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The role of the pion pair term in the theory of the weak axial meson exchange currents

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Abstract

The structure of the **weak axial pion exchange current** is discussed in various models.

It is shown how the interplay of the **chiral invariance** and the **double counting** problem restricts uniquely the form of the pion potential current, in the case when the nuclear dynamics is described by **the Schroedinger equation** with static nucleon-nucleon potential.

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Introduction

The weak axial pion potential current and the nuclear PCAC

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Results and Conclusions

INTRODUCTION

The semileptonic weak nuclear interaction

$$^3H \rightarrow {}^3He + e^- + \bar{\nu} \, ,$$

$$\mu^- + {}^3He \rightarrow {}^3H + \nu_\mu \, ,$$

$$\mu^- + d \rightarrow n + n + \nu_\mu \, ,$$

Congleton-T, PRC 53, 957 (1996)

$$\Gamma_0 = 1502 \pm 32\ \mathrm{s}^{-1}.$$

We take the range $f^2_{\pi N \Delta}/4\pi = 0.23$–$0.36$ reflecting the various models: there is a large uncertainty in the value of this parameter.

$$\frac{g_P}{g_P^{\mathrm{PCAC}}} = 1.05 \pm 0.19$$

Our result for μ_V is -2.52 ± 0.03 which agrees with the experimental value of -2.55. Our result for $M(\mathrm{GT})$ is 0.977 ± 0.013 which also agrees with the experimental value of 0.961 ± 0.003.

A.A. Vorobyov et al., Hyperfine Interact. 101/102, 413 (1996).

$$\Gamma_0^{\mathrm{expt}} = 1494 \pm 4 s^{-1}.$$

L.E. Marcucci et al., PRC 66, 054003 (2002)

TABLE III. Capture rate Γ_0 in sec^{-1}, and angular correlation parameters A_v, A_t, and A_Δ, as defined in Eqs. (2.14)–(2.18), calculated using CHH wave functions corresponding to the AV18, AV14, AV18/UIX, and AV14/TM Hamiltonian models. The theoretical uncertainties, shown in parentheses, reflect the uncertainty in the determination of the $N\Delta$ transition axial coupling constant g_A^*.

Observable	AV18	AV14	AV18/UIX	AV14/TM
Γ_0	1441(7)	1444(7)	1484(8)	1486(8)

$$R_{PS} = 0.94 \pm 0.06.$$

$$
\begin{aligned}
\nu_x + d &\longrightarrow \nu'_x + n + p\,,\\
\overline{\nu}_x + d &\longrightarrow \overline{\nu}'_x + n + p\,,\\
\nu_e + d &\longrightarrow e^- + p + p\,,\\
\overline{\nu}_e + d &\longrightarrow e^+ + n + n\,.
\end{aligned}
$$

The cornestones of this field of research:

- Chiral symmetry
- Conserved vector current (CVC)
- Partial conservation of the axial curent (PCAC)

The PCAC reads (Adler, 1965):

$$q_\mu \; < \Psi_f | j^a_{5\mu}(q) | \Psi_i > = \; i f_\pi m_\pi^2 \Delta_F^\pi(q^2) \; < \Psi_f | m^a_\pi(q) | \Psi_i >$$

The current $j^a_{5\mu}(q)$ is for the system of A nucleons
the sum of the one- and two--nucleon components

$$j^a_{5\mu}(q) = \sum_{i=1}^{A} j^a_{5\mu}(1,i,q_i) + \sum_{i<j}^{A} j^a_{5\mu}(2,ij,q)$$

Let us describe the nuclear system by the Schroedinger equation

$$H|\Psi> = E|\Psi>$$

$$H = T + V$$

For the one- and two—nucleon components of the total axial current

$$\begin{aligned}
\vec{q}_i \cdot \vec{j}^{\,a}_5(1,\vec{q}_i) &= [T_i\,,\,\rho^a_5(1,\vec{q}_i)] + if_\pi m^2_\pi \Delta^\pi_F(q^2)\, m^a_\pi(1,\vec{q}_i)\,, \quad i=1,2\,, \\
\vec{q} \cdot \vec{j}^{\,a}_5(2,\vec{q}) &= [T_1+T_2\,,\,\rho^a_5(2,\vec{q})] + ([V\,,\,\rho^a_5(1,\vec{q})] + (1 \leftrightarrow 2)) \\
&\quad + if_\pi m^2_\pi \Delta^\pi_F(q^2)\, m^a_\pi(2,\vec{q})\,.
\end{aligned}$$

Space component of the WAEC $\vec{j}_5^a(2, \vec{q})$ is of the order $\mathcal{O}(1/M^3)$

Being of a relativistic origin, it is model dependent. This component of the WAEC was derived by several authors in various models.

In the standard nuclear physics approach :

Chemtob-Rho, 1971
Ivanov-T, 1979
Towner, 1987
Adam-Hajduk-Henning-Sauer-T, 1991
T-Khanna, 1995
Schiavilla, 1991
Tsushima-Riska, 1992
Ananyan, 1998

BS currents:

Dmitrasinovic, 1996
Khanna-T, 2000

Gross equation:

Jaus, 1976

EFT's:

Park-Kubodera-Min-Rho, 1998
Butler-Chen-Kong, 2001

Accepting the chiral symmetry as the basic symmetry governing the nuclear dynamics, it is expected that the WAEC of the pion range, constructed within approaches respecting this symmetry and in conjunction with the given nuclear equation of motion, should exhibit model independence

Situation is not transparent

The WAEC of a given range has two parts:

- potential currents
- non-potential currents

The potential WAEC is such that it satisfies the part of the continuity equation containing the commutator

$$[V\ ,\ \rho_5^a(1,\vec{q})\,].$$

The pair term is one of the exchange currents that belong to the potential current. It **is obtained by the non--relativistic reduction of the negative frequency part of the nucleon Born term.** Besides, other potential currents can appear. Then the total potential WAEC is defined as the sum of all potential terms of a given range.

•Tsushima-Riska

The approach is the only one that is not based on chiral Lagrangians. It uses the relativistic nucleon Born terms and the WAEC is obtained by imbedding the nuclear potential into the negative frequency part of these terms, thus directly connecting the potentials and the WAEC.

The pion pair term is obtained using the pseudovector pi-N-N coupling.

Being of the order $\mathcal{O}(1/M^5)$, it is negligible .

•Adam-Hajduk-Henning-Sauer-T

The WAEC is derived within the extended S-matrix method, using the chiral Lagrangian model with the PV pi-N-N coupling. **The resulting potential current is of the order** $\mathcal{O}(1/M^3)$ **and it is given by the difference of the nucleon Born term and the first Born iteration**.

•T-Khanna

The same potential current is obtained from the chiral model with the PS pi-N-N coupling. In this case, **besides the pair term, the PCAC constraint term contributes.**

•Schiavilla

The pion pair term is derived from the PS pi-N-N coupling that is not chiral invariant.

•Park-Kubodera-Min-Rho

The WAEC is derived within the HBChPT approach, but **the nucleon Born term is considered as fully reducible and therefore omitted.**

•It is not clear from Sci,TR,PKMR that the constructed WAECs of the pion range satisfy a particular form of the PCAC, or in conjunction with a specific nuclear equation of motion they can be used.

•**The problem of double counting is overlooked.**

These currents do not satisfy the PCAC as stated above, if used in standard nuclear physics calculations, based on the Schroedinger equation and static nuclear potentials.

Here we discuss **the role of the weak axial pion pair term in fulfilling the nuclear PCAC** for the WAEC in conjunction with the Schroedinger equation and the static nuclear potential. Simultaneously, we consider the problem of the double counting.

In pionless EFT, the pion source is zero: **the WAEC should be conserved**

Constructing the **pion pair term** and the related **potential current** in two models**:**

•Starting from the ChPT Lagrangian of the pi-N system from which we construct in the leading order (tree approximation) the WAEC of the pion range. We explicitly show how the potential and non potential parts interplay with other components entering the nuclear PCAC so that it is satisfied.

The resulting pion potential term is the same as the one derived earlier from the hard pion Lagrangian of the N-Delta-pi-rho-a_1 system.

•Later on, we derive the pion potential term in the leading order of the HBChPT approach. We show that the obtained current is the same as the one derived within the ChPT approach.

- We compare the space component of the long-range part of the WAEC computed in various models

- We calculate the effect of the pion potential term in the deuteron weak disintegration by low energy neutrinos in the neutral current channel.

The pion pair term and the nuclear PCAC

Start from the set of **relativistic Feynman amplitudes satisfying the PCAC** equation.

In general, these amplitudes are not yet the nuclear exchange currents, because of the double counting problem: the presence of the potential term (and related pair term) in the exchange current operator is related to the equation, describing the nuclear states. If the nucleon propagator in the first Born iteration is the fully relativistic one then this iteration is equal to the nucleon Born term and the exchange currents do not contain any potential term, in order to avoid the double counting. This is the case of the **axial currents constructed in conjunction with the Bethe--Salpeter equation**. In this case, the nucleon Born term is fully reducible. In the case of the **Schroedinger equation, the nucleon Born term is not fully reducible**. The propagator of the first Born iteration contains only the positive frequencies . *If the Feynman amplitudes are constructed using the chiral model with the PV coupling*, the positive frequency part of the nucleon Born term does not coincide with the first Born iteration and the *difference should be calculated*.

Then the resulting potential current is equal to this difference, since the negative frequency part of the nucleon Born term (the pair term) is suppressed by a factor $\approx\ 1/M^2$ and therefore, negligible.

The weak axial pion pair term within the ChPT

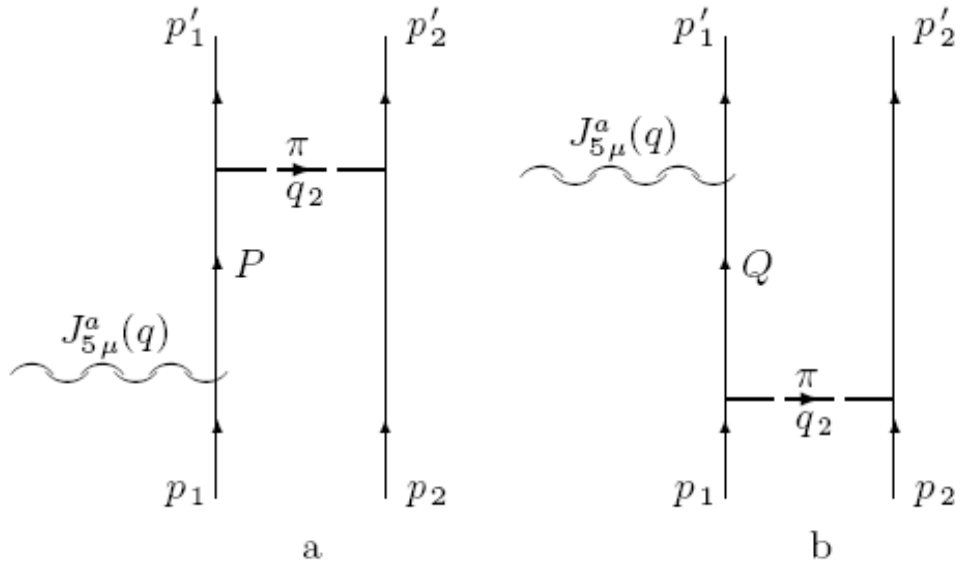


FIG. 1: The weak axial nucleon Born term of the pion range.

We need to extract from the ChPT Lagrangian the lowest order vertices:

$$\Delta\mathcal{L}_{\pi N} = -ig_A\,\bar{N}\,\gamma_\mu\gamma_5\,\frac{\vec{\tau}}{2}\,N\cdot\vec{A}_\mu \;-\; i\frac{g_A}{2f_\pi}\,\bar{N}\,\gamma_\mu\gamma_5\,\vec{\tau}\,N\cdot\partial_\mu\vec{\pi}\,.$$

The Feynman amplitude reads

$$\begin{aligned} J^a_{5\mu}(pv) \;&=\; -\bar{u}(p_1')\Big[\hat{\mathcal{O}}^\pi_1(-q_2)S_F(P)\,\hat{J}_{5\mu}(1,q)\frac{1}{2}(a^+-a^-)+\hat{J}_{5\mu}(1,q) \\ &\quad\times S_F(Q)\hat{\mathcal{O}}^\pi_1(-q_2)\frac{1}{2}(a^++a^-)\Big]u(p_1)\Delta^\pi_F(q_2^2)\bar{u}(p_2')\hat{\mathcal{O}}^\pi_2(q_2)u(p_2)\,+\,(1\leftrightarrow 2)\,, \end{aligned}$$

The contact part of the one-nucleon current is

$$\hat{J}_{5\mu}(1,c) \,=\, -ig_A\,\gamma_\mu\gamma_5\,.$$

In calculating the contribution of the amplitude $J^a_{5\mu}(pv)$ to the exchange currents, one splits the nucleon propagator into the positive- and negative frequency parts and the non-relativistic reduction is made.

The contribution to the space component of the negative frequency part of the Feynman amplitude- pion pair term- is of the nominal order $\mathcal{O}(1/M^5)$ and therefore, negligible.

In the extended S-matrix method

$$q_{20} = P_0 - p'_{10} = P_0 - E(\vec{P}) + E(\vec{P}) - p'_{10} \equiv P_0 - E(\vec{P}) + q^{st}_{20}$$

and the positive frequency part of $J^a_{5\mu}(pv)$ is

$$J^{a(+)}_{5\mu}(pv) = J^{a(+)}_{5\mu}(ps) + \Delta J^a_{5\mu}(pv)$$

$$J^{a(+)}_{5\mu}(pv) = \frac{f_{\pi NN}}{m_\pi}\bar{u}(p_1')\left[(\vec{q}_2\cdot\vec{\gamma}+iq_{20}\gamma_4)\gamma_5\frac{1}{P_0-E(\vec{P})}u(P)\bar{u}(P)\hat{J}_{5\mu}(1,q)\frac{1}{2}(a^+-a^-)\right.$$

$$\left.+\hat{J}_{5\mu}(1,q)\frac{1}{Q_0-E(\vec{Q})}u(Q)\bar{u}(Q)\,(\vec{q}_2\cdot\vec{\gamma}+iq_{20}\gamma_4)\gamma_5\frac{1}{2}(a^++a^-)\right]u(p_1)$$

$$\times\Delta^{\pi}_{F}(q_2)\bar{u}(p_2')\hat{\mathcal{O}}^{\pi}_{2}(q_2)u(p_2)+(1\leftrightarrow 2),$$

Here $J^{a(+)}_{5\mu}(ps)$ is the positive frequency part of the nucleon Born term obtained using the static PS pi-N-N coupling. **It is the current containing a contribution from the potential**, since it coincides with the first Born iteration of the Lippmann--Schwinger equation, if the static one-pion exchange potential is used. **In order to avoid the double counting, the contribution from such a graph is not included in the exchange current since it is reducible**.

The current $\Delta J^{a}_{5\mu}(pv)$ arises from the contact interaction and, in the non-relativistic approximation, its space component is given by

$$\Delta \vec{j}^{a}_{5}(pv) = g_A \frac{g^2_{\pi NN}}{(2M)^3} \left[\left(\vec{q} + i\vec{\sigma}_1 \times \vec{P}_1 \right) \tau^a_2 + \left(i\vec{P}_1 - \vec{\sigma}_1 \times \vec{q} \right) (\vec{\tau}_1 \times \vec{\tau}_2)^a \right] \Delta^{\pi}_{F}(\vec{q}_2^{\,2})(\vec{\sigma}_2 \cdot \vec{q}_2)$$

$$+ \; (1 \leftrightarrow 2).$$

This current coincides with the potential term derived earlier from the hard pion Lagrangian with the PV pi-N-N coupling and **it contributes to the space component of the WAEC in the same leading order in 1/M as other pion exchange currents**.

The well known Foldy-Dyson unitary transformation of the nucleon field can be used in the Lagrangian to obtain the PS pi-N-N coupling

$$N = exp[-i \frac{g_A}{2f_\pi} \gamma_5 (\vec{\tau} \cdot \vec{\pi})] N' .$$

$$J^a_{5\mu}(pv) = J^a_{5\mu}(ps) + J^a_{5\mu}(PCAC)$$

The powerful **representation independence** (equivalence) theorem

In order to extract the nuclear WAEC from the relativistic amplitudes in this case, the reducible part of the nucleon Born amplitude $J^a_{5\mu}(ps)$ is isolated. This is the positive frequency part $J^{a\,(+)}_{5\mu}(ps)$.

It holds

$$\Delta J^a_{5\mu}(pv) = J^{a\,(-)}_{5\mu}(ps) + J^a_{5\mu}(PCAC)$$

where $J^{a\,(-)}_{5\mu}(ps)$ is the negative frequency part of the nucleon Born term obtained with the PS pi-N-N coupling.

Explicitly, one has for the space component of the pair term

$$\vec{j}_5^a(ps) = g_A \frac{g_{\pi NN}^2}{(2M)^3} \left[\left(\vec{q} + i\vec{\sigma}_1 \times \vec{P}_1 \right) \tau_2^a - (\vec{\sigma}_1 \times \vec{q}_2)\, (\vec{\tau}_1 \times \vec{\tau}_2)^a \right] \Delta_F^\pi(\vec{q}_2^{\,2})(\vec{\sigma}_2 \cdot \vec{q}_2) + (1 \leftrightarrow 2)\,,$$

and for the PCAC constraint term

$$\vec{j}_5^a(PCAC) = g_A \frac{g_{\pi NN}^2}{(2M)^3} \left[i\vec{P}_1 - (\vec{\sigma}_1 \times \vec{q}_1) \right] (\vec{\tau}_1 \times \vec{\tau}_2)^a \, \Delta_F^\pi(\vec{q}_2^{\,2})(\vec{\sigma}_2 \cdot \vec{q}_2) + (1 \leftrightarrow 2)\,.$$

In the chiral model with the PS pi-N-N coupling, the potential current is obtained as the sum of the negative frequency part of the nucleon Born term (pair term) and the PCAC constraint term.
The result is the same current $\Delta\vec{j}_5^a(pv)$ as obtained in the chiral model with the PV pi-N-N coupling.

•In a chiral invariant model with the PS pi-N-N coupling, **additional potential term arises, that makes the resulting current equivalent** to the current of the chiral model with the PV pi- N-N coupling.
•It follows that the necessity of constructing the WAEC within the chiral models and not simply in terms of pi-N-N couplings was by Sci and T-R overlooked .

•**In order to avoid double counting, the reducible part of the potential current should be removed**, since it is taken into account already at the level of the impulse approximation calculations. **This procedure depends on the nuclear equation of motion** used for the description of nuclear states. Here the calculation is carried out for the Schroedinger equation and static one-pion exchange potentials.
• In our opinion, the problem of the double counting was by Sci,T-R and P-K-M-R omitted.
•Since the pion pair term is absent in P-K-M-R, it is concluded that those currents can be used in conjunction with the Bethe-Salpeter equation.

The continuity equation

It can be shown that the nucleon Born term due to the contact part of the one-body current of our model satisfies the continuity equation

$$q_\mu J^a_{5\mu,\pi}(B,c) = i f_\pi \, M^a_\pi(B)$$

The related nuclear continuity equation for the nuclear current reads

$$q_\mu j^a_{5\mu,\,\pi}(B,c) = i f_\pi m^a_\pi(2) + \left([V_\pi, \rho^a_5(1,c)] + (1 \leftrightarrow 2)\right)$$

Here the space part of the current $j^a_{5\mu,\,\pi}(B,c)$ is given by $\Delta \vec{j}^{\,a}_5(pv)$ with the divergence

$$\vec{q} \cdot \Delta \vec{j}^{\,a}_5(pv) = \frac{g^3_A}{8 M f^2_\pi} \left\{ \left[\vec{q}^{\,2} + i(\vec{q} \cdot \vec{\sigma}_1 \times \vec{P}_1) \right] \tau^a_2 + i(\vec{q} \cdot \vec{P}_1)\, (\vec{\tau}_1 \times \vec{\tau}_2)^a \right\} \Delta^\pi_F(\vec{q}_2^{\,2})(\vec{\sigma}_2 \cdot \vec{q}_2) + (1 \leftrightarrow 2),$$

It holds for the time component that

$$q_0 \Delta j^a_{50}(pv) \approx \mathcal{O}(1/M^5)$$

The pion absorption amplitude is obtained by the same method used above for the derivation of the current $\Delta J^a_{5\mu}(pv)$

$$\Delta j^{a\,(+)}_{5\mu,\,B} = j^a_{5\mu,\,B}(vert) + j^a_{5\mu,\,B}(ext) + j^a_{5\mu,\,B}(ret)\,.$$

Besides the contribution $m^a_\pi(2, ver)$ from the energy dependence of the pi- N-N vertex of the internal pion, the contribution $m^a_\pi(2, ext)$ from the energy dependence of the pi-N-N vertex of the external pion arises with the result

$$i f_\pi m^a_\pi(2, ver) = \vec{q} \cdot \Delta \vec{j}^a_5(pv)\,,$$

$$\begin{aligned} i f_\pi m^a_\pi(2, ext) &= \frac{g^3_A}{8M f^2_\pi} \left\{ \left[\vec{q}_2^{\,2} - i(\vec{q}_2 \cdot \vec{\sigma}_1 \times \vec{P}_1) \right] \tau^a_2 + i(\vec{q}_2 \cdot \vec{P}_1)\,(\vec{\tau}_1 \times \vec{\tau}_2)^a \right\} \Delta^\pi_F(\vec{q}_2^{\,2})(\vec{\sigma}_2 \cdot \vec{q}_2) \\ &+ (1 \leftrightarrow 2)\,. \end{aligned}$$

It is straightforward to obtain **for the static one-pion exchange potential and the one-nucleon axial charge density** that

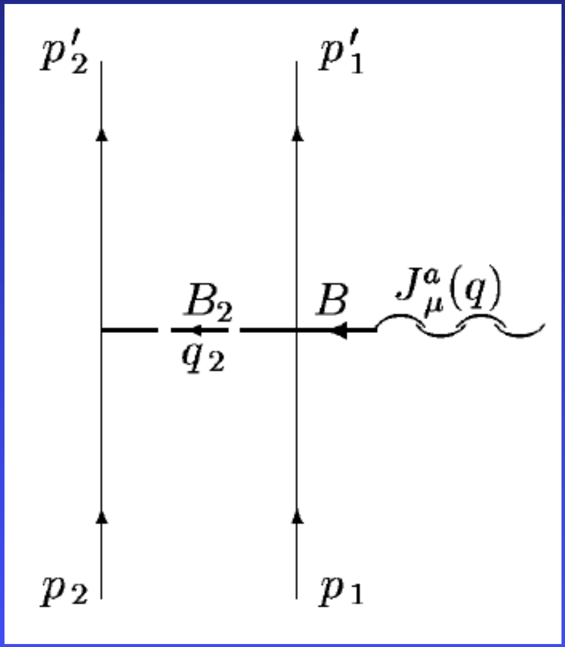
$$([V_\pi, \rho_5^a(1,c)] + (1 \leftrightarrow 2)) = -i f_\pi m_\pi^a(2, ext)\,.$$

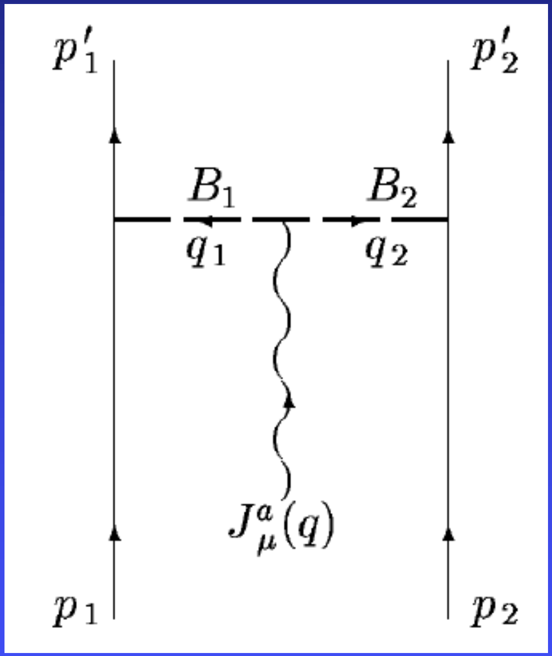
The continuity equation which is in the leading order in 1/M of the form

$$\vec{q}\cdot\Delta\vec{j}_5^{\,a}(pv) = i f_\pi\,[m_\pi^a(2, ver) + m_\pi^a(2, ext)] + ([V_\pi, \rho_5^a(1,c)] + (1 \leftrightarrow 2))\,,$$

is satisfied exactly. **The contact term** $\Delta j_{5\mu}^a(pv)$ is related to the part of the continuity equation, containing the potential and **can be called as the true potential current**.

The rho-pi current





Our model Lagrangian contains a $\mathcal{A}\pi NN$ vertex

$$\Delta\mathcal{L}_{\mathcal{A}\pi NN} = -\frac{i}{2f_\pi}\bar{N}\,\gamma_\mu\,\vec{\tau}\,N\cdot\left(\vec{\pi}\times\vec{\mathcal{A}}_\mu\right)$$

providing another contact current that is a part of the full contact term

$$\begin{aligned} j^a_{5\mu}(c) &= \frac{i}{2f_\pi}\varepsilon^{amn}\,\bar{u}(p_1')\left(\gamma_\mu - \frac{\kappa^V}{2M}\sigma_{\mu\nu}q_\nu\right)\tau^m\,u(p_1)\,\Delta^\pi_F(q^2)\,\bar{u}(p_2')\hat{\mathcal{O}}^\pi_2(q_2)\tau^n u(p_2) \\ &+ (1\leftrightarrow 2)\,. \end{aligned}$$

This current is required by the current algebra prediction for the weak pion production and **it corresponds to the well known rho-pi current**. It looks like a potential one, but **it is not connected to the potential** and it satisfies the PCAC equation

$$\begin{aligned} q_\mu j^a_{5\mu}(c) &= \frac{i}{2f_\pi}\varepsilon^{amn}\,\bar{u}(p_1')\ \not{q}_2\tau^m\,u(p_1)\,\Delta^\pi_F(q^2)\,\bar{u}(p_2')\hat{\mathcal{O}}^\pi_2(q_2)\tau^n u(p_2) \\ &+ (1\leftrightarrow 2)\equiv i f_\pi\,m^a_\pi(c)\,. \end{aligned}$$

The amplitude $m^a_\pi(c)$ is generated from the N-N-pi-pi term

$$\Delta\mathcal{L}_{NN\pi\pi} = (i/4f^2_\pi)\bar{N}\gamma_\mu\vec{\tau}N\cdot(\partial_\mu\vec{\pi}\times\vec{\pi}).$$

The non-relativistic reduction yields

$$\vec{j}^{\,3}_{5,\,\pi}(\rho\pi) = -(\frac{g}{2M})^2 \frac{1}{4Mg_A} [1 + \frac{m_\rho^2}{m_\rho^2 + \vec{q}_1^{\,2}}] [\vec{P}_1 + (1+\kappa_\rho^V)\, i\,(\vec{\sigma}_1 \times \vec{q}_1)] \times F_{\rho NN}(\vec{q}_1^{\,2})\, \Delta_F^\pi(\vec{q}_2^{\,2})\, F_{\pi NN}(\vec{q}_2^{\,2})\ (\vec{\sigma}_2 \cdot \vec{q}_2)\, i\,(\vec{\tau}_1 \times \vec{\tau}_2)^3 + (1 \leftrightarrow 2)$$

Let us note that the contact term $j^a_{5\mu}(c)$ is present in all the discussed model currents.

The weak axial pion pair term within the HBChPT formalism

•First derive the positive frequency nucleon Born term for the weak pion production amplitude on the nucleon in the leading order.
•Start from the lowest order HBChPT Lagrangian

$$\mathcal{L}^{(1)}_{\pi N} = -\bar{\mathcal{N}}_v \left[i v \cdot D + g_A S_v \cdot u \right] \mathcal{N}_v$$

Taking into account only the weak axial external interaction, $a_\mu = \mathcal{A}^a_\mu \tau^a / 2,$ we obtain in the leading order

$$g_A S_v \cdot u \approx g_A \tau^a S_v \cdot \mathcal{A}^a_\mu - \frac{g_A}{f_\pi} S_{v,\mu} (\vec{\tau} \cdot \partial_\mu \vec{\pi}) \, .$$

Then the amplitude, corresponding to Fig.1a reads

$$M_c^v = -i\frac{2g_A^2}{f_\pi} N' N \tau^b \frac{\tau^a}{2} \, \bar{u}'_v \, (S_v \cdot q_2) \, \frac{P_{v+}}{v \cdot K} \, (S_v \cdot \mathcal{A}^a) \, u_v \, .$$

$$v_\mu = p_\mu / M \, , \quad \vec{p} = \vec{P} \, , \quad p_0 = E(\vec{P})$$

$$P_\mu = p_\mu + K_\mu$$

$$v \cdot K = -v_0 \, (P_0 - E(\vec{P}))$$

$$\begin{aligned} M_c^v &= -i\frac{2g_A^2}{f_\pi} N' N \tau^b \frac{\tau^a}{2} \, \bar{u}'_v \, \left(S_v \cdot q_2^{st}\right) \frac{P_{v+}}{v \cdot K} \, (S_v \cdot \mathcal{A}^a) \, u_v \\ &- i \, \frac{2g_A^2}{f_\pi} N' N \tau^b \frac{\tau^a}{2} \frac{1}{2v_0} \, \bar{u}'_v \, \gamma_5 \gamma_4 \, P_{v+} \, (S_v \cdot \mathcal{A}^a) \, u_v \, . \end{aligned}$$

$$M^v_{c+d} = M^v_{c+d}(st) + \Delta M^v_{c+d}$$

$$\begin{aligned} \Delta M^v_{c+d} &= i\, \frac{g_A^2}{f_\pi} \frac{1}{v_0} N' N\, \bar{u}'_v \Big[\gamma_4\gamma_5\, P_{v+}(P)\;\; (S_v \cdot \mathcal{A}^a)\, \tau^b\, \frac{\tau^a}{2} \\ &- \quad (S_v \cdot \mathcal{A}^a)\;\; P_{v+}(Q)\; \gamma_4\gamma_5\, \frac{\tau^a}{2} \tau^b \Big] u_v \;. \end{aligned}$$

According to the generalized Weinberg's counting rules, such an amplitude has $\nu = -1,$ like the contact amplitude $j^a_{5\mu}(c)$.

One can obtain from the interaction ΔM^v_{c+d} a contact amplitude

$$\begin{aligned} \Delta J^a_{c+d,\mu} &= -\frac{g_A^3}{f_\pi^2} \frac{1}{v_{10}} N'_{v_1} N_{v_1} \bar{u}'_{v_1} \left[\gamma_4\gamma_5\, P_{v_1+}(P) S_{v_1,\mu} \frac{1}{2}(a^+ - a^-) - S_{v_1,\mu} P_{v_1+}(Q)\, \gamma_4\gamma_5 \frac{1}{2}(a^+ + a^-) \right] u_{v_1} \\ &\times \Delta^\pi_F(q_2^2)\, N'_{v_2} N_{v_2} \bar{u}'_{v_2}\; (S_{v_2} \cdot q_2)\; u_{v_2} \end{aligned}$$

that is of the same form as $\Delta J^a_{5\mu}(pv)$. Non-relat. reduction: $\Delta j^a_{5\mu}(pv)$

Comparison of the WAEC

The space component of the WAEC of the pion range – the long range part

•**Standard nuclear physics approach** (Ivanov-T, T-Khanna, Congleton-T, Adam et al.), based on the chiral hard pion Lagrangians

$$\begin{aligned}\vec{j}^{\,a}_{5,\pi} \;=\; & \frac{g_A}{2Mf_\pi^2}\Big\langle g_A^2\Big\{\Big(\frac{f_{\pi N\Delta}}{f_{\pi NN}}\Big)^2\frac{2M}{9(M_\Delta-M)}\,\vec{q}_2 \,+\, \frac{1}{4}[\vec{q}+i(\vec{\sigma}_1\times\vec{P}_1)]\Big\}\,\tau_2^{\,a} \\ & +\frac{1}{4}\Big\{\Big[g_A^2\Big(\frac{f_{\pi N\Delta}}{f_{\pi NN}}\Big)^2\frac{2M}{9(M_\Delta-M)} \,+\, (1+\kappa_\rho^V)\Big]\,i(\vec{\sigma}_1\times\vec{q}_2) \\ & +[g_A^2-(1+\kappa_\rho^V)]\,i(\vec{\sigma}_1\times\vec{q}) \,+\, (g_A^2-1)\vec{P}_1\Big\}\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\Big\rangle \\ & \times(\vec{\sigma}_2\cdot\vec{q}_2)\,\Delta_F^\pi(\vec{q}_2^{\,2}) \,+\, (1\leftrightarrow 2)\,.\end{aligned}$$

•The leading order HBChPT currents (Park et al., PRC 67, 2003)

with the potential current $\Delta\vec{j}^{\,a}_5(pv)$ added:

$$\vec{A}^{\,a:\nu 3}_{12}(1\pi) = \frac{g_A}{2M f_\pi^2}\Big\langle \{2\hat{c}_3\,\vec{q}_2 + \frac{g_A^2}{4}[\vec{q} + i(\vec{\sigma}_1\times\vec{P}_1)]\}\tau_2^a + \frac{1}{4}\{(4\hat{c}_4+1)\,i(\vec{\sigma}_1\times\vec{q}_2) + [g_A^2 - 1 - c_6]\,i(\vec{\sigma}_1\times\vec{q}) + (g_A^2-1)\vec{P}_1\}\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\Big\rangle(\vec{\sigma}_2\cdot\vec{q}_2)\Delta_F^\pi(\vec{q}_2^{\,2}) + (1\leftrightarrow 2)$$

The currents $\vec{j}^{\,a}_{5,\,\pi}$ and $\vec{A}^{\,a:\nu 3}_{12}(1\pi)$ have an identical structure. This was achieved by respecting the chiral invariance and solving the double counting problem in conjunction with the Schroedinger equation. In our opinion, it is the current $\vec{A}^{\,a:\nu 3}_{12}(1\pi)$ that should be used in the nuclear physics calculations with the nuclear wave functions derived using the Schroedinger equation.

•Riska-Tsushima currents

one should remove the reducible piece from the positive frequency part of the nucleon Born term and add the rest to the already derived exchange current

If the pion exchange current is constructed with the PS pi-N-N coupling, one should sum up the PCAC constraint term and the negative frequency Born term (the pair term), both with the potential imbedded. The resulting potential current will be the same as in the PV pi-N-N coupling case.

Schiavilla's currents

One needs to add the PCAC constraint term to the pair term, in order to obtain
the chiral potential current.

$$pp:\quad p+p \to d+e^{+}+\nu_e,$$

$$\text{hep:}\quad p+{}^{3}\text{He} \to {}^{4}\text{He}+e^{+}+\nu_e.$$

BS currents from HP Lagrangian

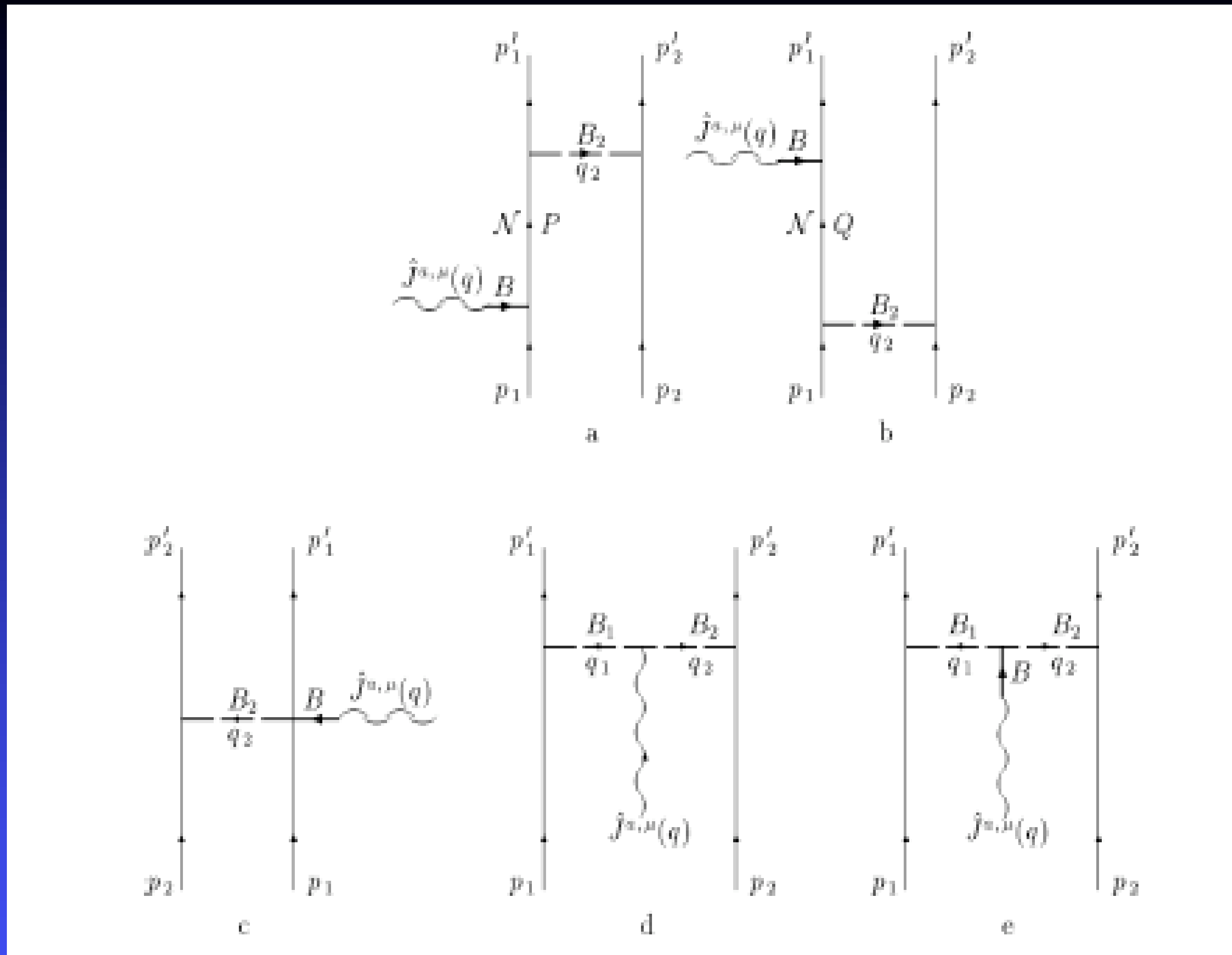
Khanna-T, NPA 673, 455 (2000)

chiral Lagrangians of the $N\Delta(1236)\pi\rho a_1\omega$ system.

$$\widehat{J}^{a\mu}(1,i) = \frac{g_A}{2} m_{a_1}^2 \Delta_{a_1}^{\mu\nu}(q)\widehat{\Gamma}_{i\nu}^{5a} + i f_\pi \Delta_F^\pi(q^2) q^\mu \widehat{\Gamma}_i^a .$$

$$\begin{aligned} q_\mu \widehat{J}^{a\mu}(1,i) &= \left[G^{-1}(p_i')\hat{e}_A(i) + \hat{e}_A(i) G^{-1}(p_i)\right] + i f_\pi m_\pi^2 \Delta_F^\pi(q^2)\widehat{\Gamma}_i^a, \\ &\equiv \left[\hat{e}_A(i), G_i^{-1}\right]_+ + i f_\pi m_\pi^2 \Delta_F^\pi(q^2)\widehat{\Gamma}_i^a . \end{aligned}$$

$$\hat{e}_A(i) = g_A \left(\gamma_5 \frac{\tau^a}{2}\right)_i, \qquad G^{-1}(p) = \not{p} - M .$$


p'_1
p'_2
B_2
q_2
N
P
$\hat{j}^{a,\mu}(q)$
B
p_1
p_2
a
Q
b
c
B_1
q_1
d
e

$$q_\mu \widehat{J}^{a\mu}_{BS\pi}(ex) = [\hat{e}_A(1) + \hat{e}_A(2), \widehat{V}_\pi]_+ + i f_\pi m_\pi^2 \Delta_F^\pi(q^2) \widehat{\mathcal{M}}^a_\pi(2),$$

$$\widehat{J}^{a\mu}_{\rm BS} = i \widehat{J}^\mu_a(1,1) G_2^{-1} + i \widehat{J}^\mu_a(1,2) G_1^{-1} + \widehat{J}^{a\mu}_{\rm BS}(ex) = \widehat{J}^{a\mu}_{\rm IA} + \widehat{J}^{a\mu}_{\rm BS}(ex),$$

$$q_\mu \widehat{J}^{a\mu}_{\rm BS} = [\hat{e}_A(1) + \hat{e}_A(2), \mathcal{G}^{-1}]_+ + i f_\pi m_\pi^2 \Delta_F^\pi(q^2) \widehat{\mathcal{M}}^a,$$

where the inverse Green function is

$$\mathcal{G}^{-1} = G_{\rm BS}^{-1} + \widehat{V},$$

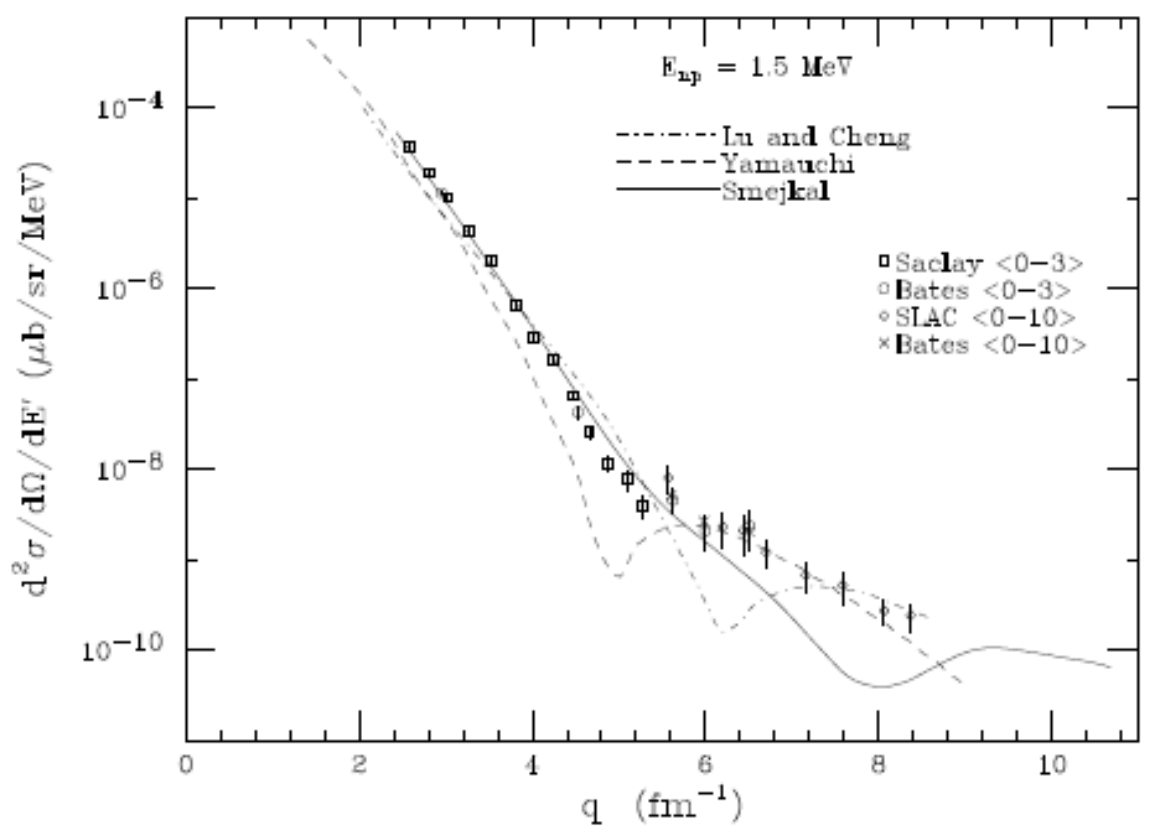
with

$$\widehat{V} = \widehat{V}_\pi + \widehat{V}_\rho + \widehat{V}_{a_1} + \widehat{V}_\omega.$$

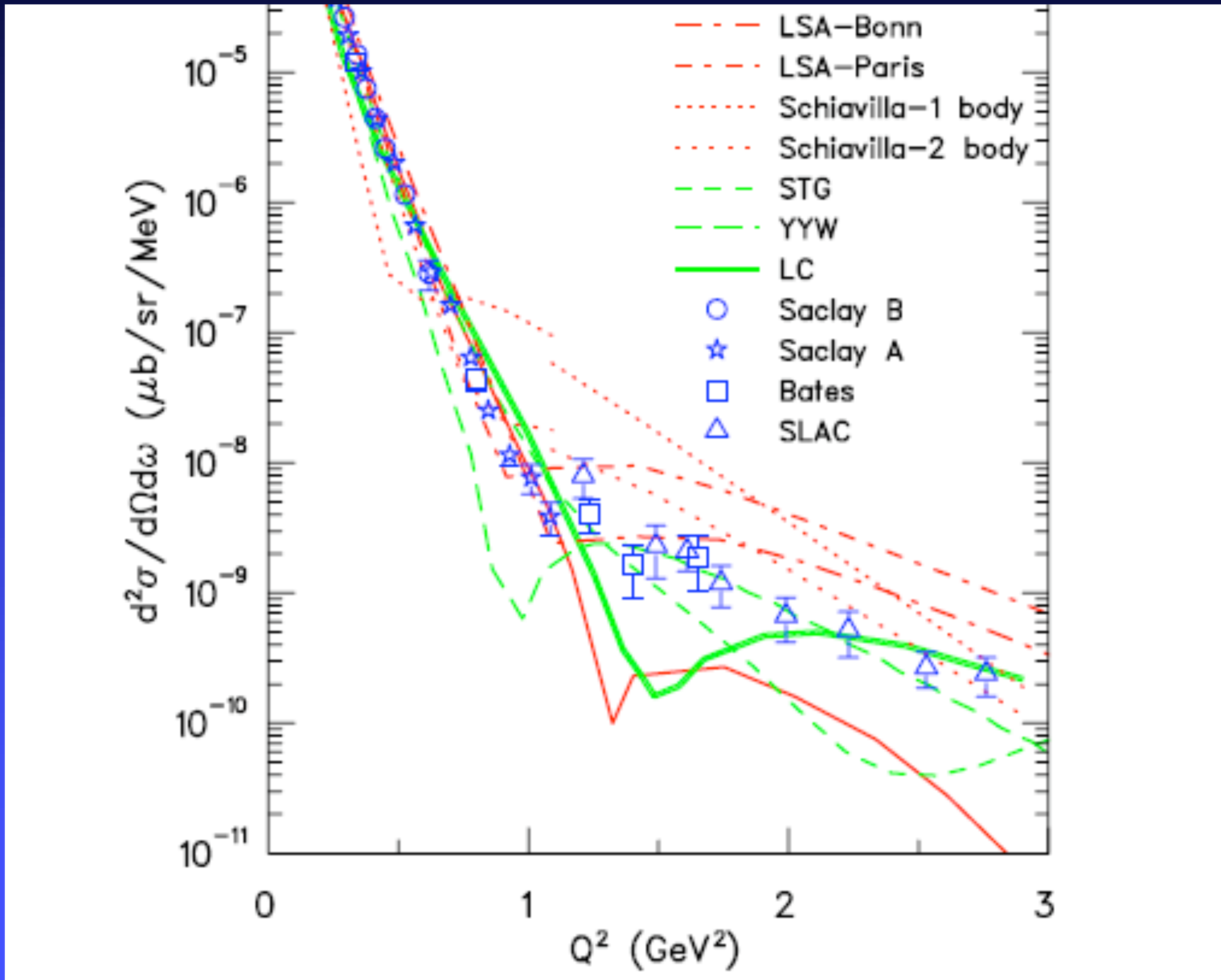
$$\mathcal{G}^{-1}|\psi\rangle = \langle\psi|\mathcal{G}^{-1} = 0,$$

$$q_\mu \langle\psi|\widehat{J}^{a\mu}_{\rm BS}|\psi\rangle = i f_\pi m_\pi^2 \Delta_F^\pi(q^2) \langle\psi|\widehat{\mathcal{M}}^a|\psi\rangle.$$

e + d ---> e' + n + p; I.Sick, Progr. Part. Nucl. Phys 47, 245 (2001)



R. Gilman, F. Gross, J. Phys. G28, R37 (2002)



Numerical estimate of the effect

We compute the contribution of the potential current to the cross section for the low energy electron neutrino-deuteron inelastic scattering in the neutral current channel

$$\nu_e + d \rightarrow \nu'_e + p + n \,.$$

This reaction is important for studying the solar neutrino oscillations and it has been intensively studied both theoretically (Butler et al., Ando et al., Nakamura et al., Mosconi et al., Ying et al.) and experimentally (SNO).

The model axial current considered contains the one—nucleon current and the one-pion WAEC , to which we add also the contribution from t Δ isobar excitation of the ρ range.

Table 1

Cumulative contributions to the cross section $\sigma_{\nu d}\ (\times 10^{-42}\ \mathrm{cm}^2)$ from the weak axial exchange currents for various neutrino energies.

E_ν [MeV]	5	10	15	20	101
IA	0.0938 (-)	1.076 (-)	3.244 (-)	6.591 (-)	147.1 (-)
IA+$\Delta(\pi+\rho)$	0.0977 (1.041)	1.126 (1.046)	3.405 (1.050)	6.935 (1.052)	158.5 (1.077)
$+\rho$-π	0.0986 (1.009)	1.137 (1.010)	3.443 (1.011)	7.016 (1.012)	161.1 (1.016)
+p(π)	0.0978 (0.992)	1.127 (0.991)	3.408 (0.990)	6.940 (0.989)	157.9 (0.980)

It is seen from Table 1 that the effects of the rho-pi and potential terms are about 1 % and they cancel each other to a large extent. Since the total effect from the space part of WAEC is at the level of a few percent, **it is important to correctly identify all the components of the WAEC that satisfy the PCAC** and contribute sensibly.

Table 2

The values of the constant $L_{1,A}$ obtained by the fit to the cross section of the reaction $\nu_e + d \to \nu_e' + p + n$ calculated using the NijmI potential.

	IA	$+\Delta(\pi+\rho)$	$+\rho$-π	$+\mathrm{p}(\pi)$
$\bar{L}_{1,A}$	0.8	4.1	4.9	4.2

The effective cross section [Butler et al., PRC 63, 035501 (2001)] is presented in the form

$$\sigma_{EFT}(E_\nu) = a(E_\nu) + L_{1,A}\, b(E_\nu)\,.$$

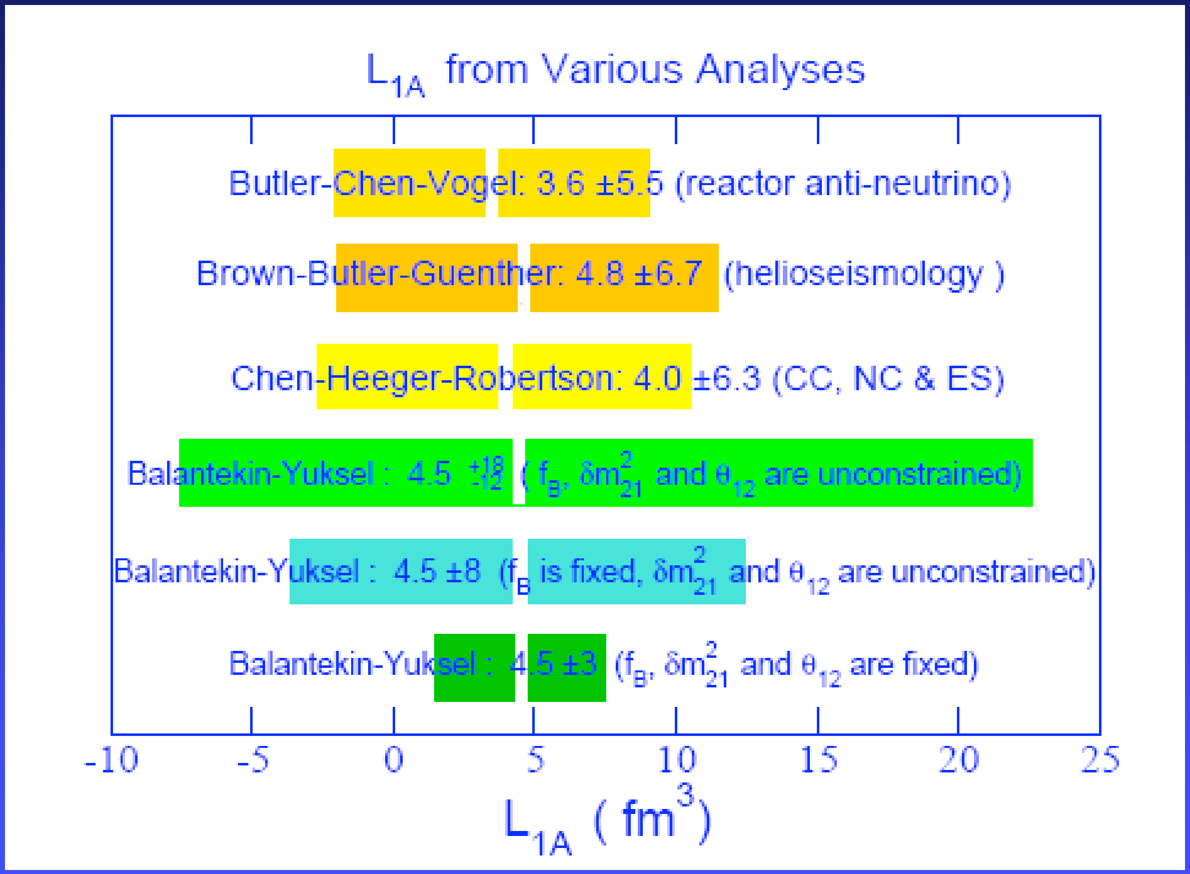
dimension of $L_{1,A}$ - fm^3.

Interactions of the solar neutrinos with the deuterons

Mosconi-Ricci-T: ***Nuclear Physics in Astrophysics, Debrecen, Hungary, May 16-20, 2005.***

reaction		NijmI	Nijm93	OBEPQG	NSGK
(10)	$L_{1,A}$	4.6	5.2	4.8	5.4
	S	1.001	1.001	1.001	1.000
(11)	$L_{1,A}$	4.9	5.5	5.1	5.5
	S	1.001	1.001	1.001	1.000
(12)	$L_{1,A}$	4.1	5.0	-	6.0
	S	1.001	1.001	-	1.002
(13)	$L_{1,A}$	4.5	5.4	6.9	5.6
	S	1.001	1.000	0.9996	0.9997

A. B. Balantekin and H. Yuksel, Phys. Rev. C68, 055801 (2003); hep-ph/0307227



M. Butler, J.W. Cheng and P. Vogel, Phys. Lett. B549, 26 (2002); nucl-th/0206026

$$\overline{L}_{1,A}^{reactor} = 3.6 \pm 5.5 \text{ fm}^3,$$

	$\overline{\sigma}_{fission} (10^{-44} cm^2/fission)$	$L_{1,A} (fm^3)$
$\overline{\nu}$CC Rovno	1.17 ± 0.16	17.4 ± 13.9
$\overline{\nu}$NC Rovno	2.71 ± 0.47	-2.0 ± 13.8
$\overline{\nu}$CC Krasnoyarsk	1.05 ± 0.12	-1.3 ± 9.5
$\overline{\nu}$NC Krasnoyarsk	3.09 ± 0.30	1.8 ± 8.1
$\overline{\nu}$CC Bugey	0.95 ± 0.20	-1.5 ± 17.2
$\overline{\nu}$NC Bugey	3.15 ± 0.40	11.1 ± 11.7

Scattering lengths and effective ranges for the 1S0 NN states

	NijmI	Nijm93	OBEPQG	AV18	EFT	exp.
a_{np}	-23.72	-23.74	-23.74	-23.73	-23.7	-23.740±0.020[1]
r_{np}	2.65	2.68	2.73	2.70	2.70	2.77 ±0.05[1]
a_{pp}	-7.80	-7.79	-	-7.82	-7.82	-7.8063±0.0026[2]
r_{pp}	2.74	2.71	-	2.79	2.79	2.794±0.014[2]
a_{nn}	-18.16	-18.11	-18.10	-18.49	-18.5	-18.59±0.40[3]
r_{nn}	2.80	2.78	2.77	2.84	2.80	2.80±0.11[4]

[1] Ref. [37]; [2] Ref. [38]; [3] Ref. [39]; [4] Ref. [40]

Table 4. Cross section and the differences in % between cross sections for the reaction (11). For notations, see table 3, only instead of E_ν, now $E_{\bar\nu}$ is the antinetrino energy in MeV.

$E_{\bar\nu}$	σ_{NijmI}	NijmI	NSGK	Δ_1	Δ_2	Δ_3
3	0.00332	0.6	0.1	-1.1	-0.5	-
4	0.0302	1.0	0.2	-0.8	-0.1	9.3
5	0.0928	1.0	0.1	-0.8	-0.1	0.9
6	0.196	1.1	0.3	-0.9	-0.1	5.7
7	0.342	0.8	0.1	-1.0	-0.2	2.0
8	0.531	1.4	0.8	-1.1	-0.3	3.1
9	0.765	0.8	0.2	-1.2	-0.4	0.9
10	1.043	0.6	0.2	-1.4	-0.5	-1.7
11	1.364	0.1	-0.2	-1.6	-0.7	-0.7
12	1.729	-0.2	-0.4	-1.7	-0.8	-2.8
13	2.136	-0.3	-0.2	-1.9	-1.0	-2.1
14	2.585	-0.5	-0.2	-2.1	-1.2	-3.9
15	3.074	-0.7	-0.2	-2.4	-1.4	-4.1
16	3.604	-0.9	-0.1	-2.6	-1.7	-5.6
17	4.173	-1.2	-0.2	-2.9	-1.9	-6.0
18	4.779	-1.6	-0.3	-3.3	-2.2	-7.6
19	5.422	-1.9	-0.3	-3.6	-2.5	-8.0
20	6.101	-2.2	-0.2	-3.9	-2.9	-9.4

Table 5. Cross section and the differences in % between cross sections for the reaction (12). For notations, see table 3.

E_ν	σ_{NijmI}	NijmI	NSGK	Δ_1	Δ_2	Δ_3
2	0.00338	-5.5	-0.6	-7.6	-6.7	-
3	0.0455	-0.5	-0.3	-3.0	-2.0	-
4	0.153	0.5	-0.6	-1.9	-0.9	1.9
5	0.340	1.5	0.1	-1.6	-0.6	2.9
6	0.613	1.9	0.4	-1.6	-0.5	3.0
7	0.978	1.9	0.4	-1.6	-0.6	3.0
8	1.438	0.0	-2.4	-1.8	-0.7	3.1
9	1.997	-0.2	-2.3	-1.9	-0.8	2.9
10	2.655	0.1	-1.7	-2.1	-1.0	3.1
11	3.415	3.3	3.3	-2.4	-1.2	2.8
12	4.277	1.0	0.3	-2.6	-1.5	2.5
13	5.243	0.7	0.2	-2.9	-1.8	2.4
14	6.311	0.4	0.2	-3.2	-2.1	2.1
15	7.484	0.0	0.2	-3.6	-2.4	1.7
16	8.760	-0.5	-0.1	-4.0	-2.8	1.4
17	10.14	-0.9	-0.1	-4.4	-3.2	1.0
18	11.62	-1.3	-0.1	-4.8	-3.6	0.1
19	13.21	-1.7	-0.0	-5.3	-4.1	-0.1
20	14.89	-2.4	-0.3	-5.8	-4.5	-0.3

Table 6. Cross section and the differences in % between cross sections for the reaction (13). For notations, see table 3.

E_γ	σ_{NijmI}	NijmI	NSGK	Δ_1	Δ_2	Δ_3
5	0.0274	-1.3	-0.9	-2.4	-1.5	9.0
6	0.116	0.1	-0.1	-2.1	-1.1	8.1
7	0.277	0.2	-0.2	-1.8	-0.7	7.4
8	0.514	0.5	-0.1	-1.7	-0.6	7.1
9	0.829	0.4	-0.2	-1.7	-0.6	6.9
10	1.224	0.9	0.4	-1.7	-0.6	6.8
11	1.697	0.7	0.2	-1.9	-0.7	6.0
12	2.249	0.6	0.1	-2.0	-0.8	6.1
13	2.876	0.4	0.0	-2.2	-1.0	5.5
14	3.578	0.4	0.2	-2.3	-1.1	5.2
15	4.353	0.0	0.0	-2.6	-1.3	4.9
16	5.200	-0.2	0.1	-2.8	-1.6	4.6
17	6.115	-0.3	0.2	-3.1	-1.9	3.5
18	7.097	-0.5	0.4	-3.4	-2.1	3.2
19	8.143	-0.9	0.2	-3.8	-2.5	2.8
20	9.251	-1.2	0.3	-4.1	-2.8	2.4

Results and Conclusions

•The question of the interplay of the chiral invariance restriction and of the double counting problem in the construction of the weak axial potential exchange currents of the pion range is discussed.

•Only the part of the nucleon Born term, that is not contained in the first Born iteration contributes to the exchange currents.

•The total potential exchange current, with the pion pair term included, satisfies the PCAC constraint.

•The resulting potential term is the same in the ChPT and HBChPT and it coincides with the potential term derived earlier from the hard pion Lagrangians.

•Numerically, the contribution of the pion potential term is at the same level as the contribution from the well-known rho-pi current and the both contributions tend to cancel each other at low energies.

Acknowledgments

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