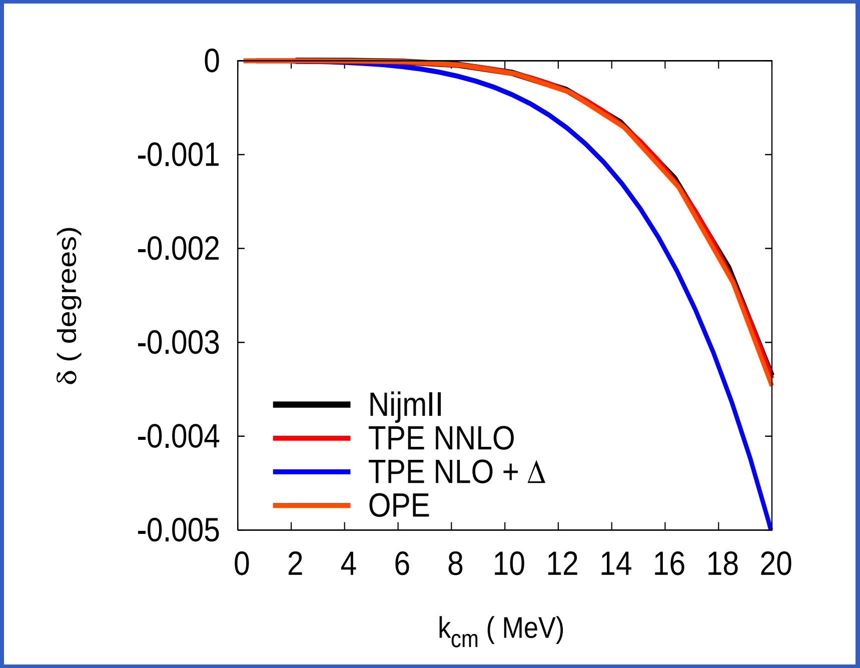




The NLO + Δ

on the $^3D_1$ wave the threshold is highly overestimated





The NLO + Δ

But there is a curious thing...

Replace the Δ-induced c_3 and c_4 by the values that Entem and Machleidt provide.

One will arrive to results almost identical to the TPE NNLO ones (although all not the TPE NNLO terms have been included).

Should TPE NNLO be promoted to NLO?

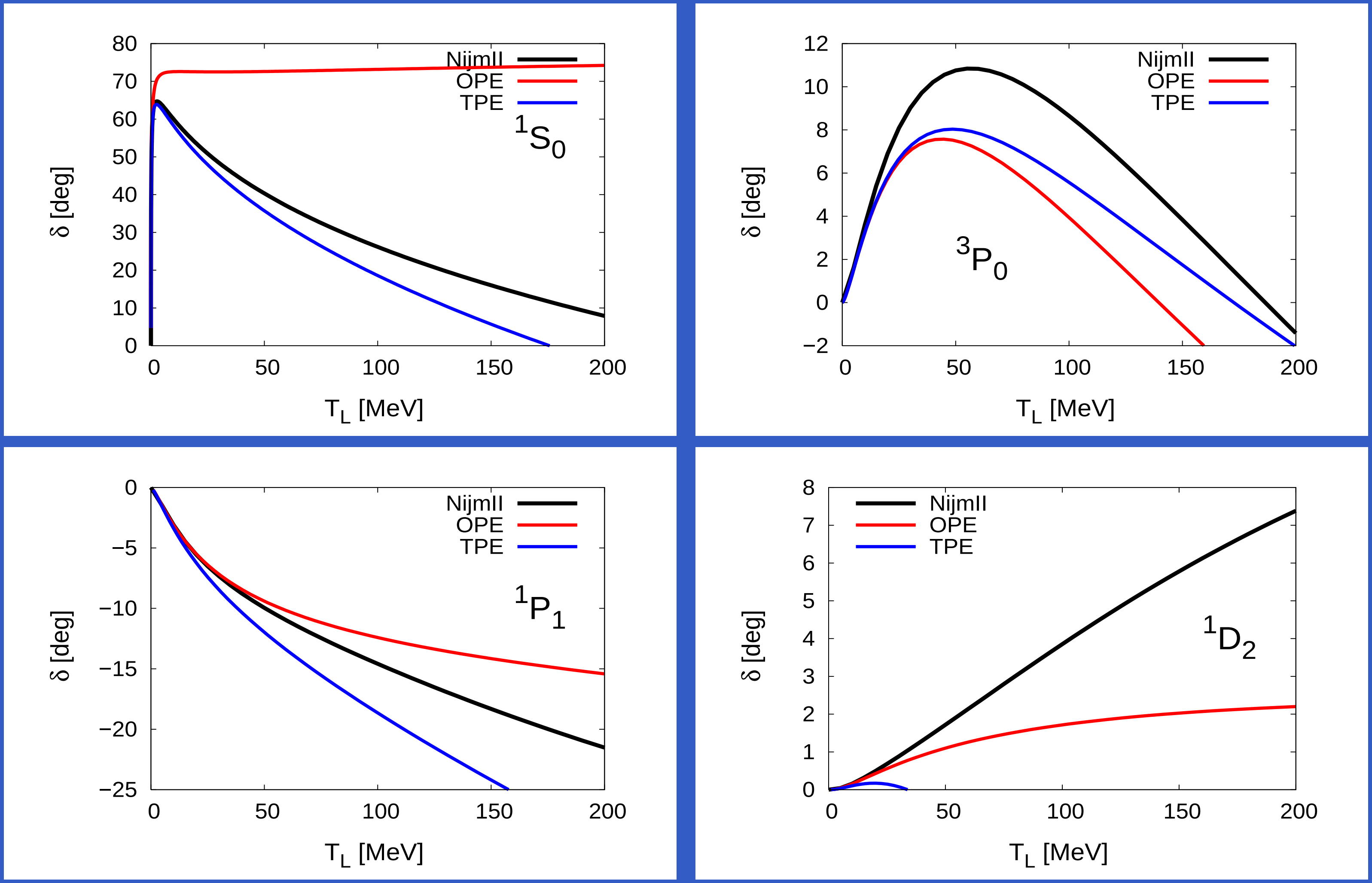
(More on the Δ in Pandharipande, Phillips,van Kolck)

Peripheral Waves

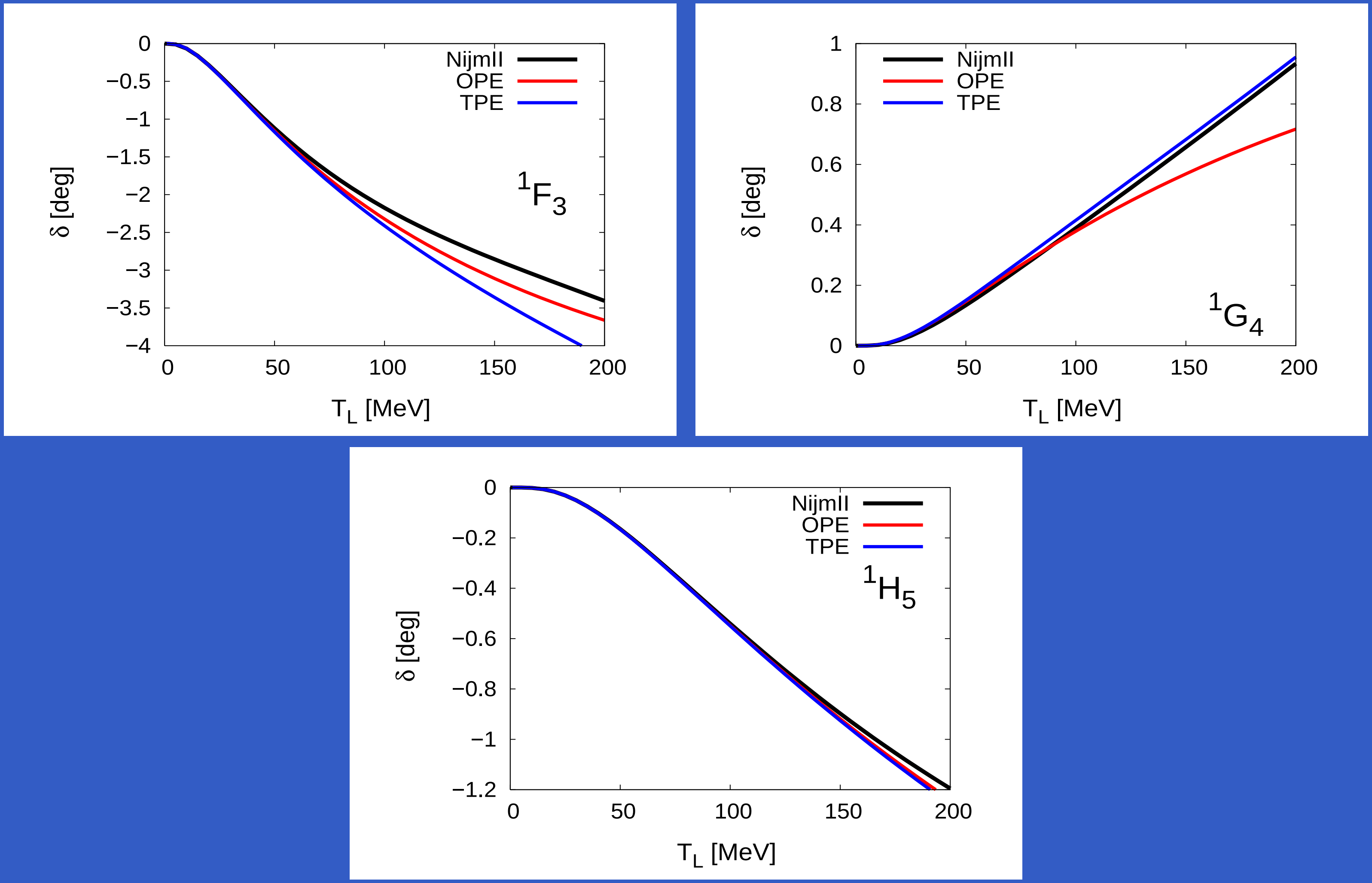
Caution: Preliminar Results

(Stapp-Ypsilantis-Metropolis parametrization)

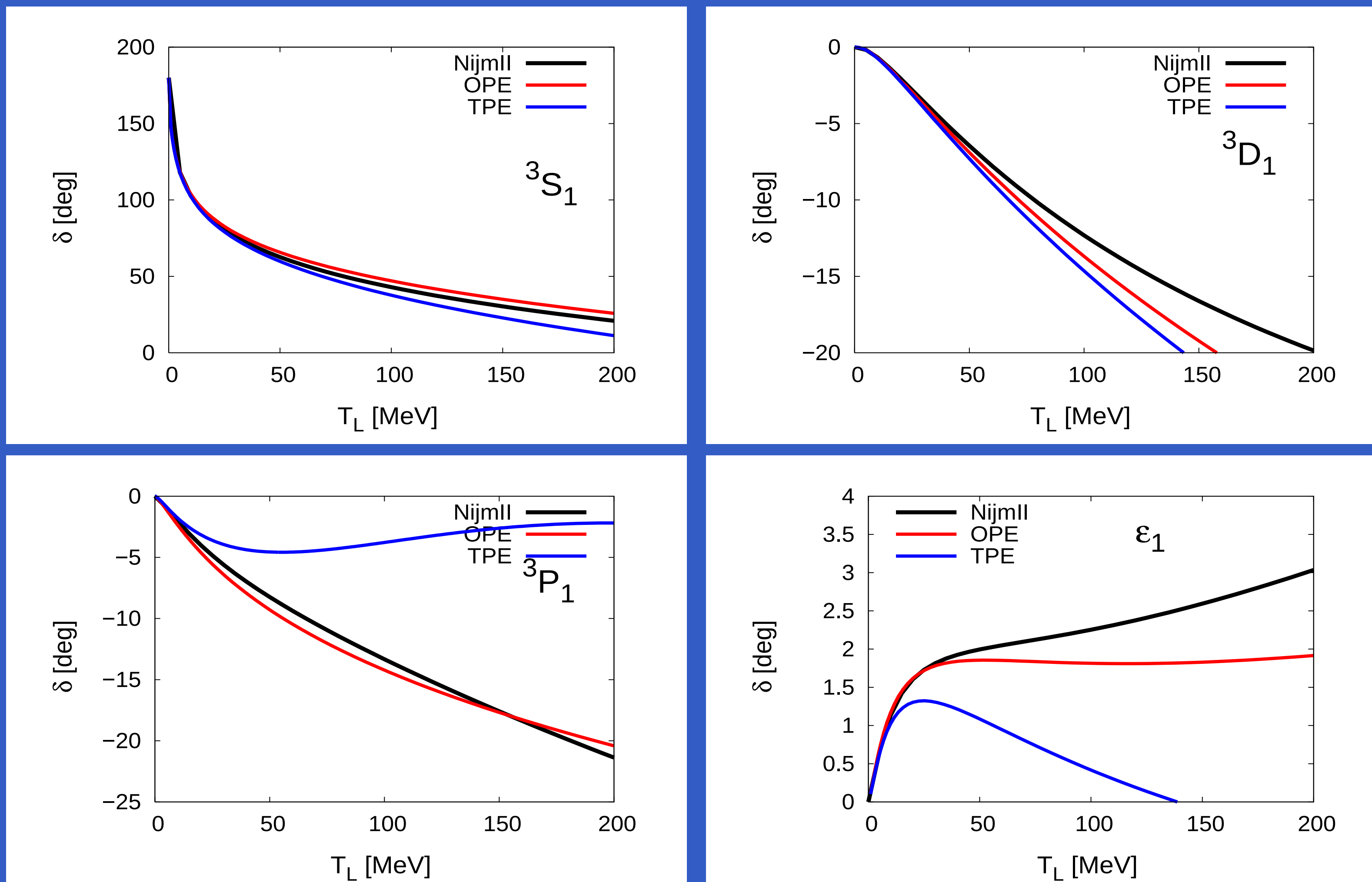
Peripheral Waves



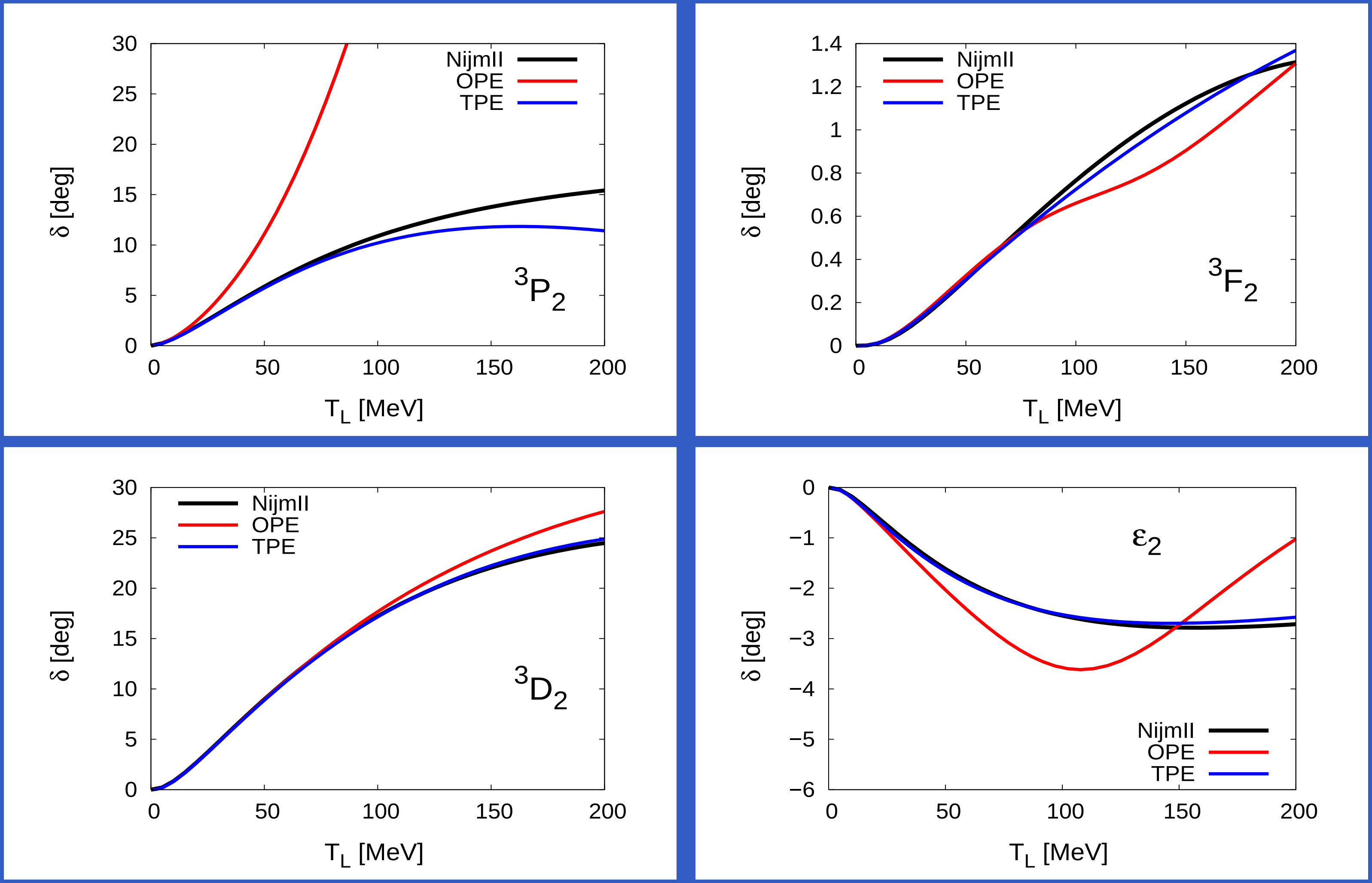
Peripheral Waves



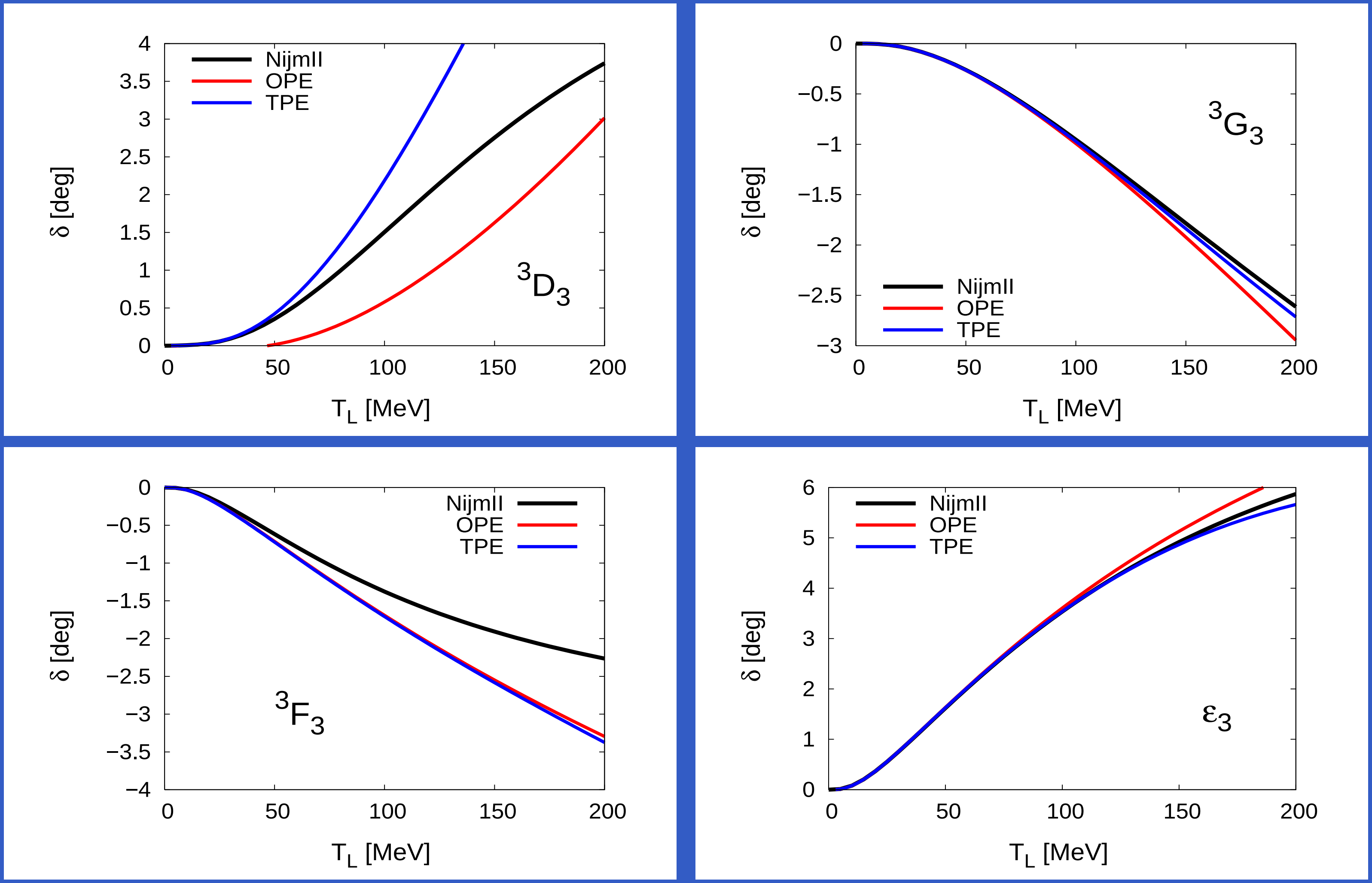
Peripheral Waves



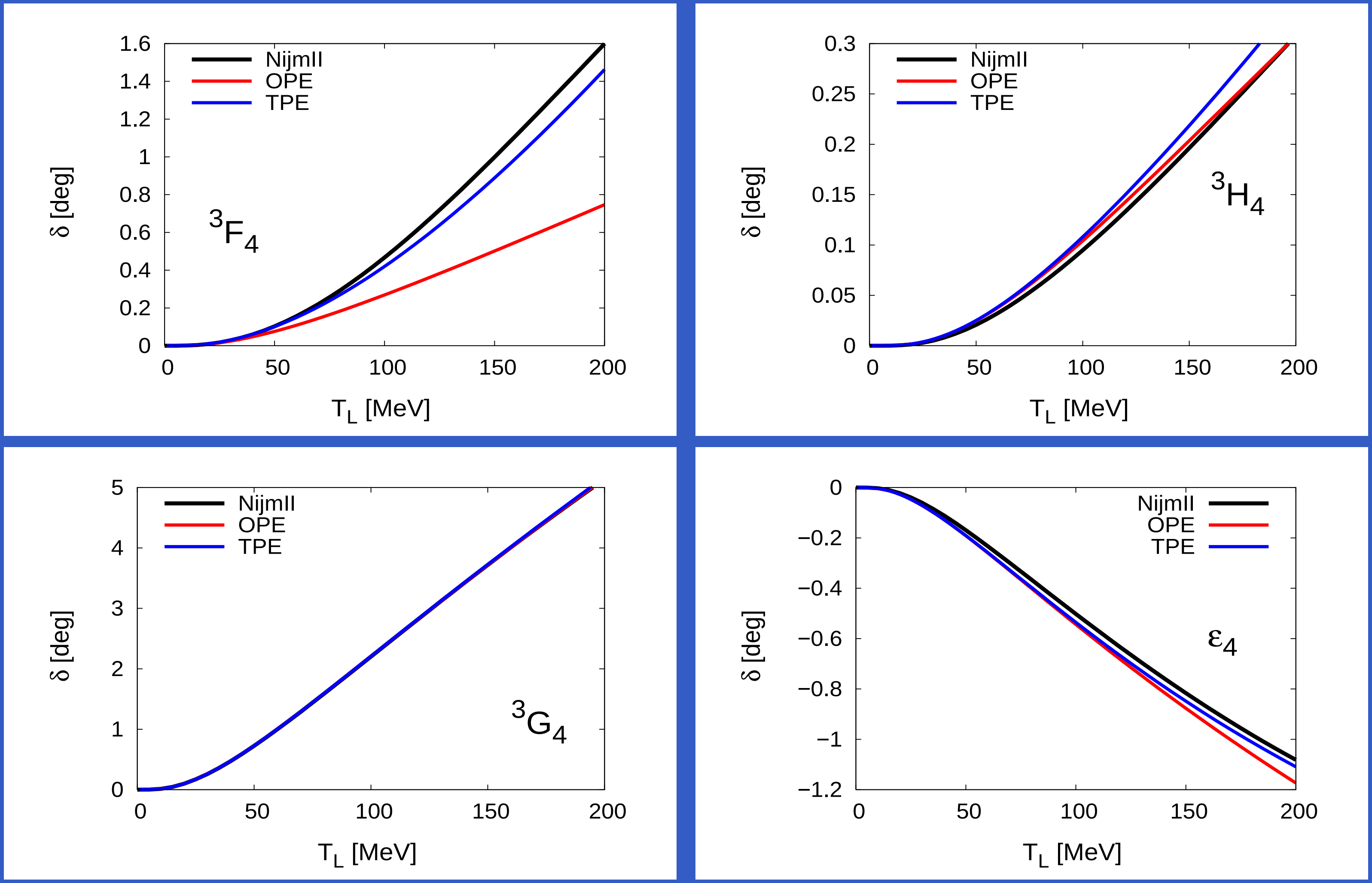
Peripheral Waves



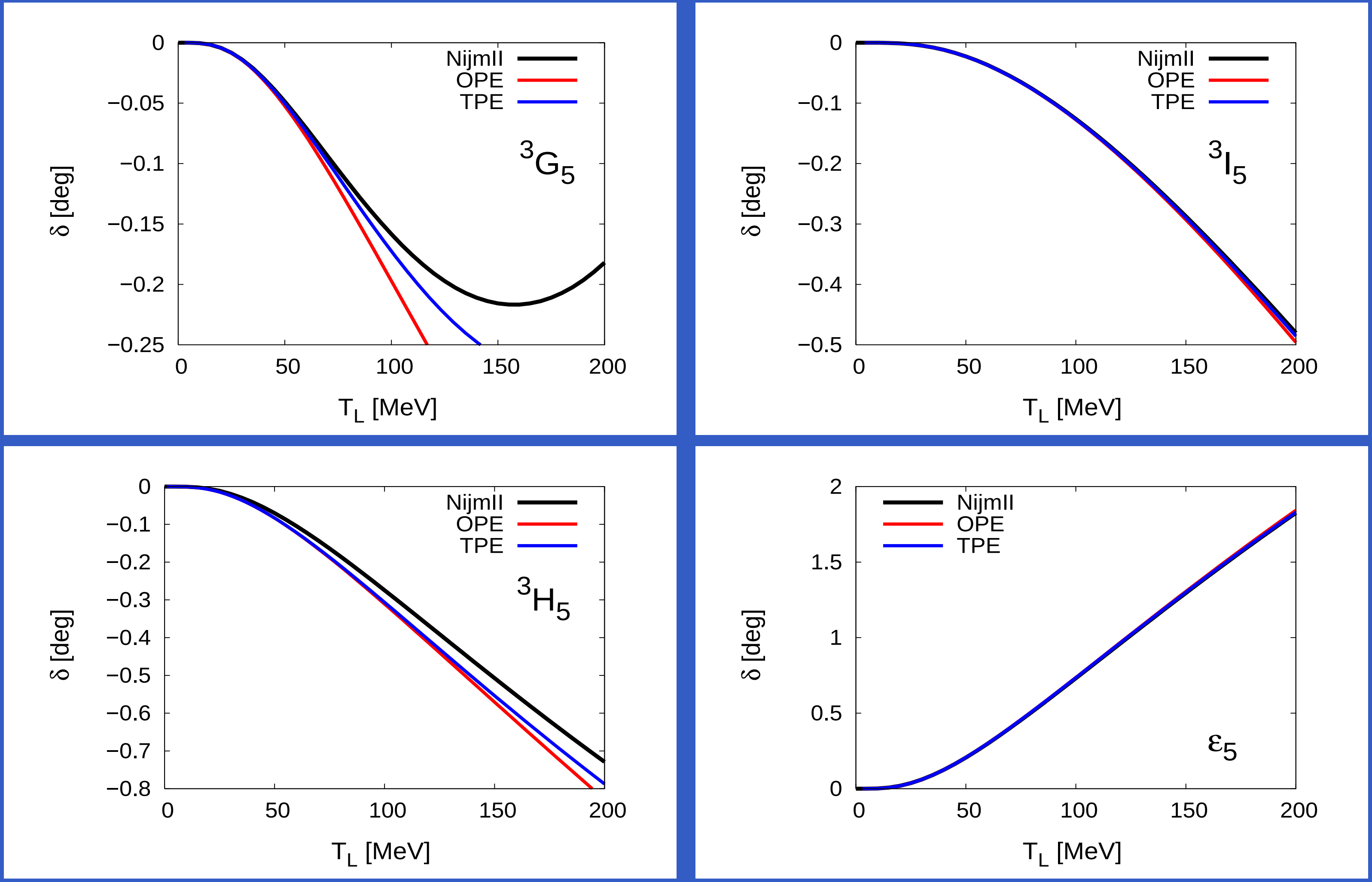
Peripheral Waves



Peripheral Waves



Peripheral Waves



Conclusions

- Singular potentials are good!
- The singularity structure of the potential determines the number of counterterms.
- OPE alone gives a good description of the Deuteron.
- TPE improves this description, although the uncertainties on the LEC's induce higher uncertainties that in the OPE case.
- Cutoff errors in the LEC's worsen TPE results for central waves with removed cutoffs. New LEC's determinations without cutoff needed.
- Δ arguments suggest "promoting" NNLO to NLO.