Advantages of low-momentum interactions
based on applications to few-nucleon systems and nuclear matter

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Nuclear Forces from QCD, ECT*, June 21, 2005

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Outline

1) Introduction and motivation
2) RG applied to nuclear interactions
3) Insights from few-nucleon systems
4) Is nuclear matter perturbative with low-momentum interactions?
5) Summary

1) Introduction and motivation

Non-relativistic Hamiltonian with inter-nucleon interactions:

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{3\mathrm{N}}(\Lambda) + V_{4\mathrm{N}}(\Lambda) + \ldots$$

Phenomenological hierarchy: NN > 3N > 4N > …

NN: fit to world 2-body scattering data, $E_{deuteron}$

3N: fit to E_{triton}, selected 3-body scattering data/A>3 spectra

4N, …: neglected, estimates very small for normal densities

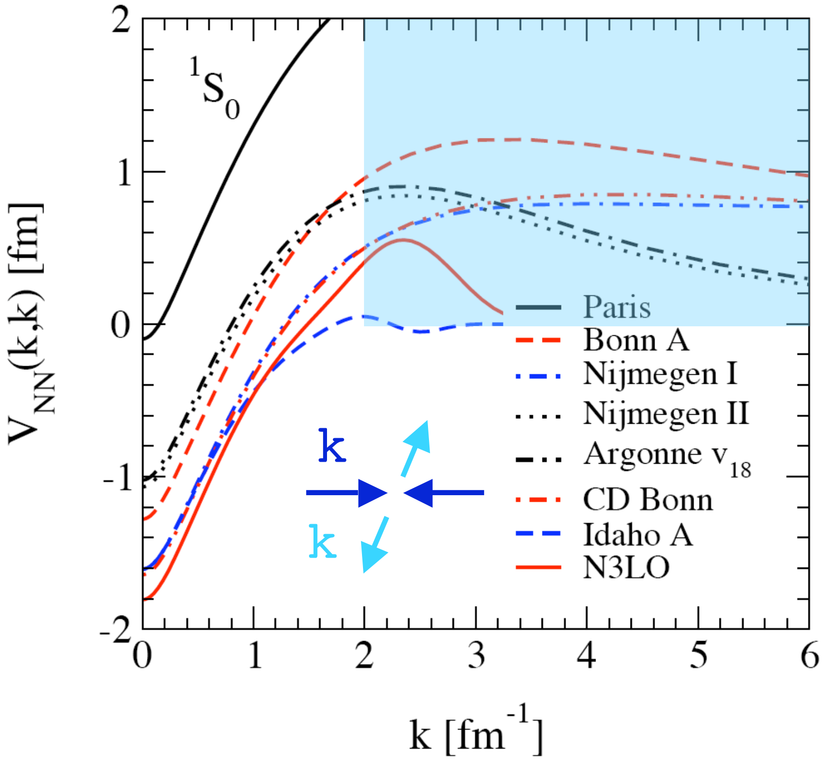
All nuclear forces are **effective** interactions with
cutoff(s) Λ: usually used as fit parameter(s), V_{NN}(600 MeV, 1GeV, …)

Most nuclear structure/matter calculations only with V_{NN}
Results thus incomplete and BIASED towards choice for cutoff Λ

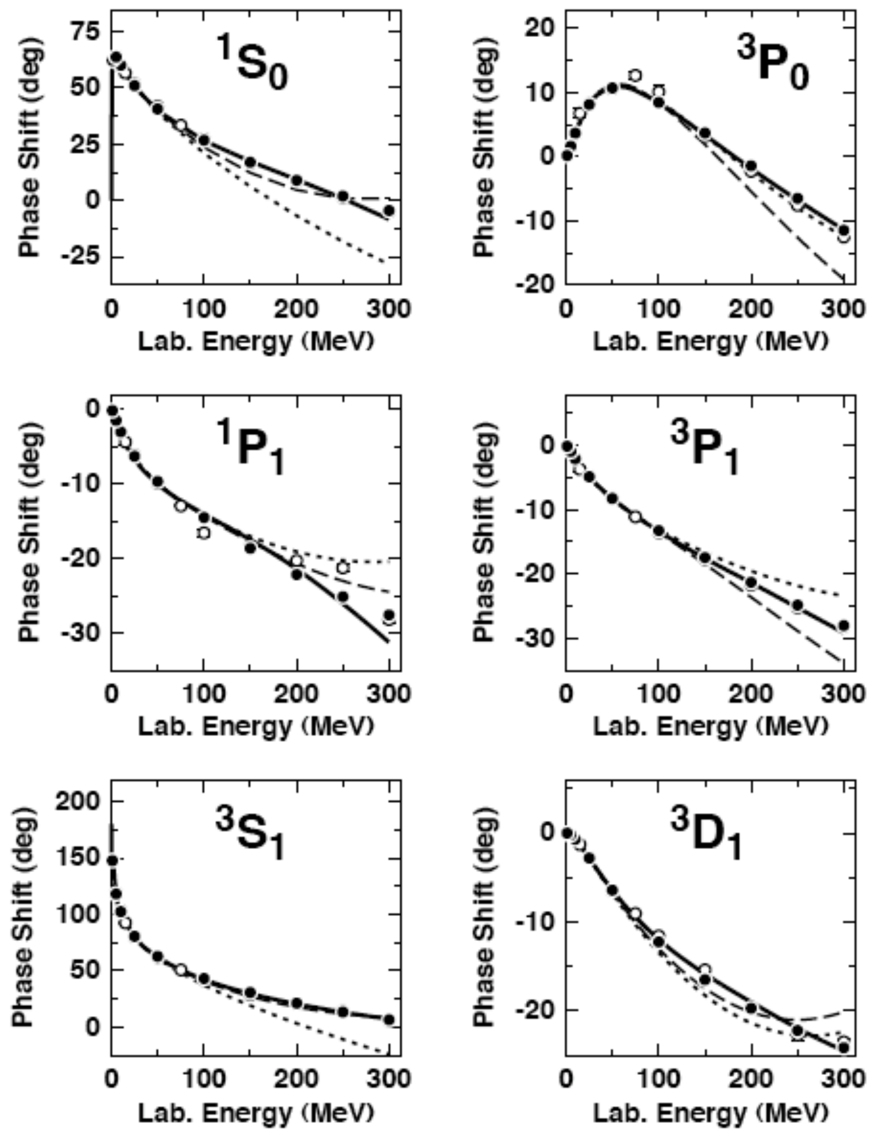
NN interactions well-constrained by NN scattering

Many different NN potentials fit to data below $E_{\mathrm{lab}} \lesssim 350\,\mathrm{MeV}$

(corresponding relative k < 2.1 fm^{-1})



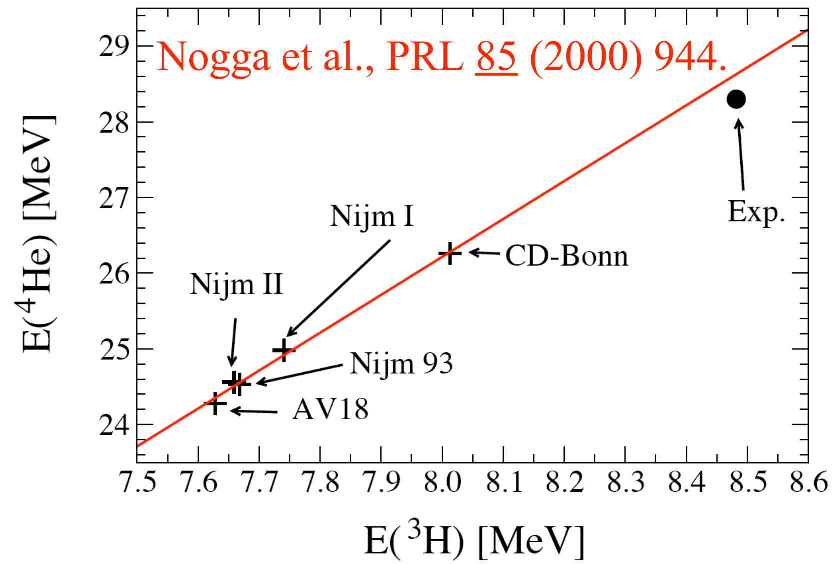
Details not constrained for momenta k > 2.1 fm^{-1}



N3LO, Entem, Machleidt, PR C68 (2003) 041001

Importance of 3N interactions

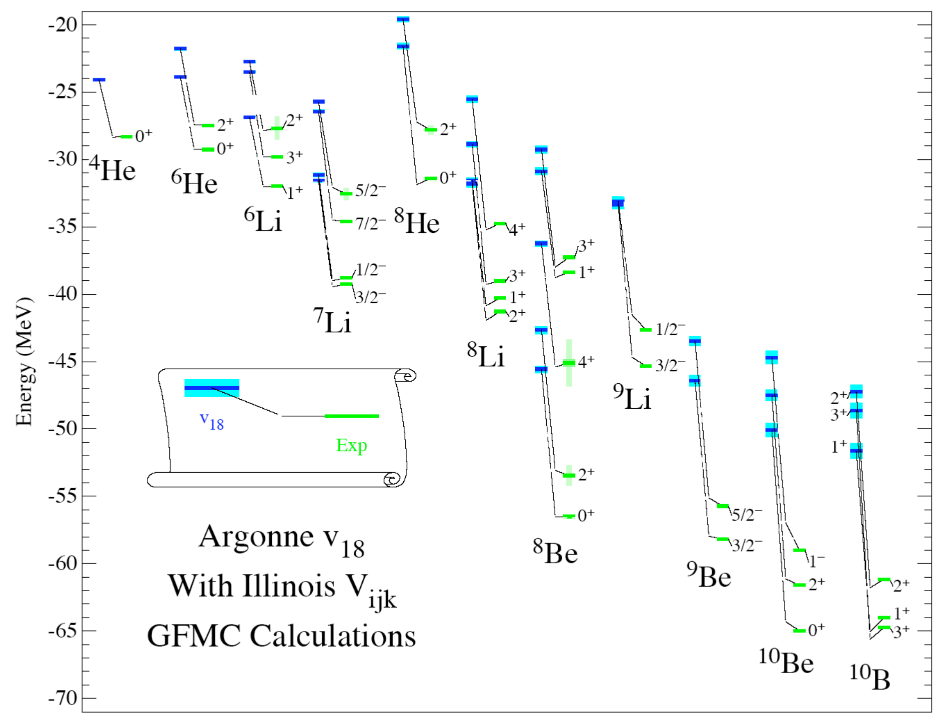
All exact calculations with only NN forces miss A=3,4 nucleon binding energies



Exact GFMC results with only NN force miss A≤10 spectra

Size of 3N depends on V_{NN}

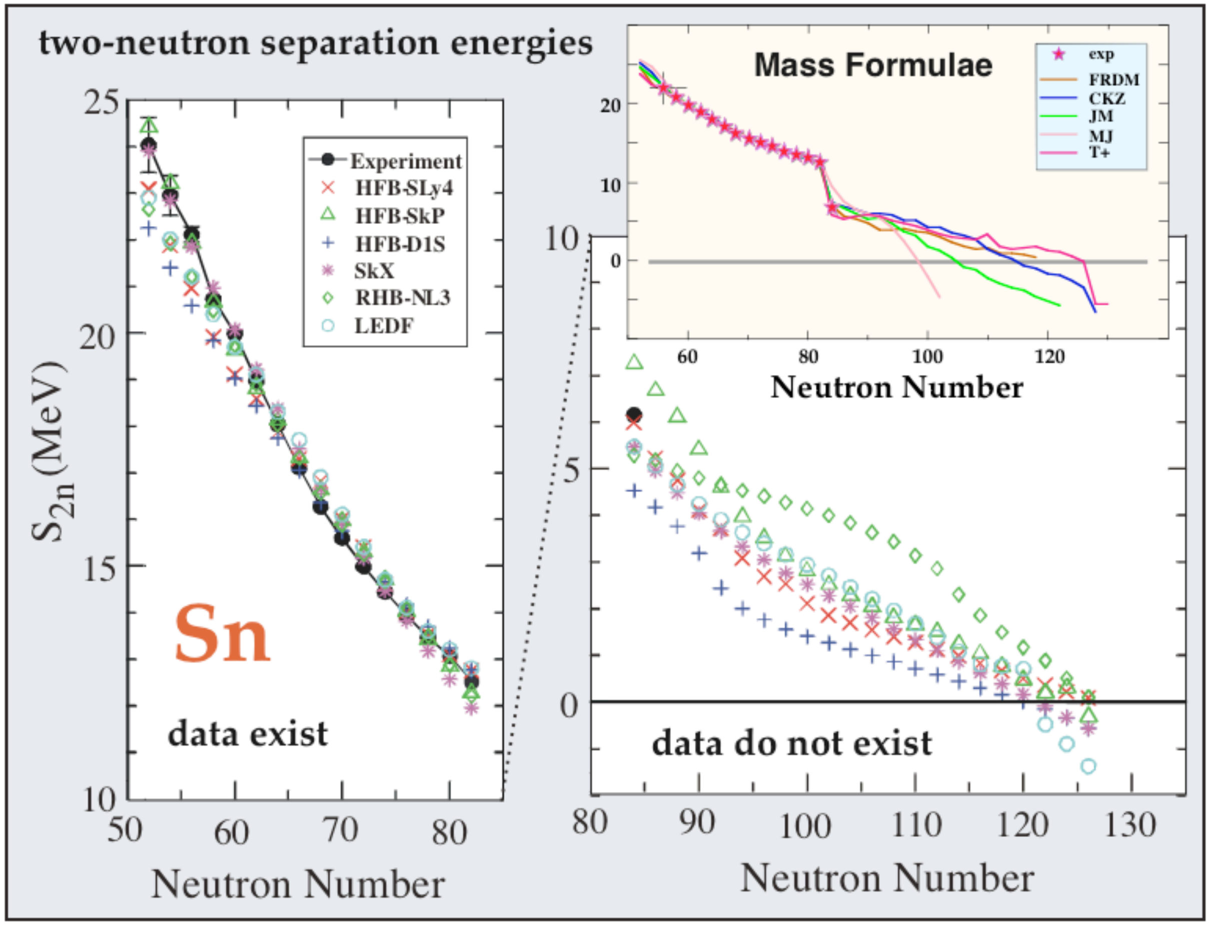
no "true" 3N force: $V_{3N}(\Lambda)$



Pieper, Wiringa, Ann. Rev. Nucl. Part. Sci. 51 (2001) 53.

Other model dependences in nuclear structure

Limited predictive power of energy functionals



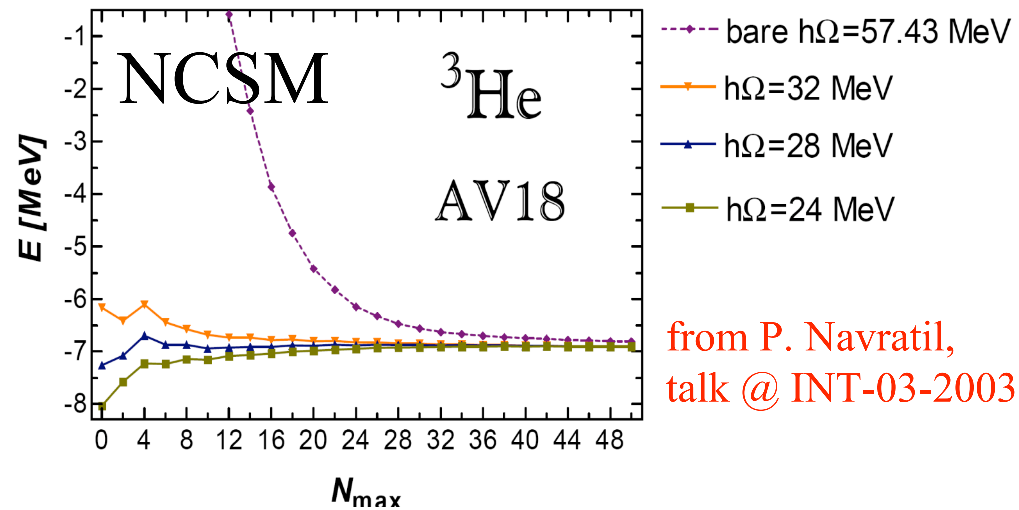
Desire model-independent, microscopic approach with theoretical error estimates for predictions: EFT/RG

Many-body applications with large-cutoff NN interactions

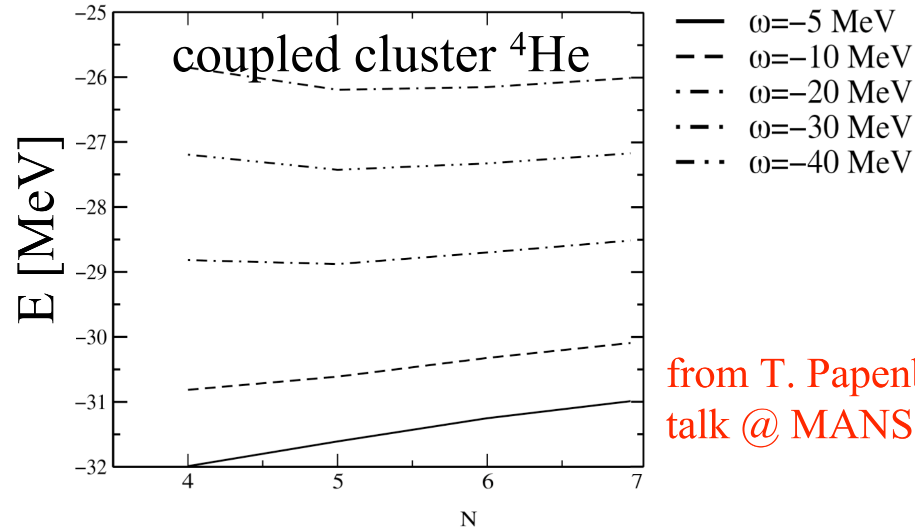
Requires resummation due to slow convergence with HO basis size

Resummation methods only rigorous for light nuclei

Otherwise resummation acquires uncontrolled dep. on “starting-energy” ω (inserted by hand to render ladders convergent)



from P. Navratil, talk @ INT-03-2003



from T. Papenbrock, talk @ MANSC 2004

Desire **low-momentum interaction for many-body applications** where resolution is low and no need for model-dep. high-mom. parts

Will use RG to lower the cutoff in chiral EFT interactions (Λ_0~3 fm^{-1}) and other NN models (Λ_0~5-20 fm^{-1})

RG generates higher-order short-range operators (Λ to Λ_0) necessary to preserve NN observables (NN phase shifts, $E_{deuteron}$)

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One could also re-fit chiral EFT interactions for lower cutoff, but varying the cutoff in Weinberg counting needs further studies

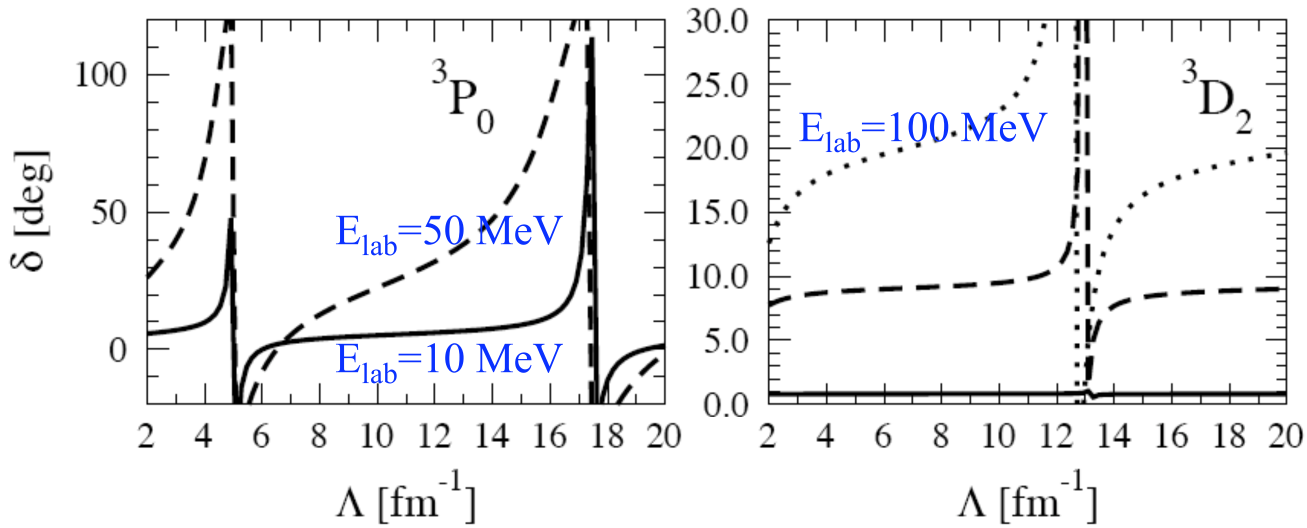
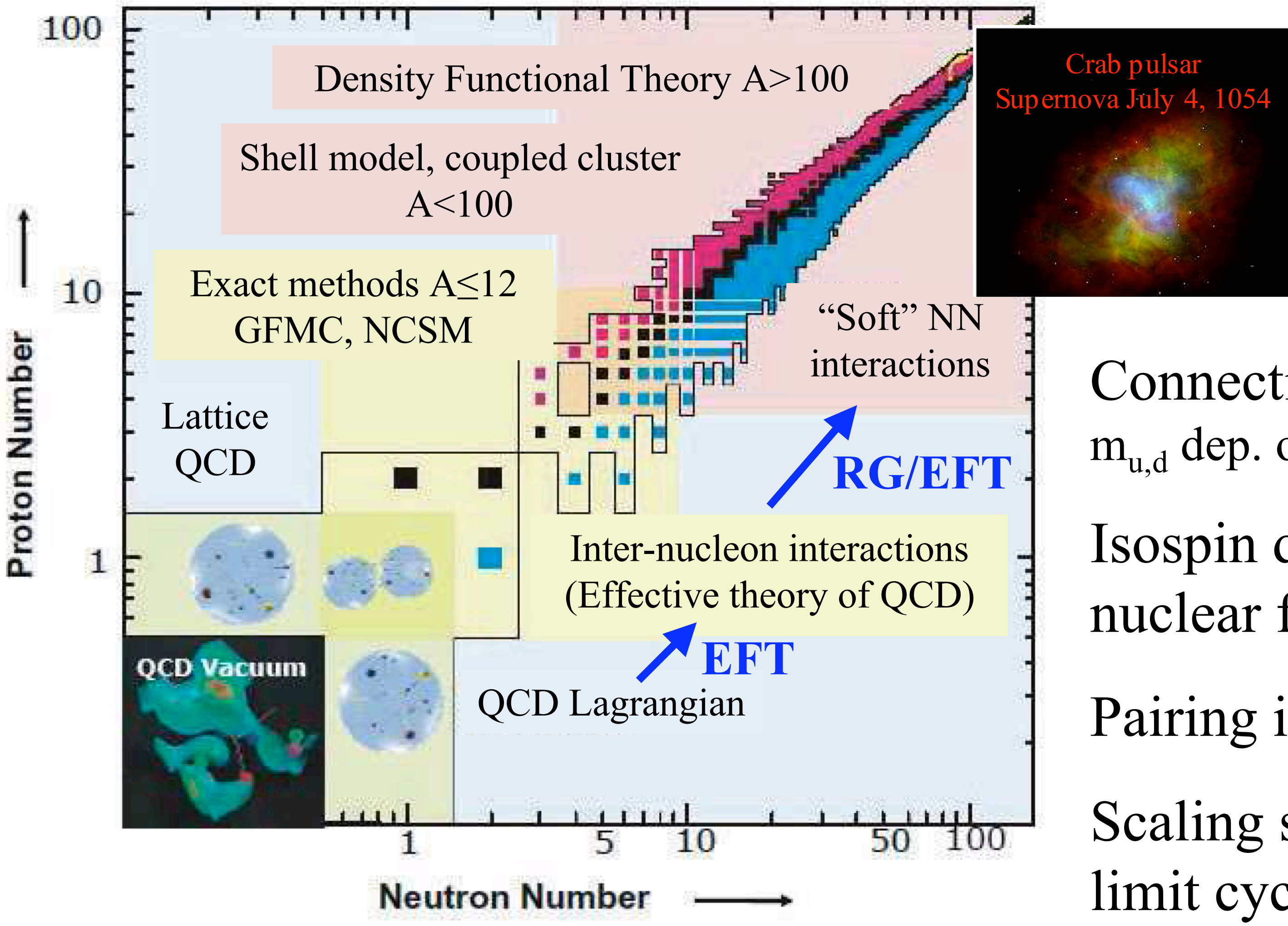


FIG. 9: Cutoff dependence of phase shifts in attractive triplet channels at laboratory energies of 10 MeV (solid line), 50 MeV (dashed line), and 100 MeV (dotted line). from Nogga, Timmermans, van Kolck, nucl-th/0506005.

due to spurious bound states from 1π-exchange tensor force ~ $-1/r^3$

Big picture



adapted from A. Richter @ INPC2004

Connection to QCD

$m_{u,d}$ dep. of shell structure?

Isospin dependence of nuclear forces

Pairing in nuclei

Scaling symmetries, limit cycle physics

…

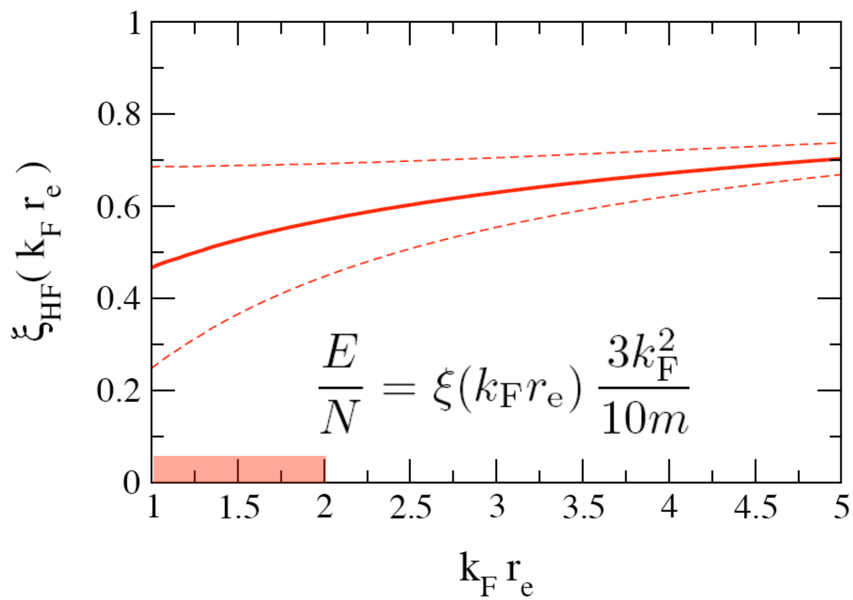
Scaling symmetry of cold neutron matter

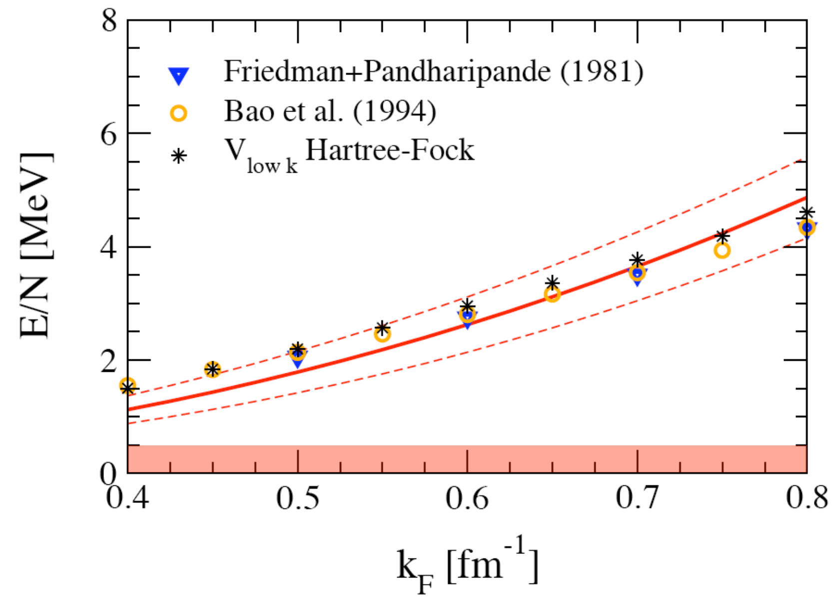
Low-density neutron matter is nearly universal $a_{\rm nn} \; = \; -18.5 \pm 0.3\,{\rm fm}$

Effective range is also appreciable $r_{\rm np} = 2.68 \pm 0.01\,{\rm fm}$

Can calculate equation of state in dibaryon EFT AS, Pethick, nucl-th/0506042.

$$\mathcal{L} \;=\; \psi^\dagger\left(i\partial_0 + \frac{\nabla^2}{2}\right)\psi - d^\dagger\left(i\partial_0 + \frac{\nabla^2}{4} - \Delta\right)d \;-\; g\left(d^\dagger\psi\psi + d\,\psi^\dagger\psi^\dagger\right)$$





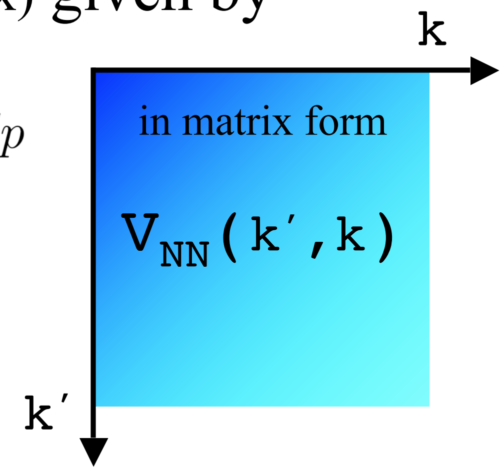
Cold low-density neutron matter is close to unitary limit (ξ≈0.44)

Conventional many-body results within errors of simple LO EFT

2) RG applied to NN interactions

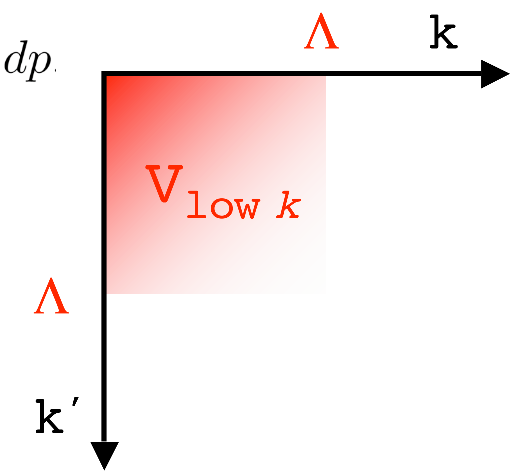
In momentum space, scattering amplitude (T matrix) given by

$$T(k',k;k^2) = V_{\rm NN}(k',k) + \frac{2}{\pi}\,\mathcal{P}\int_0^{\infty} \frac{V_{\rm NN}(k',p)\,T(p,k;k^2)}{k^2-p^2}\,p^2 dp$$



Integrating out high-momentum modes leads to effective interaction $V_{{\rm low}\,k}$ (which reproduces the low-momentum T matrix)

$$T(k',k;k^2) = V_{\rm low\,k}(k',k) + \frac{2}{\pi}\,\mathcal{P}\int_0^{\Lambda} \frac{V_{\rm low\,k}(k',p)\,T(p,k;k^2)}{k^2-p^2}\,p^2 dp$$



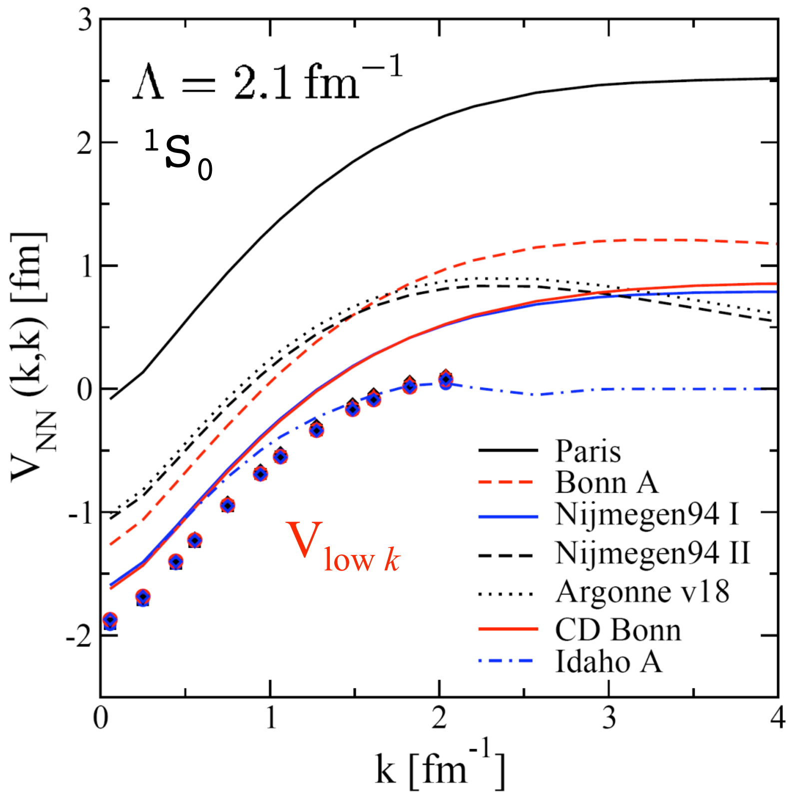
Changes of effective interaction with cutoff Λ are given by RG equation

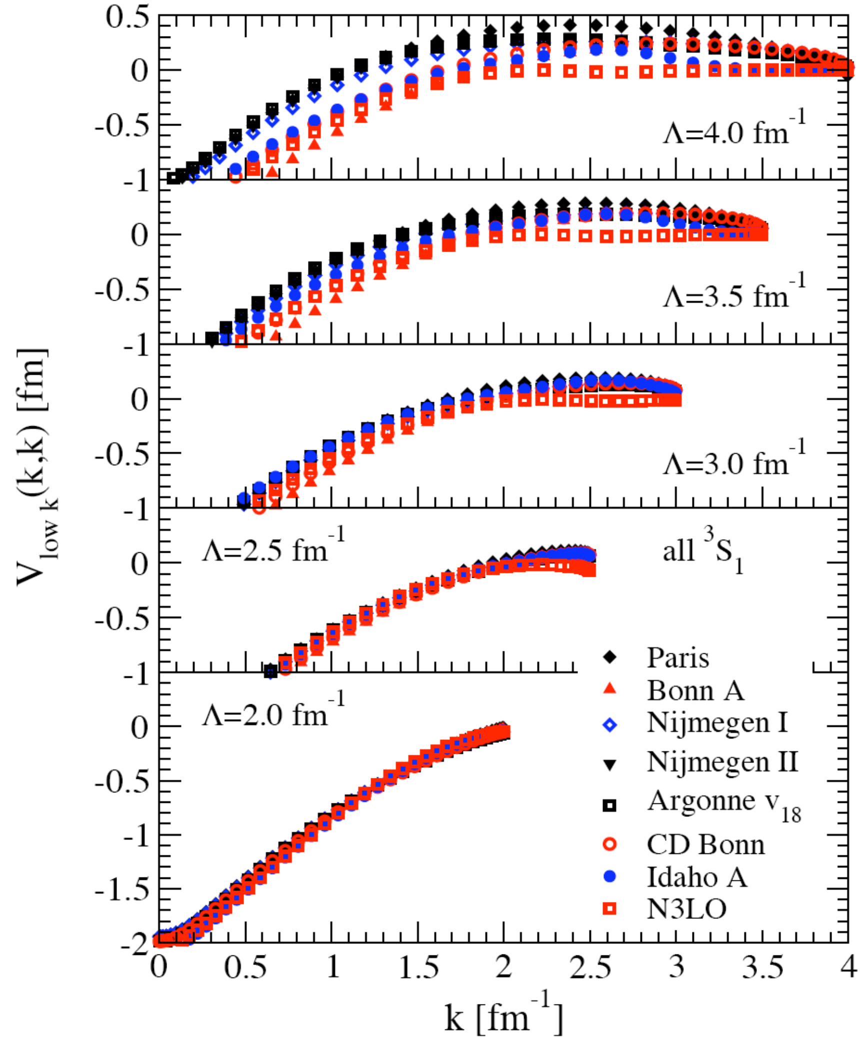
$$\frac{d}{d\Lambda} V_{\rm low\,k}(k',k) = \frac{2}{\pi}\,\frac{V_{\rm low\,k}(k',\Lambda)\,T(\Lambda,k;\Lambda^2)}{1-(k/\Lambda)^2}$$

Technically equivalent to Lee-Suzuki trafo in momentum space

How cockroaches and dinosaurs run with cutoff…

Starting from any NN interaction:
Solutions to the RG eqn. evolve
to a "universal" interaction $V_{\text{low }k}$
for cutoffs below $\Lambda \lesssim 2.1\,\text{fm}^{-1}$

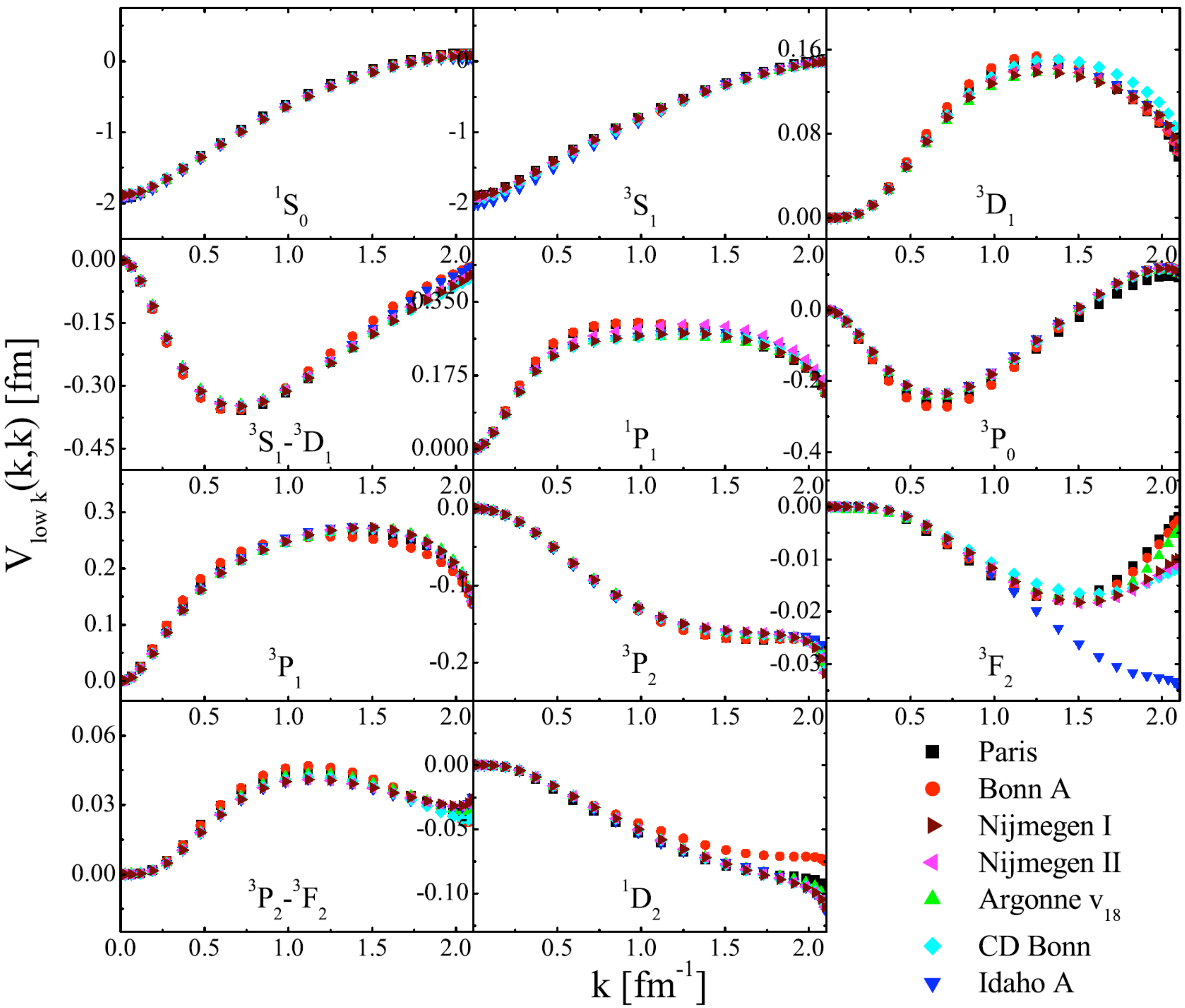




Bogner, Kuo, AS, Phys. Rep. 386 (2003) 1.

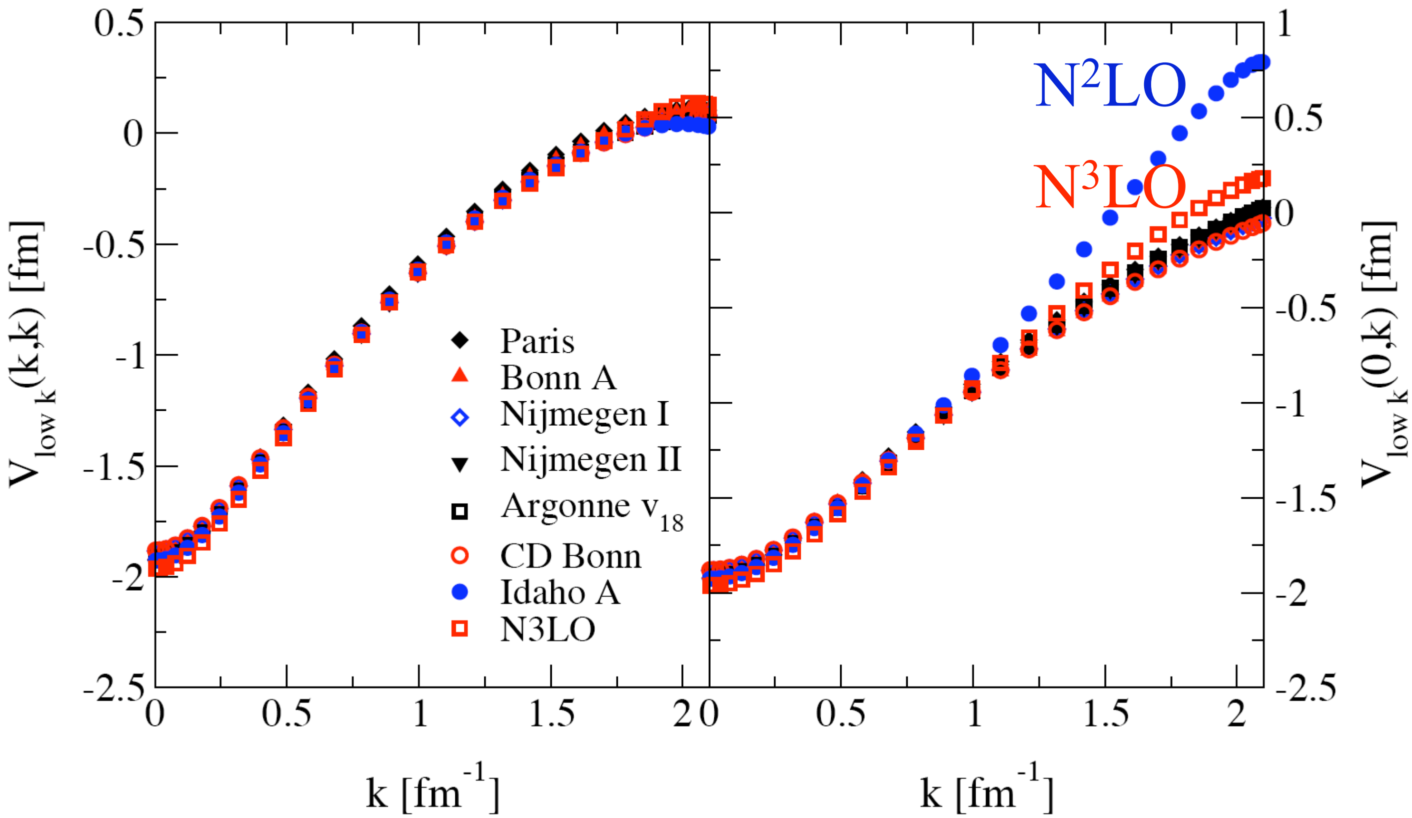
Collapse in all partial waves

due to same long-distance (π) physics and phase shift equivalence



small differences related to spread in phase shift fit (Idaho misses $^3F_2$)

Collapse of off-shell matrix elements as well



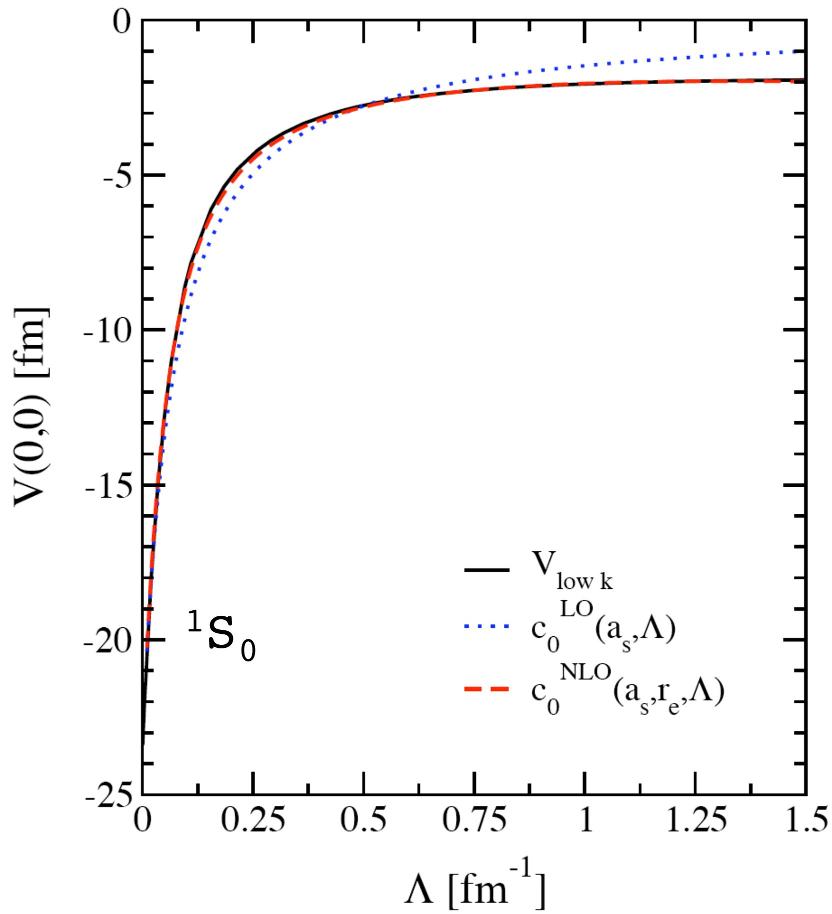
"Universal" result indicates that $V_{\mathrm{low}\ k}$ effectively parameterizes a high-order EFT interaction

Simplest EFT connection

Running of $V_{\mathrm{low}\ k}(0,0)$ vs. EFT contact interaction c_0

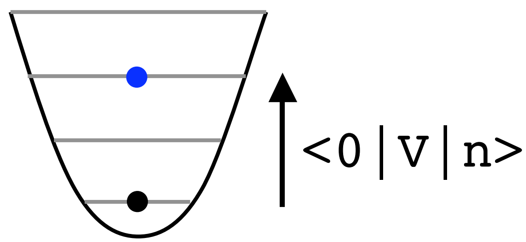
LO: c_0 fixed by scattering length a_s + RG invariance

NLO: + effective range r_e



Lower cutoffs are advantageous for nuclear structure

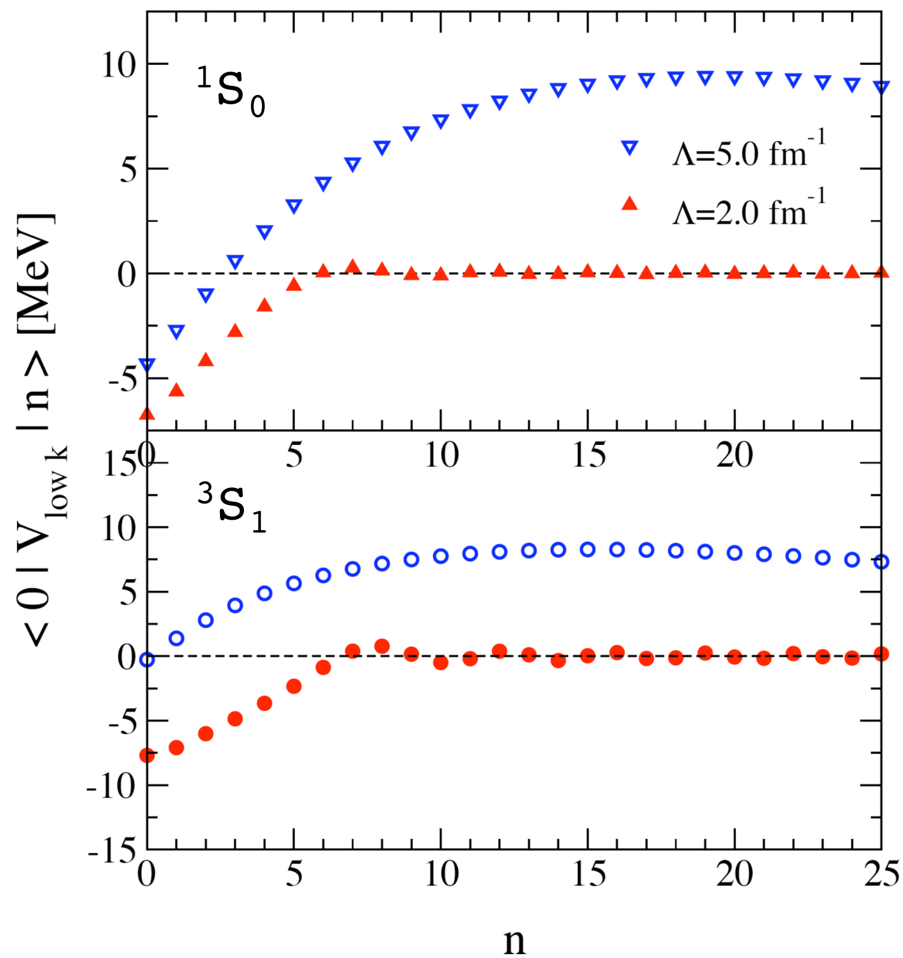
Low-momentum interactions are tractable in a HO basis



Connection to nuclei for $V_{\text{low }k}$ simple, without resummations

Convergence will not require N~50 shells

Exciting results from coupled cluster method D. Dean shell model A. Zuker



3) Insights from few-nucleon systems

By construction: For $V_{\text{low } k}$ all NN observables are cutoff-independent and reproduced with same $\chi^2 \approx 1$

Nucleons are composite objects: All NN potentials have corresponding 3N, 4N,… forces

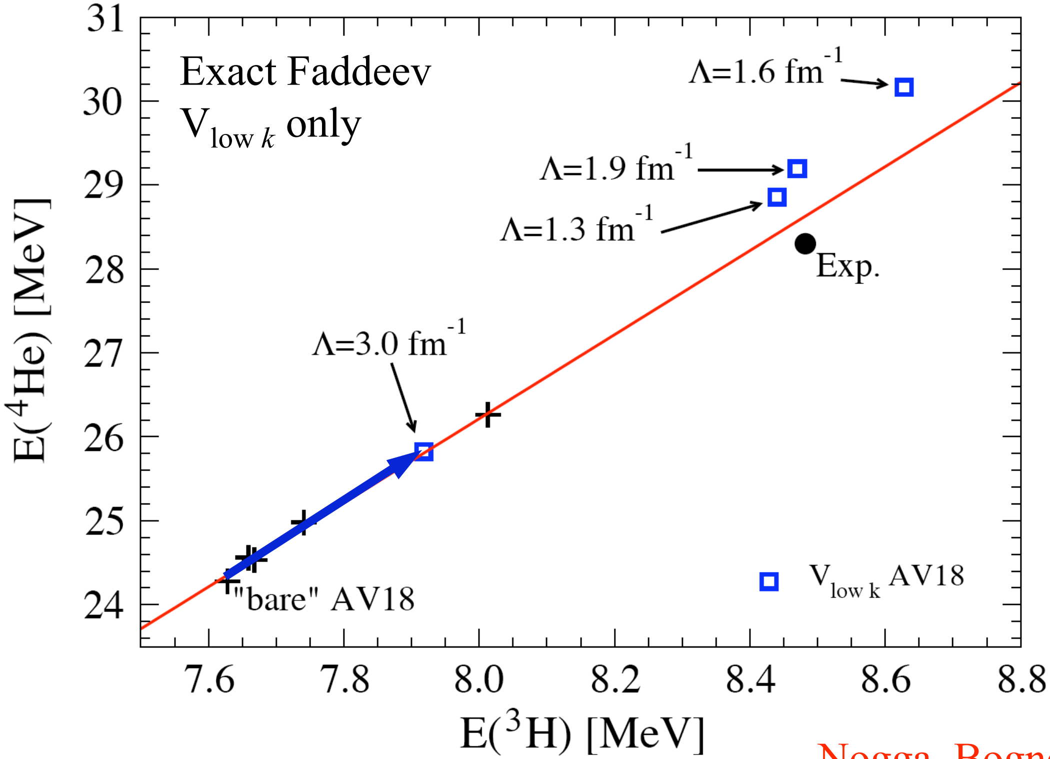
If one omits the many-body forces, calculations of low-energy 3N, 4N,… observables will be cutoff-dependent

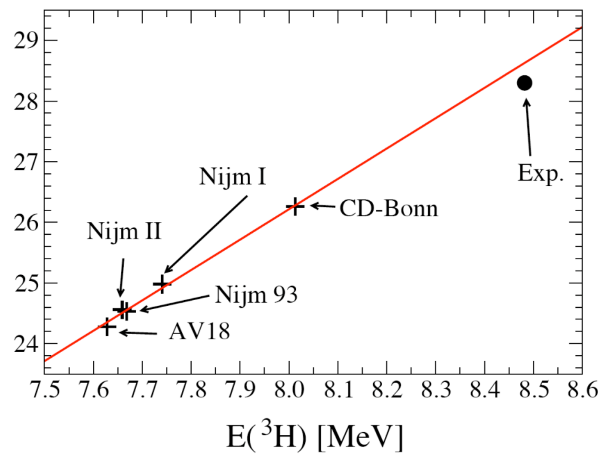
Varying the cutoff gives an estimate for effects of omitted 3N, 4N,… forces Nogga, Bogner, AS, PR C70 (2004) 061002(R).

Important for predictive extrapolations to n/p-rich systems and astrophysical densities/temperatures

Cutoff dependence due to omitted 3N forces

A=3,4 binding energies





Nogga, Bogner, AS, PR C70 (2004) 061002(R).

Cutoff dependence shows that three-body forces are inevitable

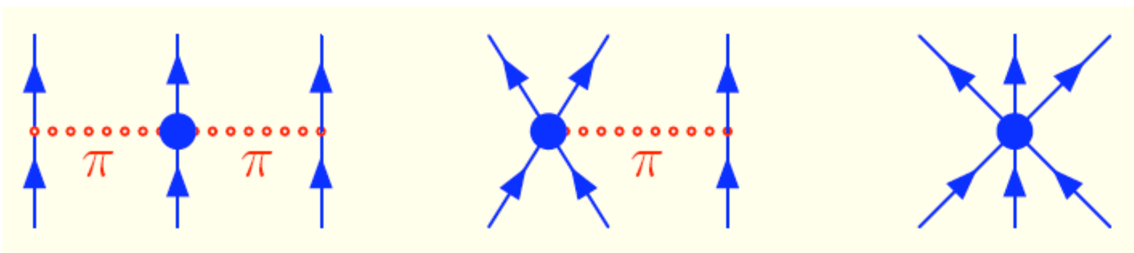
Cutoff dependence is almost linear (explains Tjon line), with 3N contributions smaller for low-momentum interactions

Low-momentum 3N forces with only pion exchanges

organized by chiral Effective Field Theory van Kolck, PR C49 (1994) 2932; Epelbaum et al., PR C66 (2002) 064001.

operator structure of any 3N force collapses at low energies to

long (2π) intermediate (π) short-range



c-terms D-term E-term

chiral counting:
$(Q/\Lambda)^3 \sim (m_\pi/\Lambda)^3$
rel. to 2N force

c_i from NN PWA with chiral 2π exchange (c_3, c_4 important for nuclear structure)

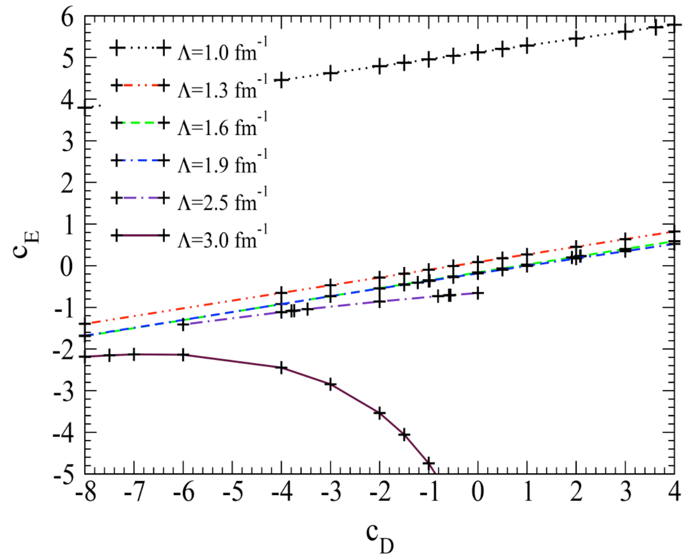
approximate 3N evolution by fitting low-energy D,E constants in leading-order chiral 3N interaction to $V_{\mathrm{low}\,k}$

Two couplings fit to ^{3}H and ^{4}He

Linear dependences in fits, consistent with perturbative 3N contributions

$$E(^3H) = \langle T + V_{\mathrm{low}\,k} + V_c \rangle + c_D \langle O_D \rangle + c_E \langle O_E \rangle$$

3N forces become perturbative for cutoffs $\Lambda \lesssim 2\,\mathrm{fm}^{-1}$



nonperturbative at larger cutoffs, with expectation values beyond expected $(Q/\Lambda)^3 \sim (m_\pi/\Lambda)^3 \approx 0.1$ of NN contribution

	^{3}H					^{4}He					max	^{4}He
Λ	T	$V_{\mathrm{low}\,k}$	c-terms	D-term	E-term	T	$V_{\mathrm{low}\,k}$	c-terms	D-term	E-term	$\lvert V_{3N}/V_{\mathrm{low}\,k}\rvert$	k_{rms}
1.0	21.06	−28.62	0.02	0.11	−1.06	38.11	−62.18	0.10	0.54	−4.87	0.08	0.55
1.3	25.71	−34.14	0.01	1.39	−1.46	50.14	−78.86	0.19	8.08	−7.83	0.10	0.63
1.6	28.45	−37.04	−0.11	0.55	−0.32	57.01	−86.82	−0.14	3.61	−1.94	0.04	0.67
1.9	30.25	−38.66	−0.48	−0.50	0.90	60.84	−89.50	−1.83	−3.48	5.68	0.06	0.70
2.5(a)	33.30	−40.94	−2.22	−0.11	1.49	67.56	−90.97	−11.06	−0.41	6.62	0.12	0.74
2.5(b)	33.51	−41.29	−2.26	−1.42	2.97	68.03	−92.86	−11.22	−8.67	16.45	0.18	0.74
3.0($*$)	36.98	−43.91	−4.49	−0.73	3.67	78.77	−99.03	−22.82	−2.63	16.95	0.23	0.80

$\approx m_\pi$

4) Is nuclear matter perturbative?

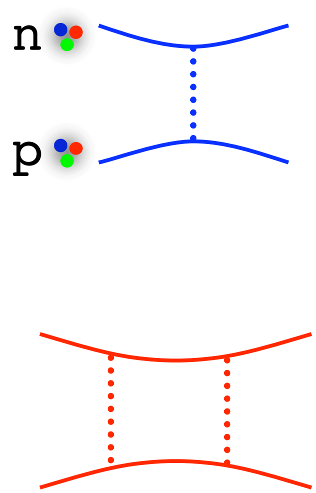
Sources of nonperturbative behavior:

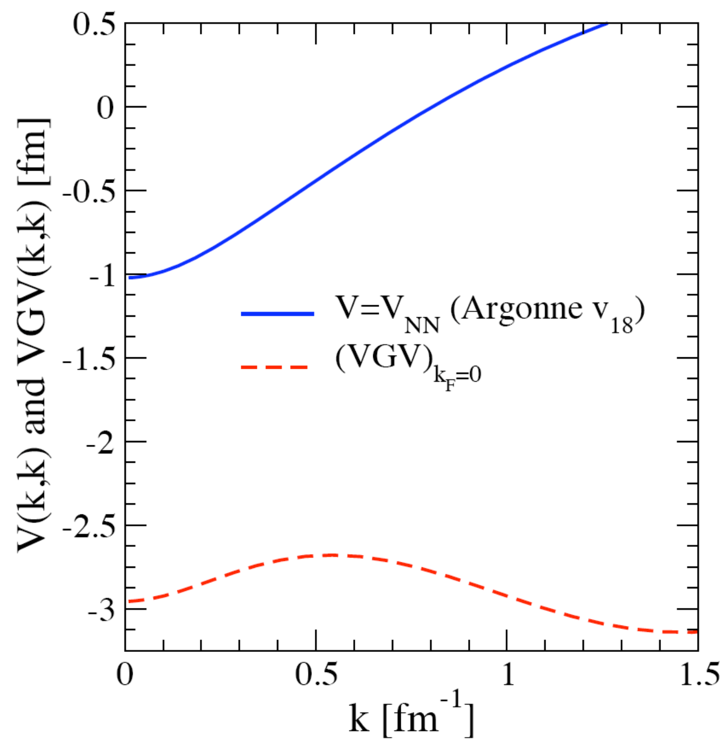
1. Iterated interactions at low momenta are nonperturbative due to near bound states $a_{^1S_0} = -23.7\,\mathrm{fm} \quad a_{^3S_1} = 5.4\,\mathrm{fm}$

2. Cores scatter strongly to high-momentum states (overwhelms 1.)

For interactions with cores:

second-order contributions larger than V_{NN}





NN scattering in free-space vs. nuclear matter

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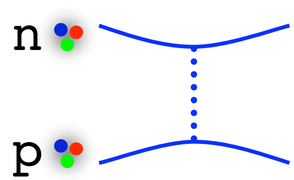
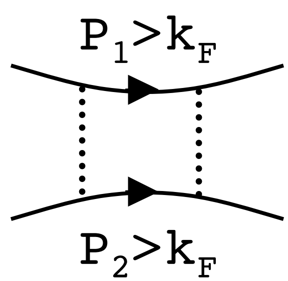
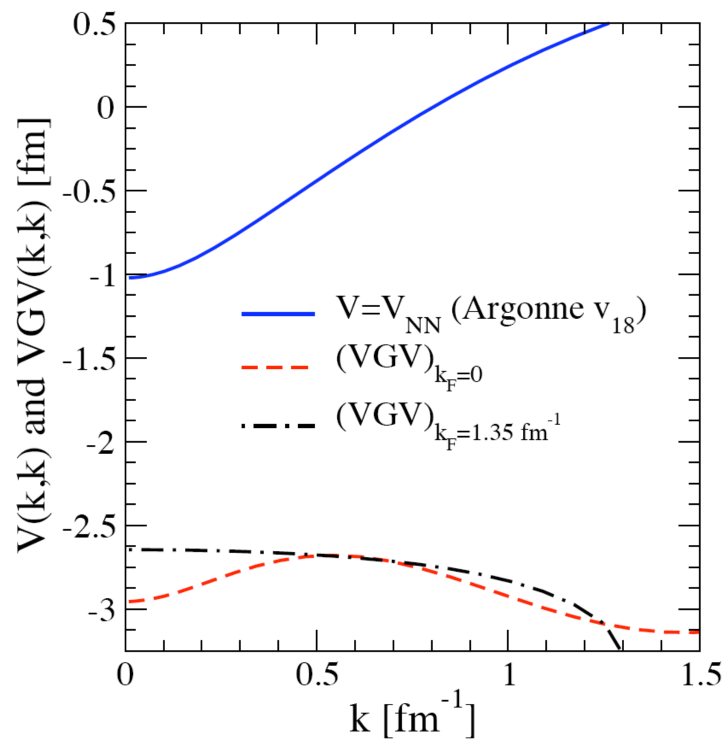


even with Pauli blocking

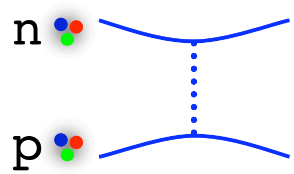


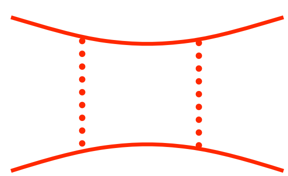


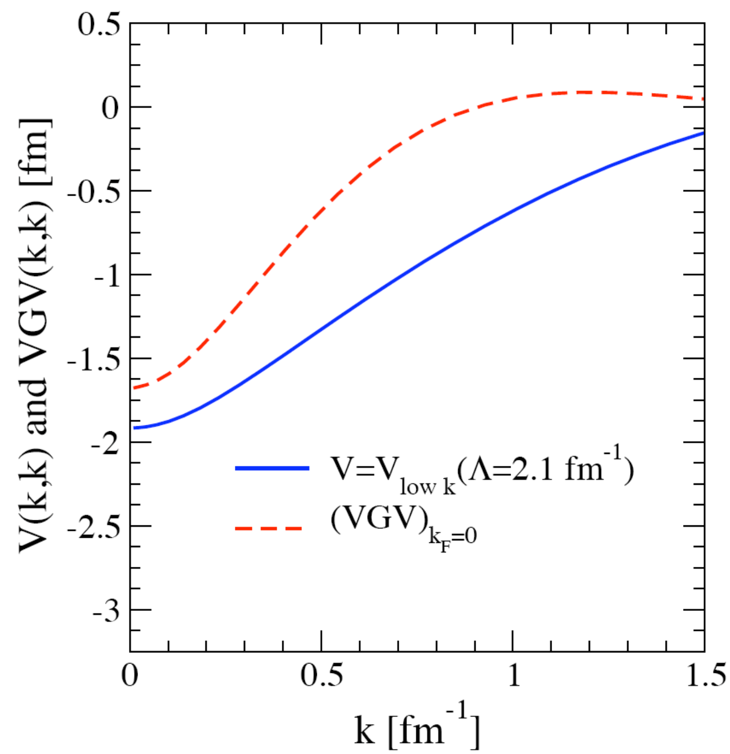
NN scattering in free-space vs. nuclear matter

For low-momentum interactions:

free-space nonperturbative due to near-bound state







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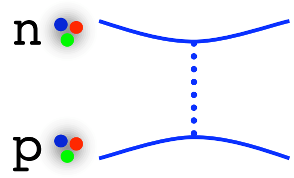
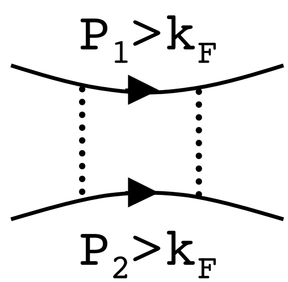
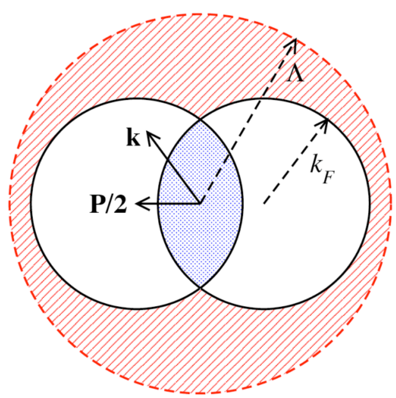
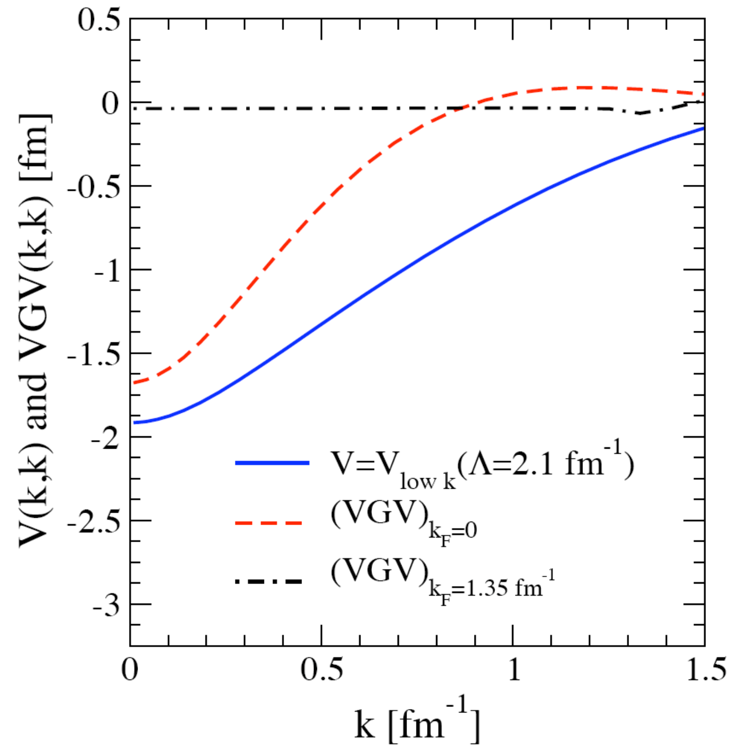


small in medium due to occupied Fermi sea

restricts phase space for two particles







mom. cutoff $k < 2.1\ \mathrm{fm}^{-1}$
= density cutoff $\rho < 4\ \rho_0$

Weinberg eigenvalues

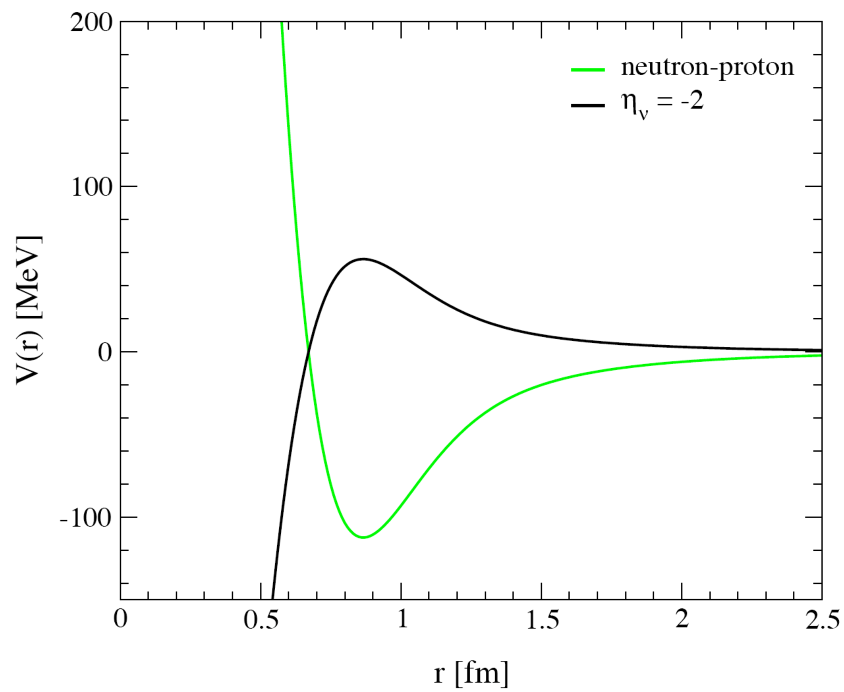
rigorously show that in-medium Born series is perturbative for $V_{\text{low }k}$ at sufficient densities Bogner, AS, Furnstahl, Nogga, nucl-th/0504043.

study spectrum of $G_0(z)V\,|\Psi_\nu(z)\rangle = \eta_\nu(z)\,|\Psi_\nu(z)\rangle$

i.e., convergence of $T(z)\,|\Psi_\nu(z)\rangle = (1+\eta_\nu(z)+\eta_\nu(z)^2+\ldots)\,V\,|\Psi_\nu(z)\rangle$

write as Schroedinger eqn. $\left(H_0 + \dfrac{1}{\eta_\nu(z)}V\right)|\Psi_\nu(z)\rangle = z\,|\Psi_\nu(z)\rangle$

large-cutoff interactions
have at least one large $\eta_\nu < 0$



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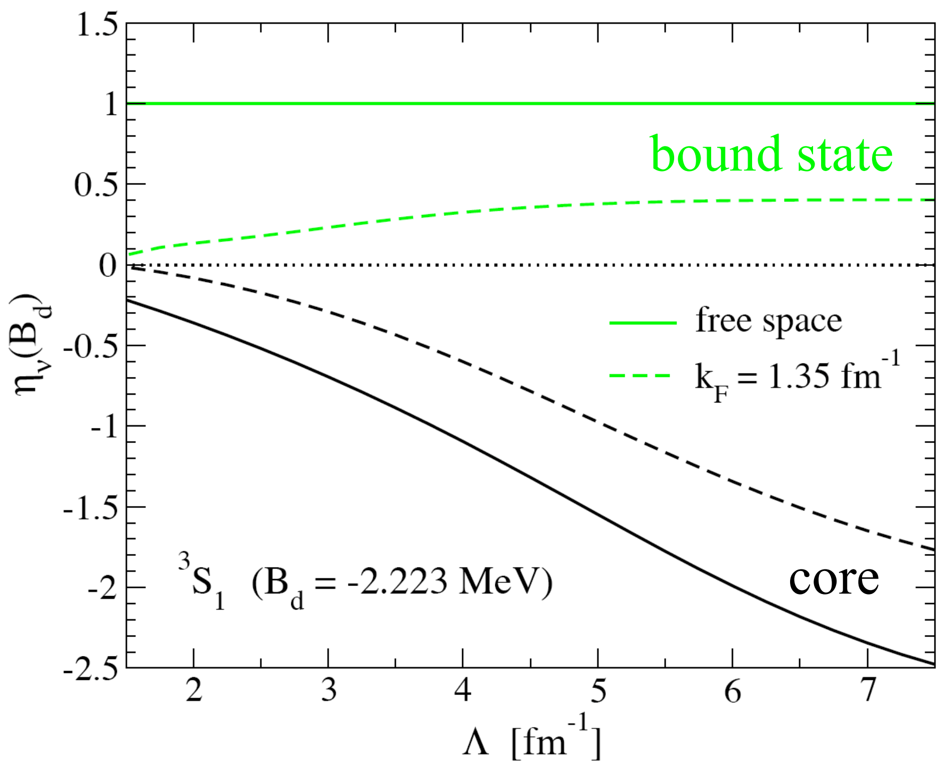
write as Schroedinger eqn.

$$\left(H_0 + \frac{1}{\eta_\nu(z)}V\right)|\Psi_\nu(z)\rangle = z\,|\Psi_\nu(z)\rangle$$

repulsive core eigenvalue small for lower cutoffs

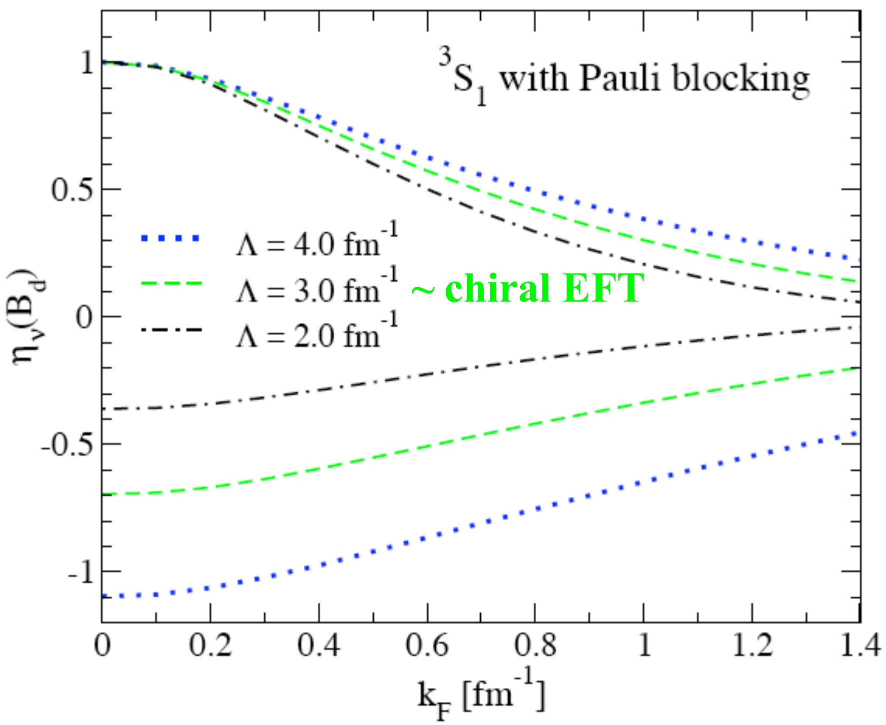
deuteron eigenvalue small in-medium (fine-tuning elim.)

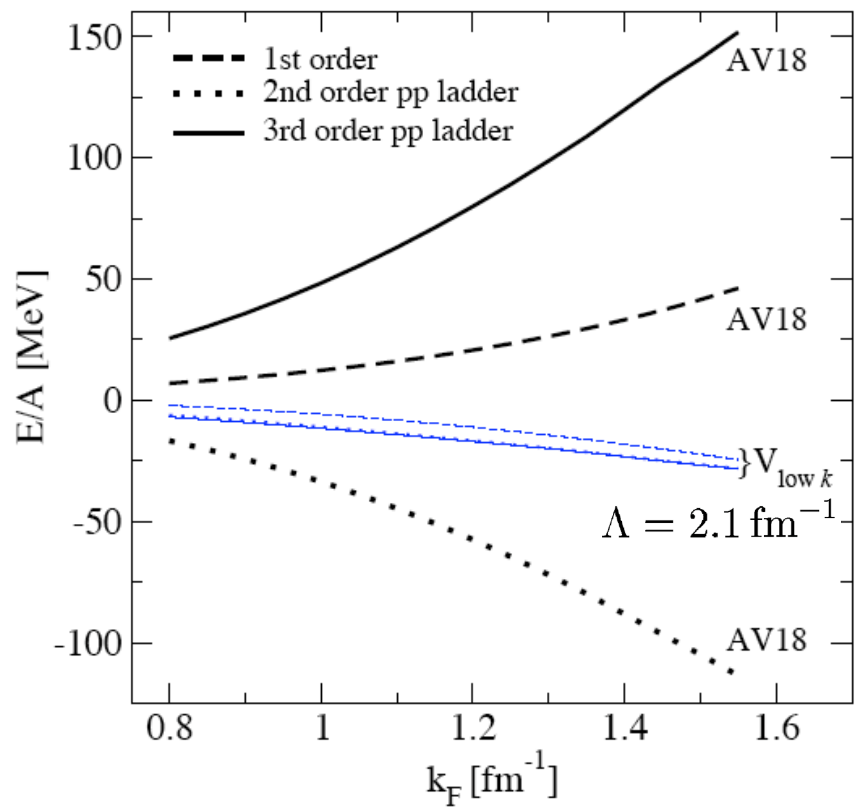


Perturbative two-body ladders

Weinberg analysis anticipates rapidly converging particle-particle contributions to nuclear matter energy for low-mom. interactions

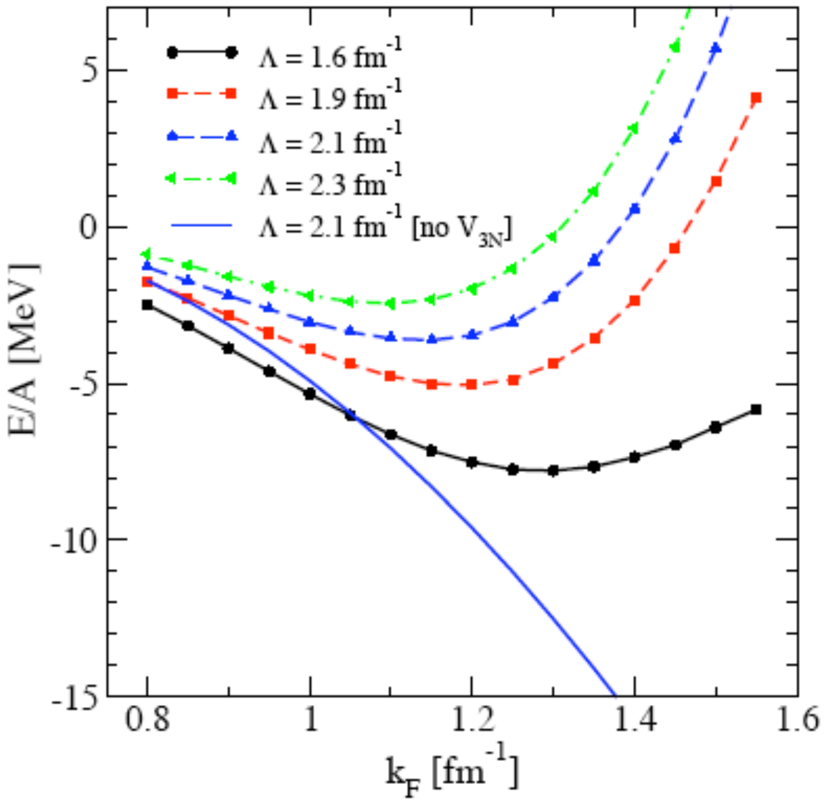
Bogner, AS, Furnstahl, Nogga, nucl-th/0504043.





Perturbation theory in place of ladder resummations!

Nuclear matter with NN and 3N



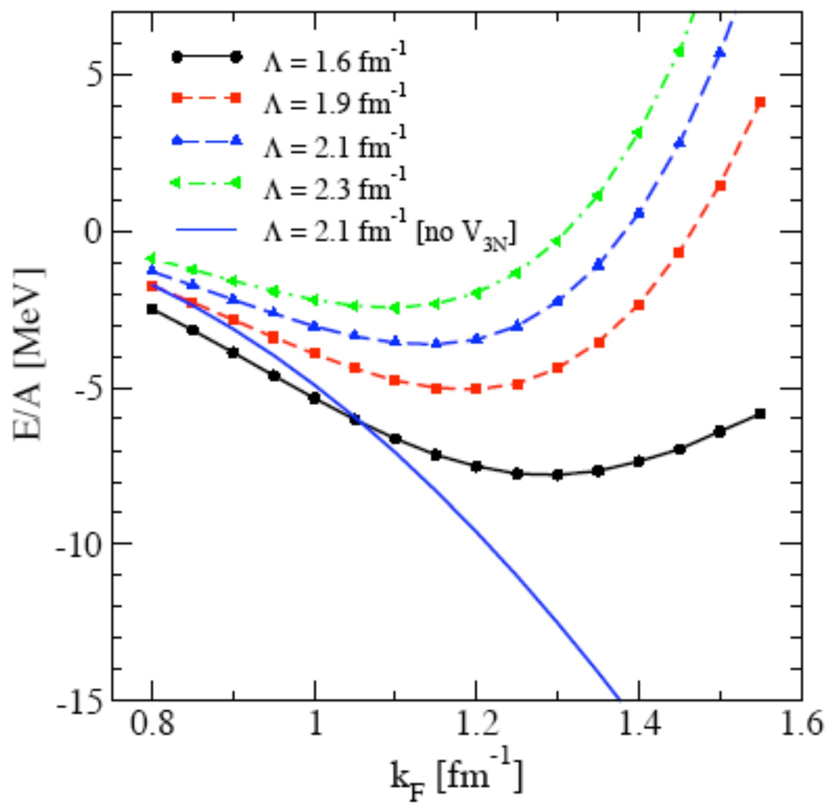
Hartree-Fock

bound, saturation from 3N force

(2π-exchange dominates, no adjustments)

Bogner, AS, Furnstahl, Nogga, nucl-th/0504043.

Nuclear matter with NN and 3N

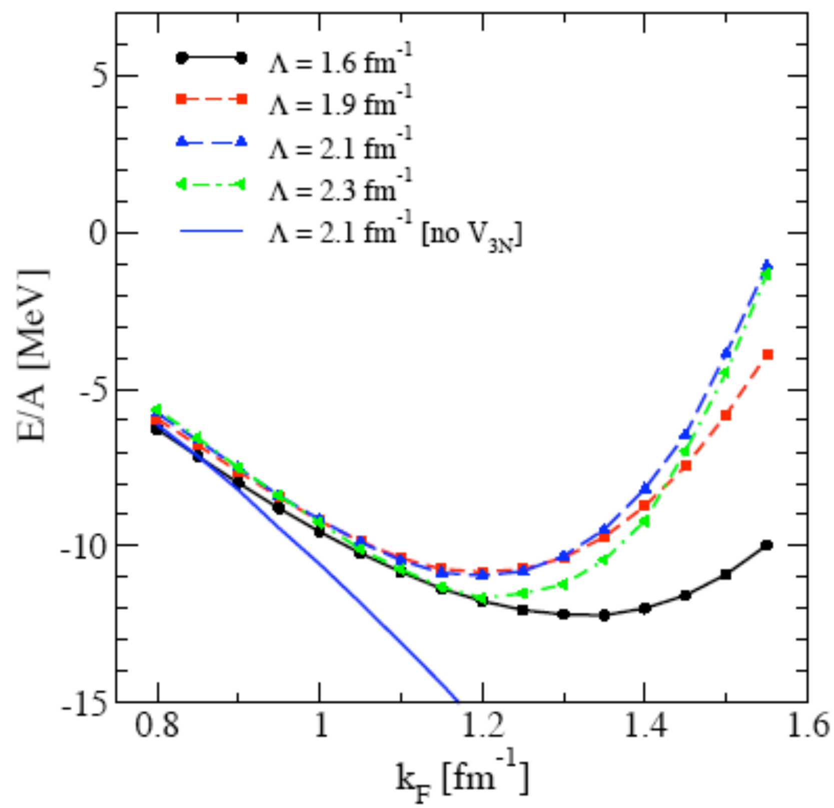


Hartree-Fock

bound, saturation from 3N force

(2π-exchange dominates, no adjustments)

Bogner, AS, Furnstahl, Nogga, nucl-th/0504043.



Hartree-Fock + 2nd-order

(approx. 2nd-order 3N treatment, continuous spectrum with m*)

cutoff dep. strongly reduced

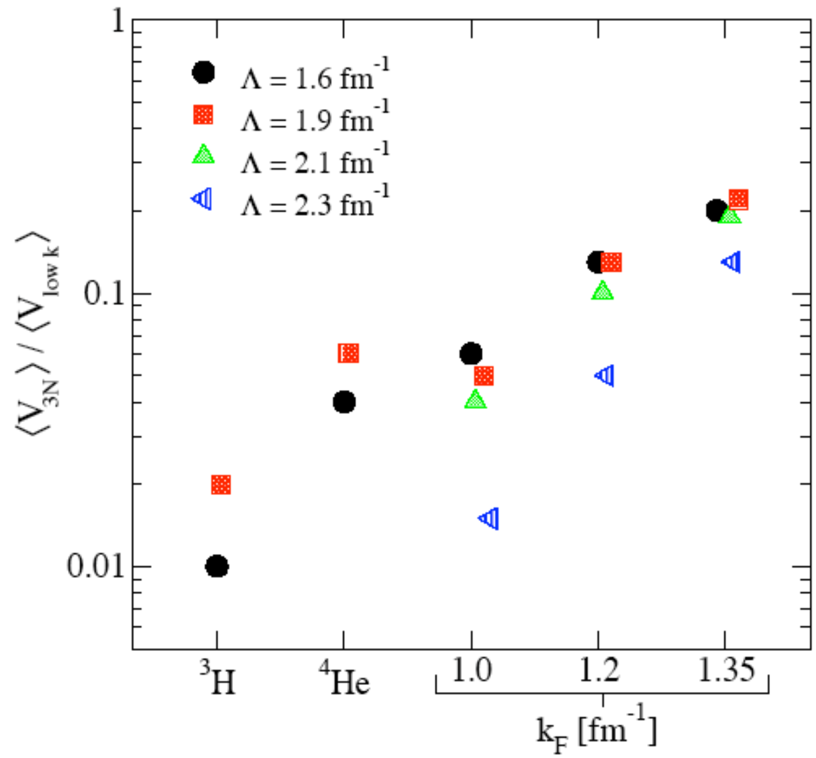
(preliminary 3rd order < 1 MeV)

3N force remains natural in nuclear matter

3N force drives saturation but expectation values not unnaturally large

consistent with chiral EFT scaling $\langle V_{3N} \rangle \sim (Q/\Lambda)^3 \langle V_{NN} \rangle$

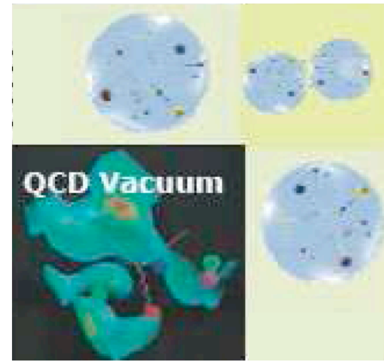
$V_{\mathrm{low}\,k}$ expectation values change by ≈ 0.5 MeV after inclusion of 3N interaction



		Hartree-Fock					Hartree-Fock + dominant second order				
k_F	Λ	T	$V_{\mathrm{low}\,k}$	V_c	V_D	V_E	T	$V_{\mathrm{low}\,k}$	V_c	V_D	V_E
1.2	1.6	17.92	−31.47	5.37	1.31	−0.64	20.86	−37.66	4.59	1.03	−0.65
	1.9	17.92	−28.95	5.61	−0.81	1.18	21.80	−37.38	3.99	−0.50	1.28
	2.1	17.92	−27.51	5.67	−1.37	1.84	22.87	−37.53	2.27	−0.37	1.82
	2.3	17.92	−26.13	5.70	−1.86	2.42	24.32	−37.95	−0.38	0.51	1.78

Nuclear structure connection to QCD

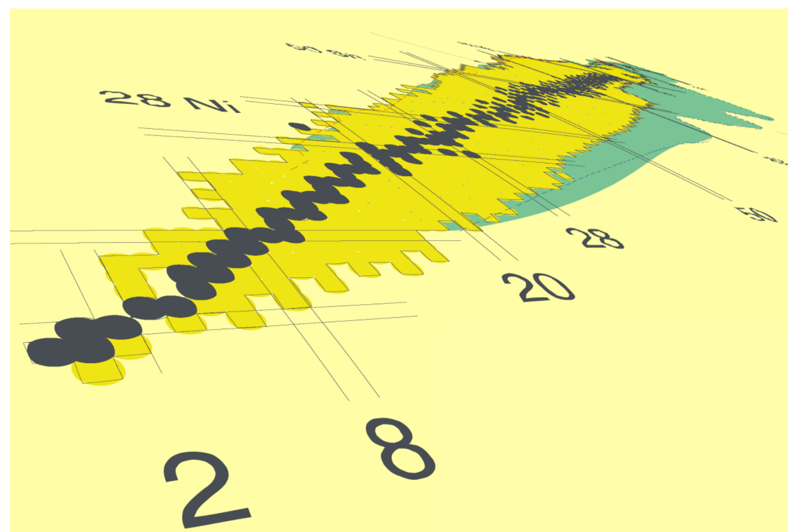
Lattice QCD determination
of low-energy parameters



Chiral EFT interactions with consistent
many-body forces and operators

RG evolution to lower cutoffs
for many-body applications

possible now: run NN and fit 3N



5) Summary - Exciting times in nuclear physics

1. RG removes model dependences from nuclear forces, results in "universal" low-momentum interaction

2. Low-momentum 3N forces are perturbative in this regime
   Very first 3N calculations for A>12 nuclei Dean, Papenbrock, AS, in prep.

3. $V_{\text{low } k}$ tractable with promising results for nuclear structure

4. Nuclear matter seems perturbative with low-mom. interactions
   Detailed calculations under way Bogner, Furnstahl, Nogga, AS, in prep.

5. Cutoff variation estimates errors + completeness of calculations
   Important for extrapolations to drip-lines, ISAC@TRIUMF, FAIR@GSI, RIA

6. Low-mom. interactions are superior for many-body applications
   To date: No microscopic many-body calc. for A>12 directly from NN (+3N)

7. Important advances in nuclear physics, some dinosaur wisdom is very model-dependent