# The Shell Model on NNN forces

A. P. Zuker. ECT* talk on 24 June 2005

The following remarks from Abzouzi, Caurier, and Zuker (1991) still provide a good introduction to the subject:

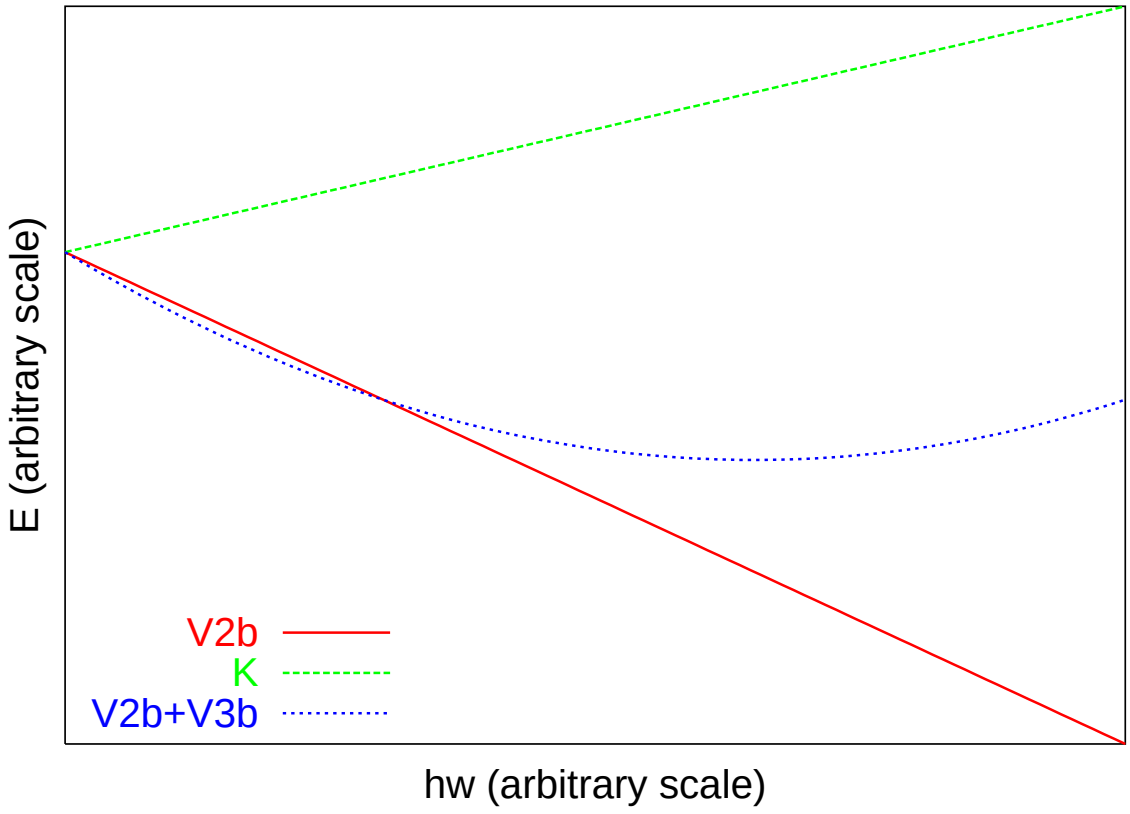
"The use of realistic potentials (*i.e.*, consistent with $NN$ scattering data) in shell-model calculations was pioneered by Kuo and Brown (1966). Of the enormous body of work that followed we would like to extract two observations. The first is that whatever the forces (hard or soft core, ancient or new) and the method of regularization (Brueckner $G$ matrix (Kahana *et al.*, 1969; Kuo and Brown, 1966), Sussex direct extraction (Elliott *et al.*, 1968) or Jastrow correlations (Fiase *et al.*, 1988)) the effective matrix elements are *extraordinarily similar* (Pasquini and Zuker, 1978; Rutsgi *et al.*, 1971). The most recent results (Jiang *et al.*, 1989) amount to a vindication of the work of Kuo and Brown. We take this similarity to be the great strength of the realistic interactions, since it confers on them a model-independent status as direct links to the phase shifts.

The second observation is that when used in shell-model calculations and compared with data these matrix elements give results that deteriorate rapidly as the number of particle increases (Halbert *et al.*, 1971) and (Brown and Wildenthal, 1988). It was found (Pasquini and Zuker, 1978) that in the $pf$ shell a phenomenological cure, confirmed by exact diagonalizations up to A=48 (Caurier *et al.*, 1994), amounts to very simple modifications of some average matrix elements (*centroids*) of the KB interaction (Kuo and Brown, 1968)."

## The Shell Model on NNN forces

We had a paradigm:
Solve Schrödinger with "hard core" NN potentials
Why "hard core"?

- NN data. Not true $\Longrightarrow V_{\text{low } k}$
- Saturation. Not true. See figure




Soft NN does not saturate. Needs quadratic piece.

Hard NN saturates at close packing. Unrealistic.

Need NNN $\Longrightarrow$ New paradigm: $V_{\text{low } k}$ + NNN

## What do we know about NNN?

Axiom: NNN must do what NN does not.

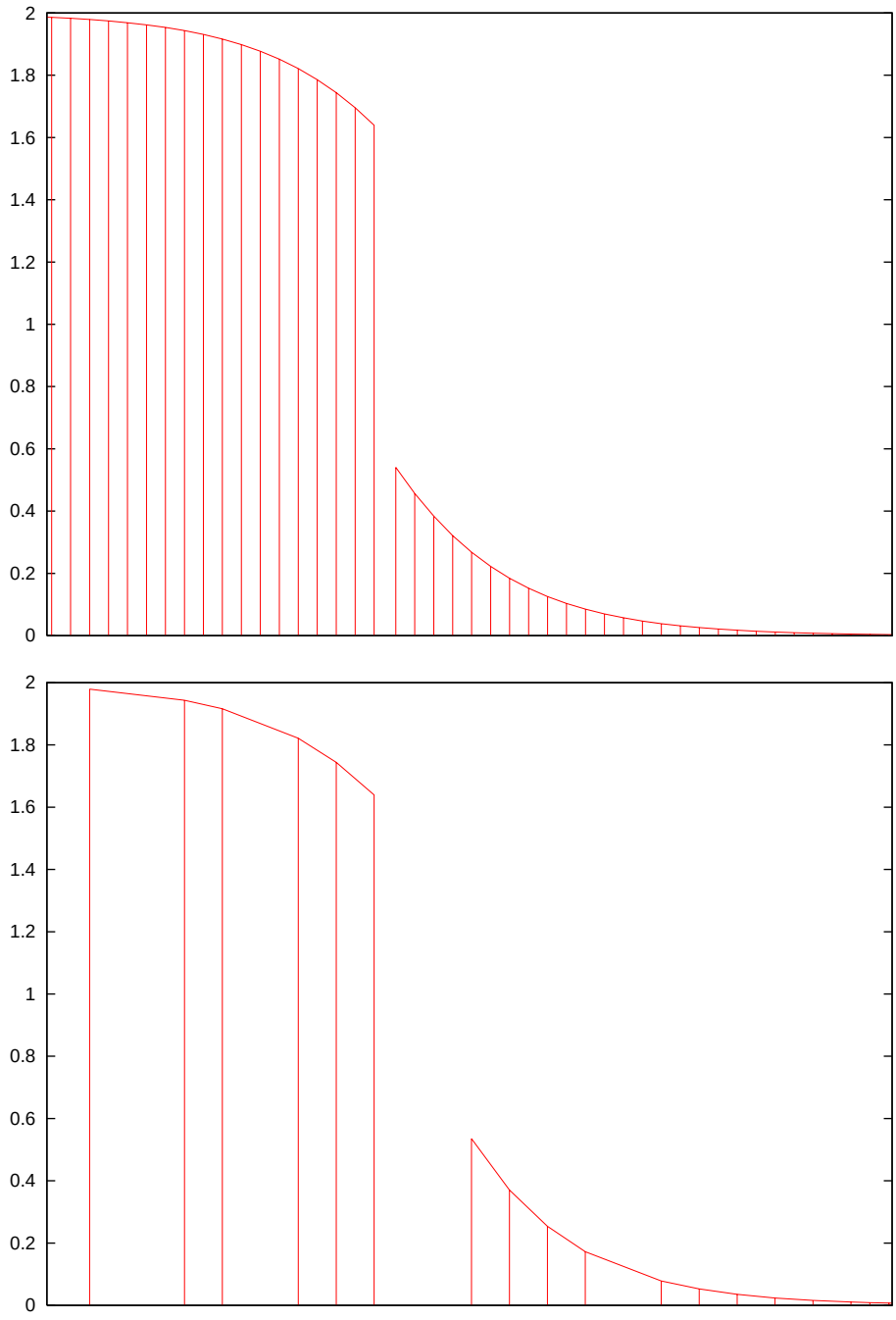
- In exact calculations NN fails in a number of details.
- There is also a smoking gun in $^{10}$B
- In SM calculations NN fails in a global way.

Aim of seminar:

- Explain what is SM.
- Exhibit NN failures: The monopole problem
- Show G-$V_{\mathrm{low}\,k}$ equivalence.
- Use $V_{\mathrm{low}\,k}$ to map full monopole problem.

Conclusion: Dealing with NNN is not (should not be) so difficult.

**What is SM?** SM is to recognize that action takes place at Fermi surface. As Landau did for $\infty$ systems (plane waves, upper plot)

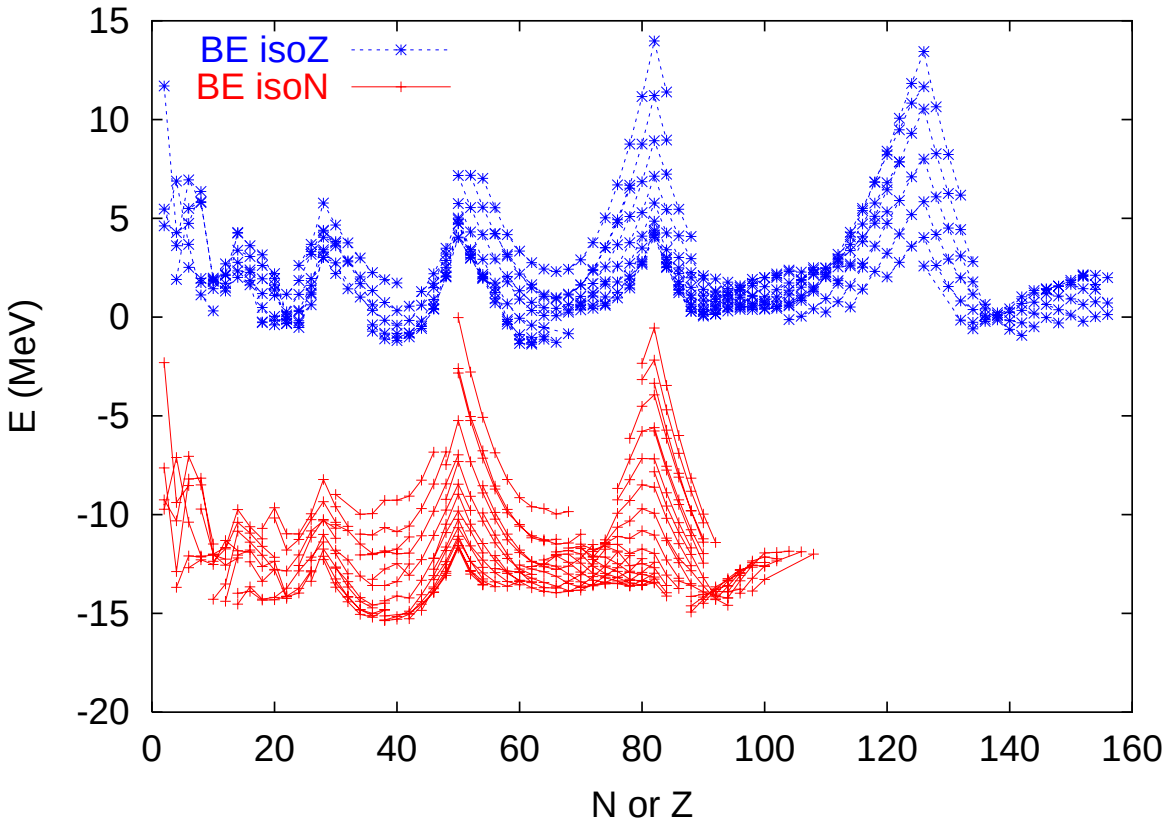



Adapting it to finite systems (harmonic oscillator, lower plot)

## SM problems. Shell effects

Subtract exp. BE from some LD

$$E(LD) = 15.5A - 17.8A^{2/3} - 28.6\frac{4T(T+1)}{A} + 40.2\frac{4T(T+1)}{A^{4/3}} - V^d_{Cm},$$




Experimental shell effects ($BE$(exp)-$E(LD)$) along isotope and isotone lines. Both plots contain the same information. The isotone lines are displaced by -14 MeV.

SM Problems

- Bypass saturation by fixing $\hbar\omega$.
- Use G matrix as pseudopotential.
- Realistic forces do not produce the right EI closures: **The monopole problem**

## Getting acquainted with $H_m$

Problem in mid-1960's:

first two excited states in $^{16}$O: 1p1h $3^-$ and nearly degenerate 4p4h $0^+$ at $\approx$6 MeV.

Conventional wisdom, $H = H_{sp} + H_{corr}$

Hence, if unperturbed $3^-$ at $\approx$ 8 Mev, unperturbed $0^+$ at $\approx$ 32 MeV. Too much

Solution: SM with $^{12}$C core and four particles in $p_{1/2} \equiv h$ and $s_{1/2}\, d_{5/2} \equiv p$ orbits some 3 MeV above.

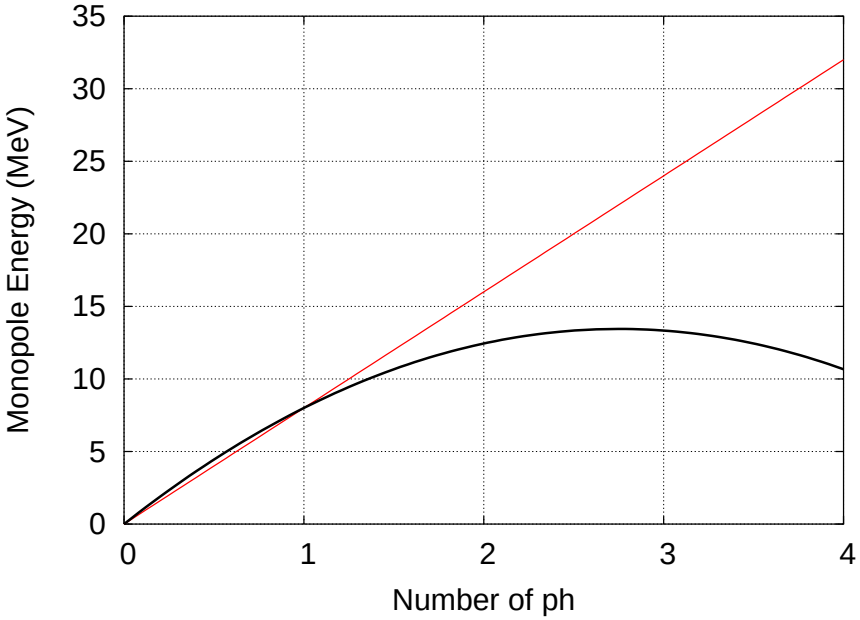
Phys. Rev. Lett. **21** (1968) 39

KEY: no $H_{sp}$, but $H_m$

Make it simple: two shells, no isospin

$$H_m = \varepsilon_p m_p + \varepsilon_h m_h + \tag{1}$$
$$-\frac{1}{2}|V_{pp}|m_p(m_p-1) - \frac{1}{2}|V_{hh}|m_h(m_h-1) - |V_{ph}|m_p m_h,$$

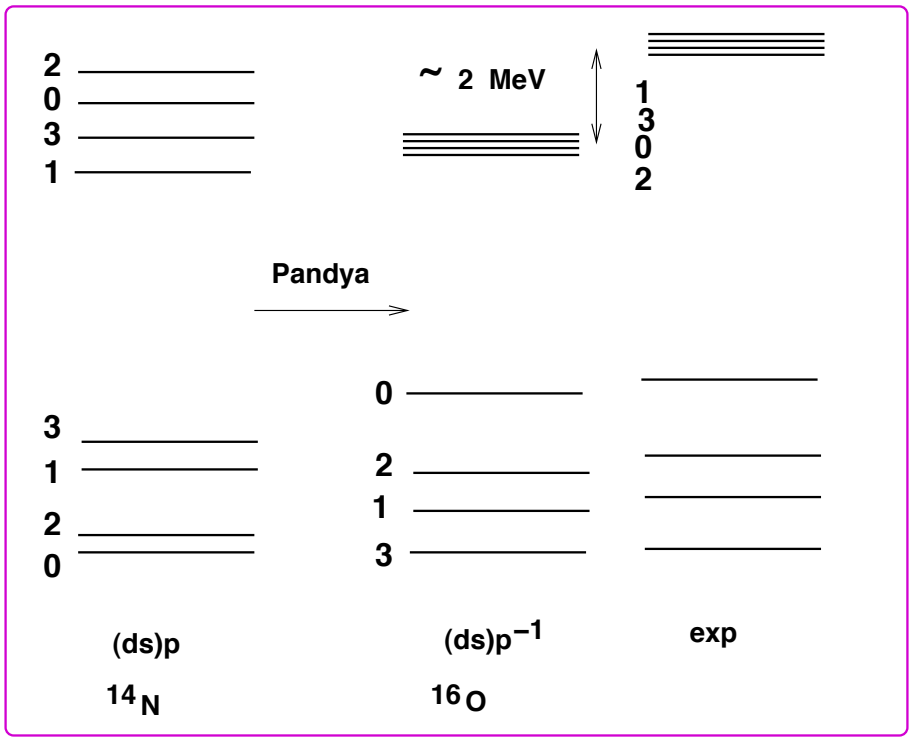
Calculate energy of $k$p$k$h. $cs$ is full with $D_h$ particles, *i.e.*, 4




$$\epsilon(k) = \left[(\varepsilon_p - \varepsilon_h) - \frac{1}{2}(|V_{pp}| - |V_{hh}|)(D_h - 1)\right] k + (\frac{1}{2}|V_{pp} + V_{hh}| - |V_{ph}|)k(D_h - k).$$

If purely realistic set very unrealistic $\varepsilon_p - \varepsilon_h = 0$,
Hence something wrong with $V_{rs}$.

**The isospin puzzle** Phys. Rev. Lett. **23** (1969) 983.




$$H_{BF} = a_{ph} m_p m_h + b_{ph} T_p \cdot T_h,$$

$$T_p \cdot T_h = \frac{1}{2}[T(T+1) - T_p(T_p+1) - T_h(T_h+1)] \qquad (2)$$

$b_{ph}$= distance between centroids.

For a 2b force $b_{ph}$, must be constant. It is not

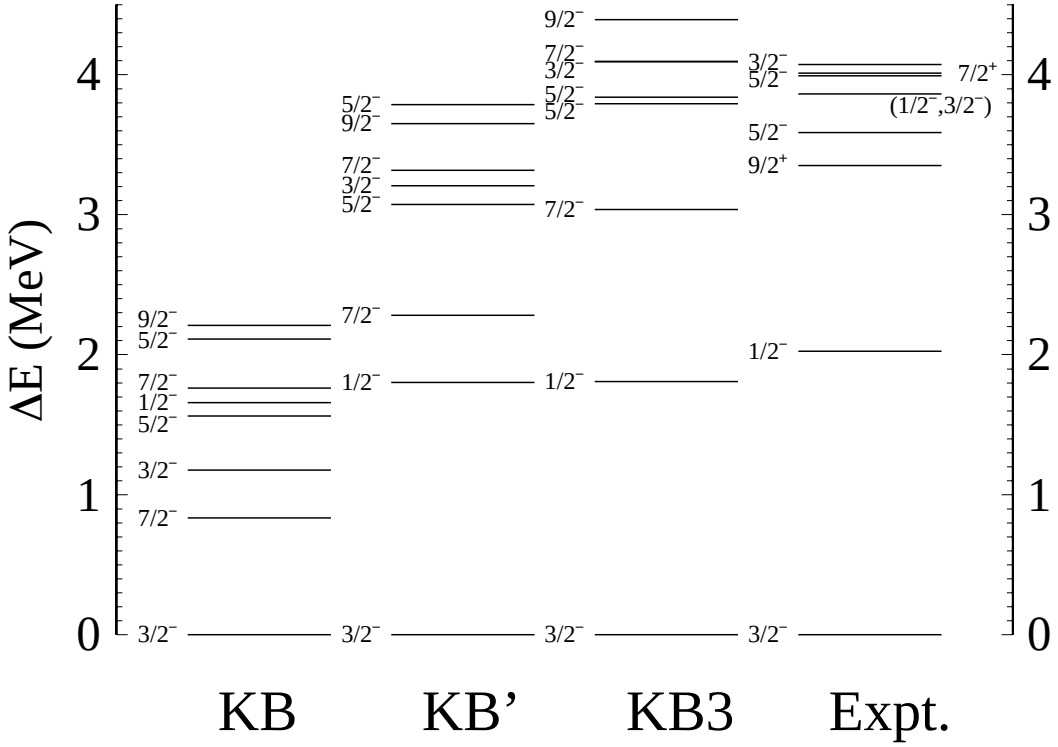
Conclusions

- Quadratic *i.e.*, 2b effects are crucial in producing monopole drift.
- Three body terms seem necessary.
- There is a monopole problem with the realistic interactions

## The SM smoking gun. *Circa* 1975

Eduardo Pasquini Ph.D thesis

The spectrum of $^{49}$Ca for the KB, KB' and KB3 interactions compared to experiment




$$V^T_{fr}(\mathrm{KB1}) = V^T_{fr}(\mathrm{KB}) - (-)^T\, 300 \text{ keV},$$
$$V^0_{ff}(\mathrm{KB1}) = V^0_{ff}(\mathrm{KB}) - 350 \text{ keV},$$
$$V^1_{ff}(\mathrm{KB1}) = V^1_{ff}(\mathrm{KB}) - 110 \text{ keV}.$$

$(f \equiv f_{7/2},\, r \equiv f_{5/2},\, p_{3/2},\, p_{1/2})$.

Therefore: NN has to be "fixed" through monopole changes.

## $H_m$ with realistic forces

Review (and improve) recipe to make $^{48}$Ca and $^{56}$Ni closed

$$V^T_{fr}(\mathrm{R}) \Longrightarrow \mathrm{V^T_{fr}(R)} - (-)^{\mathrm{T}}\,\kappa$$
$$V^T_{ff}(\mathrm{R}) \Longrightarrow \mathrm{V^T_{ff}(R)} - 1.5\,\kappa\delta_{\mathrm{T0}},$$

R=realistic, $f \equiv (p_{3/2},\, d_{5/2},\, f_{7/2})$ rest=$r$.

Fairly good for $pf$, but some problems

. No good for $sd$.

Solution

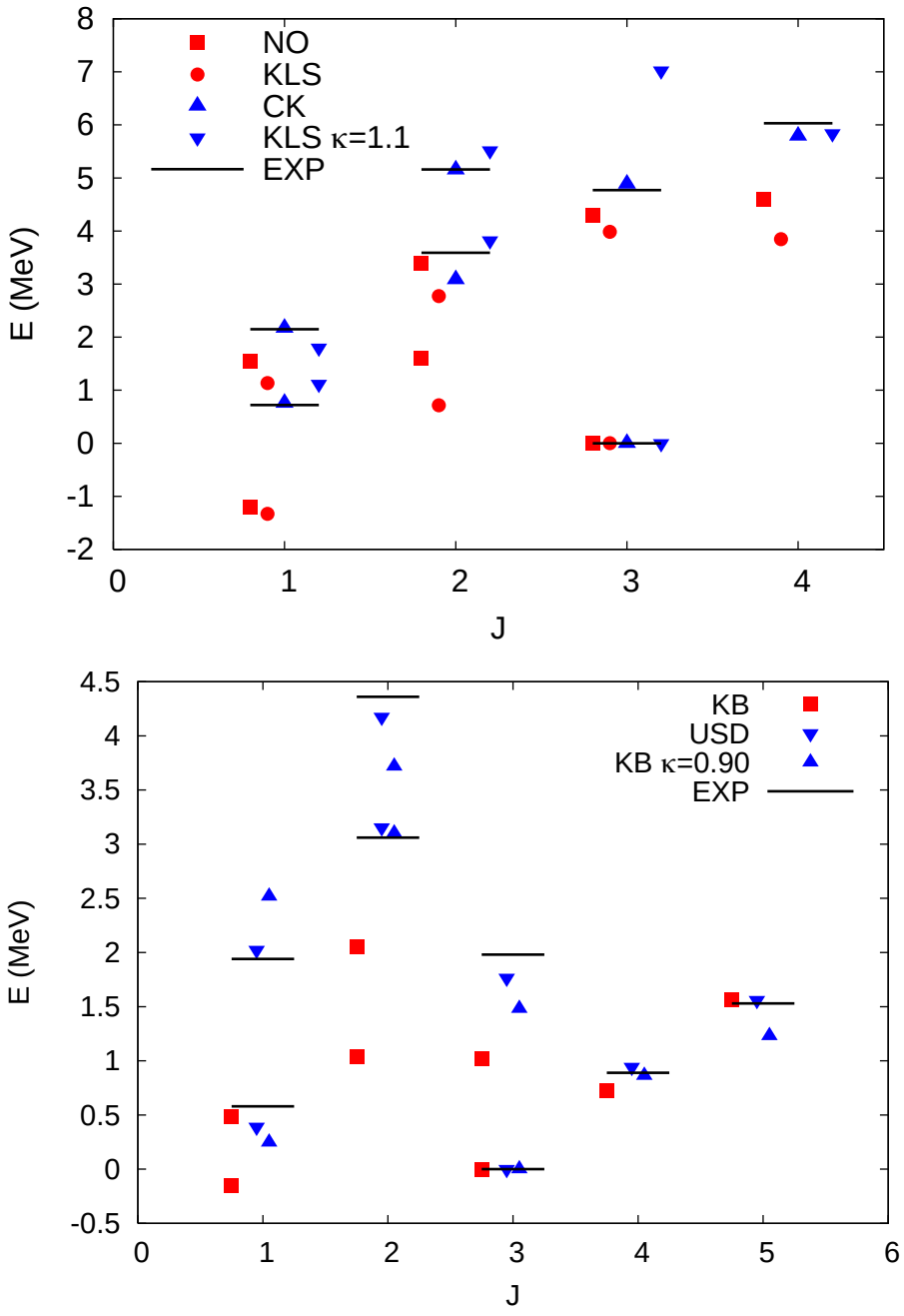
Make $\kappa = \kappa_0 + (m - m_0)\,\kappa_1$ *i.e.*, 3-body.

Phys. Rev. Lett. **90**, 042502 (2003)

See how problem arises in $p$, $sd$ and $pf$ shells.

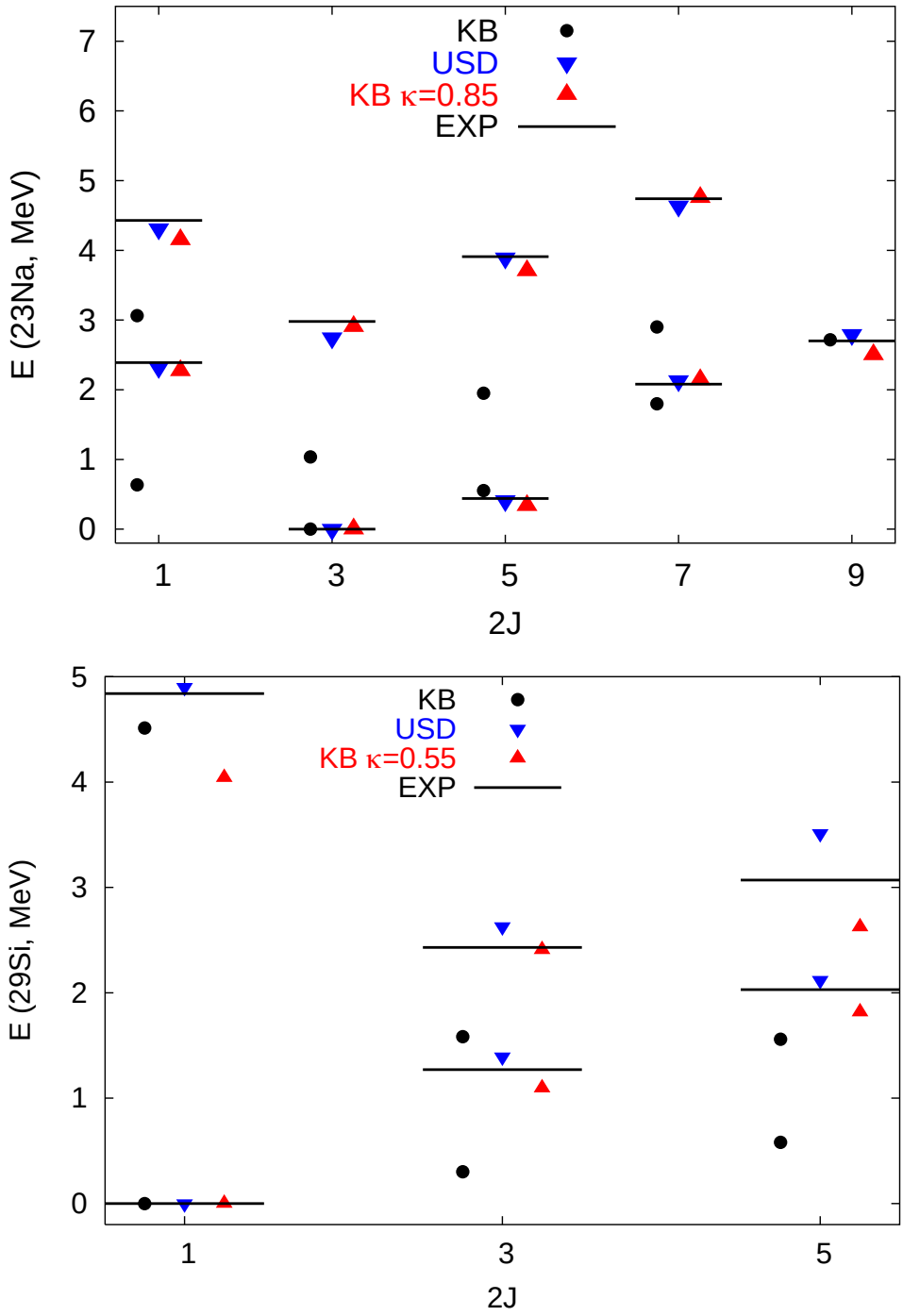
And gets solved.

**The excitation specta of $^{10}$B and $^{22}$Na for different interactions.**

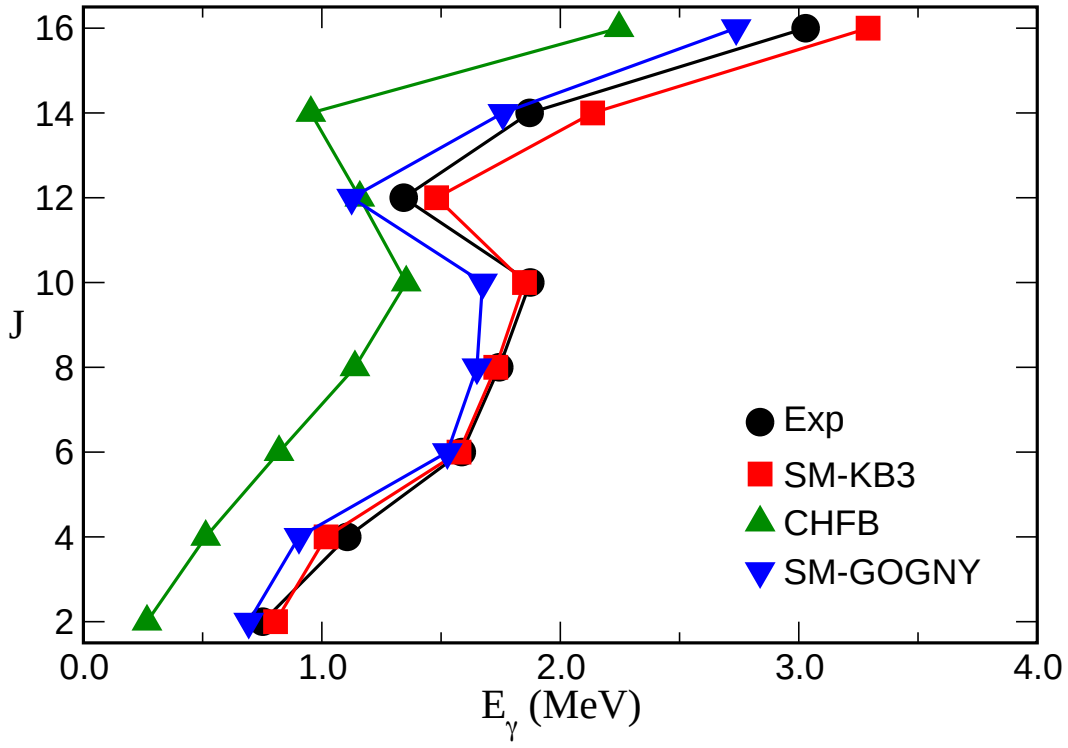


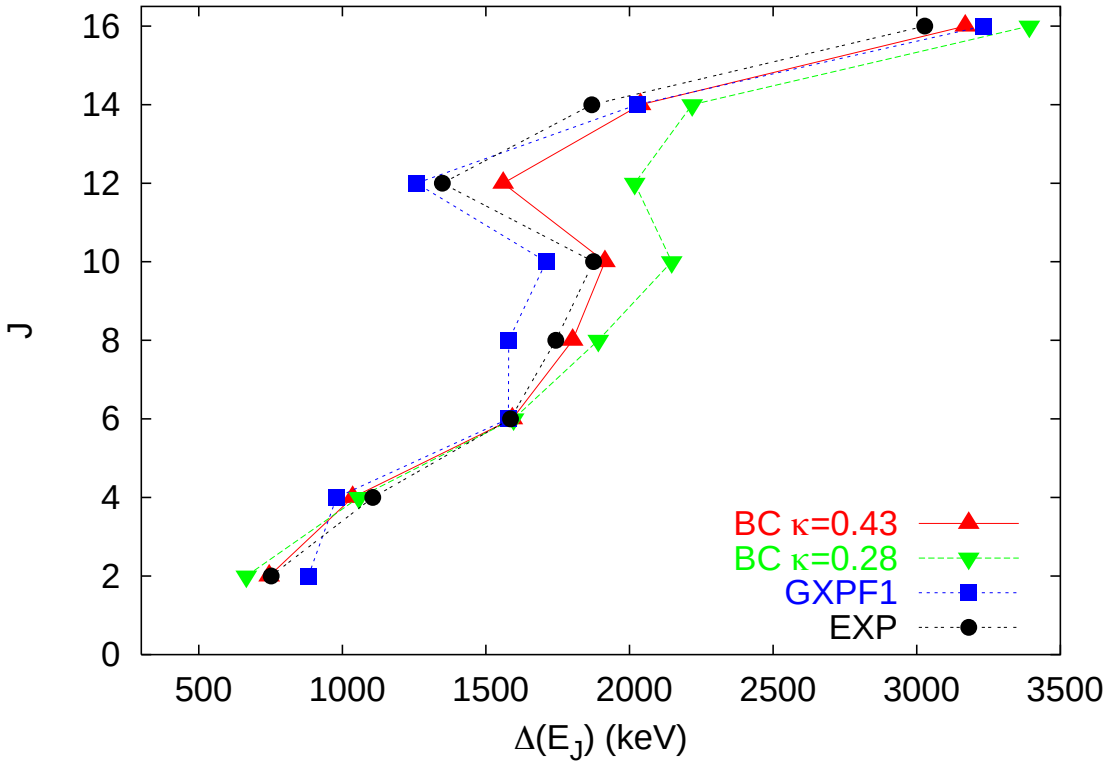

NO= P. Navratil and E. Ormand, Phys. Rev. Lett. **87** 152502 (2002). Note how simplest SM is energetically equivalent to no core SM. Note same problem in *sd* shell.

**The excitation specta of $^{23}$Na and $^{29}$Si for different interactions.**



**Backbending in $^{48}$Cr for different interactions.**




Illustrates situation in $pf$ shell. $\kappa = 0.28$ is value for $A \approx 56$.

## G and $V_{\mathrm{low}\,k}$

Usual procedure, G-matrix

$$G_{abcd} = V abcd - \sum_{\alpha\ \beta} \frac{V_{ab\alpha\beta} G_{\alpha\beta cd}}{\epsilon_{\alpha\beta} - \epsilon_{ab} + \Delta}$$

Problems:

- $\Delta$.
- No theory (except coupled cluster).
  Phys. Rept. **71** 141 (1981)
- No HF.

Advantage: it works in SM if monopole fixed.

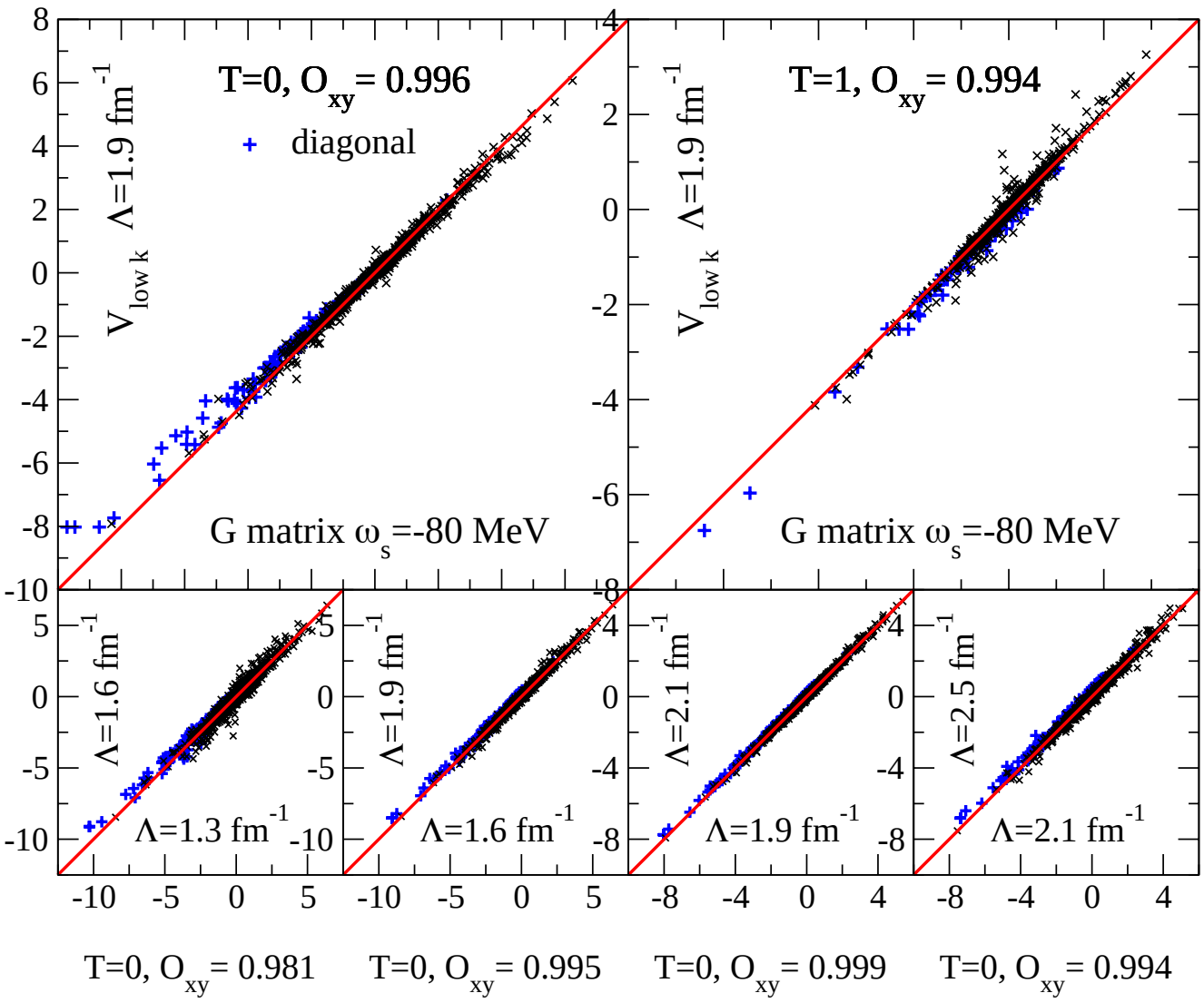
New procedure $V_{lowk} = V^{\Lambda}$
S. Bogner, T. Kuo and A. Schwenk Phys. Rept. **386** 1 (2003)

$$T(k', k; k^2) = V^{\Lambda}(k'.k) + \frac{2}{\pi} \mathcal{P} \int_0^{\Lambda} \frac{V^{\Lambda}(k'p) T(p, k; k^2)}{k^2 - p^2} p^2 dp$$

Problem: $\Lambda$, needs NNN
Advantages: soft $V$.

Program:

- Show G$\equiv V_{lowk} = V^{\Lambda}$
- Explore monopole behaviour for NNN hints (with Achim).




Top: Correlation plots between $V_{\mathrm{low}\,k}$ (Argonne $v_{18}$ )and $G$ matrix elements (Idaho A) in 4 major shells for $\hbar\omega = 14\,\mathrm{MeV}$. Note that for $\Lambda = 1.9\,\mathrm{fm}^{-1}$ the different $V_{\mathrm{NN}}$ have practically collapsed to one "universal" $V_{\mathrm{low}\,k}$. Bottom: Correlation plots between $V_{\mathrm{low}\,k}$ matrix elements for different cutoffs.

## Make it quantitative

**Universality**

$$O_{AB} = d_2^{-1} \sum_{rstu\Gamma} W^{\Gamma}_{rstuA} W^{\Gamma}_{rstuB}[\Gamma]$$

$$\overline{O}_{AB} = \frac{O_{AB}}{\sqrt{O_{AA} O_{BB}}}.$$

| $\hbar\omega$ | 18.4 | 13.9 | 11.0 | 9.0 | 7.9 | 6.9 |
|---|---|---|---|---|---|---|
| 18.4 | 12.271 | 9.420 | 7.434 | 5.841 | 5.166 | 4.436 |
| 13.9 | 9.420 | 7.391 | 5.917 | 4.703 | 4.181 | 3.609 |
| 11.0 | 7.434 | 5.917 | 4.781 | 3.829 | 3.414 | 2.958 |
| 8.8 | 5.841 | 4.703 | 3.829 | 3.084 | 2.757 | 2.395 |
| 7.9 | 5.166 | 4.181 | 3.414 | 2.757 | 2.468 | 2.146 |
| 6.9 | 4.436 | 3.609 | 2.958 | 2.395 | 2.146 | 1.869 |
| 18.4 | 1.000 | 0.989 | 0.970 | 0.949 | 0.938 | 0.926 |
| 13.9 | 0.989 | 1.000 | 0.995 | 0.985 | 0.978 | 0.971 |
| 11.0 | 0.970 | 0.995 | 1.000 | 0.997 | 0.994 | 0.989 |
| 8.8 | 0.949 | 0.985 | 0.997 | 1.000 | 0.999 | 0.997 |
| 7.9 | 0.938 | 0.978 | 0.994 | 0.999 | 1.000 | 0.999 |
| 6.9 | 0.926 | 0.971 | 0.989 | 0.997 | 0.999 | 1.000 |

**Overlaps at different $\hbar\omega$ for BonnC (G matrix).**

$$O_{AB} = d_2^{-1} \sum_{rstu\Gamma} W^{\Gamma}_{rstuA} W^{\Gamma}_{rstuB}[\Gamma]$$

$$\sigma_A^2 = O_{AA}, \qquad \overline{O}_{AB} = \frac{O_{AB}}{\sqrt{O_{AA} O_{BB}}}.$$

| $\hbar\omega$ | 18.4 | 13.9 | 11.0 | 9.0 | 7.9 | 6.9 |
|---|---|---|---|---|---|---|
| $W$ | -1.374 | -1.035 | -0.802 | -0.620 | -0.546 | -0.463 |
| $\sigma_A$ | 3.288 | 2.488 | 1.931 | 1.500 | 1.323 | 1.127 |

| $\hbar\omega$ | 18.4 | 13.9 | 11.0 | 8.8 | 7.9 | 6.9 |
|---|---|---|---|---|---|---|
| 18.4 | 1.000 | 0.992 | 0.978 | 0.961 | 0.952 | 0.941 |
| 13.9 | 0.992 | 1.000 | 0.996 | 0.987 | 0.982 | 0.975 |
| 11.0 | 0.978 | 0.996 | 1.000 | 0.997 | 0.995 | 0.990 |
| 8.8 | 0.961 | 0.987 | 0.997 | 1.000 | 0.999 | 0.998 |
| 7.9 | 0.952 | 0.982 | 0.995 | 0.999 | 1.000 | 0.999 |
| 6.9 | 0.941 | 0.975 | 0.990 | 0.998 | 0.999 | 1.000 |

**Overlaps at different $\hbar\omega$ for Av18 ($\mathbf{V}_{\mathrm{low}\,\mathbf{k}}$).**

## Scaling laws

$$V^{\Lambda_1,\hbar\omega_1}_{\mathrm{low}\,k} \approx \frac{\sigma_{\Lambda_1,\hbar\omega_1}}{\sigma_{\Lambda_2,\hbar\omega_2}} V^{\Lambda_2,\hbar\omega_2}_{\mathrm{low}\,k} \;\Rightarrow\; V^{\Lambda,\hbar\omega}_{\mathrm{low}\,k} \approx \sigma_{\Lambda,\hbar\omega}\, U\,,$$

$$\frac{\sigma_{\Lambda,\hbar\omega_1}}{\sigma_{\Lambda,\hbar\omega_2}} \approx \frac{\hbar\omega_1}{\hbar\omega_2} \qquad \frac{\sigma_{\Lambda_1,\hbar\omega}}{\sigma_{\Lambda_2,\hbar\omega}} \approx \sqrt{\frac{\Lambda_2}{\Lambda_1}}\,.$$

| | T=0 | T=1 | Total |
|---|---|---|---|
| $O_{1\,1}$ | 17.024 | 2.508 | 6.275 |
| $O_{2\,2}$ | 18.122 | 3.330 | 7.169 |
| $O_{3\,3}$ | 20.516 | 3.024 | 7.563 |
| $O_{4\,4}$ | 17.381 | 3.448 | 7.064 |
| $O_{5\,5}$ | 11.343 | 2.188 | 4.563 |
| $O_{1\,2}$ | 0.980 | 0.987 | 0.981 |
| $O_{1\,3}$ | 0.979 | 0.980 | 0.979 |
| $O_{1\,4}$ | 0.966 | 0.986 | 0.970 |
| $O_{1\,5}$ | 0.974 | 0.986 | 0.976 |
| $O_{2\,3}$ | 0.995 | 0.994 | 0.993 |
| $O_{2\,4}$ | 0.982 | 0.999 | 0.988 |
| $O_{2\,5}$ | 0.979 | 0.993 | 0.984 |
| $O_{3\,4}$ | 0.980 | 0.995 | 0.983 |
| $O_{3\,5}$ | 0.975 | 0.979 | 0.975 |
| $O_{4\,5}$ | 0.994 | 0.991 | 0.993 |

**Overlaps 1=Lee12, 2=V14_1.9, 3=GA14, 4=GB14, 5=GB11.**

$$W^{JT}_{rstu} = V^{JT}_{rstu} - \delta_{rt}\delta_{su}V^{T}_{rs}. \tag{3}$$

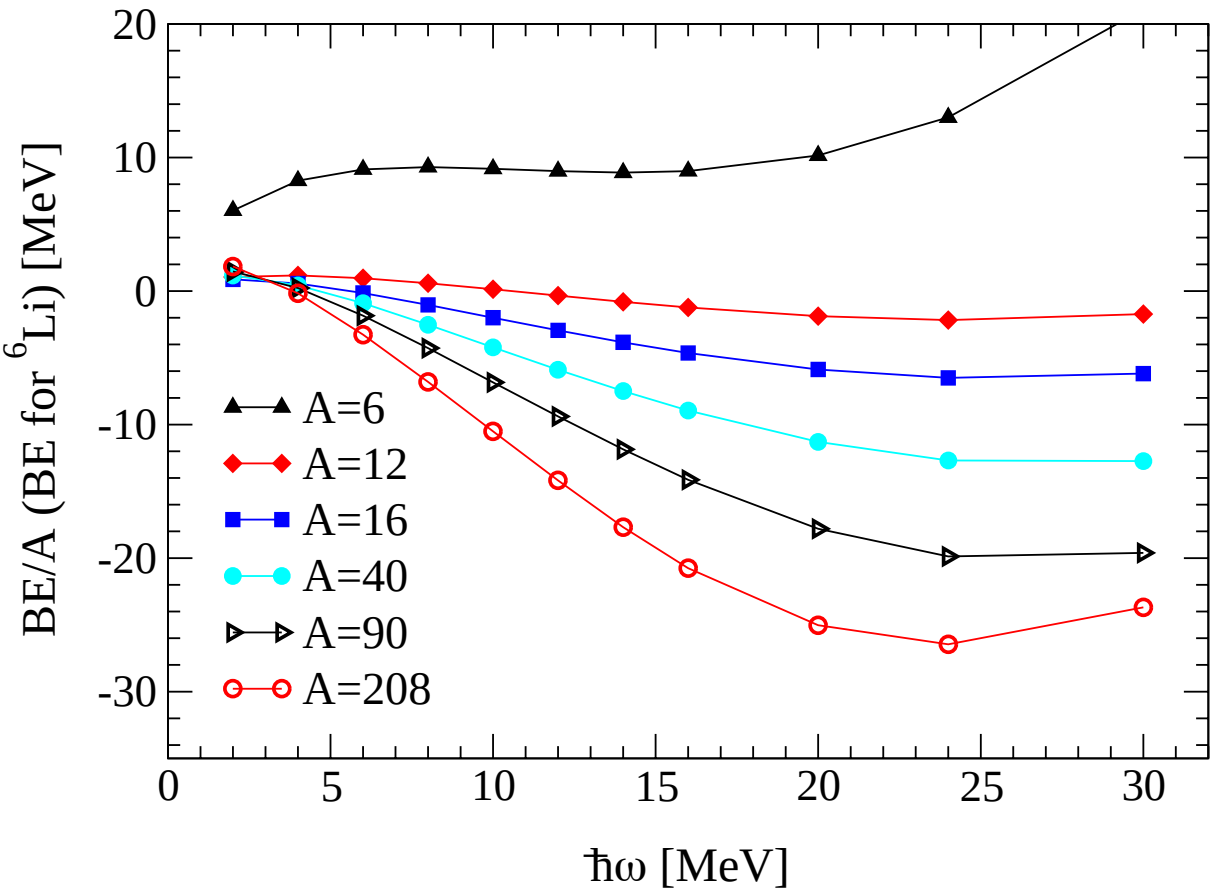
$$V^{T}_{rs} = \frac{\sum_J V^{JT}_{rsrs}[J](1-(-1)^{J+T}\delta_{rs})}{(2j_r+1)(2j_s+1+\delta_{rs}(-1)^T)} \tag{4}$$

In the np scheme each orbit $r$ goes into two $r_n$ and $r_p$ and the centroids can be obtained through ($x, y = n$ or $p$, $x \neq y$)

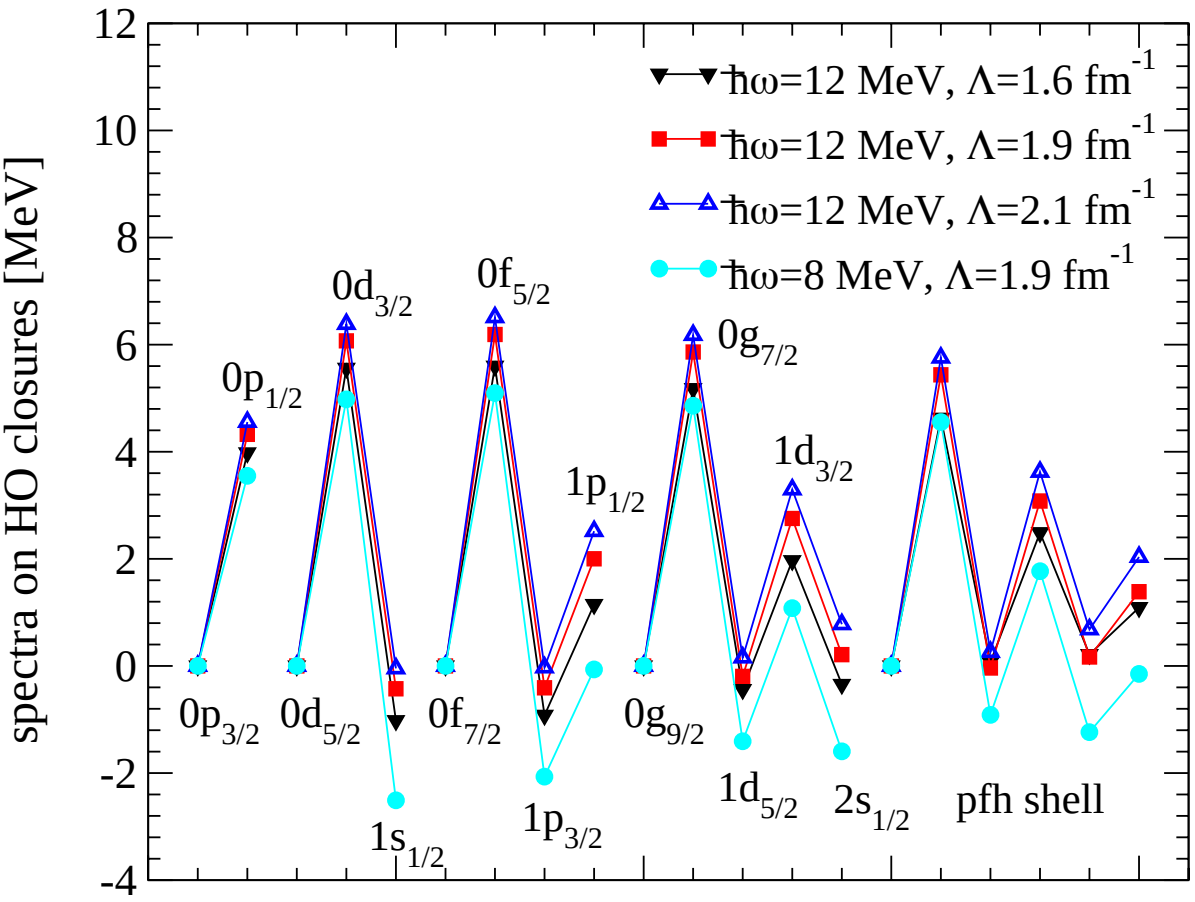
$$\begin{aligned} V_{r_x s_y} &= \frac{1}{2}\left[\mathcal{V}^1_{rs}\left(1-\frac{\delta_{rs}}{2j_r+1}\right) + V^0_{rs}\left(1+\frac{\delta_{rs}}{2j_r+1}\right)\right] \\ V_{r_x s_x} &= V^1_{rs}. \end{aligned} \tag{5}$$

$$H^d_m = K^d + \frac{1}{2}\sum_{r_x, s_y} V_{r_x s_y} m_{r_x}(m_{s_y} - \delta_{r_x s_y}\delta_{xy})\,.$$

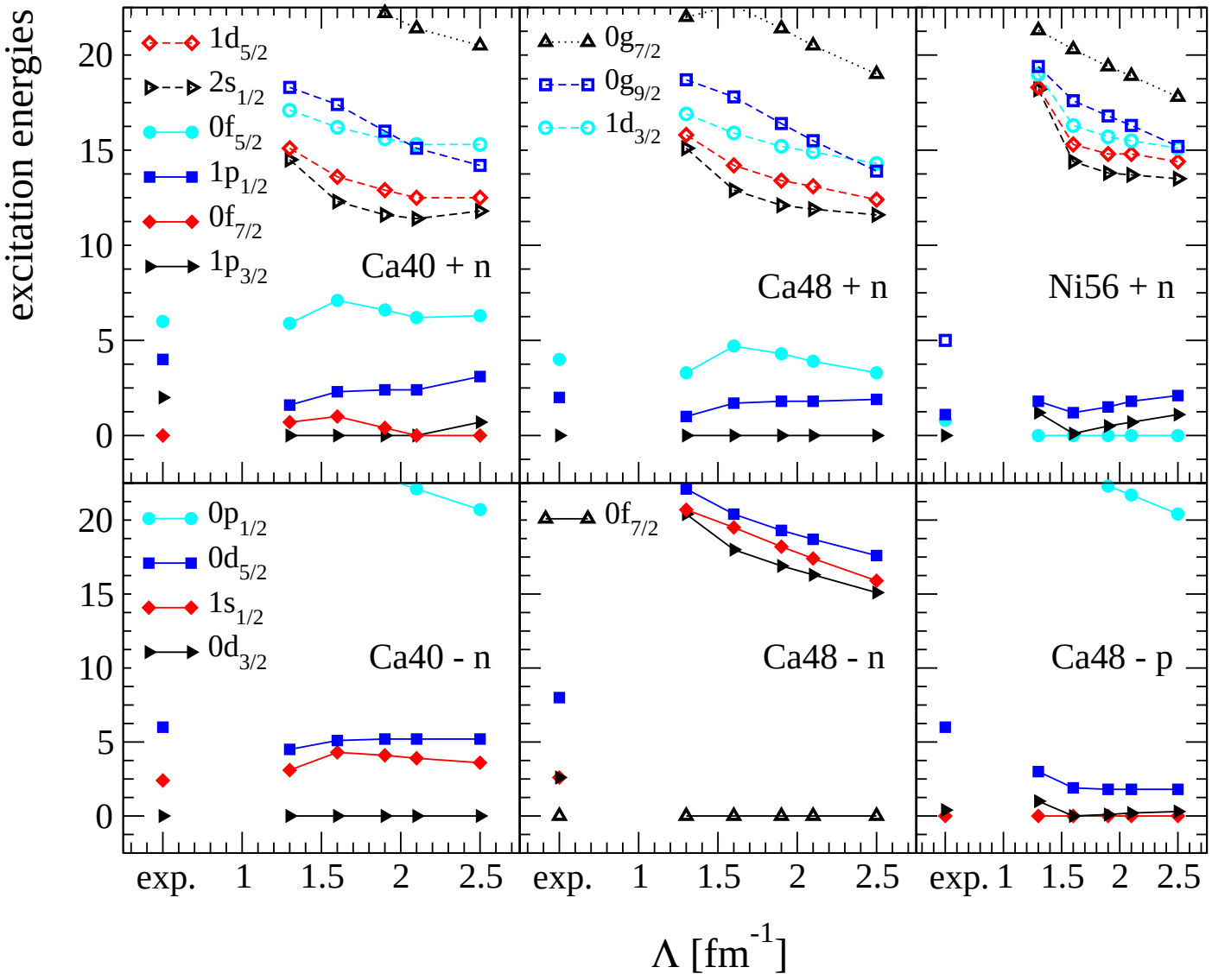
Examine




Binding energies obtained from Eq. (6) (Coulomb included schematically) by filling lowest oscillator orbits.




Single-particle spectra on harmonic oscillator closures. All matrix elements referred to $\Lambda = 1.9\,\mathrm{fm}^{-1}$. Rescaling by $(\hbar\omega/\hbar\omega_0)^2 = \left((p+2)^2[(p_0+2)^3+3]/(p_0+2)^2[(p+2)^3+3]^2\right)^2$ so mass number $A$ appropriate to each major shell with principal quantum number $p+1$. $p_0 = 2\,(4)$ correspond to the $pf$ $(pfh)$ shell and $\hbar\omega_0 = 12\,(8)\,\mathrm{MeV}$.




$cs \pm 1$ spectra in the $pf$ region for a wide cutoff range. The experimental $0g_{9/2}$ energy in $^{57}$Ni is an estimate.

Make remark on $^{15}$N

Same as plot in previous page.

| Ca | 20 21 | cs+n | Ca | 20 29 | cs+n | Ni | 28 29 | cs+n |
|---|---|---|---|---|---|---|---|---|
| 13 | 0.0 | 2.0 | 13 | 0.0 | 0.0 | 23 | 0.0 | 0.8 |
| 33 | 0.0 | 0.0 | 3 | 1.8 | 2.0 | 13 | 0.7 | 0.0 |
| 3 | 2.4 | 4.0 | 23 | 3.9 | 4.0 | 3 | 1.8 | 1.1 |
| 23 | 6.2 | 6.0 | 4 | 11.9 | | 4 | 13.7 | |
| 4 | 11.4 | | 24 | 13.1 | | 24 | 14.8 | |
| 24 | 12.5 | | 14 | 14.9 | | 14 | 15.5 | |
| 44 | 15.1 | | 44 | 15.5 | | 44 | 16.3 | |
| 14 | 15.3 | | 34 | 20.5 | | 34 | 18.9 | |
| Ca | 20 19 | cs-n | Ca | 20 27 | cs-n | K | 19 28 | cs-p |
| 12 | 0.0 | 0.0 | 33 | 0.0 | 0.0 | 2 | 0.0 | 0.0 |
| 2 | 3.9 | 2.4 | 12 | 16.3 | 2.6 | 12 | 0.2 | 0.4 |
| 22 | 5.2 | 6.0 | 2 | 17.4 | 2.6 | 22 | 1.8 | 6.0 |
| 1 | 22.1 | | 22 | 18.7 | | 1 | 21.7 | |

Notation $j - 1/2\, p$, So
1=$0p_{1/2}$, 11=$0p_{3/2}$ , 2=$1s_{1/2}$ , 12=$0d_{3/2}$ , 22=$0d_{5/2}$ , 3=$1p_{1/2}$ 13=$1p_{3/2}$ , 23=$0f_{5/2}$ , 33=$0f_{7/2}$ , 4=$2s_{1/2}$ , 14=$1d_{3/2}$ , 24=$1d_{5/2}$ , 34=$0g_{7/2}$ , 44=$0g_{9/2}$ .

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