

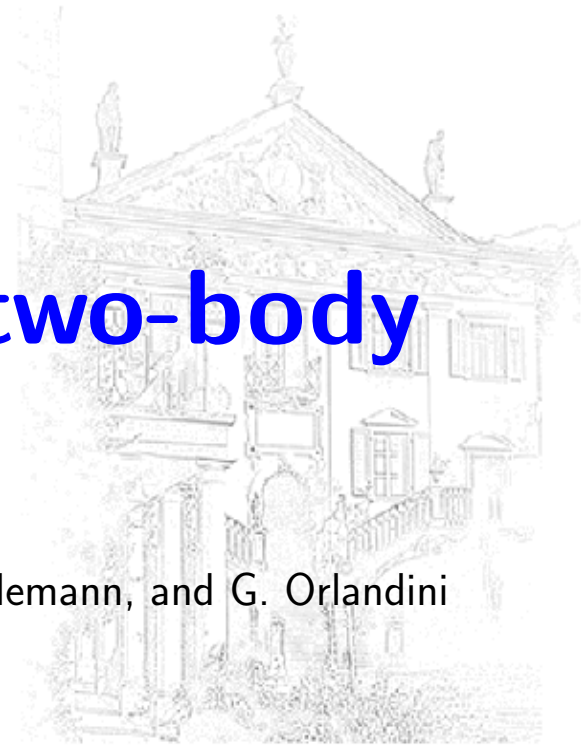


UNIVERSITÀ DEGLI STUDI
DI TRENTO

Dipartimento di Fisica

Electromagnetic two-body breakup of ${}^4\text{He}$

S. Quaglioni, D. Andreasi, V. D. Efros, W. Leidemann, and G. Orlandini

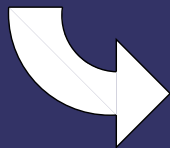
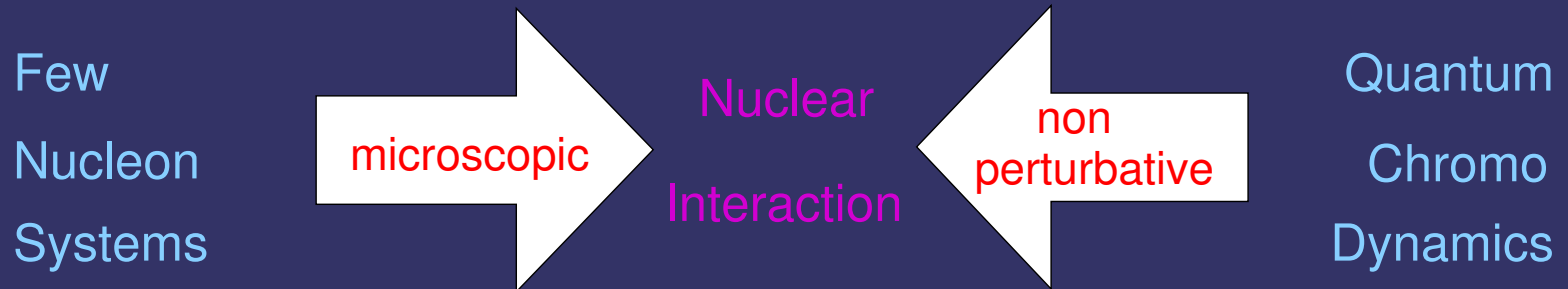


Outline of the Talk

- Quest for a fundamental NN interaction
- Two-fragment reactions through LIT
- Photodisintegration
- Electrodisintegration
- Conclusions and Outlook

Quest for a fundamental NN interaction

Nucleons are composite bound states of quarks and gluons



“Theoretical laboratory” to test nuclear force models

- structure of light nuclei
- few–nucleon scattering and reactions

A consistent treatment of both ISI and FSI is mandatory!

What have we learned?

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$A=2$ almost perfect description of $2N$ data

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A=3 binding energies \Rightarrow local V_{NN} not sufficient!

\rightsquigarrow off-shell effects!

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scattering observables \Rightarrow fundamental!

\rightsquigarrow open problems: A_y -puzzle, ...

\rightsquigarrow find reaction observables sensitive to 3NF:
inclusive or exclusive?

Total electrodisintegration of ^3He : Role of 3NF

Longitudinal
Response

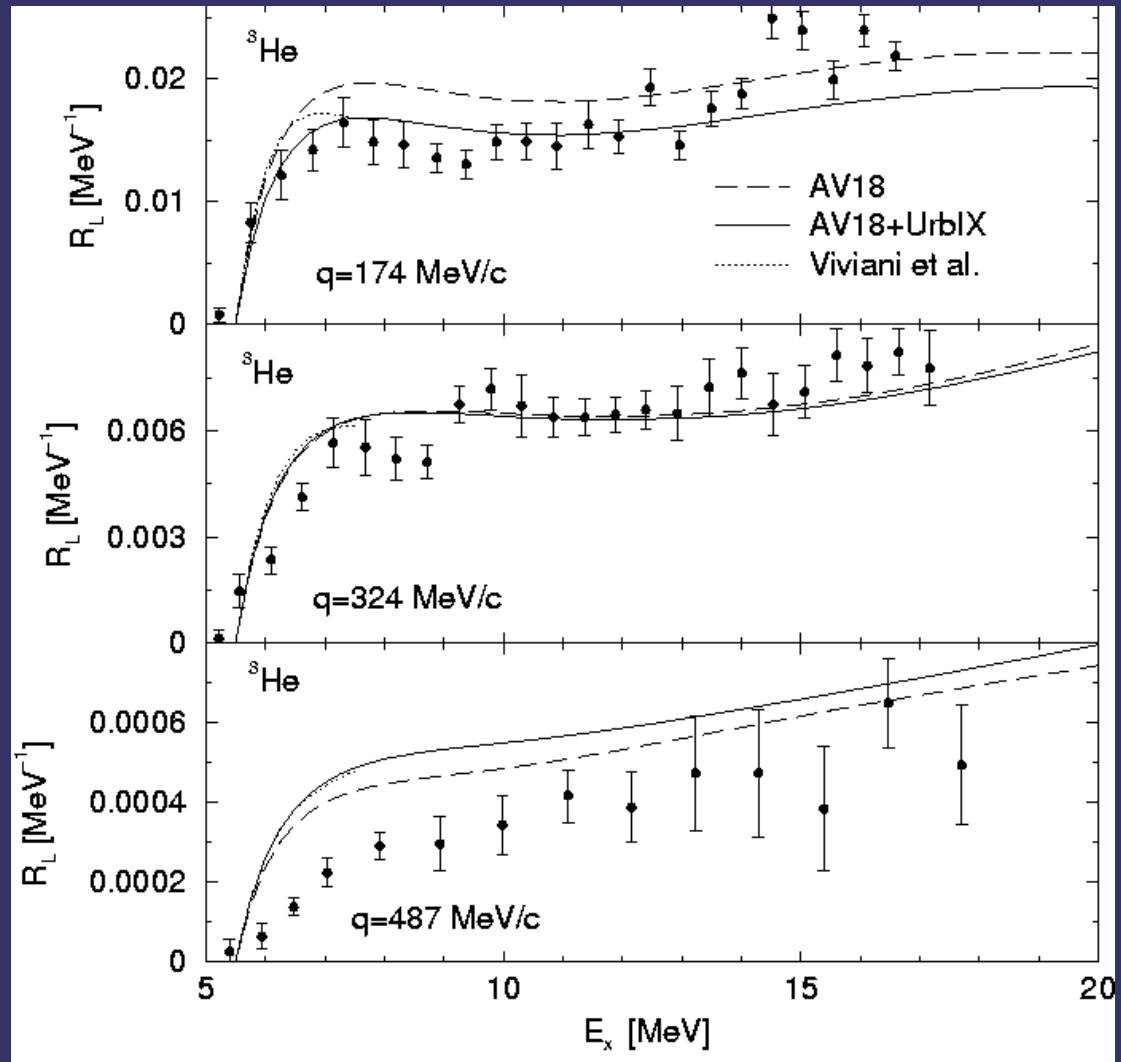
low energy

--- AV18
— AV18+UrbIX

CHH

Efros, Leidemann, Orlandini,
Tomusiak
PRC69(2004)044001

● Retzlaff *et al.*,
PRC49(1994)1263



Exclusive ${}^3\text{He}(\gamma, pp)n$ reaction

3NF effects

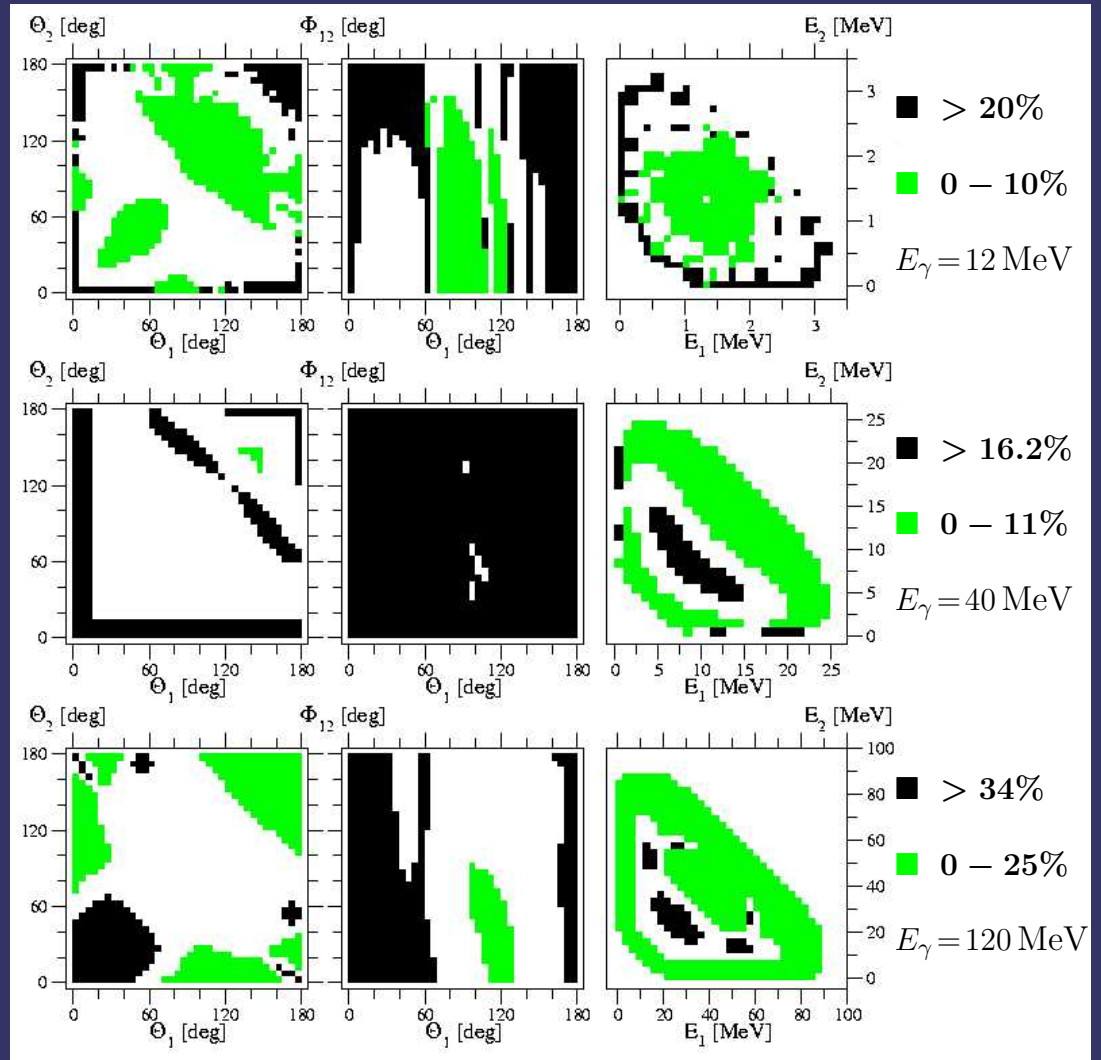
3N breakup
phase-space

AV18+UIX+MEC

versus

AV18+MEC

Golak *et al.*,
nucl-th/0505072



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$A=4$ a novel testground for nuclear interactions

Low-energy $n-{}^3\text{H}$ scattering: A novel testground for nuclear interactions

Lazauskas, Carbonell, Fonseca, Viviani, Kievsky, Rosati,

PRC71(2005)034004

“The four-nucleon system represents a qualitative jump in complexity relative to the $A = 3$ case ...”

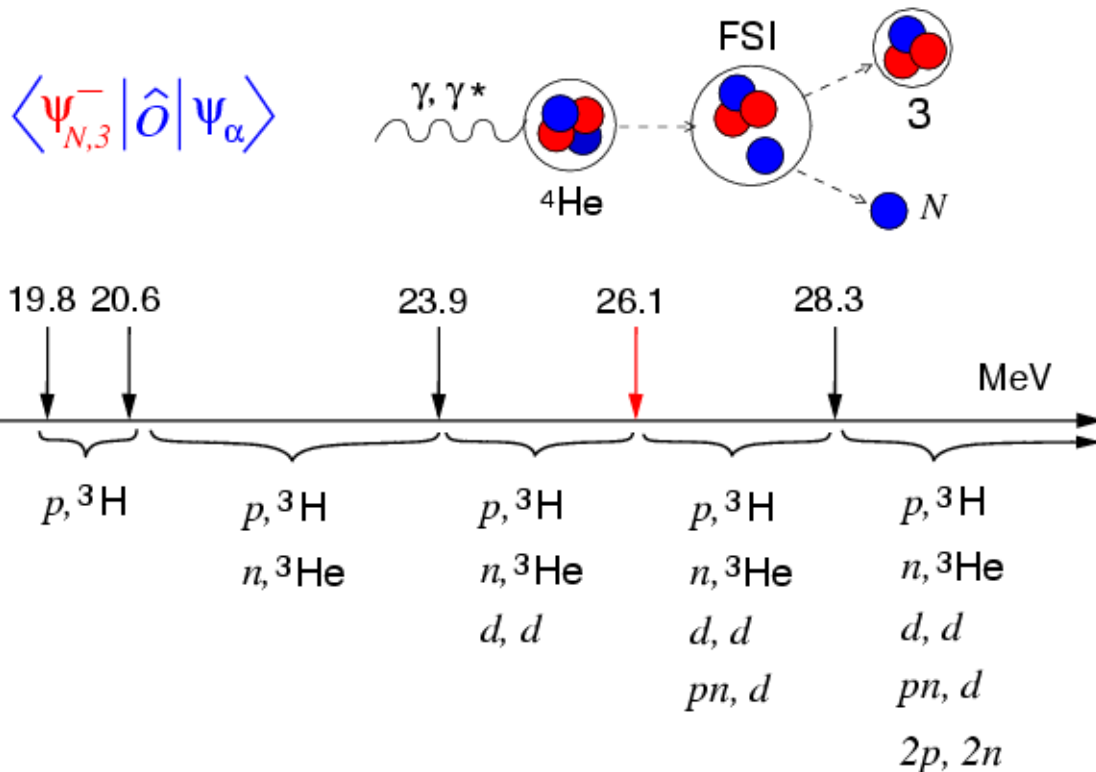
“... Unlike the $A = 3$ systems, $A = 4$ shows a delicate and rich structure of excited states in the continuum, whose position and width depends critically on the underlying NN interaction ...”

“... Therefore is of the utmost importance to have reliable few-body techniques powerful enough to deal with this system ...”

Where is the challenge?

Push *ab initio* calculation of exclusive reactions of a 4-particle system, also beyond the 3-body breakup threshold

Channels up to the π -production threshold



▲ full FSI also beyond 3bbu

▼ V_{NN} model:

MTI-III

Two-Fragment Reactions through LIT

Which theoretical technique?

The Lorentz integral transform method:

- ▷ the only feasible beyond the 3-body breakup threshold
- ▷ already applied for inclusive reactions:
 $A \leq 7$
- ▷ less known in its exclusive version: first application to the 4-nucleon system

Exclusive perturbation-induced processes

$$T_f(E_f) = \langle \Psi_f^-(E_f) | \hat{O} | \Psi_0 \rangle$$

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 $= \int \frac{F_f(E)}{E_f + i\varepsilon - E} dE$

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$$= \int \frac{F_f(E)}{E_f + i\epsilon - E} dE$$

$$F_f(E) = \sum_{\nu} \langle \Phi_f | \hat{V} | \Psi_{\nu} \rangle \langle \Psi_{\nu} | \hat{O} | \Psi_0 \rangle \delta(E - E_{\nu})$$

Integral Transform Approach

$$\Phi[F](\sigma) = \int F(E)K(\sigma, E)dE$$

Aim: determining $F(E)$ via the integral transform.
It means to calculate $\Phi[F](\sigma)$ without knowledge of $F(E)$.

$$\Phi[F](\sigma) \Rightarrow \text{inversion} \Rightarrow F(E)$$

- Laplace Transform:

$$K(\sigma, E) = e^{-E\sigma}$$

- Stiltjes Transform:

$$K(\sigma, E) = \frac{1}{E+\sigma}$$

- Lorentz Transform:

$$K(\sigma, E) = \frac{1}{(E-\sigma_R)^2 + \sigma_I^2}$$

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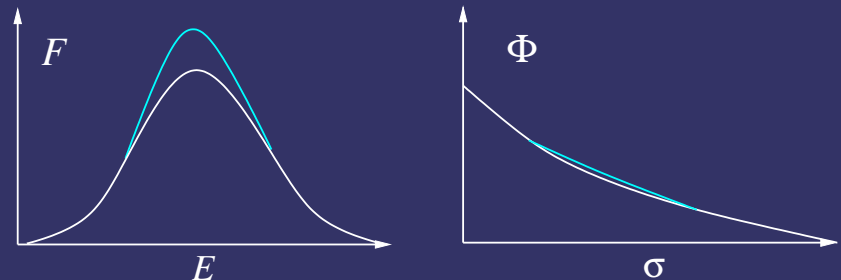
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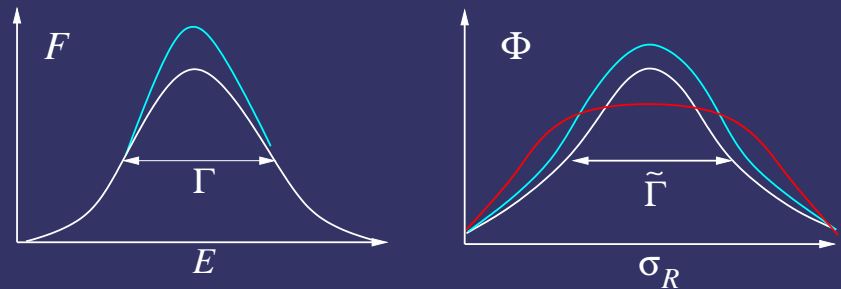
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The Lorentz Transform

$$L[F_f](\sigma) = \int F_f(E) \frac{1}{(E - \sigma_R)^2 + \sigma_I^2} dE$$

$$= \int \sum_{\nu} \frac{\langle \Phi_f | \hat{V} | \Psi_{\nu} \rangle \langle \Psi_{\nu} | \hat{O} | \Psi_0 \rangle \delta(E - E_{\nu})}{(E - \sigma_R + i\sigma_I)(E - \sigma_R - i\sigma_I)} dE$$

$$= \sum_{\nu} \left\langle \Phi_f \left| \hat{V} \frac{1}{E_{\nu} - \sigma^*} \right| \Psi_{\nu} \right\rangle \left\langle \Psi_{\nu} \left| \frac{1}{E_{\nu} - \sigma} \hat{O} \right| \Psi_0 \right\rangle$$

$$= \sum_{\nu} \left\langle \Phi_f \left| \hat{V} \frac{1}{H - \sigma^*} \right| \Psi_{\nu} \right\rangle \left\langle \Psi_{\nu} \left| \frac{1}{H - \sigma} \hat{O} \right| \Psi_0 \right\rangle$$

$$= \left\langle \Phi_f \left| \hat{V} \frac{1}{H - \sigma^*} \frac{1}{H - \sigma} \hat{O} \right| \Psi_0 \right\rangle = \langle \tilde{\psi}_2 | \tilde{\psi}_1 \rangle$$

The Lorentz Transform

$$\begin{aligned} L[F_f](\sigma) &= \int F_f(E) \frac{1}{(E - \sigma_R)^2 + \sigma_I^2} dE \\ &= \left\langle \Phi_f \left| \hat{V} \frac{1}{H - \sigma^*} \frac{1}{H - \sigma} \hat{O} \right| \Psi_0 \right\rangle = \langle \tilde{\psi}_2 | \tilde{\psi}_1 \rangle \end{aligned}$$

To calculate $L[F_f](\sigma)$ one doesn't need to know $F_f(E)$.
Solve for many σ_R and fixed σ_I :

$$\begin{aligned} (H - \sigma_R - i\sigma_I) |\tilde{\psi}_1\rangle &= \hat{O} |\Psi_0\rangle \\ (H - \sigma_R - i\sigma_I) |\tilde{\psi}_2\rangle &= \hat{V} |\Phi_f\rangle \end{aligned}$$

$F_f(E)$ from inversion of $L[F_f](\sigma)$: **FSI included**

Procedure:

1. choose an NN-interaction
2. find the excitation operator relevant to the reaction
(consistent with NN-interaction)
3. choose a good bound-state technique

- to solve A-body Schrödinger equation

$$(H - E_0)|\Psi_0\rangle = 0$$

- to solve A-body LIT equation

$$(H - \sigma_R - i\sigma_I)|\tilde{\psi}\rangle = |Q\rangle$$

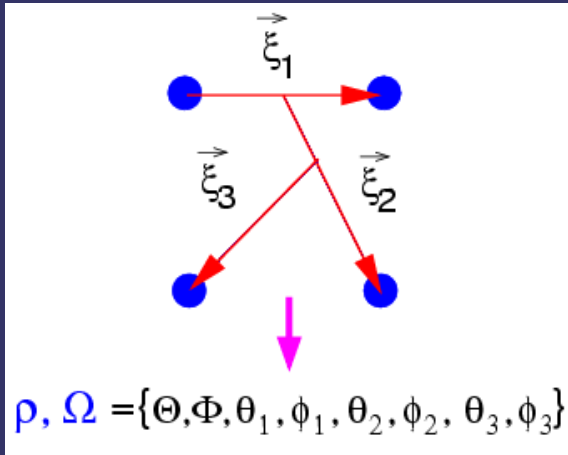
4. calculate the overlap $L[F_f](\sigma) = \langle\tilde{\psi}_2|\tilde{\psi}_1\rangle$

5. invert transform $L[F_f](\sigma) \longrightarrow F_f(E)$

6. calculate matrix element $T_f^{\text{FSI}}(E_f) = \int \frac{F_f(E)}{E_f + i\varepsilon - E} dE$

Remove CM Degrees of Freedom

$$\Phi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) = \phi(\vec{R}_{\text{cm}}) \Psi(\vec{\xi}_1, \vec{\xi}_2, \vec{\xi}_3)$$



$$H = -\frac{1}{2m} \left(\Delta_\rho - \frac{\hat{K}^2}{\rho^2} \right) + \sum_{i < j}^4 v_{ij}$$

$$\hat{K}^2 \mathcal{Y}_{[K]}^\mu(\Omega) = K(K+7) \mathcal{Y}_{[K]}^\mu(\Omega)$$

Which dynamical model?

Spin dependent S-wave interaction **MTI-III**

$$v_{ij} = V_{31}(r) \mathcal{P}_\sigma^+ \mathcal{P}_\tau^- + V_{13}(r) \mathcal{P}_\sigma^- \mathcal{P}_\tau^+$$

Total photodisintegration of ^3H :

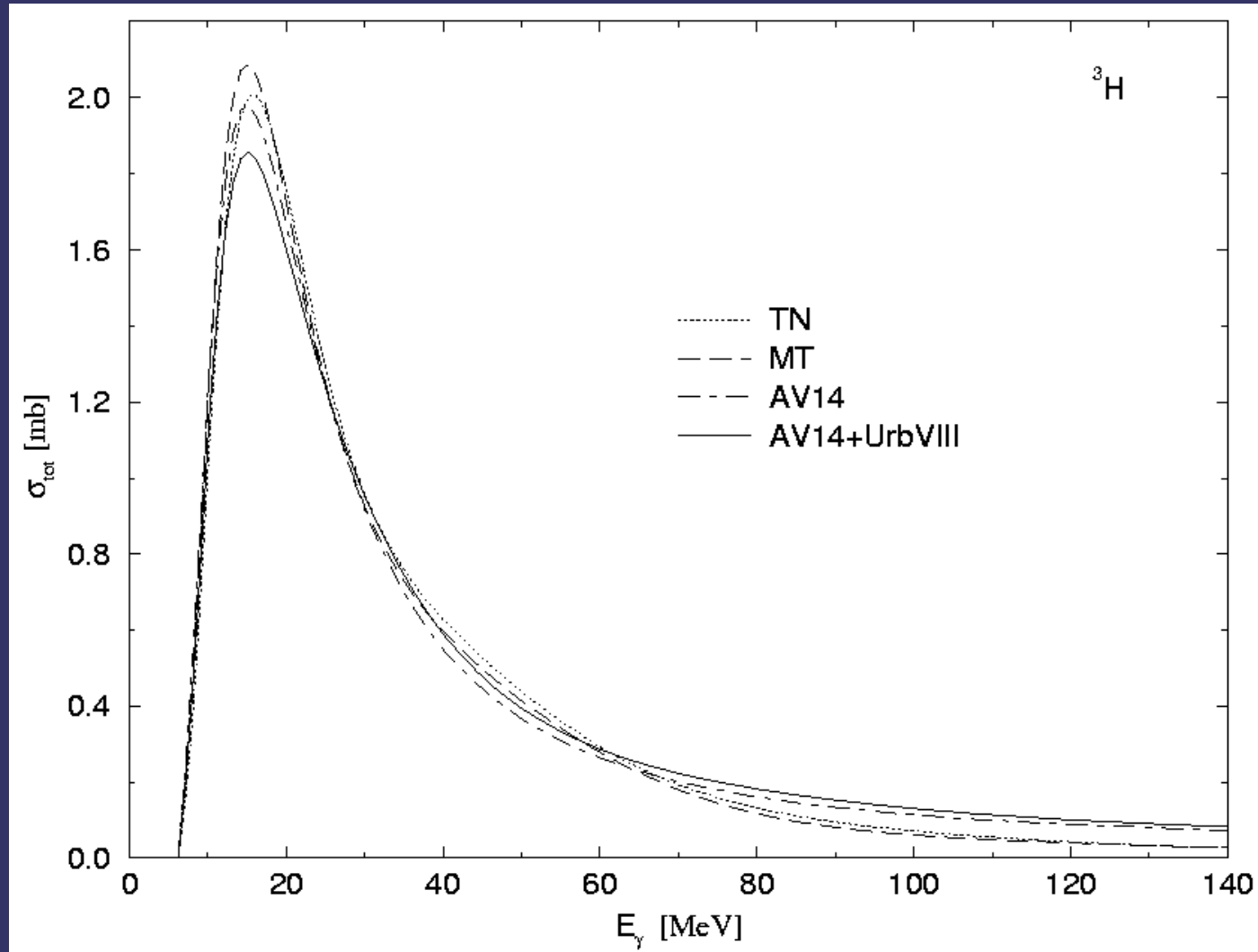
MTI-III vs AV14

Total
cross section

--- MTI-III
--- AV14

CHH

Efros, Leidemann,
Orlandini, Tomusiak
PLB484(2000)223



CHH Expansion Method

We seek the solutions of Schrödinger and LIT equations in the following form:

$$\Psi = \sum_{n, [K]}^{n_{max}, [K_{max}]} C_n^{[K]} \mathcal{J} R_n(\rho) \left[\mathcal{Y}_{[K]}^\mu(\Omega) \Theta_{ST}^{\bar{\mu}} \right]_{JT}^a$$

$$\mathcal{J} = \prod_{i < j} f(r_{ij}) \longrightarrow \text{Jastrow factor}$$

$$R_n(\rho) \sim L_n^\nu(\rho/\rho_0) \exp[-\rho/2\rho_0]$$

\hookrightarrow hyperradial functions

$$\mathcal{Y}_{[K]}^\mu(\Omega) \longrightarrow \text{hyperspherical harmonics}$$

$$\Theta_{ST}^{\bar{\mu}} \longrightarrow \text{spin — isospin functions}$$

Photodisintegration:



Total cross section

$$\sigma_{N,3} \propto \int |\langle \Psi_{N,3}^- | \mathbf{D} | \Psi_\alpha \rangle|^2 d\Omega$$

The $n, {}^3\text{He}$ channel ►

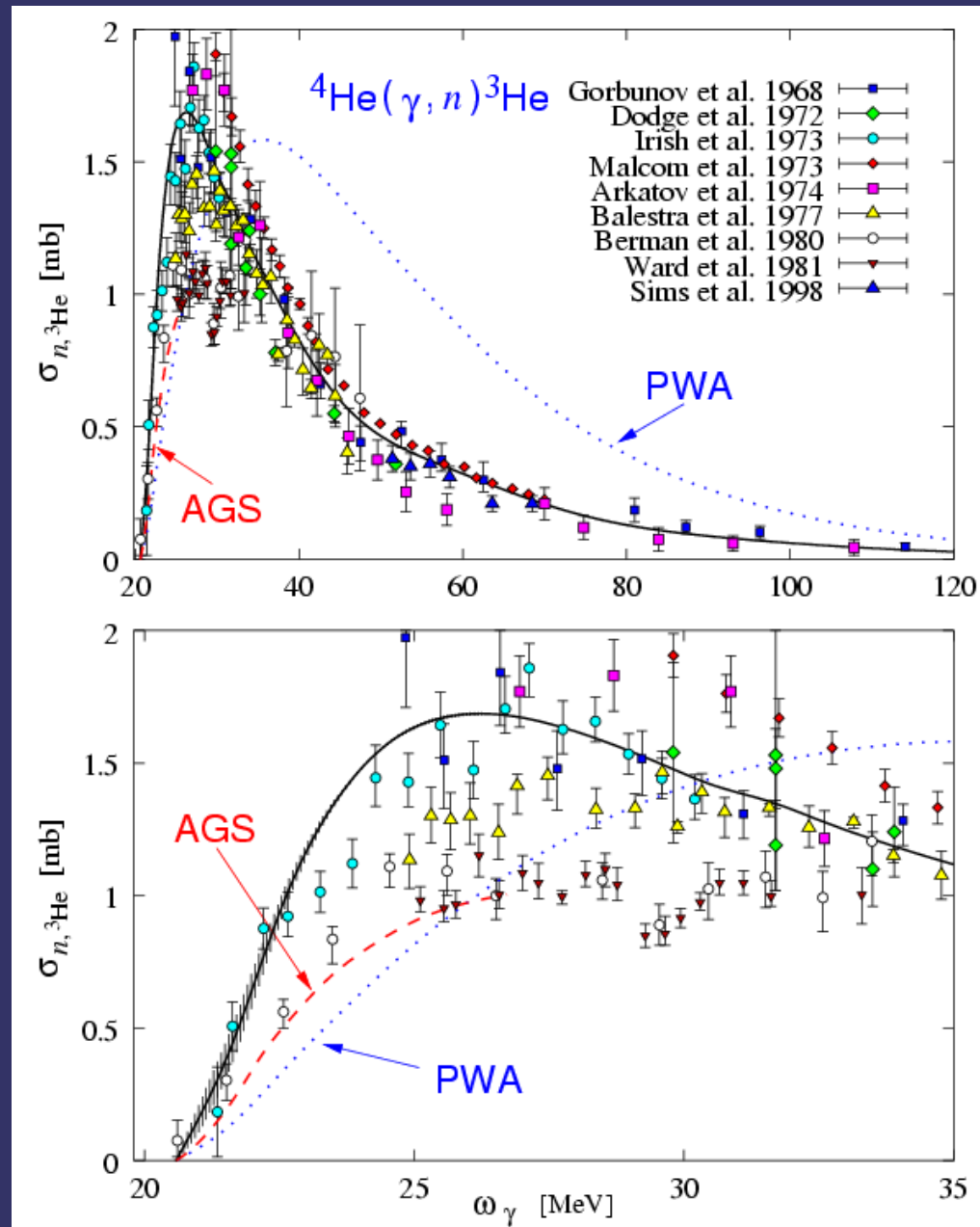
..... PWA

—— FULL

MTI-III

CHH

Quaglioni, Barnea, Efos, Leidemann,
Orlandini,
PRC69(2004)044002



Test the PWA Approximation

A simple model: Gaussian Wave Functions

$$\Psi_A^{\text{GWF}} = \left(\frac{2b_A}{\pi}\right)^{\frac{3(A-1)}{4}} \prod_{i=1}^A e^{-b_A |\mathbf{r}_i - \mathbf{R}_{cm}^{(A)}|^2} \Theta_{T_A S_A}^a$$

Analytical result:

$$\sigma_{n, {}^3\text{He}}^{\text{GPWA}} = 4\pi^2 \alpha \omega_\gamma k \mu \left(\frac{4}{3}\right)^{\frac{9}{2}} \frac{4 b_3^3 k^2 e^{-\frac{2k^2}{3b_4}}}{\sqrt{2\pi b_4} (b_3 + b_4)^6}$$

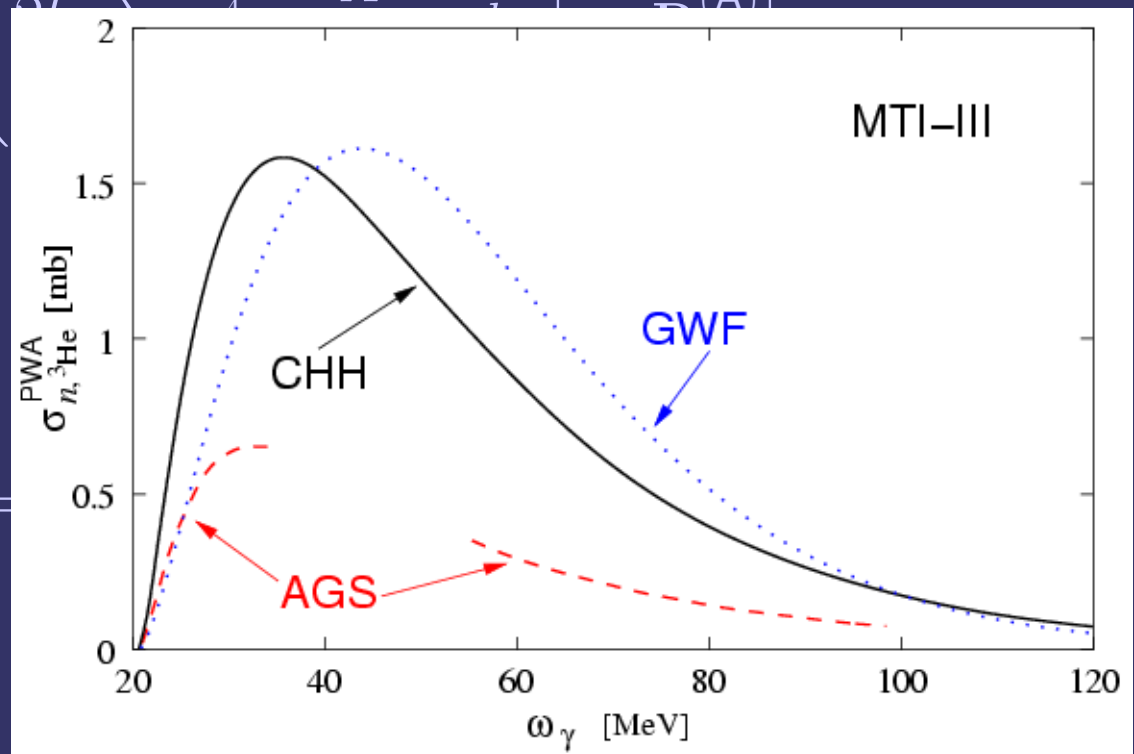
Test the PWA Approximation

A simple model: Gaussian Wave Functions

$$\Psi_A^{\text{GWF}} = \left(\frac{3(A-1)}{4\pi A} \right)^{3/4} \exp\left(-\frac{3}{4A} |\mathbf{r}|^2\right)$$

Analytical result:

$$\sigma_{n,^3\text{He}}^{\text{GPWA}} =$$



Photodisintegration:



Total cross section

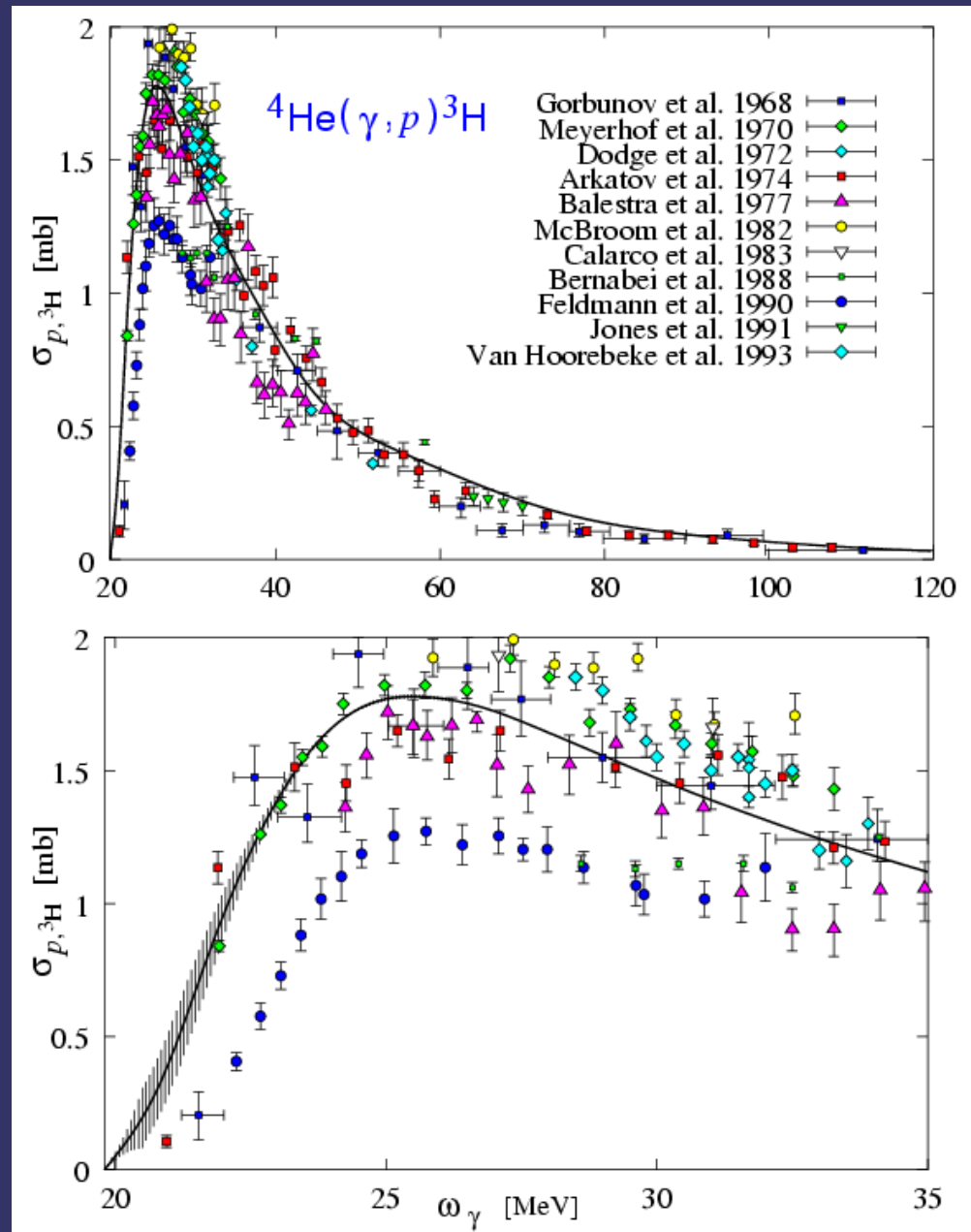
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The $p, {}^3\text{H}$ channel ►

— MTI-III

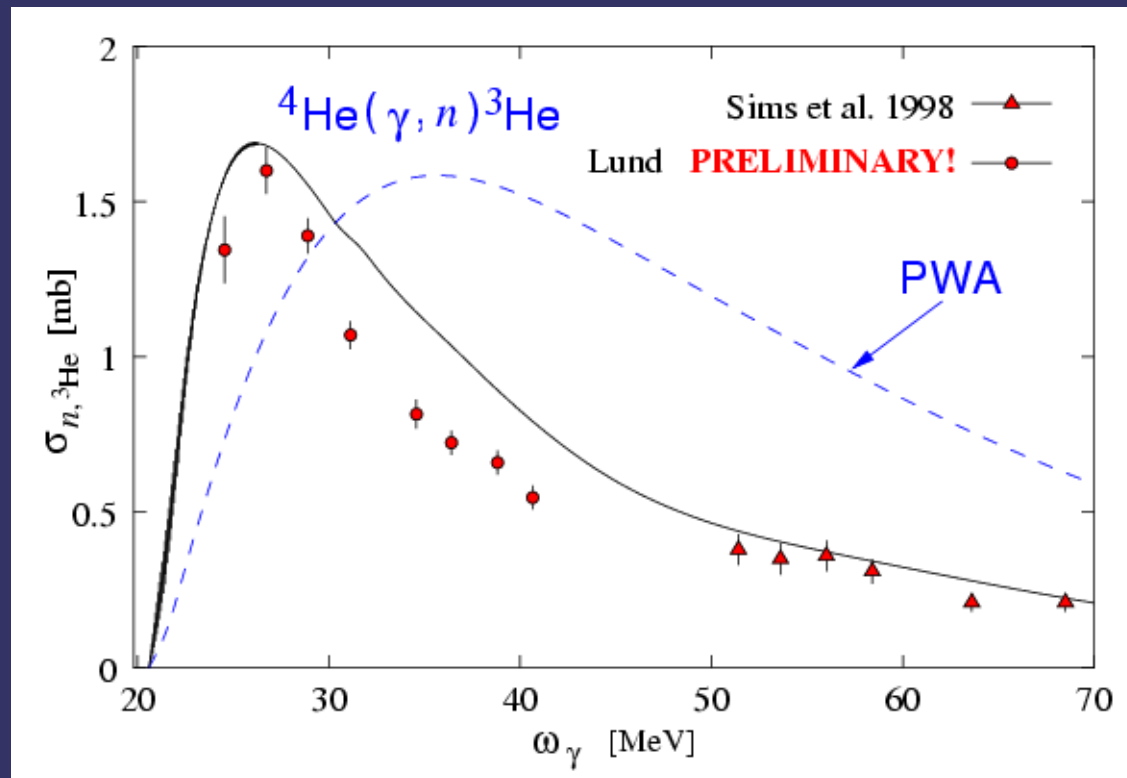
CHH

Quaglioni, Barnea, Efos, Leidemann,
Orlandini,
PRC69(2004)044002



New Experimental Data

The availability of 4-body *ab initio* calculations also beyond the 3-body breakup threshold has given birth to a new experimental interest



Tagged
Photons

Total photodisintegration of ^4He :

Total
cross
section

EIHH

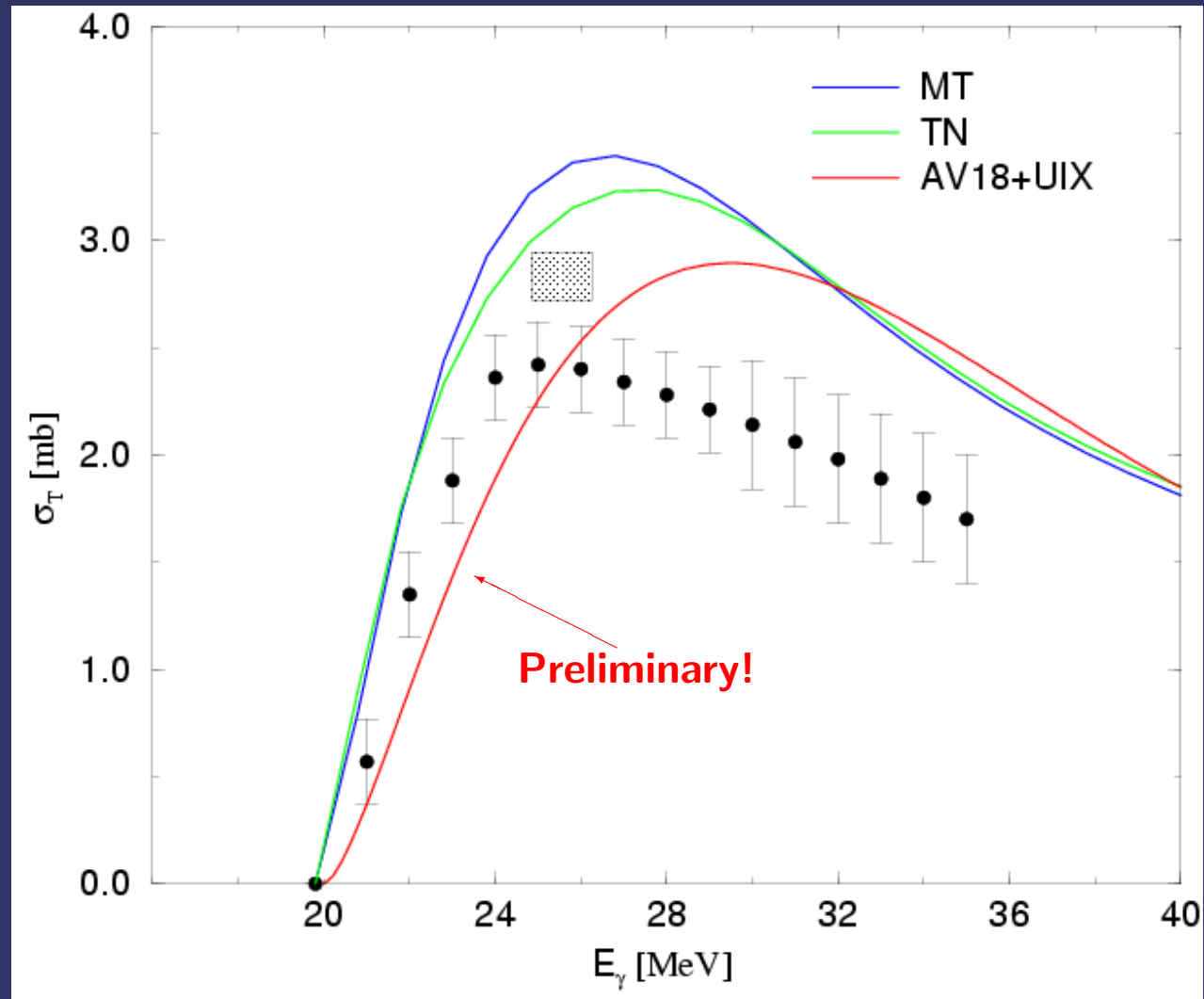
Barnea, *et al.*

—●—

(γ, n) Berman *et al.*
(1980)

+

(γ, p) Feldman *et al.*
(1990)



Electrodisintegration: $e + {}^4\text{He} \rightarrow e' + x + Y$

$$\frac{d^5\sigma}{dE' d\Omega_{e'} d\Omega_x} \propto [V_L F_L + V_T F_T + V_{LT} F_{LT} \cos \phi + F_{TT} F_{TT} \cos 2\phi]$$

${}^4\text{He}(e, e'x)Y$ Longitudinal Response

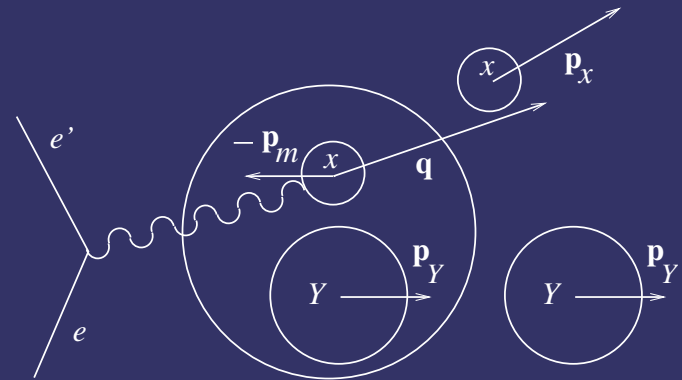
$$F_L(\mathbf{q}, \omega, \theta_x) \propto |\langle \Psi_{x,Y}^- | \hat{\rho}(\mathbf{q}) | \Psi_\alpha \rangle|^2$$

Investigate validity of PWIA:

- direct knock-out of x
- Y is a spectator



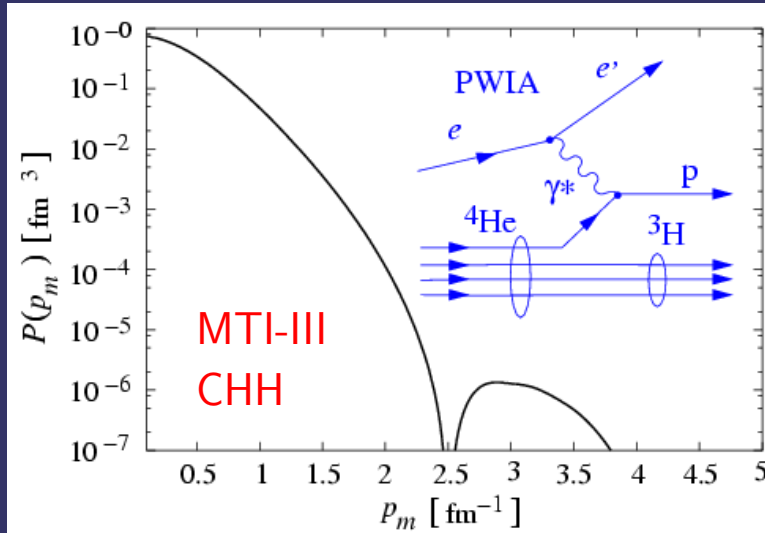
- NO** Antisymmetrization
- NO** Final State Interaction



The $p, {}^3\text{H}$ channel

The **PWIA** longitudinal response turns out to be proportional to the proton-triton **momentum distribution**

$$F_L = Z_\alpha (G_E^p)^2 P(p_m)$$



- ▷ access to g.s. quantities
- ▷ get informations on **short range correlations** from the high p_m region

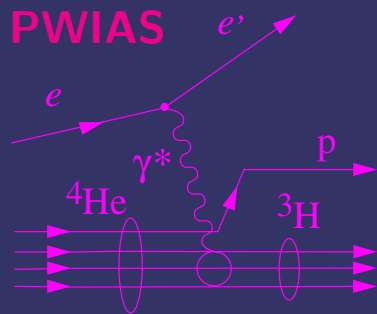
check!

Longitudinal Response

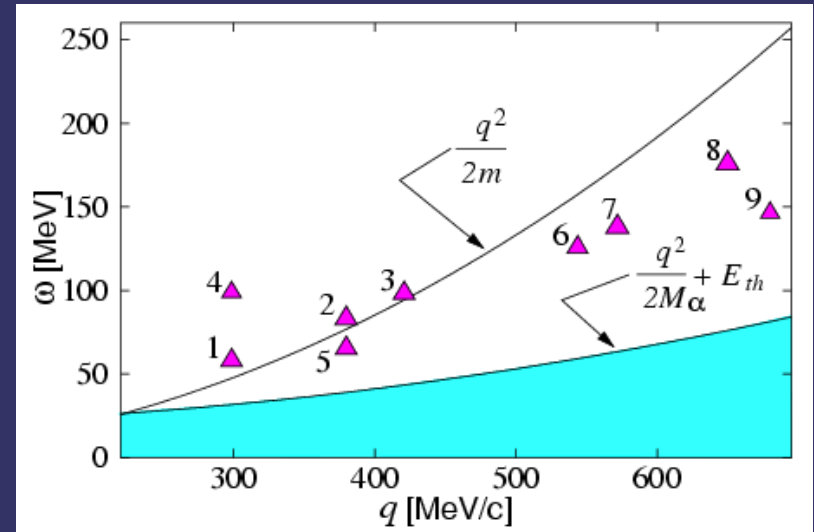
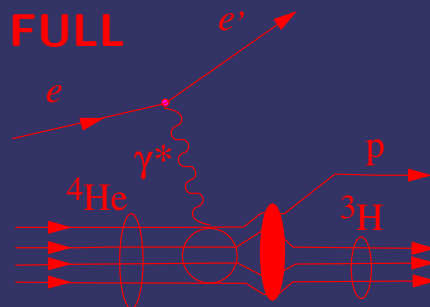
parallel kinematics $\rightarrow p_p \rightarrow q$

Deviations from PWIA

+ antisymmetrization



+ FSI



Kin. No.	PWIA $F_L / (\tilde{G}_{E_p}^2)$ [$(\text{GeV}/c)^{-3} \text{sr}^{-1}$]	Δ_{PWIAS} (%)	Δ_{FULL} (%)
1	185.2	+ 9.3	- 39.6
2	185.2	+ 1.2	- 20.1
3	185.2	+ 0.0	- 12.8
4	100.0	+ 4.5	- 43.4
5	100.0	+ 3.9	- 16.6
6	100.0	- 1.1	+ 11.4
7	100.0	- 1.9	+ 11.5
8	100.0	- 1.7	+ 10.8
9	14.63	- 4.4	+ 52.1

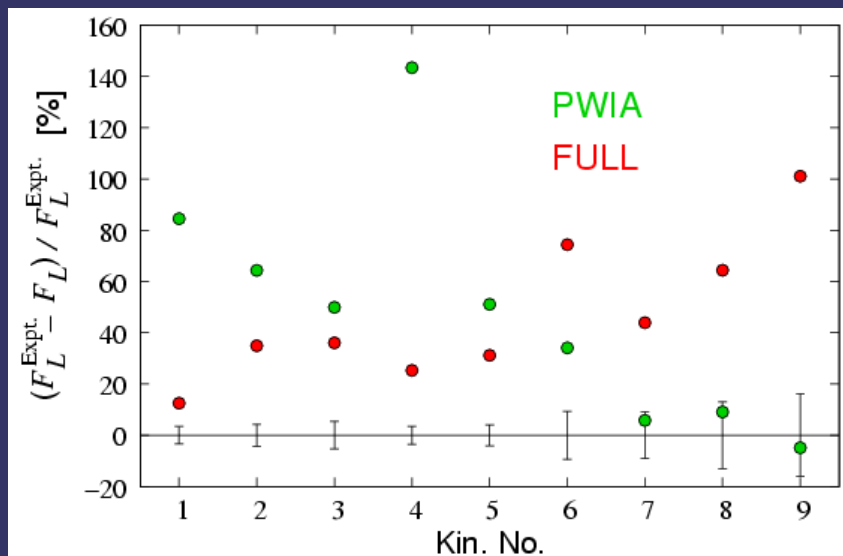
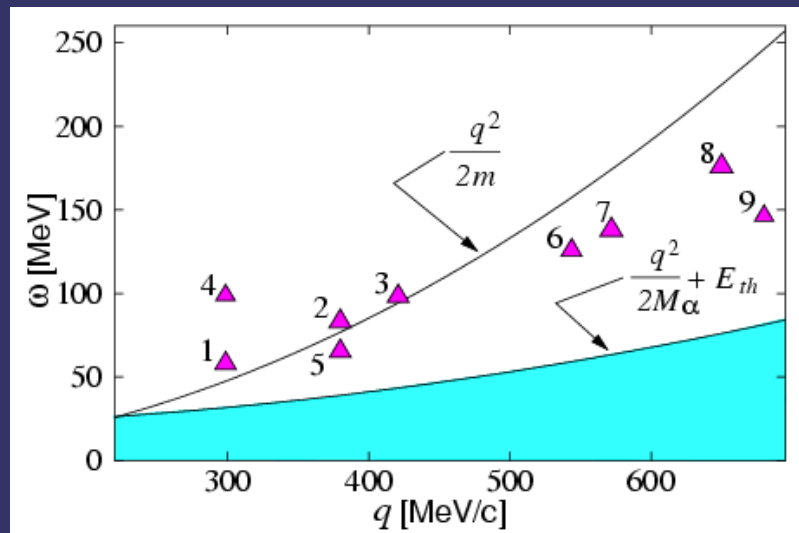
Longitudinal Response

parallel
kinematics \rightarrow \mathbf{p}_p \rightarrow \mathbf{q}

$$\frac{d^5\sigma}{dE' d\Omega_e' d\Omega_p} \propto [V_L F_L + V_T F_T]$$

Comparison with experiment

J.E. Ducret et al. NPA556(1993)373



Kin. No.	$F_L [(\text{GeV}/c)^{-3} \text{sr}^{-1}]$		
		Expt.	FULL
1	59.0	± 2.0	± 2.2
2	49.6	± 2.1	± 2.1
3	46.2	± 2.5	± 2.2
4	27.8	± 1.0	± 1.2
5	28.4	± 1.2	± 1.3
6	14.8	± 1.4	± 1.2
7	16.0	± 1.5	± 1.3
8	9.96	± 1.29	± 1.15
9	1.35	± 0.22	± 0.22

Longitudinal Response

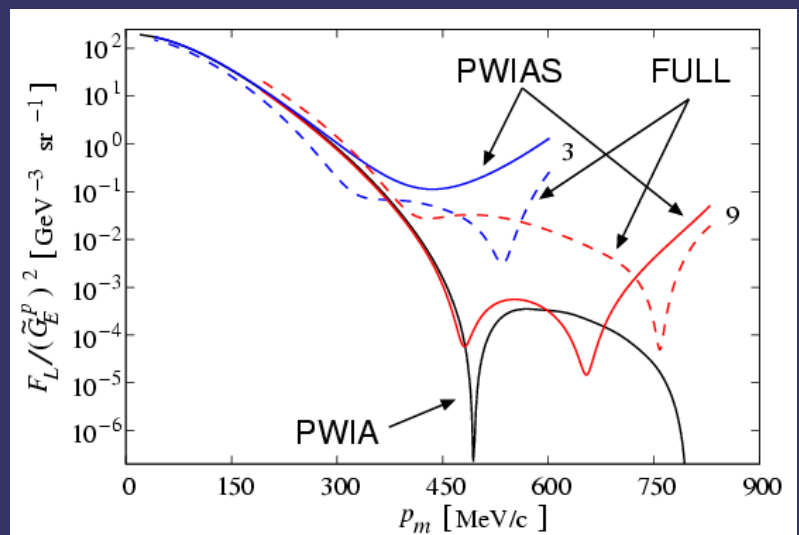
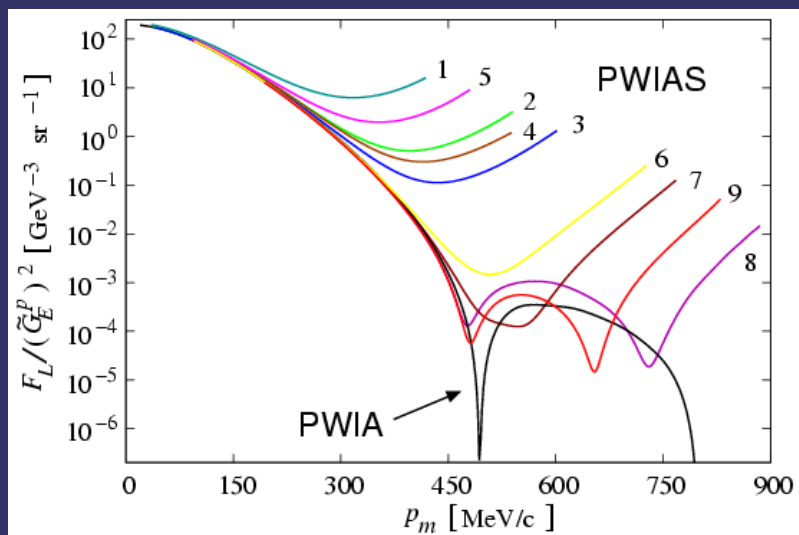
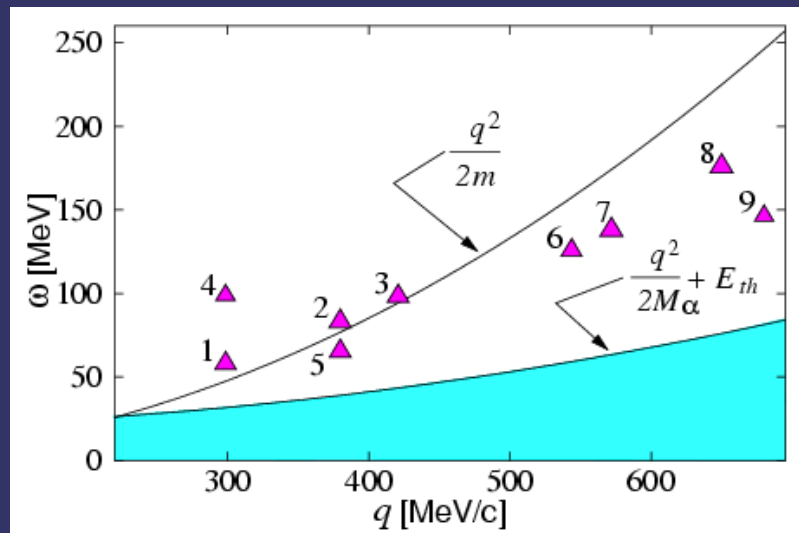
nonparallel
kinematics



Deviations from PWIA

+ antisymmetrization \Rightarrow PWIAS

+ FSI \Rightarrow FULL



The d, d channel: Longitudinal Response

Deviations from PWIA

+ antisymmetrization \Rightarrow PWIAS

+ FSI \Rightarrow FULL

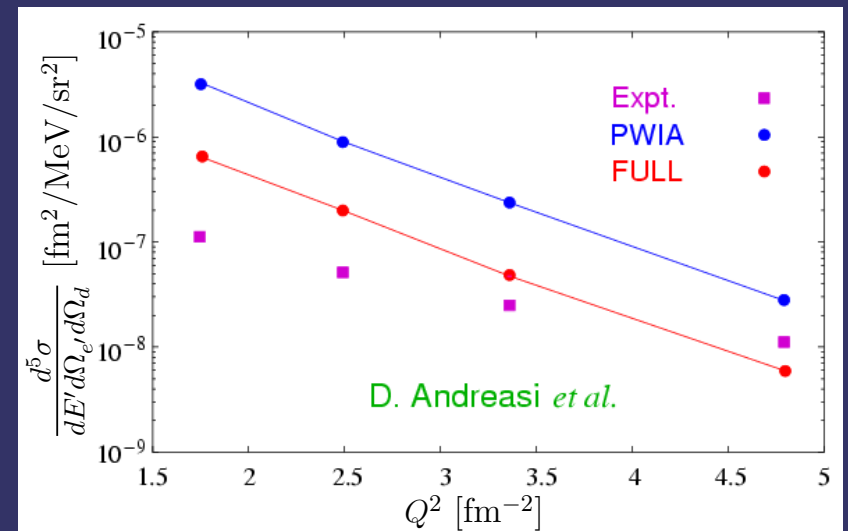
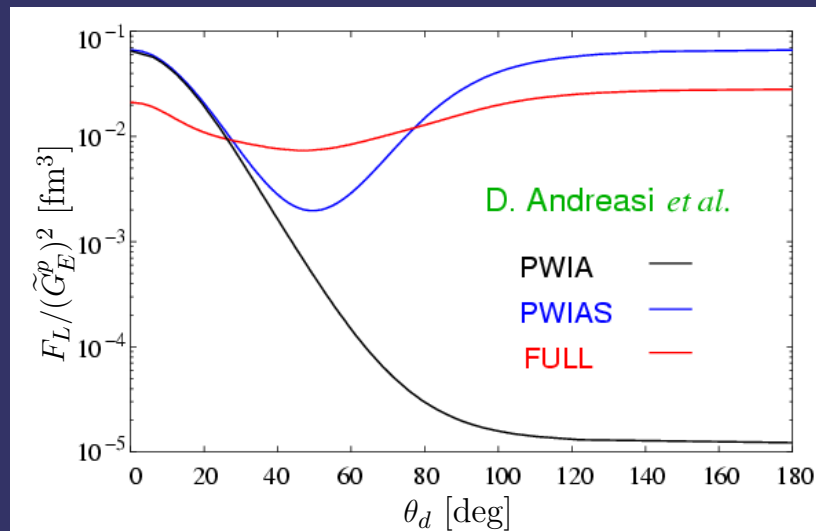
$Q^2 = 4.93 \text{ fm}^{-2}$, $E_{cm} = 35 \text{ MeV}$

Comparison with experiment

R. Ent et al. PRL67(1991)18

$Q^2 = 1.75, 2.49, 3.36, 4.93 \text{ fm}^{-2}$

$E_{cm} = 35 \text{ MeV}$, $p_m = 125 \text{ MeV}/c$



Hadronic Processes

Two fragments both in the initial and in the final state

on-shell T-matrix

$$\begin{aligned} T_{fi}(E) &= \langle \Psi_f^-(E) | V_i | \Phi_i(E) \rangle = \langle \Phi_f(E) | V_f | \Psi_i^+(E) \rangle \\ &= \langle \Phi_f(E) | V_f | \Phi_i(E) \rangle + \int \frac{F_{fi}(E')}{E + i\epsilon - E'} dE' \end{aligned}$$

Hadronic Processes

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$$= \langle \Phi_f(E) | V_f | \Phi_i(E) \rangle + \int \frac{F_{fi}(E')}{E + i\epsilon - E'} dE'$$

$$F_{fi}(E') = \sum_{\nu} \langle \Phi_f | V_f | \Psi_{\nu} \rangle \langle \Psi_{\nu} | V_i | \Phi_i \rangle \delta(E' - E_{\nu})$$

Hadronic Processes

Two fragments both in the initial and in the final state

on-shell T-matrix

$$\begin{aligned} T_{fi}(E) &= \langle \Psi_f^-(E) | V_i | \Phi_i(E) \rangle = \langle \Phi_f(E) | V_f | \Psi_i^+(E) \rangle \\ &= \langle \Phi_f(E) | V_f | \Phi_i(E) \rangle + \int \frac{F_{fi}(E')}{E + i\epsilon - E'} dE' \end{aligned}$$

$$L[F_{fi}](\sigma) = \langle \Phi_f | V_f \frac{1}{H - \sigma^*} \frac{1}{H - \sigma} V_i | \Phi_i \rangle = \langle \tilde{\psi}_f | \tilde{\psi}_i \rangle$$

$$(H - \sigma_R - i\sigma_I) | \tilde{\psi}_{i/f} \rangle = V_{i/f} | \Phi_{i/f} \rangle \longrightarrow \text{full FSI!}$$

Hadronic Processes

Two fragments both in the initial and in the final state

on-shell T-matrix

$$T_{fi}(E) = \langle \Psi_f^-(E) | V_i | \Phi_i(E) \rangle = \langle \Phi_f(E) | V_f | \Psi_i^+(E) \rangle$$
$$= \langle \Phi_f(E) | V_f | \Phi_i(E) \rangle + \int \frac{F_{fi}(E')}{E + i\epsilon - E'} dE'$$

$$L[F_{fi}](\sigma) = \langle \Phi_f | V_f \frac{1}{H - \sigma^*} \frac{1}{H - \sigma} V_i | \Phi_i \rangle = \langle \tilde{\psi}_f | \tilde{\psi}_i \rangle$$

$$(H - \sigma_R - i\sigma_I) | \tilde{\psi}_{i/f} \rangle = V_{i/f} | \Phi_{i/f} \rangle \longrightarrow \text{full FSI!}$$

Conclusions

Summary

- First *ab initio* calculation of **exclusive** reactions in the **continuum** of a 4–particle system also beyond the 3–body breakup threshold
- **FSI** has been rigorously taken into account via the **LIT** method: first application to exclusive processes with $A > 2$
- The 2–fragment breakup of ${}^4\text{He}$ induced by **real** or **virtual** photons has been studied

Photodisintegration

- The ${}^4\text{He}(\gamma, p){}^3\text{H}$ and ${}^4\text{He}(\gamma, n){}^3\text{He}$ total cross sections show a **pronounced** dipole peak at 27 MeV, which favours the high-peaked experimental data
- The **FSI** plays a fundamental role in the cross section evaluation

Electrodisintegration

- We have investigated the reliability of direct knock-out hypothesis and the role of **FSI** for both ${}^4\text{He}(e, e'p){}^3\text{H}$ and ${}^4\text{He}(e, e'd)d$ reactions

Outlook

- More quantitative predictions with **realistic** two-body interactions and **three-body** forces
- Application of the method to the study of other exclusive reactions: ${}^6\text{Li}(\gamma, {}^3\text{He}){}^3\text{H}$
- Application of the method to hadronic processes: four-body **phase shifts**