The role of the Delta-isobar in nuclear EFT

DANIEL PHILLIPS

Department of Physics and Astronomy

Ohio University, Athens, Ohio

Never the twain shall meet

Oh, East is East, and West is West, and
never the twain shall meet,
Till Earth and Sky stand presently at
God’s great Judgment Seat;
But there is neither East nor West, Border,
nor Breed, nor Birth,
When two strong men stand face to face,
tho’ they come from the ends of the earth!

Rudyard Kipling, “The Ballad of East and West”

The ballad of nuclear forces

Oh, quarks are quarks, and a σ a σ, and
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When data's compared to a nuclear force,
with a χ^2 that's suitably low

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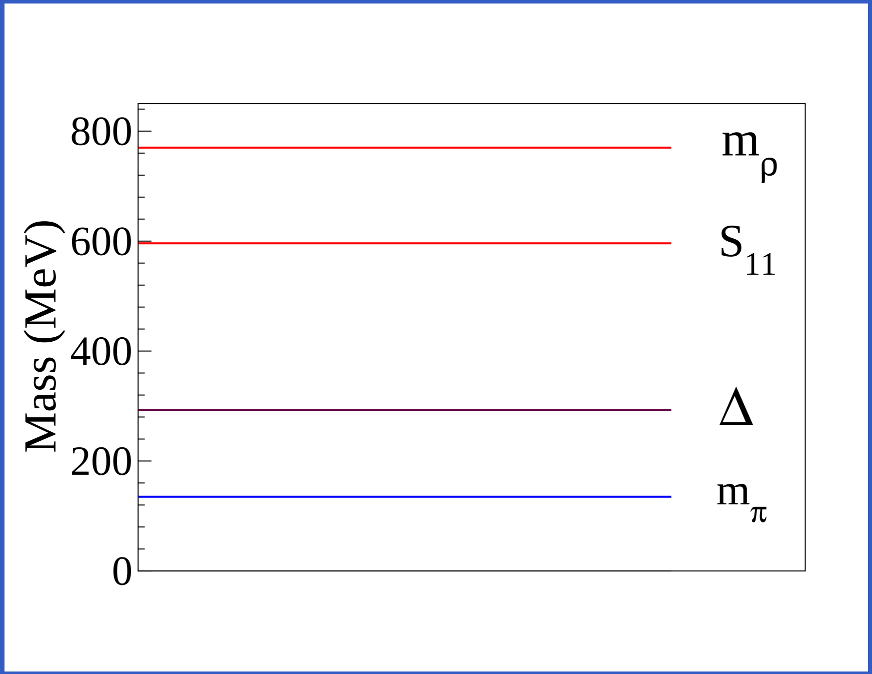
EFTs and low-energy QCD scales



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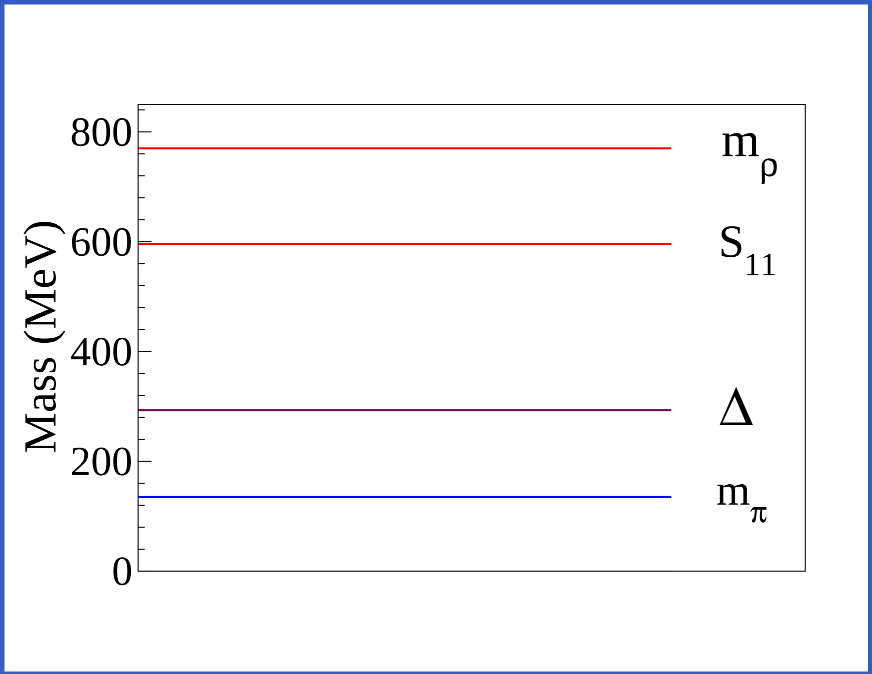

Three possible EFTs:

- EFT($\not\pi$): $\omega < m_\pi$;
- χPT ($\not\Delta$): $\omega \sim m_\pi < \Delta$;
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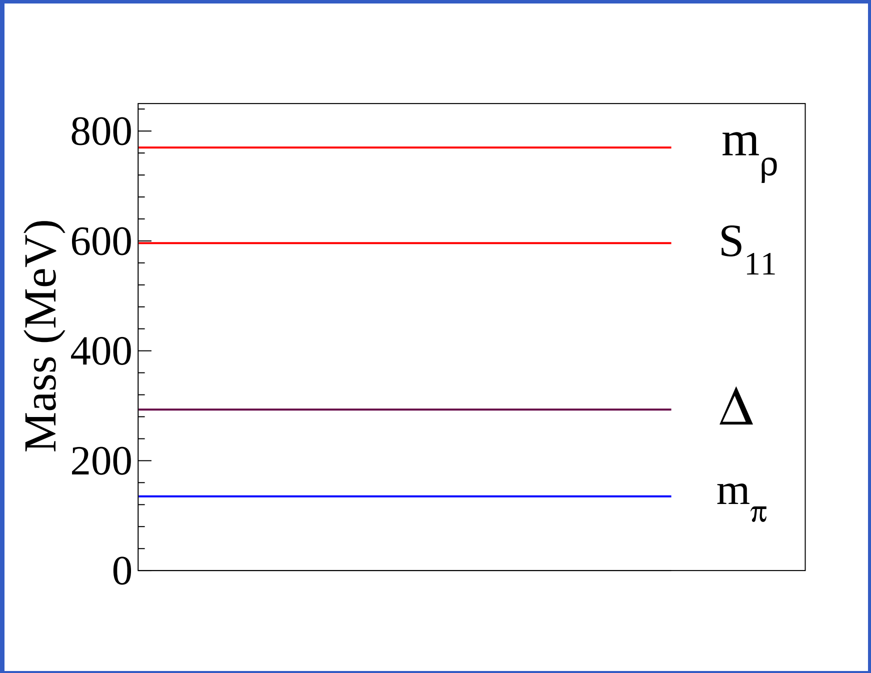

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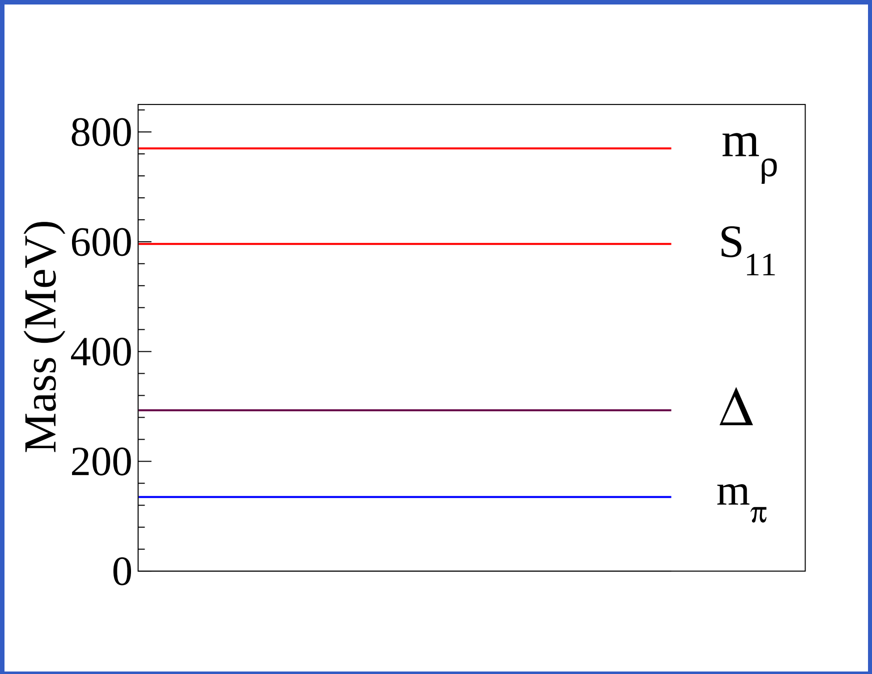

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Each can be applied in A=1 AND A=2 AND A=3 . . .

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χPT is the most general $\mathcal{L}(N,\pi,\gamma)$ consistent with the symmetries of QCD and the pattern of their breaking, up to a given order in the small expansion parameter:

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p/M expansion employed: (usually) useful, not essential.

Unknown coefficients at a given order need to be determined.

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χPT without explicit $\Delta \Rightarrow \omega, |\mathbf{q}| < \Delta$

χPT and light nuclei

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Weinberg (1990): employ chiral expansion for NN potential and solve Schrödinger equation for nuclear wave function:

$$(E - H_0)|\psi\rangle = V|\psi\rangle$$

$$V = V^{(0)} + V^{(2)} + V^{(3)} + \ldots$$

i.e. expanded in powers of P using χPT.

$V_{\chi\mathrm{PT}}$ at leading order

$$V = C + \frac{g_A^2}{4f_\pi^2}\frac{\sigma_1\cdot\mathbf{q}\,\sigma_2\cdot\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\tau_1^a\tau_2^a$$

Singular, requires regularization and renormalization.

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- $^3\mathrm{S}_1$–$^3\mathrm{D}_1$ $\surd$ (BBSvK)
- Higher partial waves: problems (Nogga, Timmermans, van Kolck)
- Phenomenological success up to $E_{\mathrm{lab}} = 250$ MeV for N^{3}LO V (Machleidt, Entem; Epelbaum, Meißner, Glöckle)

Gains over the traditional approach

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- Understand sizes of long-range parts of NN force
- Chiral expansion applies to reactions with (soft) pions and photons, e.g. ed, πd, γd$\longrightarrow \pi^0$d, γd

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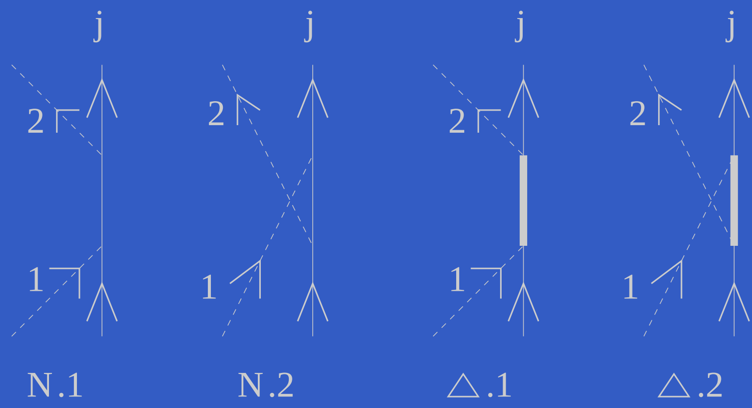
- Conclusions

A toy model with explicit Deltas

V. R. Pandharipande, D. P., U. van Kolck, Phys. Rev. C **71** 064002 (2005)



$$O_j^{\pi N} = -\frac{f^2_{\pi N\Delta}}{m^2_\pi}\frac{4}{9}\left[\mathbf{q}_1\cdot\mathbf{q}_2\mathbf{t}_1\cdot\mathbf{t}_2 - \frac{1}{4}\boldsymbol{\sigma}_j\cdot\mathbf{q}_1\times\mathbf{q}_2\boldsymbol{\tau}_j\cdot\mathbf{t}_1\times\mathbf{t}_2\right]\left(\frac{2\Delta}{\Delta^2-\omega^2}\right)$$
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with ω the pion energy, and $\mathbf{q}_{1,2}$ the pion momenta

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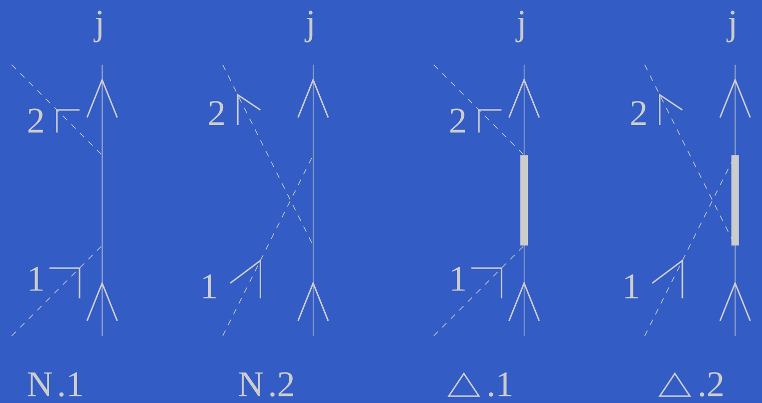

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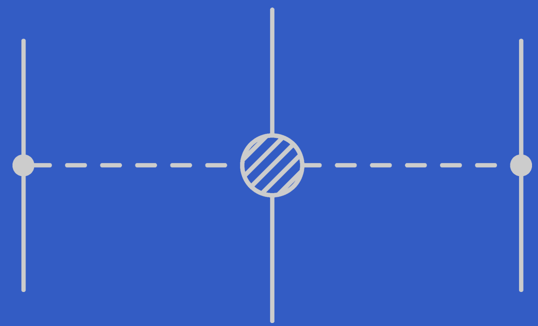
As $\omega\longrightarrow 0$, $O_j^{\pi N}\longrightarrow b\,\mathbf{q}_1\cdot\mathbf{q}_2\,\mathbf{t}_1\cdot\mathbf{t}_2 + d\,\boldsymbol{\sigma}_j\cdot\mathbf{q}_1\times\mathbf{q}_2\,\boldsymbol{\tau}_j\cdot\mathbf{t}_1\times\mathbf{t}_2$

The strength of the TPE3NI



$$\bar{V}^{2\pi}_{ijk} = \frac{f^2_{\pi NN}}{m^2_\pi} \frac{1}{\omega_1^2 \omega_2^2} \, \sigma_i \cdot \mathbf{q}_1 \, \sigma_k \cdot \mathbf{q}_2 \, (\text{Isospin}) \, \mathbf{O}^{\pi N}_j$$

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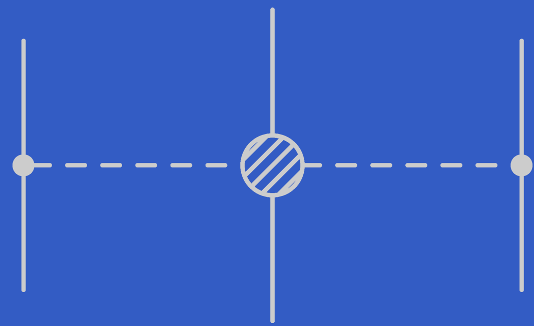

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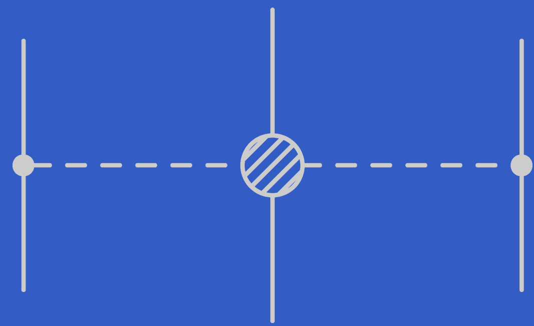

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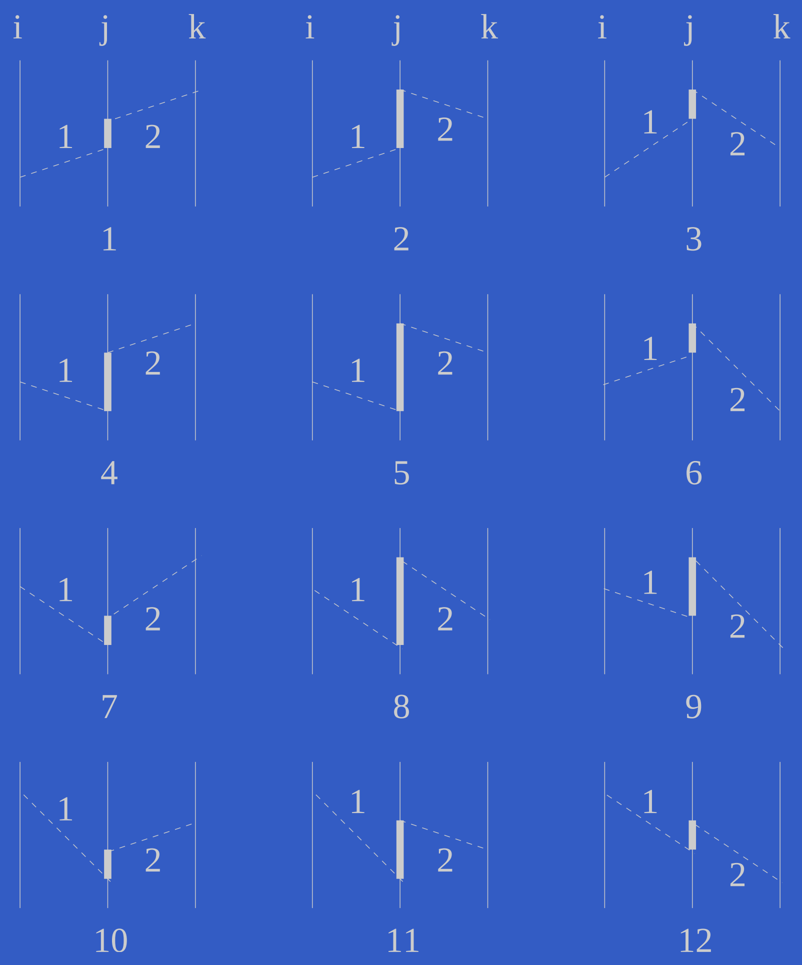
Fit b and d at πN threshold

$$b = 4d = -\frac{f^2_{\pi N\Delta}}{m^2_\pi}\,\frac{4}{9}\left(\frac{2\Delta}{\Delta^2 - m^2_\pi}\right)$$

“Delta-less” result

The “actual” strength of the TPE3NI



$$V_{ijk}^{2\pi,FM} = \frac{f_{\pi NN}^2}{m_\pi^2}\left(\frac{1}{\omega_1^2\omega_2^2}\right)\boldsymbol{\sigma}_k\cdot\mathbf{q}_2\boldsymbol{\sigma}_i\cdot\mathbf{q}_1$$

$$\left(\mathbf{q}_1\cdot\mathbf{q}_2\mathbf{t}_1\cdot\mathbf{t}_2 - \frac{1}{4}\boldsymbol{\sigma}_j\cdot\mathbf{q}_1\times\mathbf{q}_2\boldsymbol{\tau}_j\cdot\mathbf{t}_1\times\mathbf{t}_2\right)$$

$$(\text{Isospin})\left(-\frac{f_{\pi N\Delta}^2}{m_\pi^2}\frac{4}{9}\frac{2}{\Delta}\right)$$

Fujita, Miyazawa (1957)

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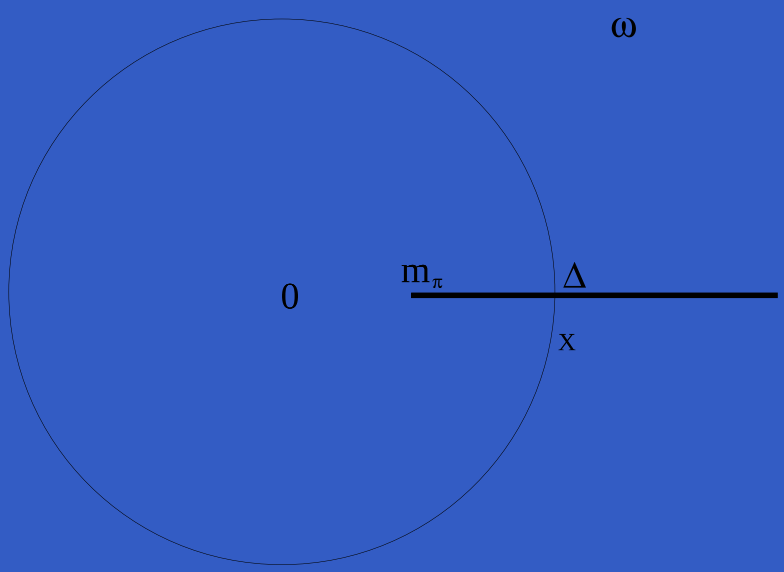
- Remedied by inclusion of ω^{2n} corrections in $\mathrm{O}_j^{\pi N}$
- $\frac{1}{m_\pi} = \left(1 - \frac{1}{3} + \frac{1}{9} + \ldots\right) \frac{4}{3m_\pi}$

Extrapolation



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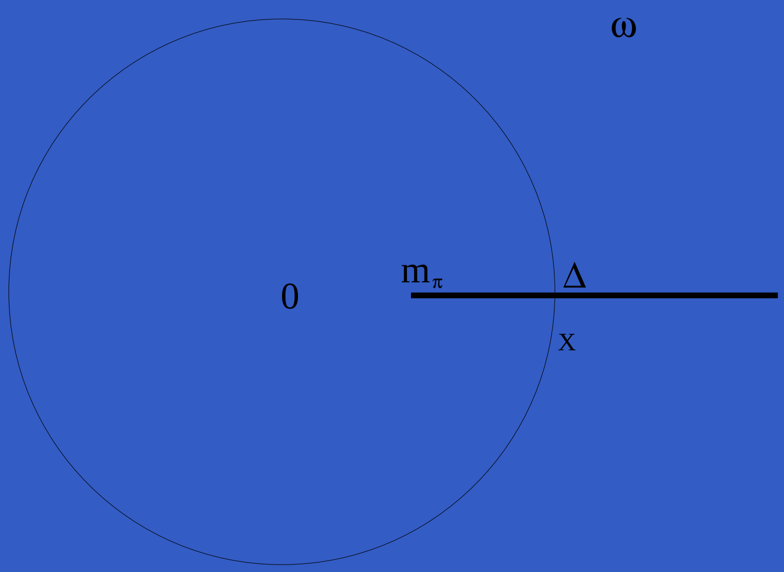

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$$b = -\frac{4}{9}\frac{f^2_{\pi N\Delta}}{m^2_\pi}\frac{4}{3m_\pi}\left[1 \pm \left(\frac{m_\pi}{\Delta M}\right)^2\right]$$

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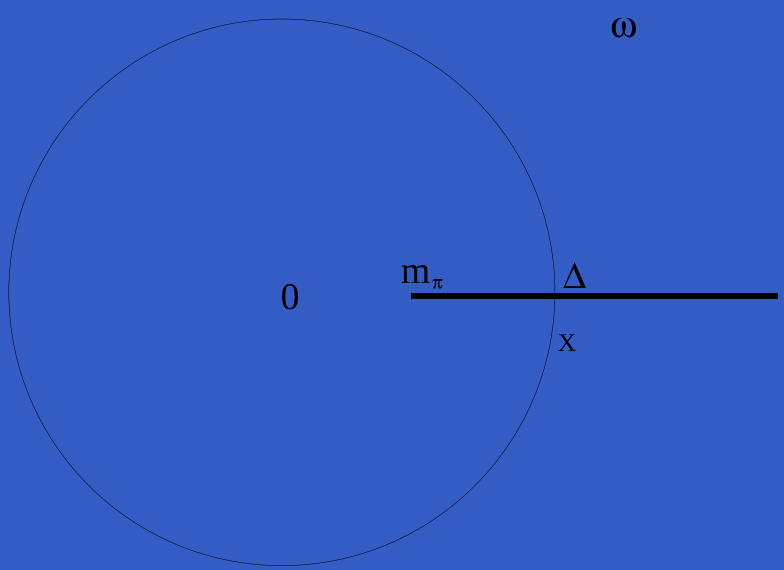

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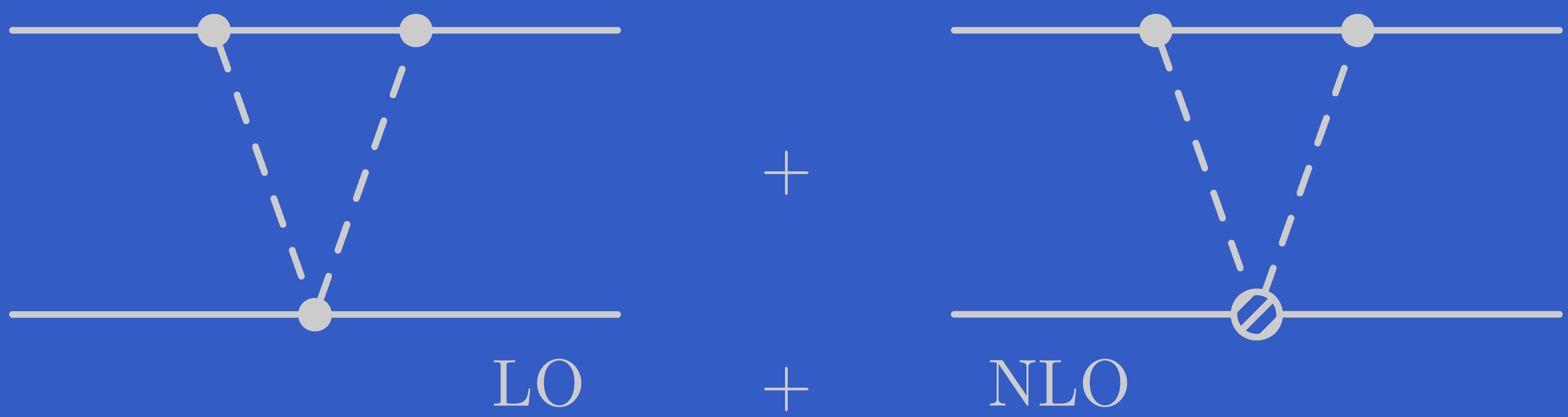
- Slow convergence: Delta-ful theory more efficient?

Central two-pion exchange



Central two-pion exchange

$$V_C(q) = \frac{3g_A^2}{16\pi f_\pi^4}\{(\text{Polynomial})$$

$$+ \left[2m_\pi^2(2c_1 - c_3) - q^2\left(c_3 + \frac{3g_A^2}{16M}\right)\right](2m_\pi^2 + q^2)A(q)\}$$

$$A(q) = \frac{1}{2q}\arctan\left(\frac{q}{2m_\pi}\right)$$

Brockmann, Kaiser, Weise 1997

Co-ordinate space $\tilde{V}_C(r)$

$$\tilde{V}_C(r) = -\frac{1}{2\pi^2 r}\int_{2m_\pi}^{\infty} d\mu\, e^{-\mu r}\,\mu \mathrm{Im} V_C(-i\mu)$$

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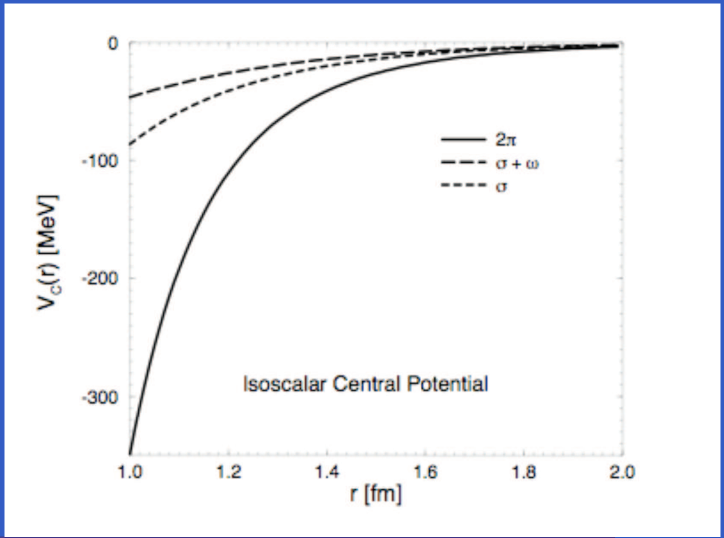

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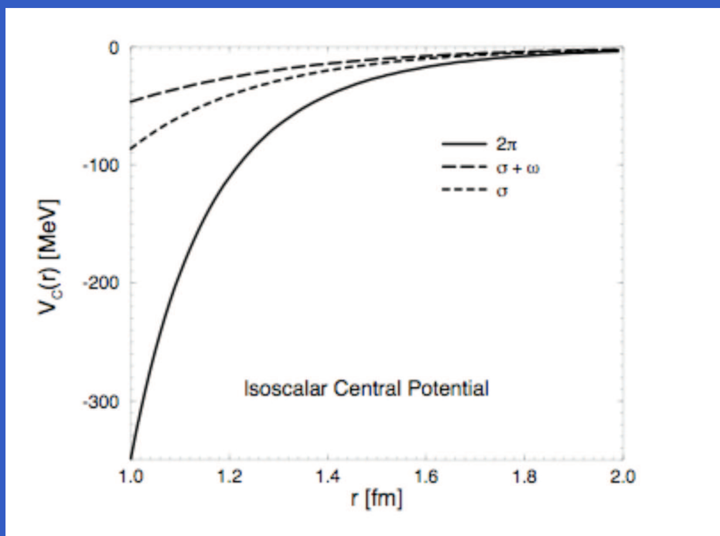
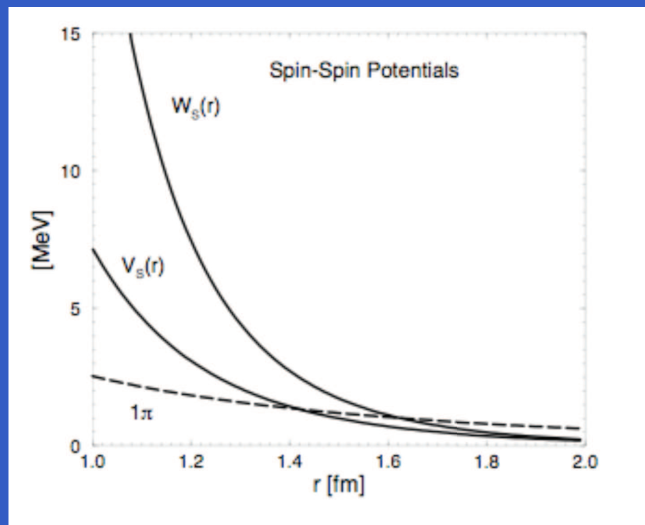




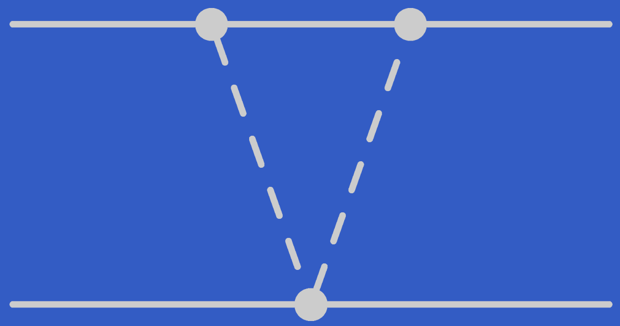
Extractions of c_3 and c_4

$\tilde{V}_C$ sensitive to c_3; $\tilde{V}_S$ and $\tilde{W}_T$ sensitive to c_4

		c_1	c_3	c_4
πN	Buttiker/Meißner	$-0.81(12)$	$-4.70(1.16)$	$3.40(4)$
πN	Fettes *et al.*	$-1.23(16)$	$-5.94(9)$	$3.47(5)$
pp	R'meester *et al.*	$-0.76(7)$	$-5.08(28)$	$4.70(70)$
NN	R'meester *et al.*	$-0.76(7)$	$-4.78(10)$	$3.96(22)$
NN	Entem/Machleidt	-0.81	-3.4	3.4

All LECs in units of GeV^{-1}

χPT + Δ for V_χ



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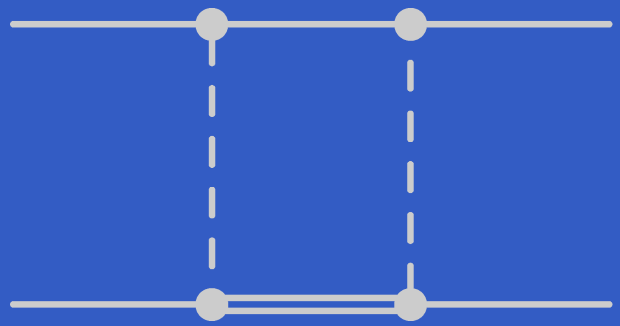

LO

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V_C and W_T can be obtained in χPT($\not\Delta$) p.v. we identify:

$$c_3 = -2c_4 = -\frac{g_A^2}{2\Delta} = -2.71\ \mathrm{GeV}^{-1}$$

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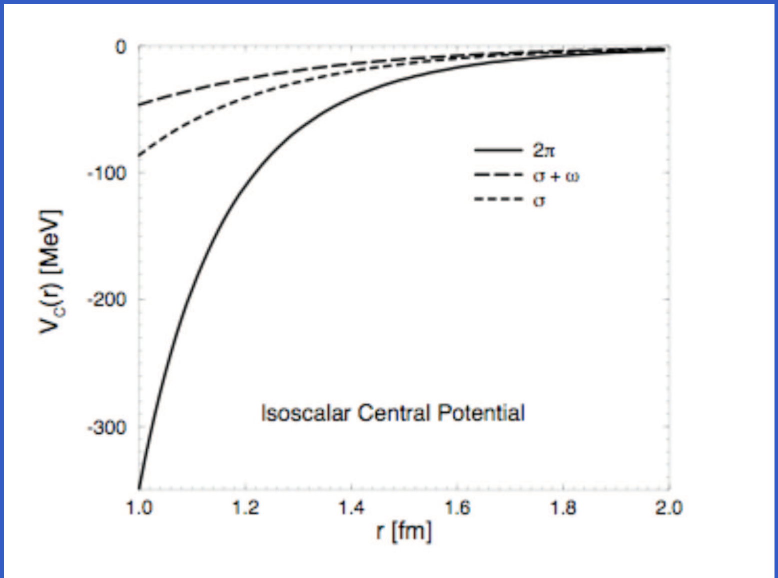
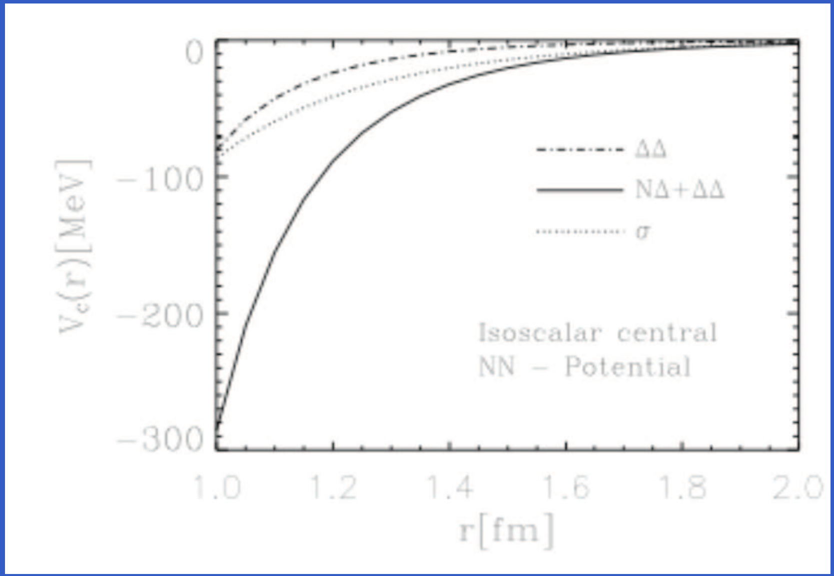
C.f. $c_3 = -2c_4 = -\frac{g_A^2 \Delta}{2(\Delta^2 - m_\pi^2)} = -3.83 \text{ GeV}^{-1}$ — BKM, NPA 1997

25% discrepancy

The Delta and ordering V: evidence





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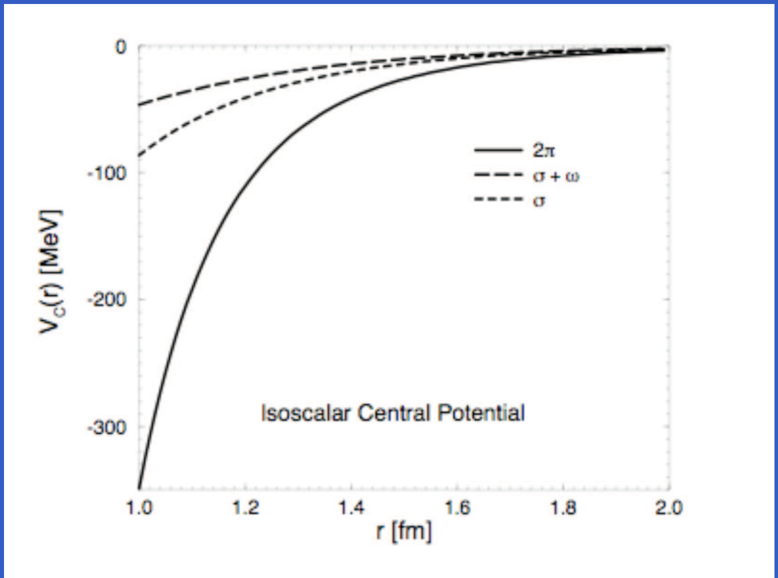

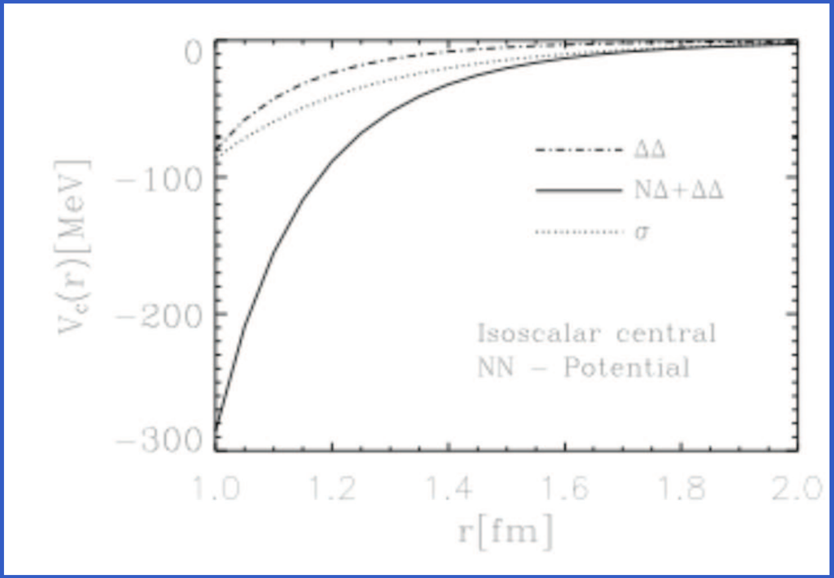

	#BC	χ^2_{min}	#BC	χ^2_{min}
OPE	31	2026.2	29	1956.6
OPE + TPE(l.o.)	28	1984.7	26	1965.9
OPE + χTPE	23	1934.5	22	1937.8

Rentmeester *et al.*, PRL 1999

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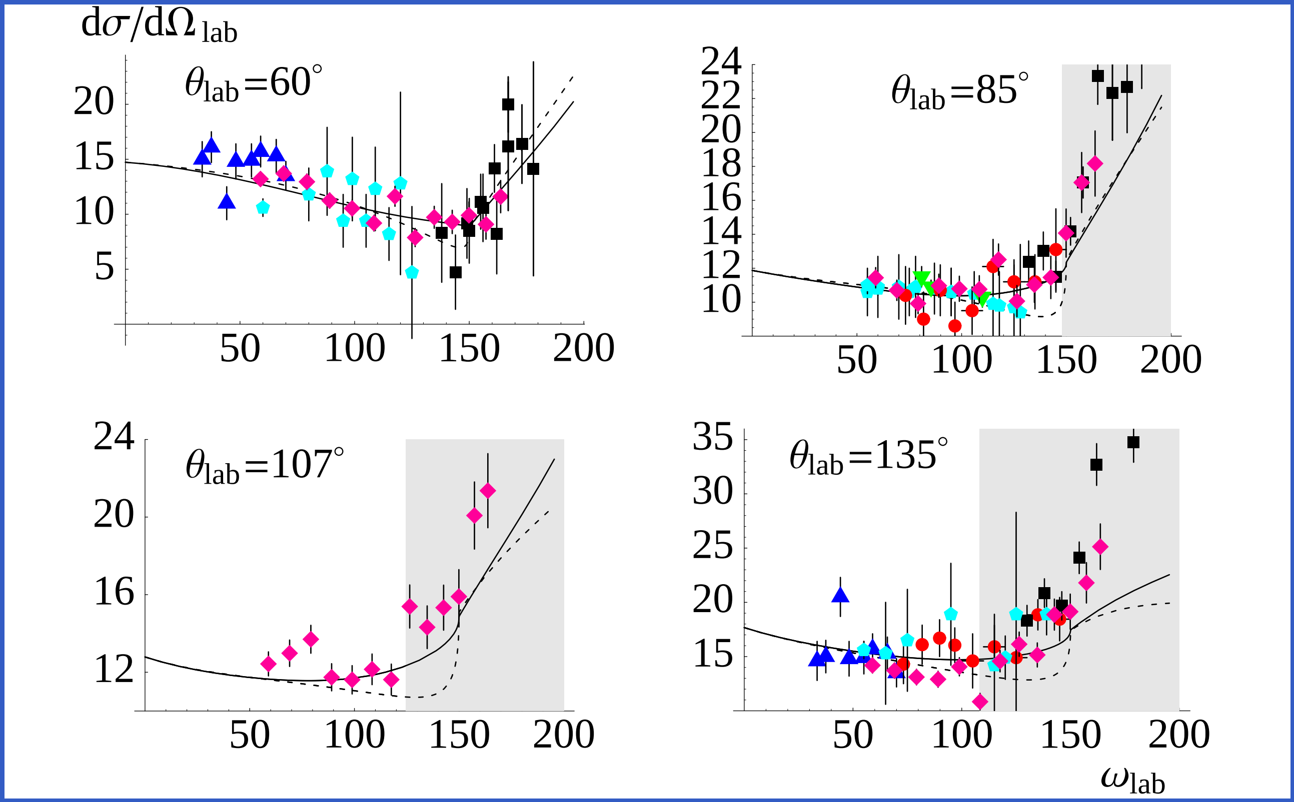
γp results within χPT($\not\Delta$) at N^{2}LO



χ^2/d.o.f.=170/131

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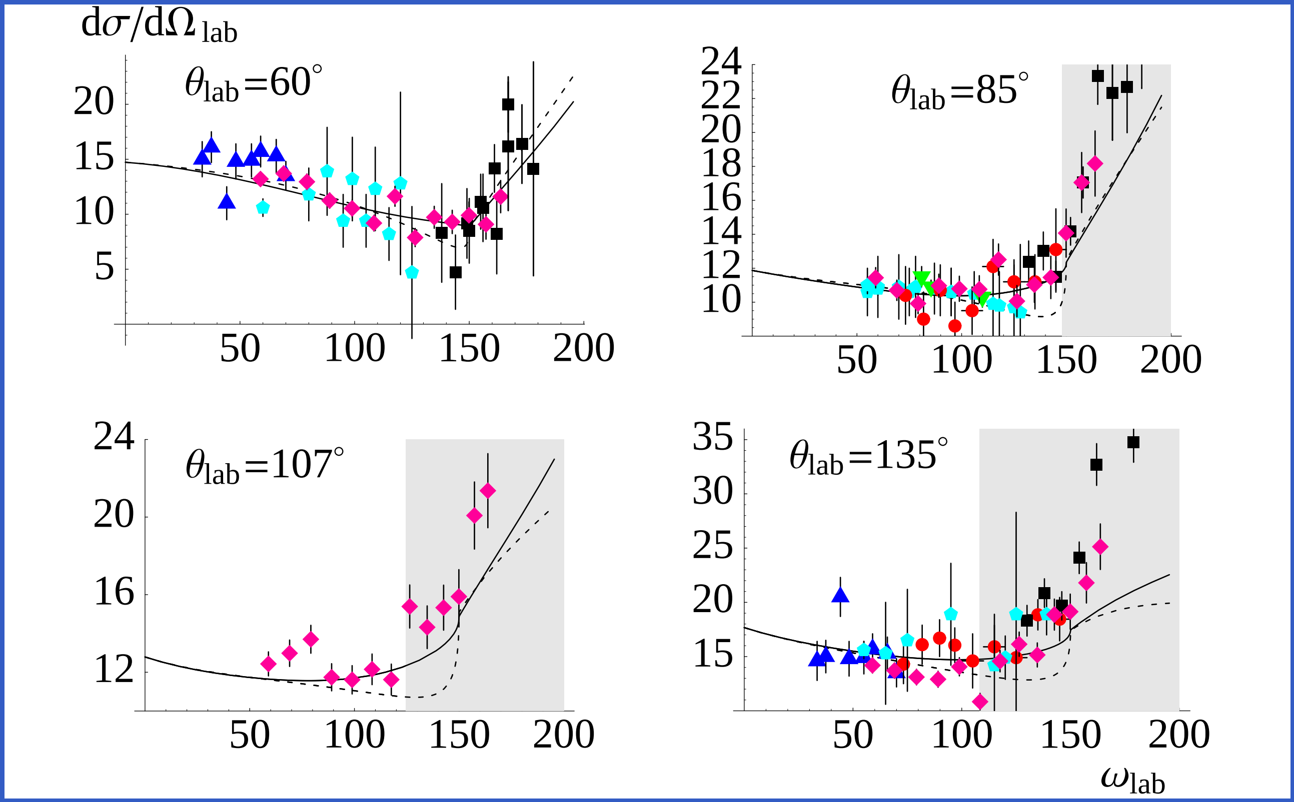

χ^2/d.o.f.=170/131

$$\alpha_p = (12.1 \pm 1.1)^{+0.5}_{-0.5} \times 10^{-4}\ \mathrm{fm}^3$$

$$\beta_p = (3.4 \pm 1.1)^{+0.1}_{-0.1} \times 10^{-4}\ \mathrm{fm}^3$$

S. R. Beane, J. McGovern, M. Malheiro, D. P., U. van Kolck, PLB, **567**, 200 (2003).

Adding the Delta to χPT

$$\mathcal{L}(N, \pi) \rightarrow \mathcal{L}(N, \pi, \Delta_\mu)$$

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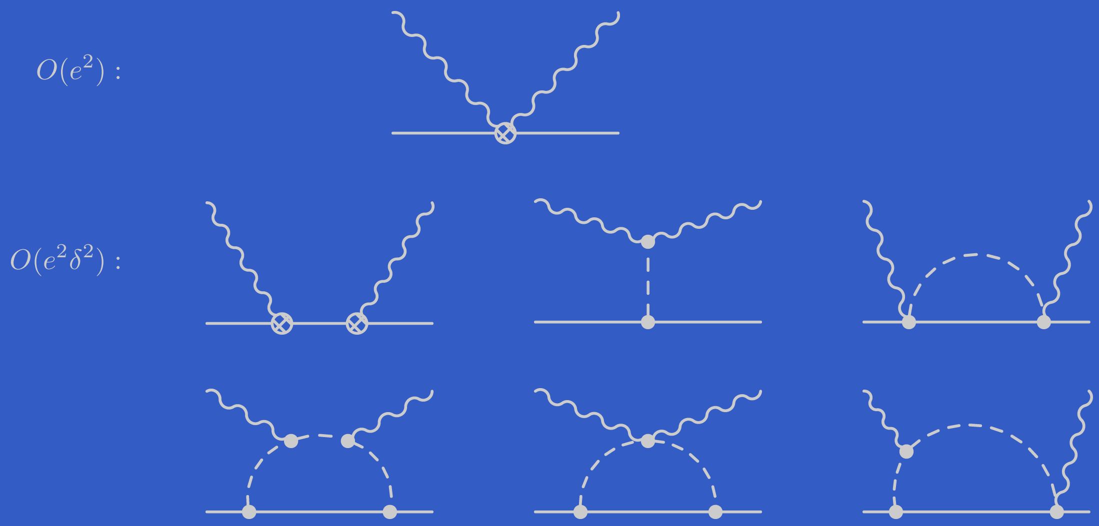
- Consider two kinematic regions for γp scattering;
- Keep track of m_π's and Δ's , then get overall counting index of graph via $m_\pi \sim \delta^2$, $\Delta \sim \delta$.

δ-counting in γp for $\omega \sim m_\pi$



$\sim e^2 \frac{\omega^2}{m_\pi} \sim e^2\delta^2$

if $\omega \sim m_\pi \sim \delta^2$.

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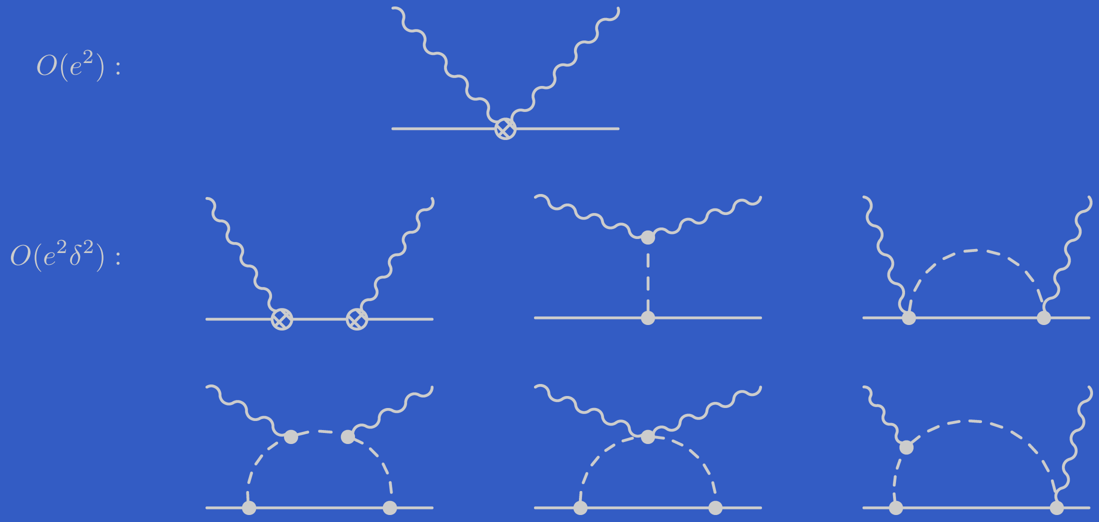

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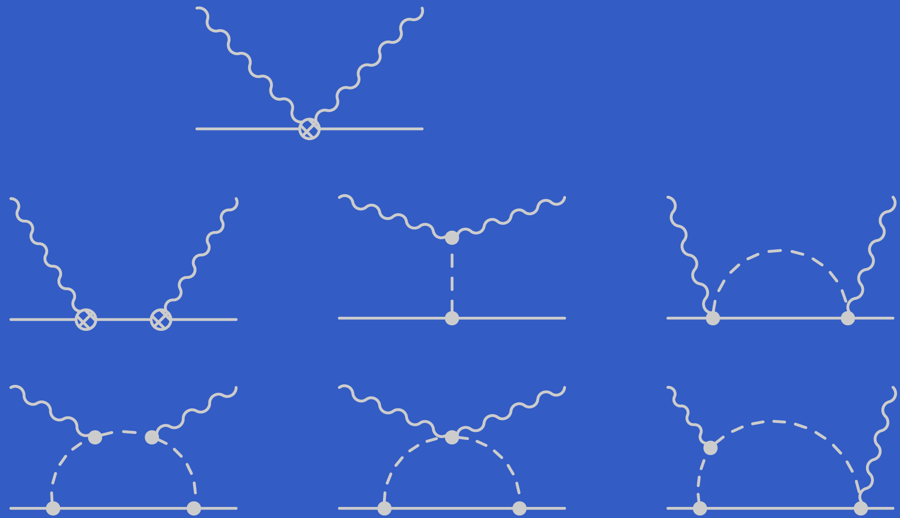

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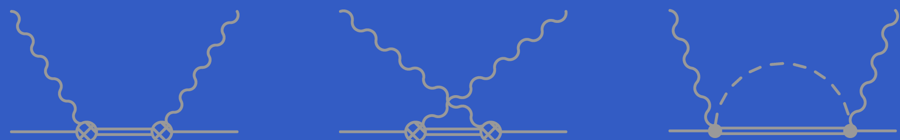

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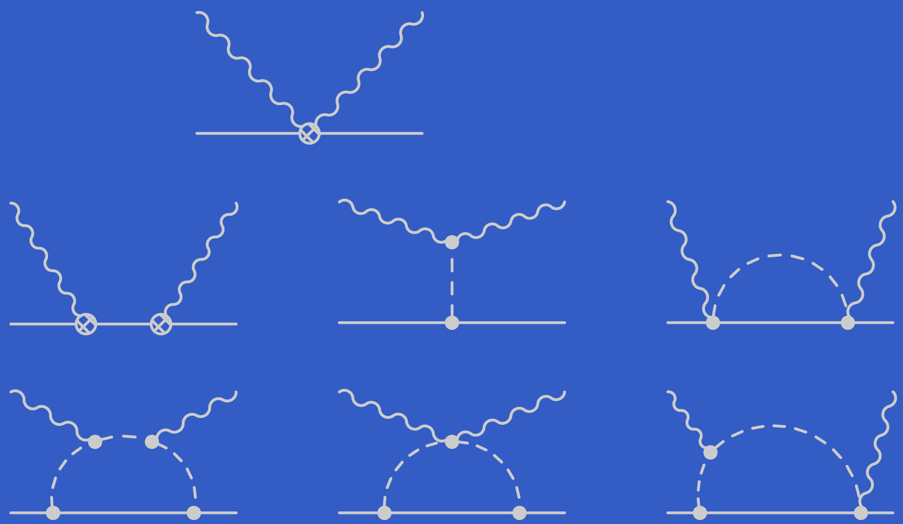

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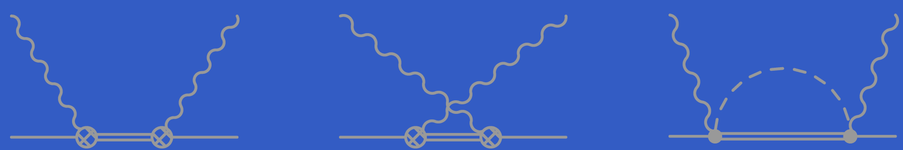
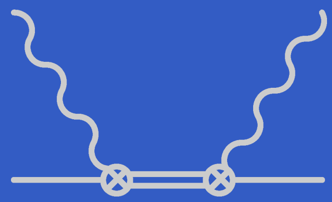


$\sim e^2 \frac{\omega^2}{\Delta} \sim e^2\delta^3$
if $\omega \sim m_\pi \sim \delta^2$.

First counterterms: $4\pi\Delta\alpha_N \mathbf{E}^2$, $4\pi\Delta\beta_N \mathbf{B}^2$, at $O(e^2\delta^4)$

$\omega \sim \Delta$: redcuible diagrams



$$\sim \frac{1}{\omega - \Delta}$$

Diverges for $\omega = \Delta$. Problem with all reducible diags.

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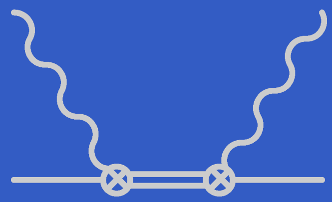

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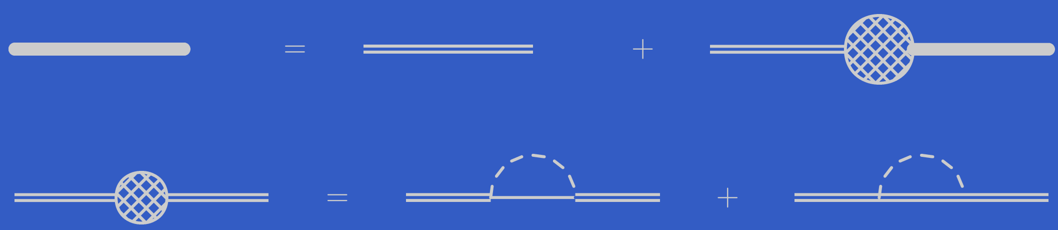

Σ begins with $\Sigma^{(3)}$

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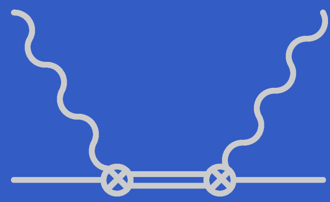

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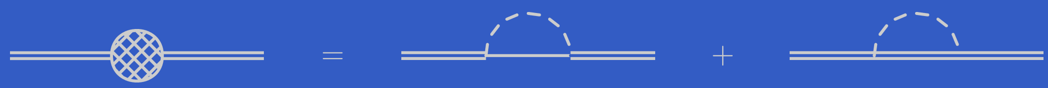
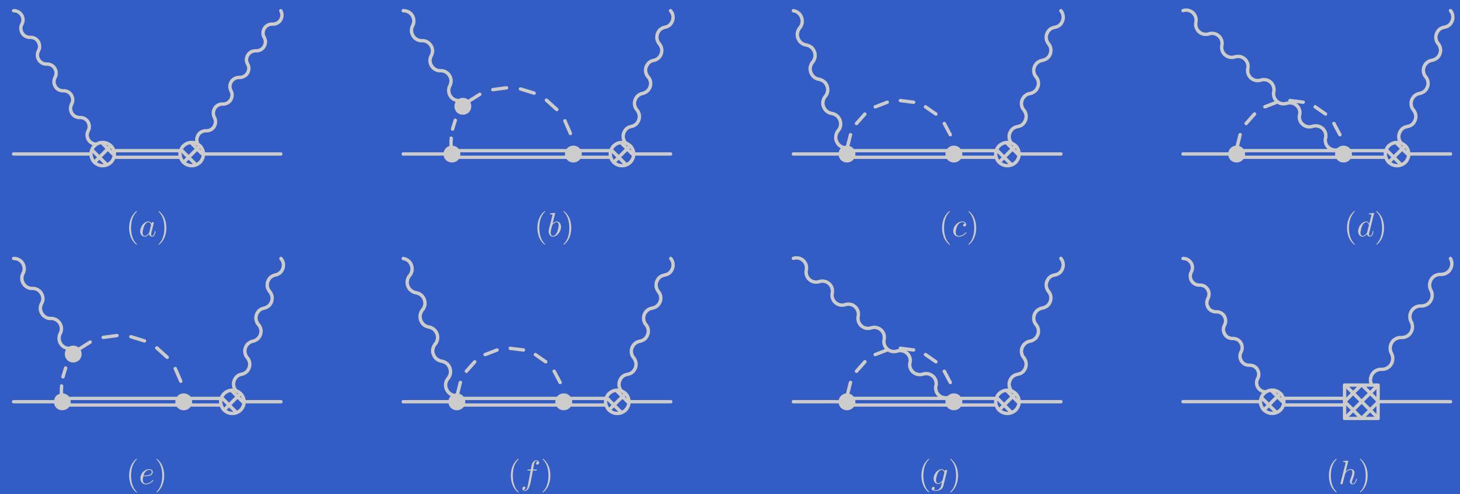


Σ begins with $\Sigma^{(3)}$

$|\omega - \Delta| \sim \frac{\Delta^3}{\Lambda^2} \Rightarrow$ all terms $\sim \delta^{-3} \Rightarrow$ Dress propagator.

$\omega \sim \Delta$: power counting

LO and NLO reducible diagrams:



These + Thomson term give NLO: $O(e^2\delta^{-1})$ + $O(e^2)$

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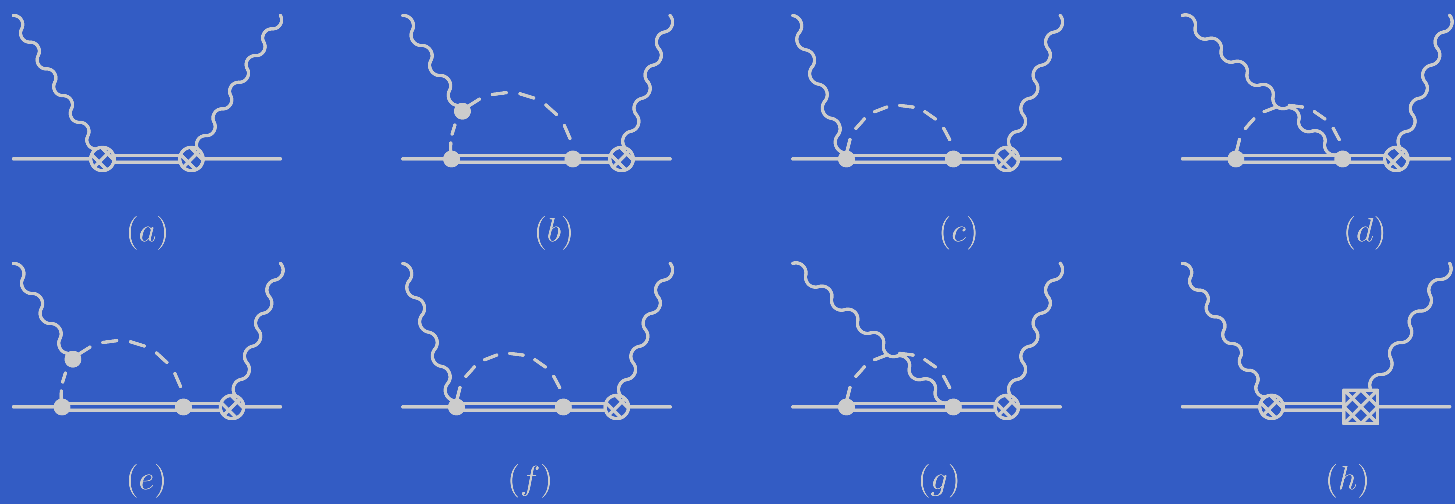

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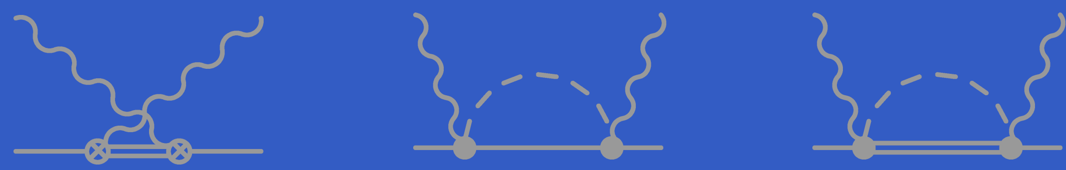
N^{2}LO, $O(e^2\delta)$:



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V. Pascalutsa and D. R. Phillips Phys. Rev. C **67**, 0552002 (2003).

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- Power counting $\Rightarrow$ error estimates

Results: parameters

Fit to γp data from threshold to ~ 300 MeV

Free parameters: h_A, g_M, g_E

$$\Gamma(M_\Delta^2) = 111 \text{ MeV} \rightarrow h_A\text{=2.81}$$

$$g_M = 2.6 \pm 0.2, g_E = -6.0 \pm 0.9$$

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NLO HBχPT	12.2	1.2
NLO δ	$10.2^{+4.2}_{-2.0}$	$3.9^{+2.7}_{-0.4}$
NLO SSE	16.4	9.1
PDG average	12.0 ± 0.7	1.6 ± 0.6
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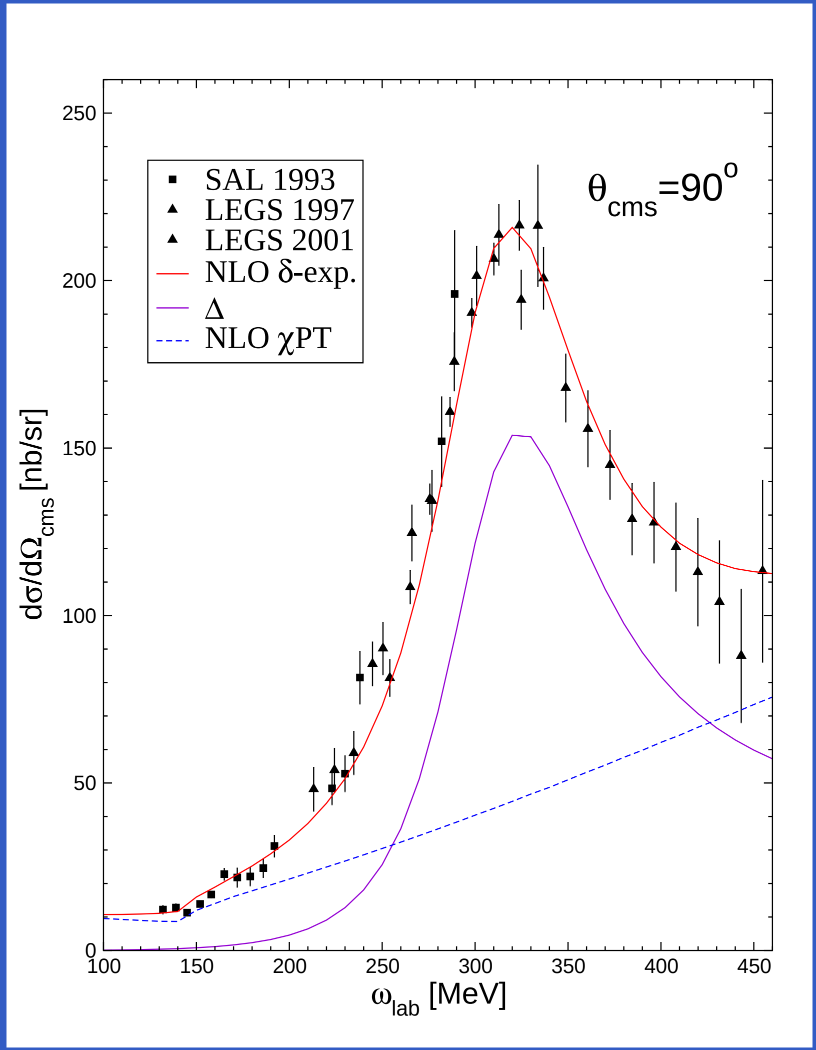
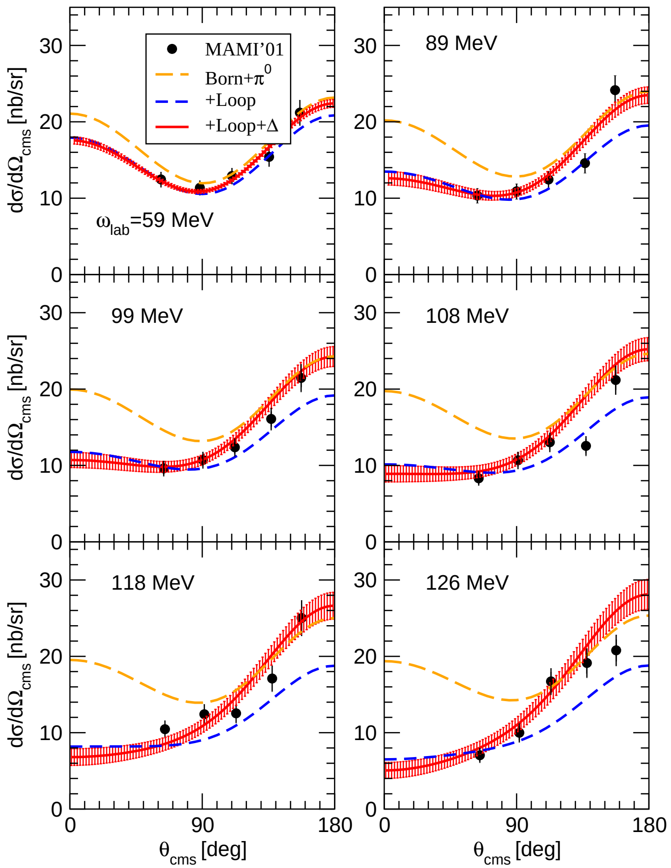
Large Δ/M corrections to spin polarizabilities, Pascalutsa and D.P., PRC **68**, 055205.

Results: I



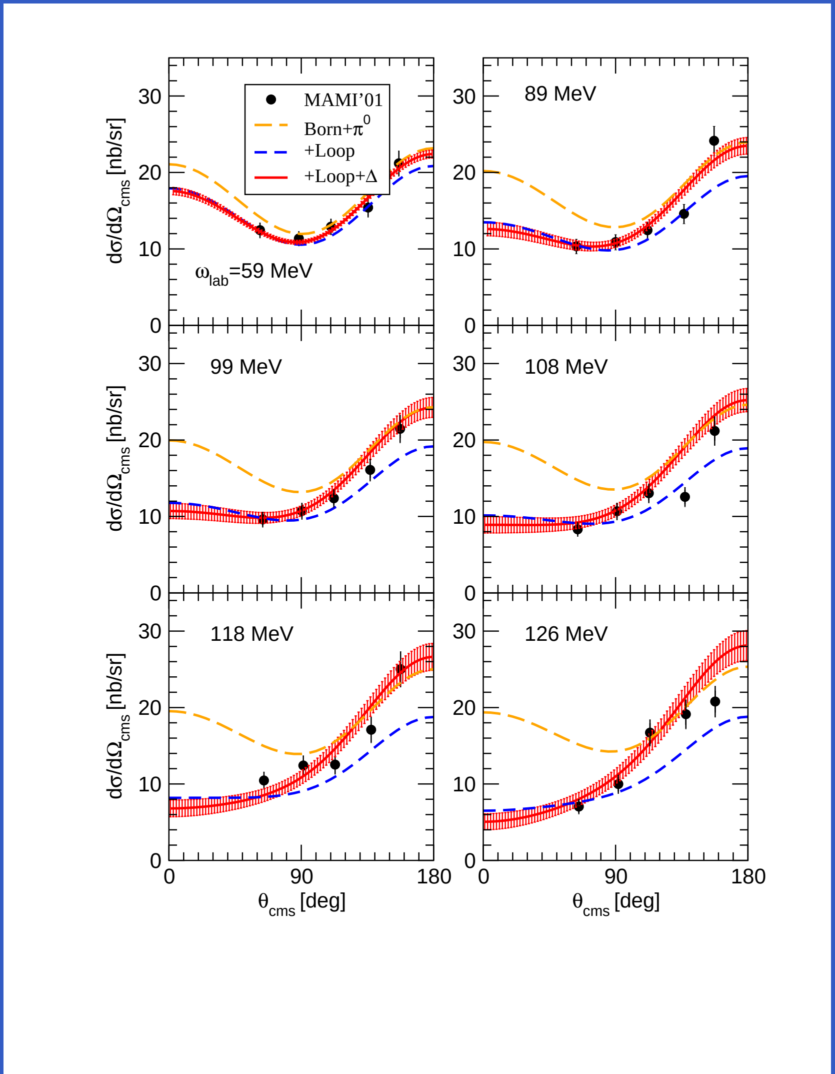
SAL 1993
LEGS 1997
LEGS 2001
NLO δ-exp.
Δ
NLO χPT
θcms=90°
dσ/dΩcms [nb/sr]
ωlab [MeV]
0
50
100
150
200
250
100
150
200
250
300
350
400
450

Results: II



MAMI'01
Born+π^0
+Loop
+Loop+Δ
ω_lab=59 MeV
89 MeV
99 MeV
108 MeV
118 MeV
126 MeV
dσ/dΩ_cms [nb/sr]
θ_cms [deg]

Results: II



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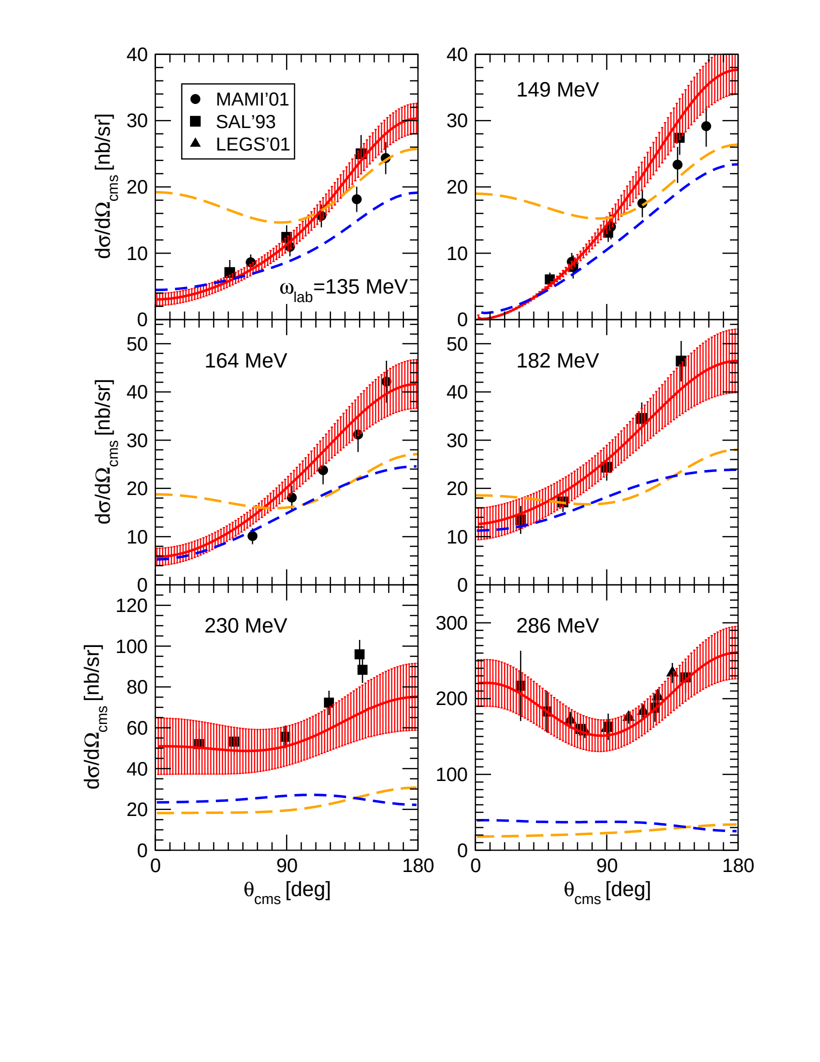
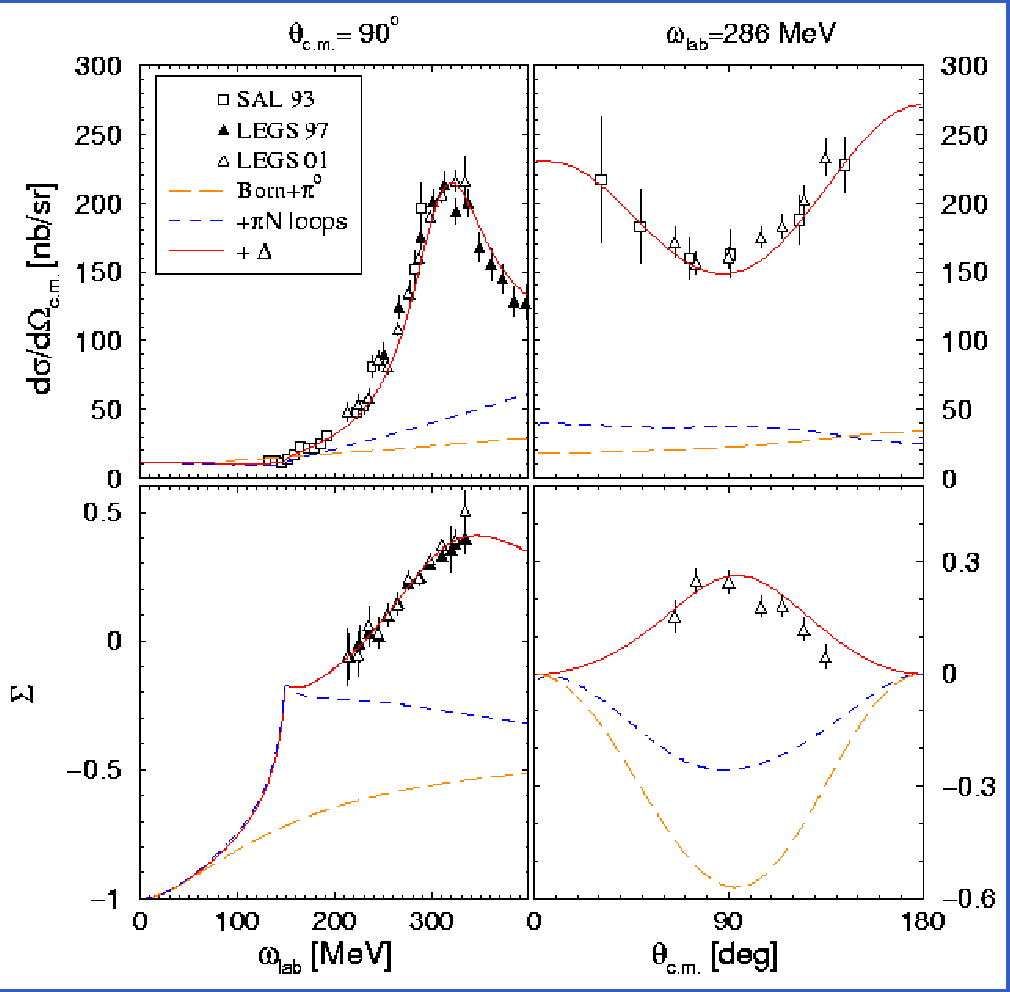


MAMI'01
SAL'93
LEGS'01
ω_{lab}=135 MeV
149 MeV
164 MeV
182 MeV
230 MeV
286 MeV
$d\sigma/d\Omega_{cms}$ [nb/sr]
θ_{cms} [deg]

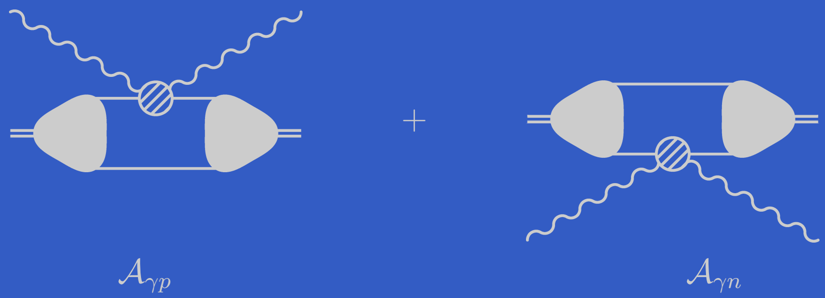
Results: III



$\theta_{c.m.} = 90^\circ$
ω_{lab}=286 MeV
SAL 93
LEGS 97
LEGS 01
Born+π^0
+πN loops
+ Δ
$d\sigma/d\Omega_{c.m.}$ [nb/sr]
Σ
ω_{lab} [MeV]
$\theta_{c.m.}$ [deg]

Compton scattering on deuterium

Want to determine α_N and β_N. Naive idea:



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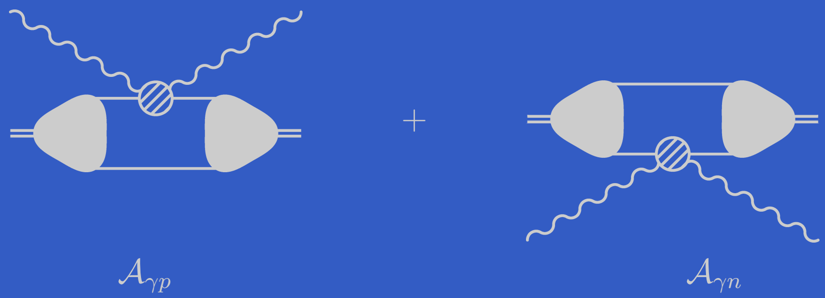

INCORRECT

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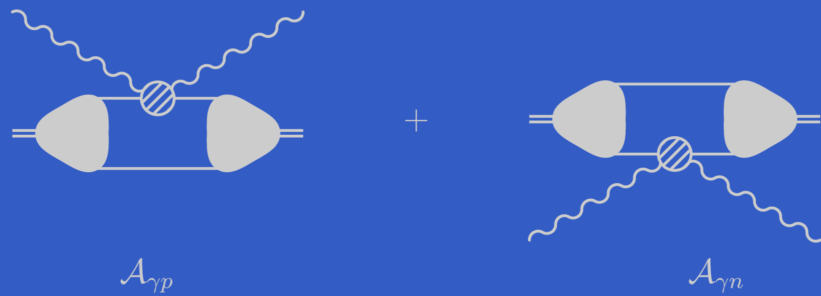

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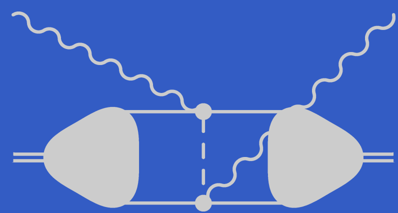

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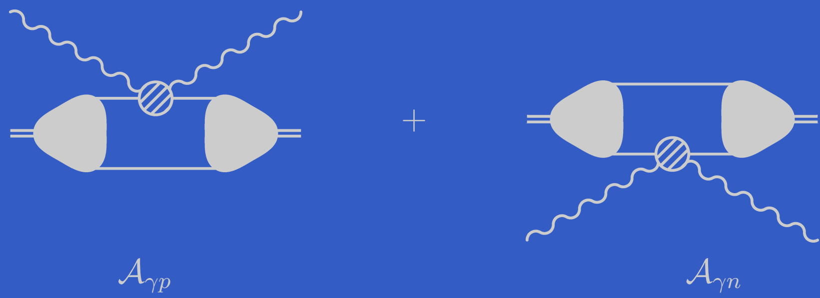

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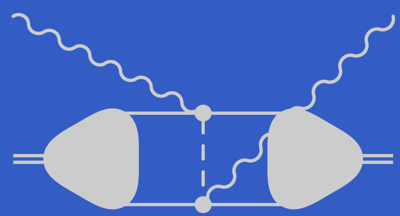
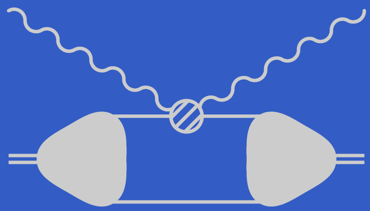


Possible to extract α_N and β_N from γd$\rightarrow$ γd data, but need to treat 2B effects SYSTEMATICALLY.

γd in χPT to $O(e^2 P)$

S. R. Beane, M. Malheiro, D. P., U. van Kolck, Nucl. Phys. **A656**, 367 (1999)

$O(e^2)$:



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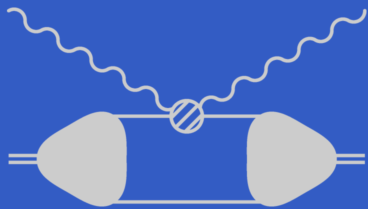

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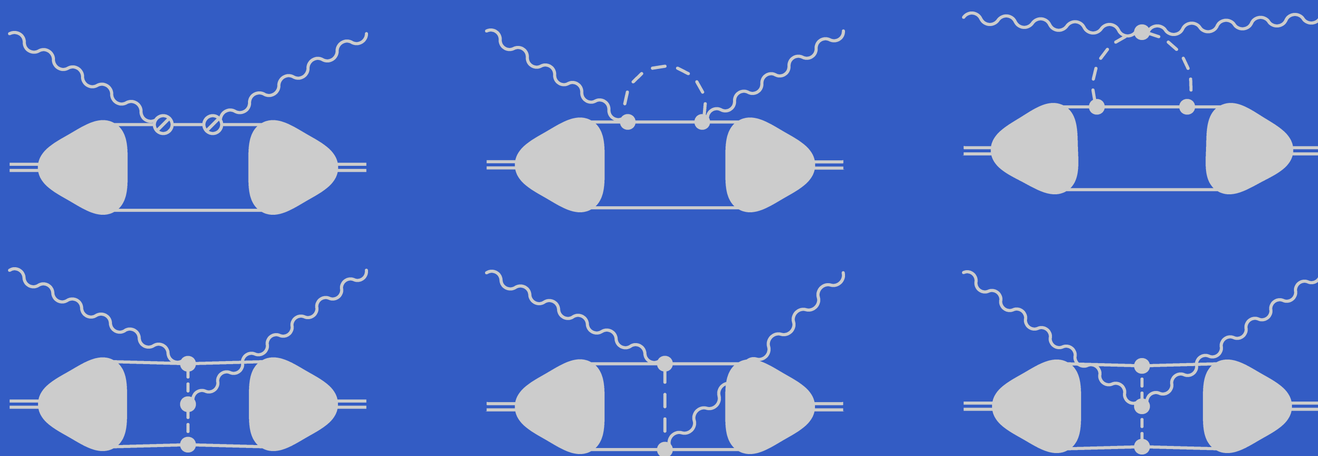
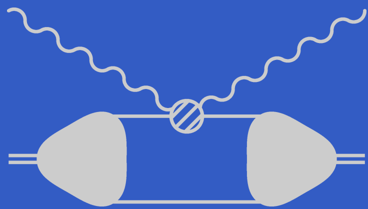


γd in χPT to $O(e^2P)$

S. R. Beane, M. Malheiro, D. P., U. van Kolck, Nucl. Phys. **A656**, 367 (1999)

$O(e^2)$:



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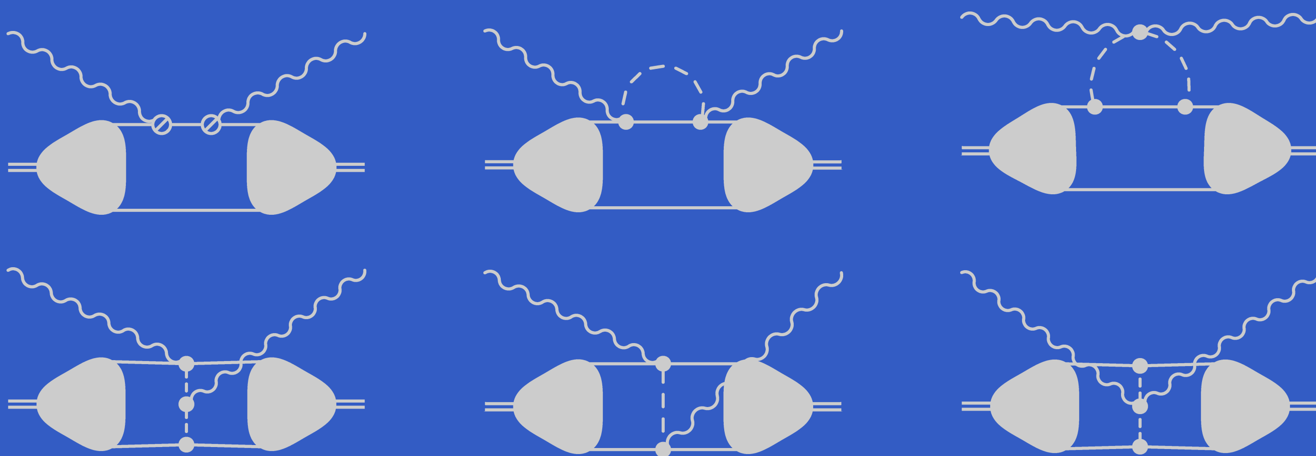
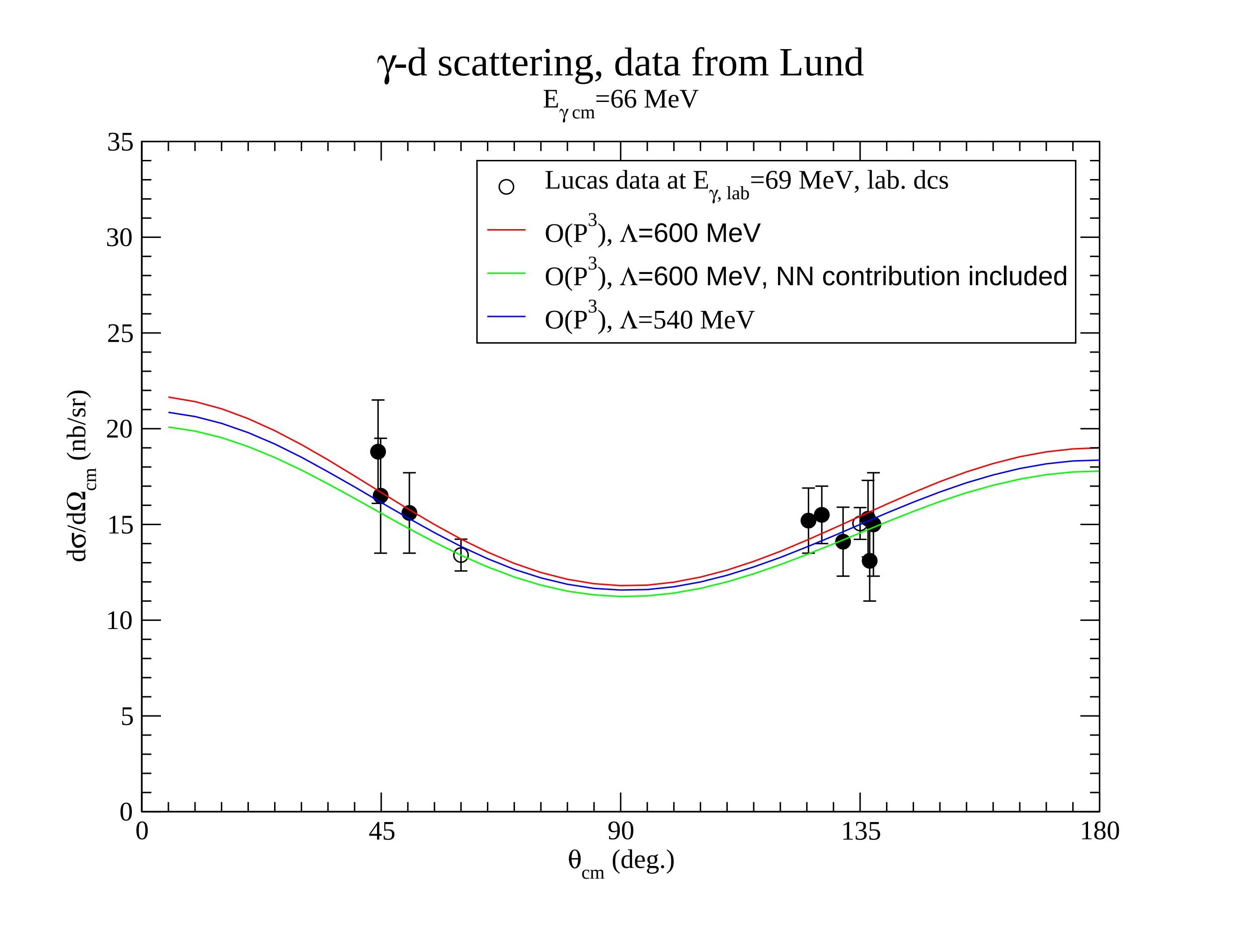


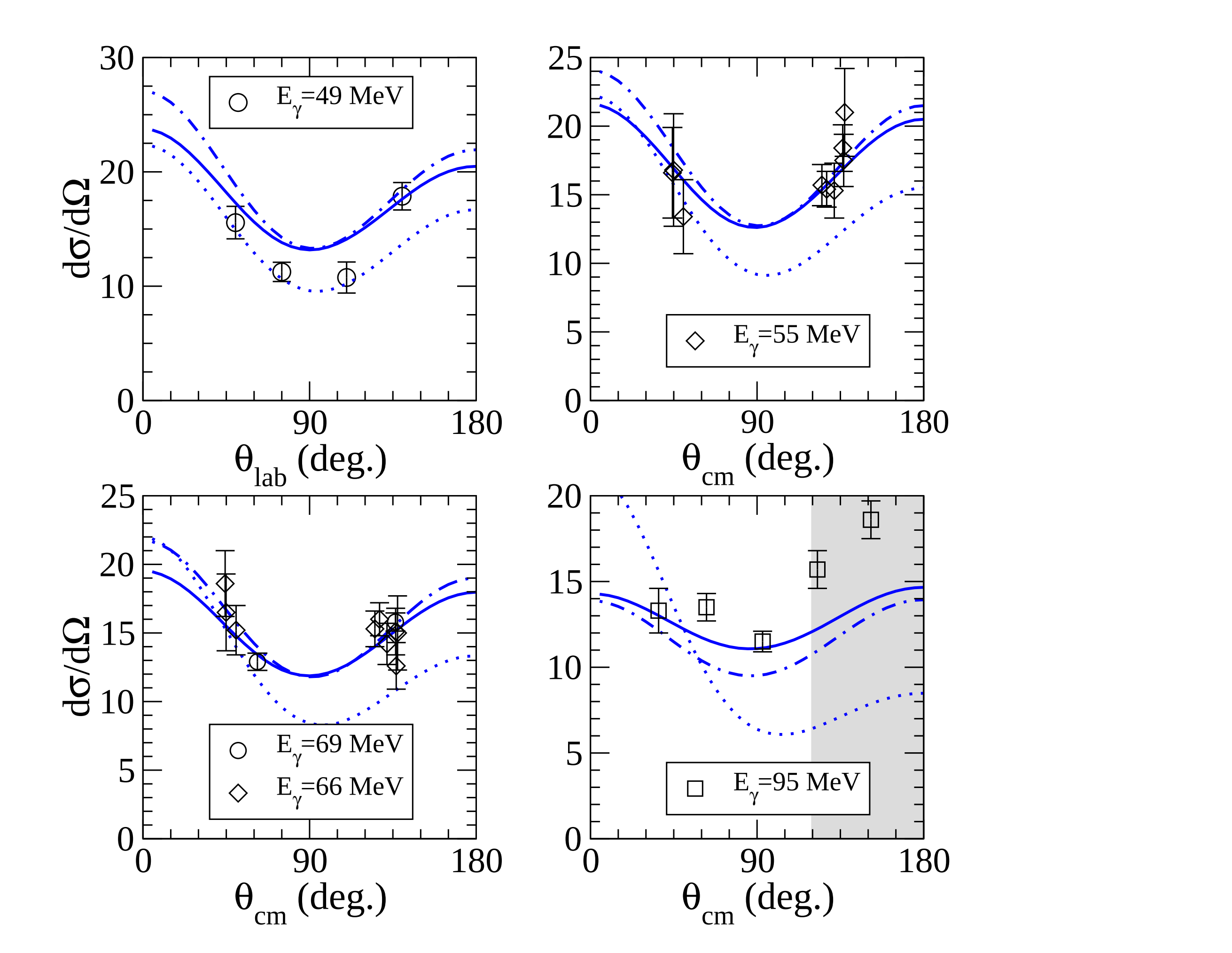
No free parameters at $O(e^2P) \Rightarrow$ PREDICTION

Results



Wave-function dependence
≈ theory error.

Best-fit results at $O(e^2 P^2)$



Fit to data with $\omega, \sqrt{|t|} \leq 160$ MeV shown.

Wave fn. error in $\frac{d\sigma}{d\Omega}$ of order 10%.

γd with explicit Deltas

R. Hildebrandt, H. Grießhammer, T. Hemmert, D.P., Nucl. Phys. A (2005)

- Calculation to NLO in χPT + Δ's
- α_{high} and β_{high} promoted by one order, use values from fit to γp scattering

$$\alpha_p = 11.04 \times 10^{-4}\ \mathrm{fm}^3, \quad \beta_p = 2.76 \times 10^{-4}\ \mathrm{fm}^3$$

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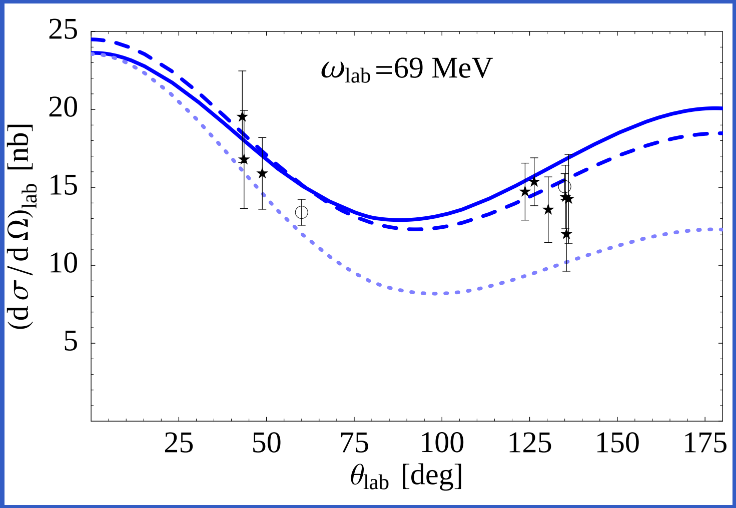

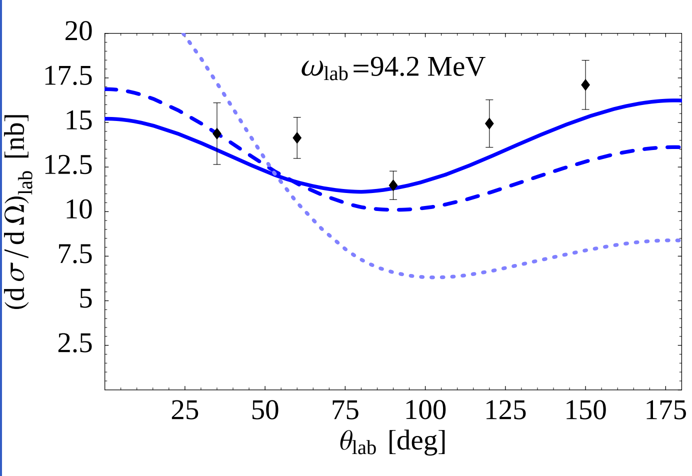

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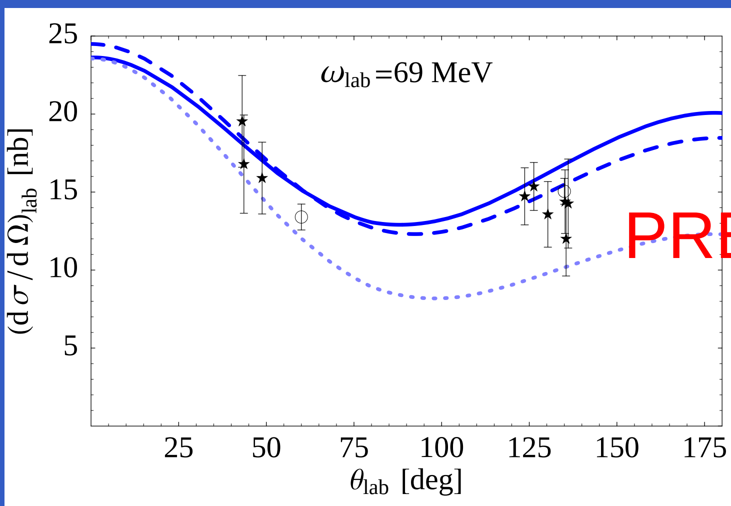
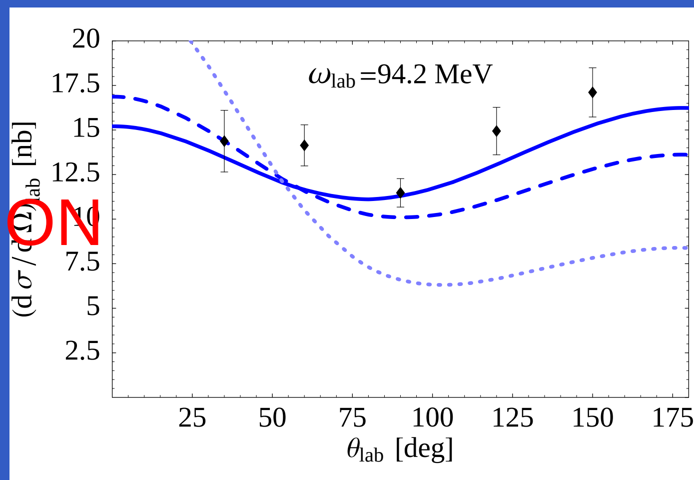
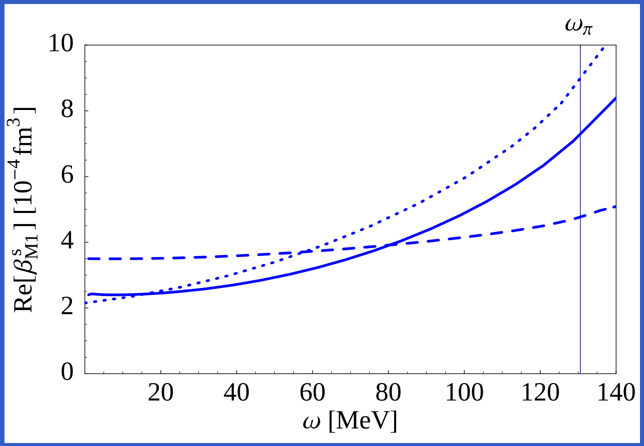


PREDICTION



A couple of pertinent details



$O(e^2P)$ fits in χPT($\not\Delta$) tend to overestimate β_N

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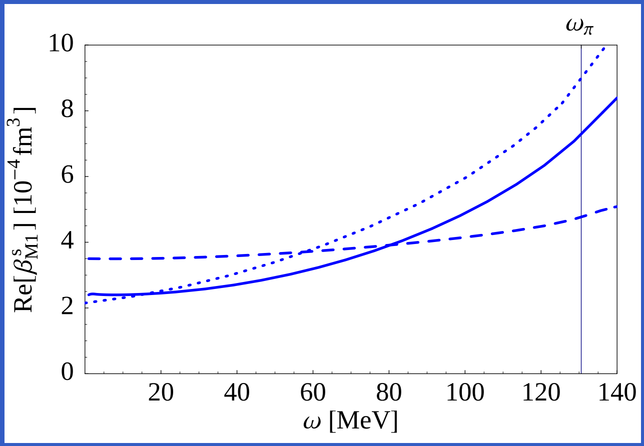
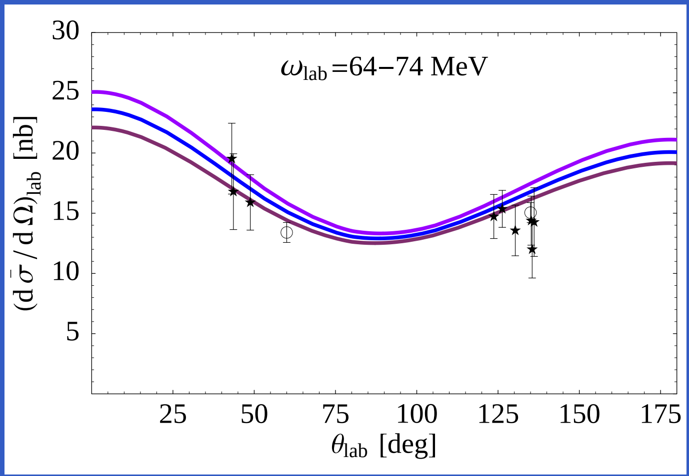
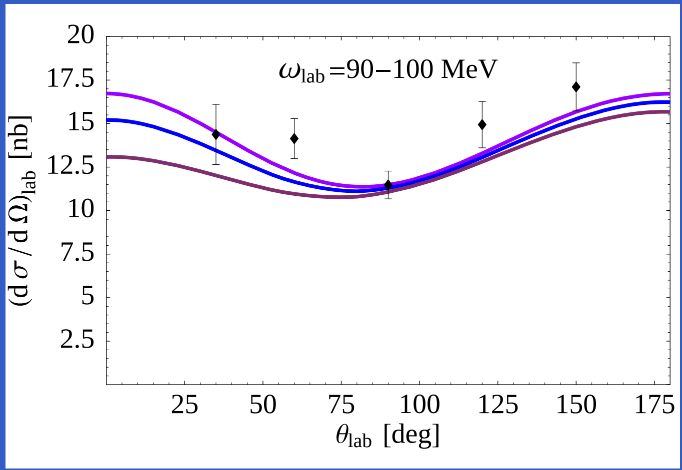


$O(e^2P)$ fits in χPT($\not\Delta$) tend to overestimate β_N

Energy dependence is an issue (esp. at fwd. angles):





Conclusions

- χPT($\not\Delta$) expansions may converge only slowly if the Delta is not explicitly included;
- Sometimes it's 2Δ vs. Δ: ed scattering $\surd$
- Connections of πN parameter extractions in $A = 1, 2, 3$?
- SSE: $m_\pi \sim \Delta \Rightarrow$ Delta-effects are perturbative
- δ-expansion $\Rightarrow$ Resum for $\omega \sim \Delta$: works well for γp.
- To do: photoproduction, πN, . . .
- Applicable to NN at higher energies?

Thanks to the US DoE for financial support

Dressing the Δ propagator

$$S^{(0)}_{\mu\nu}(p) = -\frac{\mathcal{P}^{(3/2)}_{\mu\nu}(p)}{\not{p} - M_\Delta} + \text{non spin} - 3/2 \text{ pieces}$$

$$\Sigma_{\mu\nu} = \Sigma^{(3)}_{\mu\nu} + \Sigma^{(4)}_{\mu\nu} + \ldots$$

Treat $\Sigma^{(4)}$ etc. in perturbation theory

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Treat $\Sigma^{(4)}$ etc. in perturbation theory

Consistent couplings $\Rightarrow \Sigma_{\mu\nu}(p) = \Sigma(p)\mathcal{P}^{(3/2)}_{\mu\nu}(p)$

Resum *renormalized* third-order self-energy

$$\tilde{S}_{\mu\nu}(p) = -\frac{Z(p^2)}{\not{p} - M(p^2)}\mathcal{P}^{(3/2)}_{\mu\nu}(p)$$

$$= -\frac{Z(M_\Delta^2)}{\not{p} - M_\Delta - i\,\text{Im}\,M(p^2)}\mathcal{P}^{(3/2)}_{\mu\nu}(p) + O\left(\frac{1}{\Lambda}\right)$$

Consistent couplings

$\mathcal{L}$ invariant under $\Delta_\mu \to \Delta_\mu + \partial_\mu \epsilon$

$\Rightarrow \Delta$ has correct number of spin degrees of freedom

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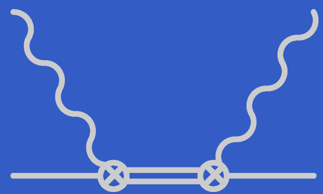

$$\Gamma^\mu S_{\mu\nu}(p)\Gamma^\nu = -\frac{\Gamma^\mu \mathcal{P}^{(3/2)}_{\mu\nu}(p)\Gamma^\nu}{\not{p} - M_\Delta}$$

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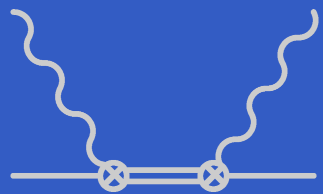

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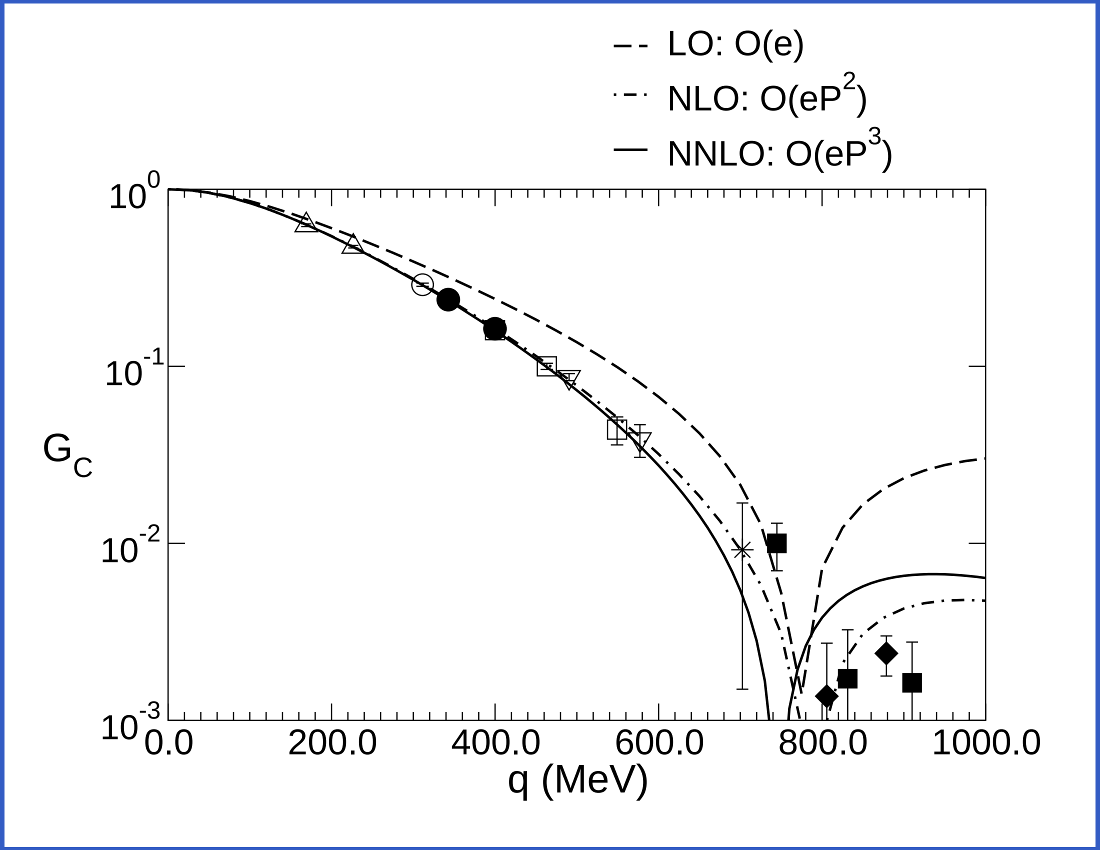
Unphysical spin-1/2 degrees of freedom do not enter any physical amplitude

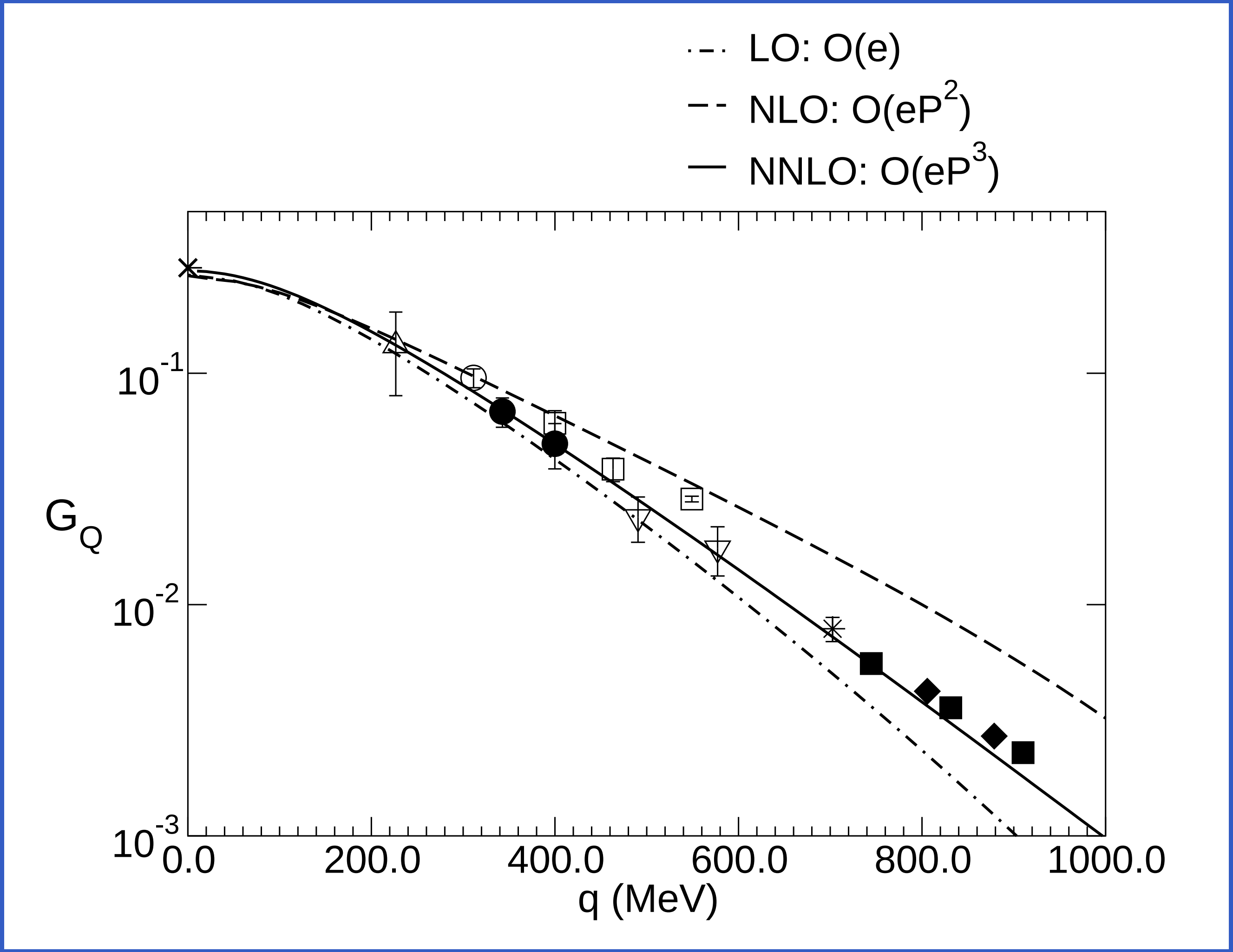
G_C using factorization

$$\frac{G_C}{G_E^{(s)}} = \langle\psi|e|\psi\rangle + \langle\psi|J_0^{(3)}|\psi\rangle + O(eP^4)$$



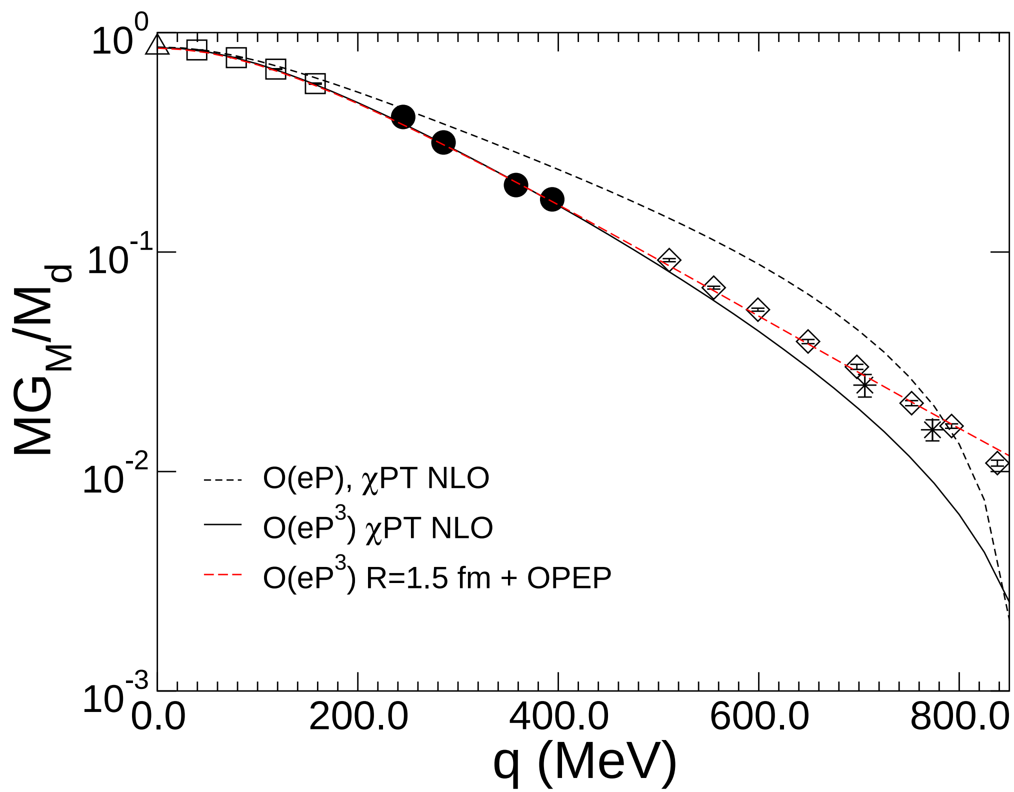
Parameter-free prediction: tests χPT's deuteron.

G_Q



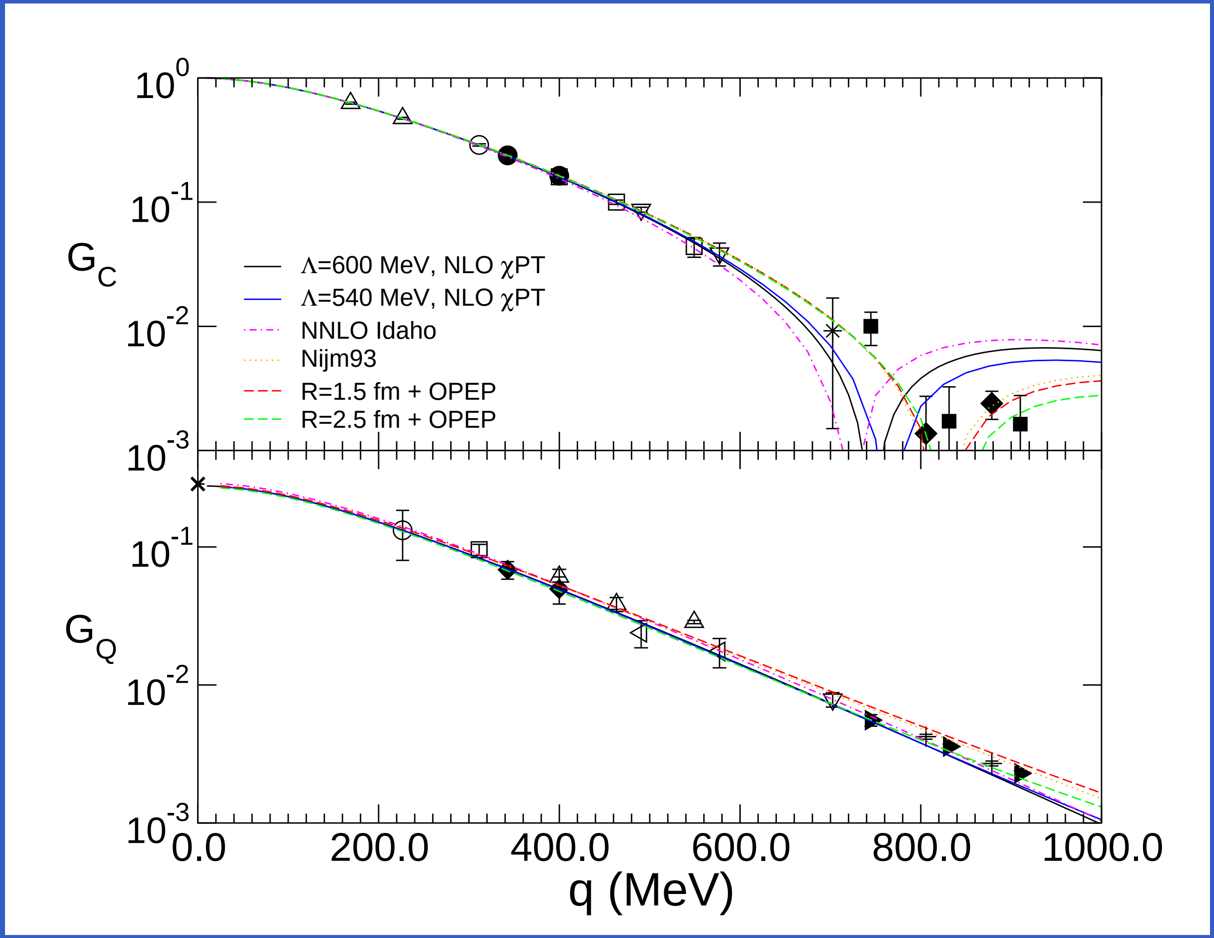
LO: O(e)
NLO: O(eP^2)
NNLO: O(eP^3)
G_Q
10^-1
10^-2
10^-3
0.0
200.0
400.0
600.0
800.0
1000.0
q (MeV)

G_M



J^+: more sensitive to short-distance contributions than J^0.

Wave-function dependence



Wave-function sensitivity → estimate higher-order effects.